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One-loop renormalization of the NMSSM in SloopS: The neutralino-chargino and sfermion sectors

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We have completed the one-loop renormalisation of the Next-to-Minimal Supersymmetric Standard Model (NMSSM) allowing for and comparing between different renormalisation schemes. A special attention is paid to on-shell schemes. We study a variety of these schemes based on alternative choices of the physical input parameters. In this paper we present our approach to the renormalization of the NMSSM and report on our results for the neutralino-chargino and sfermion sectors. We will borrow some results from our study of the Higgs sector whose full discussion is left for a separate publication. We have implemented the setup for all the sectors of the NMSSM within `SloopS`, a code for the automatic computation of one-loop corrections initially developed for the standard model and the MSSM. Among the many applications that allows the code, we present here the one-loop corrections to neutralino masses and to partial widths of neutralinos and charginos into final states with one gauge boson. One-loop electroweak and QCD corrections to the partial widths of third generation sfermions into a fermion and a chargino or a neutralino are also computed.

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I. INTRODUCTION

Supersymmetry has long been considered as the most natural extension of the standard model that can address the hierarchy problem while providing a dark matter candidate. The discovery of a Higgs boson with a mass of 125 GeV whose properties are compatible with those of the Standard Model is a great achievement of the first run of the LHC [1,2] and in some sense supports supersymmetry. Indeed, one can argue that a Higgs with a mass below 130 GeV is a prediction of the minimal supersymmetric standard model (MSSM). However, the fact that the observed Higgs mass is so close to the largest value that can be achieved in the MSSM, a value obtained by requiring a rather heavy supersymmetric spectrum, raises the issue of naturalness [3,4]. Another issue with the MSSM is the μ problem [5]. Namely why μ, a supersymmetry preserving mass parameter as it appears in the superpotential through the operator mixing the two (superfield) Higgs doublets $\mu\hat{H}_d \cdot \hat{H}_u$, should be, for a viable phenomenology, small, i.e., of the order the electroweak scale, whereas one expects its value to be rather of order the cutoff scale. Both these problems are solved in the singlet extension of the MSSM, the Next-to-Minimal Supersymmetric Standard Model (NMSSM) where the μ parameter is generated dynamically through the vacuum expectation value of the scalar component of the additional singlet superfield. Moreover, as a bonus new terms in the superpotential are now present and give a contribution to the quartic Higgs couplings beside the gauge induced quartic coupling of the MSSM. These new contributions can lead to an increase of the tree-level mass of the lightest Higgs, thus more easily explaining the observed value of the Higgs mass [6,7] without relying on very large corrections from the stop/top sector. Although fine-tuning issues remain [8–11] they are not as severe as in the MSSM.

The Higgs discovery has thus led to a renewed interest in the NMSSM both at the theoretical and experimental level with new studies of specific signatures of the NMSSM Higgs sector [12,13] and/or of the neutralino and sfermion sectors [14–16] being pursued at the LHC. With the exciting possibility of discovering new particles at the second run of the LHC, it becomes even more important for a correct interpretation of a future new particle signal to know precisely the particle spectrum as well as to make precise predictions for the relevant production and decay processes.

The importance of loop corrections to the Higgs mass in supersymmetry cannot be stressed enough. After all, it is because of radiative corrections that the MSSM has survived. The large radiative corrections from the top and stop sector are necessary to raise the Higgs mass beyond the bounds imposed by LEP and to bring it in the range compatible with the LHC. Higher-order corrections are also of relevance for supersymmetric particles, higher-order SUSY-QCD and electroweak corrections to the full SUSY spectrum have been computed for some time in the

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MSSM and are incorporated in several public codes [17–20]. More recently higher-order corrections to Higgs and sparticle masses have been extended to the NMSSM [21,22]. Several public codes incorporate these corrections with different scopes and approximations, `NMSSMTools` [23,24], `SPheno` [25,26], `SoftSUSY` [27], `NMSSMCalc` [28] and `FlexibleSUSY` [29]. See also the recent work [30] on the corrections to the Higgs masses in the NMSSM. Moreover, higher-order corrections to decays have also been computed with some of these codes [31–34].

The code `SloopS` was developed for the MSSM with the objective of computing one-loop corrections for collider and dark matter observables in supersymmetry. The complete renormalization of the model was performed in [35,36] and several renormalization schemes were implemented. This code relies on an improved version of LanHEP [37–39] for the generation of Feynman rules and counterterms. The model file generated is then interfaced to `FeynArts` [40], `FormCalC` [41] and `LoopTools` for the automatic computation of one-loop processes [42]. One-loop corrections to masses, two-body decays and production cross sections at colliders were realized together with one-loop corrections for various dark matter annihilation [43–47] and coannihilation processes [44]. `SloopS` has first been extended to include the NMSSM for one-loop processes not requiring renormalization, such as the rates for gamma-ray lines relevant for Dark Matter indirect detection [48,49] and Higgs decays to photons at the LHC [50,51].

The present paper is the first in a series that describes the implementation of the one-loop corrections for all sectors of the NMSSM. We will concentrate in this first paper on the details and issues having to do mainly with the neutralino/chargino sector since the addition of a singlet brings new features compared to the MSSM. We will be brief on the setup of the renormalization in the sfermion sector since the particle content is the same as within the MSSM. For this sector, we therefore adhere to the approach given in [36] for the MSSM. The chargino-neutralino sector, in particular through the singlet superfield, is quite tied up with the Higgs sector. We will, therefore, have to borrow some elements from our study of the Higgs sector which we will go over in more detail in a follow-up paper [52]. For the neutralino/chargino sector, different renormalization schemes are defined. In particular, we have aimed at studying different on-shell, OS, schemes. The latter are based on choosing a minimal set of observables, namely masses of physical particles in the NMSSM spectrum. These will define the set of input parameters and necessary counterterms which will allow to get rid of all ultra-violet divergences in all calculated observables. Finding the minimal set of necessary counterterms requires solving a system of coupled equations. For the case of the NMSSM, where mixing between different components occurs and where the same parameters appear in different sectors, the system of equations can be large. Moreover, some choices of the minimal set (and, therefore, the relevant coupled equations) will lead to solutions that are extremely sensitive to a particular choice of a parameter which may, in some process, induce large radiative corrections. It is also possible, when a renormalization scale $\bar{\mu}$ has been chosen, to follow a simpler implementation of the counterterms, *à la* $\overline{\rm DR}$, where these counterterms are pure divergent terms. In some instances, these can also lead to splitting a large system of coupled equations to a smaller and more manageable system of equations. The renormalization of the ubiquitous t_β which, at tree level, represents the ratio of the vacuum expectation values (vev) of the 2 Higgs doublets is a case in point. We will also study mixed schemes where some parameters are $\overline{\rm DR}$ while others are OS. The study of different renormalization schemes is very important. First it can provide an estimate on the theoretical uncertainty due to the truncation to one-loop of the perturbative prediction and may also point at a bad choice of a renormalization scheme. Second, for the NMSSM where a large part of the spectrum has not been seen it is difficult to predict which, from the point of view of an OS scheme, are the input parameters that one can use or which are the masses that will be discovered and measured (precisely) first. It is, therefore, wise to be open and prepare for different possibilities. In particular, our discussion will touch on some important issues regarding the relationship between the underlying parameters at the level of the Lagrangian and the physical parameters. This will bring up the issue of the reconstruction of the underlying parameters which is very much tied up to the renormalization scheme and the differences in how we define the counterterms.

One of our goals has been to implement our approach in a code for the automatic generation of one-loop corrected observables and for an easy implementation of the counterterms. We have relied on `SloopS`. Therefore, this work is also a natural extension of the work performed in [35,36] for the MSSM. Taking advantage of this automation we are able to provide and discuss a series of applications, pertaining to corrections to masses and various decays involving charginos, neutralinos and sfermions.

The paper is organized as follows. Section II contains a brief description of the NMSSM. Our general approach to the renormalization of the NMSSM and its implementation in `SloopS` as well as how we handle infra-red divergences is explained in Sec. III. The renormalization of the neutralino and chargino sector is detailed in Sec. IV. We also give a rather extensive presentation of the different choices for the on-shell schemes and the problematic of the choice of the input parameters. The renormalization of the sfermion sector follows the one of the MSSM. It is briefly reviewed in Sec. V. We are then ready to apply the general approach and principles to specific observables. We start in Sec. VI by defining a set of five benchmark points. In

Sec. VII, we first start by giving results for different schemes for the one-loop corrected masses of the neutralinos before presenting results for the one loop corrected two-body decays of charginos and neutralinos into gauge bosons. This is performed for all five benchmark points and for different schemes. We then turn in Sec. VIII to the one-loop two-body decays of third generation sfermions into a fermion and chargino or neutralino. Section IX contains our conclusions.

II. DESCRIPTION OF THE NMSSM

The NMSSM contains all the superfields of the MSSM as well as one additional gauge singlet superfield $\hat{S}$. Thus, the Higgs sector consists of two SU(2) Higgs doublets superfields $\hat{H}_d$,$\hat{H}_u$ and the singlet superfield,

$$\hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \qquad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix}, \qquad \hat{S}. \tag{1}$$

The interaction Lagrangian can be decomposed in terms derived from the superpotential and from the soft SUSY breaking Lagrangian. In the $\mathbb{Z}_3$-invariant NMSSM that we consider here, the superpotential can be split into two parts [5]. The first one depends only on the Higgs superfields $\hat{H}_d$, $\hat{H}_u$, $\hat{S}$ via two dimensionless couplings λ and κ,

$$W_{\mathrm{NMSSM}} = -\lambda \hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{1}{3}\kappa \hat{S}^3, \tag{2}$$

where $\hat{H}_d \cdot \hat{H}_u = \epsilon_{ab}\hat{H}_d^a \hat{H}_u^b$ and ϵ_{ab} is the two dimensional Levi-Civita symbol with $\epsilon_{12} = 1$. The second part corresponds to the Yukawa couplings between Higgs and quarks or leptons superfields,

$$W_{\mathrm{Yukawa}} = -y_u \hat{H}_u \cdot \hat{Q}\hat{U}_R^c + y_d \hat{H}_d \cdot \hat{Q}\hat{D}_R^c + y_e \hat{H}_d \cdot \hat{L}\hat{E}_R^c, \tag{3}$$

where

$$\hat{Q}_i = \begin{pmatrix} \hat{U}_{iL} \\ \hat{D}_{iL} \end{pmatrix}, \quad \hat{L}_i = \begin{pmatrix} \hat{\nu}_{iL} \\ \hat{E}_{iL} \end{pmatrix}, \quad \hat{U}_{iR}, \quad \hat{D}_{iR}, \quad \hat{E}_{iR}, \tag{4}$$

are respectively the superfields associated with the left-handed (LH) quark doublets, LH lepton doublets, right-handed (RH) quark and lepton singlets. The index i = 1…3 indicates the generation. In what follows, this index will be omitted and a sum over the three generations will be implicit. No generation mixing is assumed in our study. These supersymmetric scalar partners will be denoted as $\tilde{Q} = \binom{\tilde{u}_L}{\tilde{d}_L}$ and $\tilde{L} = \binom{\tilde{\nu}_L}{\tilde{e}_L}$ for the LH states and $\tilde{u}_R$, $\tilde{d}_R$ and $\tilde{e}_R$ for the partners of the RH states. In an abuse of language, we will also refer to these partners as LH and RH. Let us keep in mind, at this point, that parameters from the superpotential will find their way into the Lagrangian of the particle and the superparticles. For example, the same λ, κ enter both the Higgs sector and the neutralino (Higgsino) sector, thus offering ways to extract these parameters from different sectors. The soft SUSY breaking Lagrangian reads,

$$\begin{aligned} -\mathcal{L}_{\mathrm{soft}} &= m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 \\ &+ \left(\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3}\kappa A_\kappa S^3 + \mathrm{h.c}\right) \\ &+ m_{\tilde{Q}}^2|\tilde{Q}|^2 + m_{\tilde{u}}^2|\tilde{u}_R^2| + m_{\tilde{d}}^2|\tilde{d}_R^2| + m_{\tilde{L}}^2|\tilde{L}^2| + m_{\tilde{e}}^2|\tilde{e}_R^2| \\ &+ (y_u A_u \tilde{Q} \cdot H_u \tilde{u}_R^c - y_d A_d \tilde{Q} \cdot H_d \tilde{d}_R^c - y_e A_e \tilde{L} \cdot H_d \tilde{e}_R^c) \\ &- \frac{1}{2}(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}_i \tilde{W}_i + M_3 \tilde{G}^a \tilde{G}^a), \end{aligned} \tag{5}$$

(i) The first two lines belong to the Higgs sector with the first two terms in the first line representing the soft mass terms for the Higgs doublets and the third, not present in the MSSM, of the singlet. The second line, not present in the MSSM either, represents the NMSSM trilinear Higgs couplings A_κ, A_λ.

(ii) The third and fourth lines belong to the sfermion sector with a structure and a content exactly the same as in the MSSM with first the soft sfermion masses ($m_{\tilde{Q}/\tilde{L}}$ for the doublet squark/slepton, and $m_{\tilde{u},\tilde{d},\tilde{e}}$ for the RH singlets) followed by the MSSM-like trilinear A terms for squarks and sleptons A_u, A_d and A_e. We have only written the terms for one generic generation since we are not considering inter-generation mixing.

(iii) The last line contains the soft mass terms for, respectively, the $U(1)$, $SU(2)$ and $SU(3)$ gauginos, also called bino, winos and gluinos.

We consider the NMSSM with CP conservation so that all parameters are taken to be real.

The neutral components of the Higgs doublets, H_u and H_d, contain both a CP even and a CP odd part. After expanding around their vacuum expectation values, their scalar neutral component reads

$$\begin{aligned} H_d^0 &= v_d + \frac{1}{\sqrt{2}}(h_d^0 + i a_d^0), \\ H_u^0 &= v_u + \frac{1}{\sqrt{2}}(h_u^0 + i a_u^0), \\ S^0 &= s + \frac{1}{\sqrt{2}}(h_s^0 + i a_s^0) \end{aligned} \tag{6}$$

The vacuum expectation values, v_u, v_d, s are chosen to be real and positive. As in the MSSM we define $\tan\beta \equiv t_\beta = v_u/v_d$ and $v^2 = v_u^2 + v_d^2$ such that the W mass comes out to be $M_W^2 = g^2 v^2/2$.

The so-called Higgsino mass parameter in the MSSM is now a derived parameter. μ is generated dynamically from the vev of the singlet field,

$$\mu = \lambda s. \tag{7}$$

It is convenient to keep μ as an independent parameter, comparison with the MSSM will then be easier. With μ, we take λ and κ as independent parameter while s is kept as a shorthand notation for μ/λ in the same way as we use c_W as a short-hand notation for M_W/M_Z.

The particle content of the NMSSM has extra particles in the neutralino and Higgs sector than what constitutes the MSSM. The physical scalar fields consist of 3 neutral CP-even Higgs bosons, h_1^0, h_2^0, h_3^0, 2 CP-odd Higgs bosons, A_1^0, A_2^0 and a charged Higgs boson, $H^\pm$. The fermionic component of $\hat{S}$ is a neutralino called singlino. It mixes with the two Higgsinos. With the two gauginos (from U(1) and SU(2)) the NMSSM has five neutralinos.

To summarize, the parameters that will be relevant for the present paper which covers the neutralino, chargino and sfermion sector and which need to be renormalized (apart from the SM parameters) are

$$\underbrace{t_\beta, \lambda, \kappa, \mu}_{\text{in Higgs also}}, M_1, M_2; \quad m_{\tilde{Q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, A_u, A_d; \quad m_{\tilde{L}}, m_{\tilde{e}_R}, A_e, \tag{8}$$

The first six of these parameters enter the chargino/neutralino sector. t_β, λ, κ, μ also enter the Higgs sector. In fact, t_β and μ are also present in the sfermion sector. The second group corresponds to the squark sector while the last group corresponds to the sleptons.

Other parameters not listed in Eq. (8), such as A_κ and A_λ, enter only the Higgs sector. They will be studied in a separate publication detailing the treatment of the Higgs sector. Because of the supersymmetric nature of the model, in particular the origin of the μ parameter, the neutralino/chargino and the Higgs sectors share parameters in common as was presented in Eq. (8). Since it may be advantageous to use inputs from the Higgs sector to extract one or all of the parameters t_β, λ, κ, μ in Eq. (8), their extraction will be influenced by how all the parameters of the Higgs sector are extracted. Let us, therefore, list the nine parameters of the Higgs sector:

$$\underbrace{t_\beta, \lambda, \kappa, \mu}_{\text{in } \tilde{\chi} \text{ sector also}}, A_\lambda, A_\kappa, m_{H_d}, m_{H_u}, m_S. \tag{9}$$

Finally since we concentrate on electroweak corrections and do not consider gluino production or decay, the renormalization of M_3 is not needed.

III. FULL ONE-LOOP CORRECTIONS: GENERAL APPROACH

A. Renormalization: Our general approach

The renormalization procedure follows the same approach as the one adopted in SloopS for the SM and the MSSM. Namely we aim primarily at an on-shell renormalization of all parameters [35,36]. Other realizations of on-shell renormalization schemes for the chargino/neutralino sector have also been performed both in the MSSM [31,53], the complex MSSM [54] and the NMSSM [31].

OS schemes mean that one uses as inputs physical observables which are, therefore, defined when particles taking part in these observables are physical and on their mass shell. Technically, the easiest and most obvious set of this type of observables are the masses of the particles themselves. In this case, one only exploits the pole structure of two-point self-energy functions and require that the residue at the pole be unity. One difficulty occurs when we have mixing between particles sharing the same quantum numbers and, therefore, transitions from one to the other are possible. This will occur for Higgses, charginos, neutralinos and sfermions. The OS conditions mean also that when these physical particles are on their mass shell these (nondiagonal) transitions vanish. From another technical point of view this means that in the calculation of scattering amplitudes and decays we should not worry about corrections on the external legs, the wave functions will be automatically normalized. Recall that at tree level one starts with the underlying parameters of a Lagrangian in terms of current/gauge fields where mixing between these fields is present. We then move to the physical basis where the physical fields are defined. This is achieved by some diagonalizing matrices. At one-loop each underlying parameter is shifted by the addition of a counterterm. There is then a minimum set of conditions to restrict the form and the value of the counterterm. This shifting of parameters will at one-loop mix some particles. To perform a full definition of a physical particle at one-loop, in our approach, we introduce a matrix of wave functions with the conditions that when these transitions (containing one-loop plus counterterms) are evaluated OS, all transitions vanish. It is important to stress that it is unnecessary to introduce shifts in the diagonalizing matrix that was used at tree level.

Related to mixing also is the fact that one physical parameter, for example the mass of one neutralino in the NMSSM, depends on a large number of independent underlying parameters contained, in this case, in the 5×5 mixing matrix. For instance, besides the SM parameters, six parameters [the first set in Eq. (8)] contribute to the neutralino mass matrix. In this particular case, one needs to solve a system of six coupled equations. This is the reason why the reconstruction of the parameters, or in other words the necessary counterterms, requires finding the solution to a (large) system of coupled equations. Finding the solutions can be extremely difficult and sometimes impossible from a partial or even total knowledge of the physical parameters. For example, the chargino masses can furnish M_2, μ but with a $M_2 \leftrightarrow \mu$ degeneracy. If the system of coupled equations can be split into different independent subsystems of equations, the extraction of the parameters will be much easier and their evaluations less subject to uncertainties in the sense of being less sensitive to small variations in the input parameters. Therefore, by combining different sectors one can work with smaller, independent blocks which are easier or more efficiently solved. For example, take the set in Eq. (8), t_β originates from the Higgs sector and finds it way in all sectors of the NMSSM. As we will illustrate, it is much easier to get the counterterm for t_β from the Higgs sector for which we could revert to a $\overline{\rm DR}$ scheme. In this case, this involves a one-to-one mapping between the required counterterm for t_β and some simple evaluation of 2-point functions involving the Higgs. Reverting to the Higgs sector for this particular parameter is, therefore, technically much easier than trying to extract all the six parameters in the first set of Eq. (8) solely from the neutralino/chargino sector. Moreover by extracting t_β from the Higgs sector we can choose a scheme where one further decomposes the remaining system of the 5×5 coupled equations into two blocks: two equations from the chargino sector that will then furnish μ, M_2 and the rest can be determined from the neutralino sector. Another advantage is that we have a much better handle on the extraction of t_β. Indeed, as we stressed and as we will see explicitly, the effect of t_β on the neutralino/chargino is quite small. In a nutshell, a physical mass M_χ of a neutralino/chargino is essentially given by a soft mass M with a small correction ϵ_m which is proportional to t_β, such that $M_\chi = M + t_\beta \epsilon_m$, then $t_\beta \propto 1/\epsilon_m$. Although we will propose to use the Higgs sector for a definition of t_β, we will in this first paper be very brief about the renormalization of the Higgs, the full renormalization of the Higgs sector will be detailed in a forthcoming publication [52]. In order to facilitate the comparison with other computations, we will also use a $\overline{\rm DR}$ scheme in which the six parameters of the neutralino/chargino sector are taken as $\overline{\rm DR}$ while on-shell conditions are used for the SM parameters.

Leaving aside the issue of t_β (where it is defined from), the chargino/neutralino sector through the masses of the seven particles it contains, could furnish enough input to constrain the set of six parameters. There are various choices for the minimal set of inputs. We will propose a few. The most appropriate choice of input may depend on the observable considered. For example, imagine a scenario where M_1 is much larger than all other masses. The scheme with the three lightest neutralinos will be quite insensitive to M_1 and its counterterm. As long as we concentrate on correcting observables that are not sensitive to the bino component, this should be fine but clearly within this scheme we should not expect to make a good prediction to any observable where the bino component plays a role. Similar issues occur with the singlino. The mention of the bino and singlino component, or any other component for that matter, raises the issue of how can one weigh any of these components from a knowledge of masses only. In general, this is not possible. This is one of the shortcomings of the OS approach based solely on masses that we will present here. Schemes where one can use a particular decay of a neutralino which is sensitive to a particular coupling and hence component, in lieu of a mass, are possible but they are technically challenging (use of three-point function) and we will not implement this approach in this first publication.

To be complete, let us recall that the fermion and gauge sector of the SM is renormalized on-shell which means that the gauge boson masses are defined from the pole masses and that the electromagnetic coupling, α, is defined in the Thomson limit. One should keep in mind that the scale of the latter, $q^2 = 0$, is far smaller than the electroweak scale or the masses of the various supersymmetric particles we are dealing with. A running of α, from $q^2 = 0$ to $q^2 = M_Z^2$ brings in about a 7% correction.

If a complete and proper renormalization procedure has been achieved, all observables should be ultra-violet finite. We always perform this stringent test and check for the absence of ultraviolet divergences. Such divergences arise in loop integrals and are encoded in the parameter $C_{\rm UV}$ defined in dimensional reduction as $C_{\rm UV} = 2/\epsilon - \gamma_E + \ln(4\pi)$ where $\epsilon = 4 - d$, d being the number of dimensions and γ_E is the Euler constant[1]. Since physical processes must be finite, we simply check that the numerical results, for one-loop corrections to masses or to decay processes, are independent of $C_{\rm UV}$ by varying the numerical value of $C_{\rm UV}$ from 0 to 10^7. We require that the numerical results agree up to five or seven digits (recall that SloopS uses double precision). Such tests have proven extremely useful in testing the code at each step of its implementation. In schemes where at least one parameter is taken to be $\overline{\rm DR}$, a dependence on the renormalization scale $\bar{\mu}$ also appears. For all decay processes, we have set this scale to the mass of the decaying particle and in calculating corrected masses this scale is set at the tree-level mass of the particle.

B. Infrared and real corrections

A second test concerns infrared finiteness. Infrared divergences arise in processes involving charged particles in external legs. The regularization of the divergence from the pure loop contribution is done in FormCalc by adding

[1]In SloopS, we apply the constrained differential renormalization scheme which has been shown to be equivalent to the SUSY conserving dimensional reduction scheme [41].

a fictitious mass to the photon (λ). After adding the real photon emission, the divergence associated with the soft photon emission will exactly cancel that of the pure loop contribution ($1V$)

$$\sigma_{1V+\text{soft}}(s,k_c) = \sigma_{1V}(s,\lambda) + \sigma_{\text{soft}}(s,\lambda,k_c) \quad (10)$$

where k_c is a cut on the energy of the photon introduced to separate the soft and hard part when performing the phase space integral for the real emission,

$$\sigma_{1\text{soft+hard}}(s,\lambda) = \sigma_{\text{soft}}(s,\lambda,k_c) + \sigma_{\text{hard}}(s,k_c). \quad (11)$$

To check the convergence we modify the value of λ. Note that the k_c dependence should disappear when calculating the sum of the soft and hard part. This check is not automatized in SloopS, one has to calculate the sum of soft and hard emission for different values of k_c until a plateau is reached.

In our calculations of the decays of squarks, we have also considered QCD corrections. In all examples we have considered in the present paper, the genuine non-Abelian structure of QCD is not present. For all these cases, we adopt the same procedure for taming the infrared divergences concerning gluons as the one we apply to infrared photons. For these applications, we give the gluon a mass.

IV. RENORMALIZATION OF THE CHARGINO AND NEUTRALINO SECTOR

A. Implementing our general considerations

Before entering into the details of the chargino/neutralino sector let us review our setup for the renormalization of the fermions as fit for the neutralinos and charginos. We will follow, almost verbatim, the implementation in the MSSM carried out in [35,36]. We reproduce the different steps so the reader can follow exactly how we impose our conditions on the different counterterms.

For a Dirac fermionic field $\psi = \binom{\psi^L}{\psi^{R\dagger}}$ with a bare mass M_0, the kinetic and mass terms of the Lagrangian can be written at tree level as

$$\begin{aligned}\mathcal{L}_0^{\text{Dirac}} = {} & i(\psi_0^R \sigma^\mu \partial_\mu \psi_0^{R\dagger} + \psi_0^{L\dagger}\bar{\sigma}^\mu \partial_\mu \psi_0^L) \\ & - (\psi_0^{RT} M_0 \psi_0^L + \psi_0^{L\dagger} M_0^\dagger \psi_0^{R\dagger}). \end{aligned} \quad (12)$$

When several fermions mix, the mass term M_0 simply becomes a matrix. M_0 can involve a large number of underlying parameters. The mass eigenstates are obtained after diagonalizing the mass matrix with two unitary matrices D^R and D^L,

$$\chi_0^R = D^R \psi_0^R, \qquad \chi_0^L = D^L \psi_0^L,. \quad (13)$$

such that

$$\tilde{M}_0 = D^{R*} M_0 D^{L\dagger} = \tilde{M}_0^\dagger = \text{diag}(m_{\chi_1}, m_{\chi_2}, \ldots). \quad (14)$$

We now shift M_0 by shifting the parameters of its elements and proceed to shift fields through wave function normalization,

$$M_0 = M + \delta M \quad (15)$$

$$\chi_{i_0}^{R,L} = \left(\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{R,L}\right)\chi_j^{R,L} \quad (16)$$

M and χ_i are the renormalized matrix and fields respectively and $\chi_i^{R,L} = P_{R,L}\chi_i$ where $P_{R,L} = (1 \pm \gamma_5)/2$ are projection operators. For a Majorana fermion, as will be the case for the neutralinos, $\psi_0^L = \psi_0^R = \psi$, only one counterterm matrix is required; likewise, one unitary matrix is needed for the diagonalization of the mass matrix. Following [36] the renormalized two-point function describing the $i \to j$ transition can be written in a compact notation,

$$\begin{aligned}\hat{\Sigma}_{ij}(q) = {} & \Sigma_{ij}(q) - P_L \delta m_{ij} - P_R \delta m_{ji}^* \\ & + \frac{1}{2}(\not{q} - m_{\chi_i})[\delta Z_{ij}^L P_L + \delta Z_{ij}^{R*} P_R] \\ & + \frac{1}{2}[\delta Z_{ji}^{L*} P_R + \delta Z_{ji}^R P_L](\not{q} - m_{\chi_j}) \end{aligned} \quad (17)$$

including the one-loop self-energy $\Sigma_{ij}(q)$ and the counterterms δm_{ij} that represent the correction to the element $\tilde{M}_{ij}$, i.e., $\delta m_{ij} = D^{R*}\delta M D^{L\dagger}$. We stress again that we are using the same diagonalizing matrices $D^{R,L}$ as those used at tree level. This formula makes it clear that the mass and wave-function counterterms can be obtained separately from on-shell (OS) conditions.

Using one of the masses m_{χ_i} one can impose one of the OS conditions on the physical pole mass

$$\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{\chi}_i\tilde{\chi}_i}(q) u_{\chi_i}(q) = 0 \quad \text{for } q^2 = m_{\chi_i}^2. \quad (18)$$

$\widetilde{\text{Re}}$ means that the imaginary dispersive part of the loop function is discarded so as to maintain Hermiticity at one-loop. m_{χ_i} is the tree-level mass. Using a mass m_{χ_i} as an input means that the tree-level mass that is used in Eq. (18) receives no correction at one-loop. This gives a direct constraint on the δm_{ii} element which will be used as one condition to solve for the system of equations that define the full set of counterterms. When this full set of counterterms is solved, Eq. (18) is used to calculate the pole mass for the particles that were not used as input, see [36] for the algebraic details. Considering the number of coupled equations, finiteness of the mass(es) derived at one-loop is a highly nontrivial test and shows the robustness of our code. We always perform this finiteness test.

Wave-function renormalization constants are derived by requiring that

(i) the diagonal renormalized 2-point self-energies for $i \to i$ transitions have residue of 1 at the pole mass. This pole mass may get a one-loop correction. For our treatment at one-loop, it is sufficient to impose the residue condition by taking the tree-level mass. This translates into

$$\lim_{q^2\to m^2_{\chi_i}} \frac{\not{q}+m_{\chi_i}}{q^2-m^2_{\chi_i}} \widetilde{\mathrm{Re}}\hat{\Sigma}_{\chi_i\chi_i}(q) u_{\chi_i}(q) = u_{\chi_i}(q) \quad \text{and}$$

$$\lim_{q^2\to m^2_{\chi_i}} \bar{u}_{\chi_i}(q) \widetilde{\mathrm{Re}}\hat{\Sigma}_{\chi_i\chi_i}(q) \frac{\not{q}+m_{\chi_i}}{q^2-m^2_{\chi_i}} = \bar{u}_{\chi_i}(q). \tag{19}$$

(ii) To avoid any $i \to j$, $i \neq j$, transition we impose

$$\widetilde{\mathrm{Re}}\hat{\Sigma}_{\chi_i\chi_j}(q) u_{\chi_j}(q) = 0 \quad \text{for } q^2 = m^2_{\chi_j}, \qquad (i \neq j). \tag{20}$$

B. Specializing to the case of the charginos and neutralinos

The new fermions in the electroweak sector of the NMSSM are the two charginos, combination of charged winos and Higgsinos as in the MSSM, and the five neutralinos, combination of bino, wino, neutral Higgsinos and the singlino. In the basis

$$\psi^R_c = \begin{pmatrix} -i\tilde{W}^- \\ \tilde{H}^-_d \end{pmatrix}, \qquad \psi^L_c = \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{H}^+_u \end{pmatrix}, \tag{21}$$

the mass matrix for the charginos reads,

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix}, \tag{22}$$

while for the neutralinos in the basis

$$\psi^{RT}_n = \psi^{LT}_n = \psi^{0T} = (-i\tilde{B}^0, -i\tilde{W}^0_3, \tilde{H}^0_d, \tilde{H}^0_u, \tilde{S}^0) \tag{23}$$

the mass matrix reads

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu & -\lambda v s_\beta \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 & -\lambda v c_\beta \\ 0 & 0 & -\lambda v s_\beta & -\lambda v c_\beta & 2\kappa s \end{pmatrix}, \tag{24}$$

The charginos and neutralinos eigenstates are obtained with the help of two unitary matrices U and V for charginos and one unitary matrix N for neutralinos [U, V, N are particular manifestations of the matrices $D^{L,R}$ introduced in Eq. (13)],

$$\chi^R = U\psi^R_c, \qquad \chi^L = V\psi^L_c, \qquad \chi^0 = N\psi^0, \tag{25}$$

leading to the mass eigenstates

$$\tilde{X} = U^* X V^\dagger = \mathrm{diag}(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}), \qquad \tilde{Y} = N^* Y N^\dagger = \mathrm{diag}(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}, m_{\tilde{\chi}^0_5}). \tag{26}$$

Following our program, we proceed to shift the underlying parameters. This results in introducing counterterms to the mass matrices

$$\delta X = \begin{pmatrix} \delta M_2 & \delta X_{12} \\ \delta X_{21} & \delta\mu \end{pmatrix}, \qquad \delta Y = \begin{pmatrix} \delta M_1 & 0 & \delta Y_{13} & \delta Y_{14} & 0 \\ 0 & \delta M_2 & \delta Y_{23} & \delta Y_{24} & 0 \\ \delta Y_{13} & \delta Y_{23} & 0 & -\delta\mu & \delta Y_{35} \\ \delta Y_{14} & \delta Y_{24} & -\delta\mu & 0 & \delta Y_{45} \\ 0 & 0 & \delta Y_{35} & \delta Y_{45} & \delta Y_{55} \end{pmatrix}, \tag{27}$$

with, in the chargino case,

$$\begin{cases} \delta X_{12} = \sqrt{2} s_\beta \delta M_W + \sqrt{2} M_W s_\beta c^2_\beta \frac{\delta t_\beta}{t_\beta}, \\ \delta X_{21} = \sqrt{2} c_\beta \delta M_W - \sqrt{2} M_W s^2_\beta c_\beta \frac{\delta t_\beta}{t_\beta}, \end{cases} \tag{28}$$

and for the neutralino counterterms

$$\begin{cases} \delta Y_{13} &=& -M_Z s_W c_\beta\left[\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{1}{2}\frac{\delta s_W^2}{s_W^2}\right]+M_Z s_W\frac{t_\beta^2}{(1+t_\beta^2)^{3/2}}\frac{\delta t_\beta}{t_\beta}, \\ \delta Y_{14} &=& +M_Z s_W s_\beta\left[\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{1}{2}\frac{\delta s_W^2}{s_W^2}\right]+M_Z s_W\frac{t_\beta}{(1+t_\beta^2)^{3/2}}\frac{\delta t_\beta}{t_\beta}, \\ \delta Y_{23} &=& +M_Z c_W c_\beta\left[\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{1}{2}\frac{\delta c_W^2}{c_W^2}\right]-M_Z c_W\frac{t_\beta^2}{(1+t_\beta^2)^{3/2}}\frac{\delta t_\beta}{t_\beta}, \\ \delta Y_{24} &=& -M_Z c_W s_\beta\left[\frac{1}{2}\frac{\delta M_Z^2}{M_Z^2}+\frac{1}{2}\frac{\delta c_W^2}{c_W^2}\right]-M_Z c_W\frac{t_\beta}{(1+t_\beta^2)^{3/2}}\frac{\delta t_\beta}{t_\beta}, \\ \delta Y_{35} &=& -v s_\beta\delta\lambda-\lambda v s_\beta c_\beta^2\frac{\delta t_\beta}{t_\beta}-\lambda s_\beta v\left(\frac{\delta M_W}{M_W}-\frac{\delta e}{e}+\frac{\delta s_W}{s_W}\right), \\ \delta Y_{45} &=& -v c_\beta\delta\lambda+\lambda v s_\beta^2 c_\beta\frac{\delta t_\beta}{t_\beta}-\lambda c_\beta v\left(\frac{\delta M_W}{M_W}-\frac{\delta e}{e}+\frac{\delta s_W}{s_W}\right), \\ \delta Y_{55} &=& 2(\kappa\delta s+s\delta\kappa). \end{cases} \tag{29}$$

with the constraint $\delta c_W^2=-\delta s_W^2=\delta(M_W^2/M_Z^2)$ and $\delta\mu=\delta(\lambda s)$ [v is also defined from α, M_W, M_Z, a constraint which is implemented explicitly in Eq. (29)].

As we have shown in the general presentation, the renormalized self energies lead to corrections, δm_{χ_i}, to the tree-level masses. Imposing that some of these corrections vanish will put constraints on δX, δY or else will give finite one-loop correction to the mass. Note again that since after the shifts on the parameters are made we still keep the same diagonalizing matrices, we have for the corrections on the physical masses

$$\begin{aligned} \mathrm{diag}(\delta m_{\tilde{\chi}_i^\pm}) &= \delta\tilde{X}=U^*\delta X V^\dagger, \\ \mathrm{diag}(\delta m_{\tilde{\chi}_i^0}) &= \delta\tilde{Y}=N^*\delta Y N^\dagger. \end{aligned} \tag{30}$$

C. Issues in the reconstruction of the counterterms of the chargino and neutralino sector

To fully define the chargino/neutralino sector one needs, besides the SM parameters α and $M_{W,Z}$, to reconstruct and define the six parameters listed in Eq. (8), namely, t_β, λ, κ, μ, M_1, M_2. This set defines the matrices X, Y; see Eqs. (22) and (24). Three of these parameters are common to both the neutralino sector and the chargino sector; these are t_β, μ, M_2 while M_1, λ, κ are present only in the neutralino sector. Clearly, the sole knowledge of two chargino masses is not sufficient to constrain μ, M_2 and t_β. However, if t_β is provided from some other source then input from the two chargino masses can reconstruct M_2, μ. In this case, three neutralino masses are sufficient to define M_1, λ, κ. For this, one needs to solve a system of three equations.

In principle, the chargino/neutralino sector by providing seven physical masses can furnish enough constraint to define the set of the six counterterms. However, apart from assuming that one is in the lucky situation that as many as six (or seven) masses in the chargino/neutralino sector have been measured, a cursory look at the tree-level mass matrices X in Eq. (22) and Y in Eq. (24) already reveals the problems encountered in reconstructing the fundamental parameters of these mass matrices from the masses of the charginos and neutralinos only. First of all, we see that in the chargino sector, the t_β contribution is quite small. In the neutralino sector, the situation as concerns this parameter is not much better since either its contribution vanishes in the gaugeless limit ($g\to 0$ or M_W, $M_Z\to 0$), as in the chargino case or it is very much tangled up with the parameter λ. Moreover both t_β, λ represent mixing effects that may be difficult to extract from masses only. This is different from the extraction of M_1 for example where, if an almost binolike neutralino mass, $m_{\tilde{\chi}_i^0}$, is used as input, we would have an almost one-to-one mapping $M_1\sim m_{\tilde{\chi}_i^0}$. This said one must not forget that the problematic t_β, λ are also present in the Higgs sector and, in view of the observations we have made, it is worth studying whether some input from the Higgs sector may not be a better way of extracting t_β, λ. However, other parameters enter the Higgs sector but not the chargino/neutralino sector; see Eq. (9). Hence combining the Higgs and the chargino/neutralino sectors as many as 11 parameters should be reconstructed and we would, therefore, need as many inputs.

We would also like to point out an important conceptual issue having to do with the reconstruction of the underlying parameters from the sole knowledge of the physical masses, in particular from the chargino and neutralino sector. As is clear from the chargino mass matrix in Eq. (22) there is a $M_2\leftrightarrow\mu$ symmetry. Although the system can be solved by giving the two physical chargino masses, it is impossible to unambiguously assign the value of μ or M_2 to the correct "position" in the mass matrix. In other words, the Higgsino/wino content is not unambiguously assigned. This would, however, be important to know when we want to solve for the other remaining parameters in the neutralino sector.

Even without this caveat a similar problem occurs if one wants to unambiguously extract M_1 for example. A good reconstruction would require knowing not only the mass but the bino or singlino content of that mass. This is a challenging problem even in the (simpler) MSSM, [53,55,56]. We will assume that some knowledge of the content is available from a measurement of some decay or cross section and from comparing the chargino and neutralino mass spectrum, see [36] for a discussion on this issue.

Setting aside these issues and remarks, let us return to the problem of defining and reconstructing the underlying parameters and counterterms. Since, for the chargino/neutralino system, we need to define and solve for six counterterms, we need a trade-off that supplies six inputs or conditions, $\texttt{input}_1, ..., \texttt{input}_6$. Different choices of the $n = 6$ inputs correspond to a renormalization scheme. We have also discussed that we may have to revert to a larger set that includes the Higgs sector, in this case solving for both the Higgs and chargino/neutralino we may have to extend the six needed inputs to as many as $n = 11$; see Eq. (9).

Therefore, in all generality, one needs to invert a system such as

$$\begin{pmatrix} \delta\texttt{input}_1 \\ \cdots \\ \cdots \\ \delta\texttt{input}_n \end{pmatrix} = \mathcal{P}_{n,\mathrm{param}} \begin{pmatrix} \delta\mu \\ \delta M_2 \\ \delta\kappa \\ \delta M_1 \\ \delta\lambda \\ \delta t_\beta \\ \cdots \end{pmatrix} + \mathcal{R}_{n,\mathrm{residual}}, \qquad (31)$$

Where $\mathcal{R}_{n,\mathrm{residual}}$ contains other counterterms, such as gauge couplings, that are defined separately. Using the physical mass of one of the neutralinos/charginos as an input [see Eq. (18)] is a possible choice in an OS scheme. Not all inputs need to be OS. In fact, it is perfectly legitimate to adopt a fully $\overline{\mathrm{DR}}$ scheme. In this particular case, the counterterms can be simply read off from an external code such as NMSSMTools or any code based on the solution of the renormalization group equation (RGE), at one-loop. In passing, let us add that we have checked systematically that the C_{UV} part of our counterterms are the same, independently of how we extract them and we checked that they are consistent with the values extracted from NMSSMTools.

To make the system Eq. (31) manageable, one should strive to reduce the rank of the matrix $\mathcal{P}_{n,\mathrm{param}}$ by breaking it into independent blocks, such that

$$\mathcal{P}_{n,\mathrm{param}} = \mathcal{P}_{m,\mathrm{param}} \oplus \mathcal{P}_{p,\mathrm{param}} \oplus \cdots, \quad m + p + \ldots = n. \qquad (32)$$

We will compare a few schemes and implementations. In what we will call the mixed $\overline{\mathrm{DR}}$ on-shell schemes, we work to reconstruct the six parameters of the chargino/neutralino sector, therefore $n = 6$. t_β will be extracted from a $\overline{\mathrm{DR}}$ condition on t_β (from the Higgs sector), M_2, μ from the charginos and the rest of the three parameters solely from the neutralinos. In this case, we have

$$\mathcal{P}_{6,\mathrm{param}} = \mathcal{P}_{1,\mathrm{param}} \oplus \mathcal{P}_{2,\mathrm{param}} \oplus \mathcal{P}_{3,\mathrm{param}}. \qquad (33)$$

As with all resolutions of a system of equations, the inversion of the matrix $\mathcal{P}$ could introduce the inverse of a small determinant. We have already encountered such an example with t_β and the division by the small ϵ_m in Sec. III A. Another case concerns M_1 that can only be reconstructed precisely using the neutralino that is dominantly bino. This can easily be seen from the first term in Eq. (30), $\delta m_{\tilde{\chi}_i^0} = N_{i1}^{*2}\delta M_1 + \ldots$ If the mass of the dominantly bino neutralino is not chosen as an input parameter, then the extraction of δM_1 involves a division by a small number since N_{i1} is suppressed, hence can induce numerical instabilities. This is the reason we have brought up the issue of the content of the particle when its mass is used as input.

A second set of schemes, full OS-scheme, is a full $\mathcal{P}_{6,\mathrm{param}}$ where all inputs are masses from the chargino/neutralino sector. We have pointed at some of the shortcomings of this approach, lack of sensitivity to t_β and to λ to some extent. To achieve a better determination of the parameters, in particular the problematic t_β, we get help from the Higgs sector but this time all parameters are defined OS. In this case, among the inputs we will take some Higgs masses. This will be done at the expense of having a larger system, $\mathcal{P}_{8,\mathrm{param}}$., the extra two parameters that come into play are $A_{\lambda,\kappa}$.

D. Mixed $\overline{\mathrm{DR}}$ on-shell schemes

This setup is done along the decomposition $\mathcal{P}_{1,\mathrm{param}} \oplus \mathcal{P}_{2,\mathrm{param}} \oplus \mathcal{P}_{3,\mathrm{param}}$ where $\mathcal{P}_{1,\mathrm{param}}$ gets its source in the Higgs sector, implementing a $\overline{\mathrm{DR}}$ condition for t_β.

1. t_β from the Higgs sector

The renormalization of the Higgs sector is done within the same spirit as the one followed for the neutralino sector by the introduction of wave function renormalization constants, details will be given in a separate paper. The $\overline{\mathrm{DR}}$ condition calls for the wave function renormalization constants of the Higgs doublets. It is an extension of the DCPR scheme [57,58] used in the context of the MSSM to the NMSSM[59],

$$\delta t_\beta = \left[\frac{t_\beta}{2}(\delta Z_{H_u} - \delta Z_{H_d})\right]_\infty, \qquad (34)$$

where δZ_{H_u} and δZ_{H_d} are the wave function renormalization constants of the H_u and H_d doublets. The infinity symbol indicates that we take the divergent part of the expression. δZ_{H_u} and δZ_{H_d} are related to the wave function renormalization constants $Z_{h_i h_i}$ of the physical CP-even eigenstates h_1^0, h_2^0 and h_3^0. The latter are obtained from the CP-even neutral elements of H_u and H_d through the diagonalizing matrix S_h

$$(h_1^0, h_2^0, h_3^0) = (h_d^0, h_u^0, h_s^0) S_h^T \tag{35}$$

Explicitly,

$$\begin{aligned}\delta Z_{H_d} &= \frac{1}{R} \sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h,j3} S_{h,k2} \delta Z_{h_i h_i},\\ \delta Z_{H_u} &= \frac{1}{R} \sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h,j1} S_{h,k3} \delta Z_{h_i h_i},\end{aligned} \tag{36}$$

with

$$\delta Z_{h_i h_i} = \Sigma'_{h_i h_i}(m_{h_i}^2), \qquad R = -\sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h_i 1}^2 S_{h,j2}^2 S_{h,k3}^2, \tag{37}$$

where ϵ_{ijk} is the fully antisymmetric rank-3 tensor with $\epsilon_{123} = 1$ and $\Sigma'_{h_i h_i}(m_{h_i}^2)$ is the derivative of the self-energy of the Higgs h_i (with respect to its external momentum), evaluated at its mass m_{h_i}. This condition is such that the residue of the Higgs propagator is unity. The same requirement was imposed on the charginos and neutralinos.

In a $\overline{\mathrm{DR}}$ scheme, only the divergent part of the counterterm is defined, i.e., any finite term is set to 0. Nonetheless, the scheme and the one-loop result is still not fully defined unless one specifies the renormalization scale $\bar{\mu}$. The latter is the remnant scale introduced by the regularization procedure, dimensional reduction. Varying $\bar{\mu}$ can give some estimate on the theoretical uncertainty of the calculation due to the truncation at one-loop. In the numerical results obtained using a $\overline{\mathrm{DR}}$ scheme, the default value of $\bar{\mu}$ is fixed to be equal to the mass of the decaying particle or to the (tree-level) mass of the particle whose one-loop correction is calculated.

2. The charginos

Having solved for t_β, the chargino system, $\mathcal{P}_{2,\mathrm{param}}$ provides the simplest setup for defining μ, M_2 from the masses of both charginos as input. Exactly the same approach and the same expressions are found for the MSSM

$$\begin{aligned}\delta M_2 &= \frac{1}{M_2^2 - \mu^2}\Bigg((M_2 m_{\tilde{\chi}_1^+}^2 - \mu \det X) \frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} \\ &\quad + (M_2 m_{\tilde{\chi}_2^+}^2 - \mu \det X) \frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} \\ &\quad - M_W^2 (M_2 + \mu s_{2\beta}) \frac{\delta M_W^2}{M_W^2} - \mu M_W^2 c_{2\beta} s_{2\beta} \frac{\delta t_\beta}{t_\beta}\Bigg),\\ \delta\mu &= \frac{1}{\mu^2 - M_2^2}\Bigg((\mu m_{\tilde{\chi}_1^+}^2 - M_2 \det X) \frac{\delta m_{\tilde{\chi}_1^+}}{m_{\tilde{\chi}_1^+}} \\ &\quad + (\mu m_{\tilde{\chi}_2^+}^2 - M_2 \det X) \frac{\delta m_{\tilde{\chi}_2^+}}{m_{\tilde{\chi}_2^+}} \\ &\quad - M_W^2 (\mu + M_2 s_{2\beta}) \frac{\delta M_W^2}{M_W^2} - M_2 M_W^2 s_{2\beta} c_{2\beta} \frac{\delta t_\beta}{t_\beta}\Bigg).\end{aligned} \tag{38}$$

The explicit solutions shown in Eq. (38) give us the opportunity to go over the ambiguity on the *true* reconstruction of M_2, μ. In fact, Eq. (38) corresponds to four solutions since M_2, μ are given up to a sign and we have a $M_2 \leftrightarrow \mu$ ambiguity. This issue was discussed at some length, and some suggestions were given on how to lift the degeneracy [36]. By looking at the values of some decays (or cross sections) involving a chargino, for example, we can check that only one of the solutions is compatible with the value of the decay rate. This is a limitation on using only the value of the physical masses as input. Having chosen the correct $\delta\mu$, δM_2 we can now pass them to the neutralino sector.[2]

3. Three neutralino masses as input

We are now left with determining δM_1, $\delta\kappa$ (or $\delta(\kappa s)$) and $\delta\lambda$ using three neutralino masses, this is the $\mathcal{P}_{3,\mathrm{param}}$. Out of the five possible neutralino masses, assuming they have all been measured, one must pick up three masses that give the best reconstruction of the remaining parameters. As we pointed out, technically we should avoid having $\mathrm{Det}(\mathcal{P}_{3,\mathrm{param}}) \to 0$. Obviously the best extraction of M_1 would, ideally, need the binolike neutralino, whereas $\delta\kappa$ (or $\delta(\kappa s)$) is most directly tied up with the singlino component. A winolike neutralino as a third input will not do since this is essentially sensitive to M_2 with only feeble mixing with the λ contribution. The third neutralino to use as input is necessarily a Higgsino-like neutralino, again this is evident since λ in the NMSSM is intimately related to μ, the Higgsino parameter. One can also look at the mass matrix [Eq. (24)] to see that λ enters only in the singlino-Higgsino off-diagonal element. Therefore, the subset to choose calls for $\delta m_{\tilde{\chi}^0_{\text{“singlino”}}}$,

[2] Numerical problems may arise in the limit $\mu = M_2$, see [36] for a more thorough discussion.

$\delta m_{\tilde{\chi}^0_{\text{“bino”}}}$ and $\delta m_{\tilde{\chi}^0_{\text{“Higgsino”}}}$. We see again that a judicious choice calls for a knowledge of the *identity* of the particle apart from knowing the value of the corresponding mass exactly.

Having implemented the $\mathcal{P}_{1,\text{param}} \oplus \mathcal{P}_{2,\text{param}} \oplus \mathcal{P}_{3,\text{param}}$ approach this way, one can calculate the one-loop corrections to two neutralinos, the remaining winolike and the remaining Higgsino-like neutralinos.

E. Full OS schemes

1. The neutralino/chargino sector

Since all six parameters t_β, λ, κ, μ, M_1, M_2 are necessary to describe the chargino/neutralino sector which provides seven physical masses, one could entertain defining all these parameters from this sector. We have pointed out at the shortcomings of this extraction which has to do with the fact that the dependence on t_β is very weak and that the dependence on λ is complicated. From the technical point of view the reconstruction is also involved as it requires inverting a 6×6 system, $\mathcal{P}_{6,\text{param}}$. The best choice for $\mathcal{P}_{6,\text{param}}$ builds up on the remarks we have just made in picking up the three most appropriate neutralinos in the previous paragraph. Based on those arguments the $\mathcal{P}_{6,\text{param}}$. OS scheme uses the following set of inputs:

$$\begin{pmatrix} \delta m_{\tilde{\chi}_1^\pm} \\ \delta m_{\tilde{\chi}_2^\pm} \\ \delta m_{\tilde{\chi}^0_{\text{“singlino”}}} \\ \delta m_{\tilde{\chi}^0_{\text{“bino”}}} \\ \delta m_{\tilde{\chi}^0_{\text{“Higgsino”}}} \\ \delta m_{\tilde{\chi}^0_{\text{“Higgsino”}}} \end{pmatrix} = \mathcal{P}_{6,\text{param}} \begin{pmatrix} \delta\mu \\ \delta M_2 \\ \delta\kappa \\ \delta M_1 \\ \delta\lambda \\ \delta t_\beta \end{pmatrix} + \mathcal{R}_6, \qquad (39)$$

In the above, we have ordered the inputs in correspondence with the countertems they affect most directly, with the proviso that the Higgsinos do not reconstruct t_β and λ efficiently.

2. The neutralino/chargino and Higgs sectors

To improve the determination of λ and possibly t_β, while keeping with a full OS scheme, one has to get help from the Higgs sector. In that sector, the nature of the mixing between the scalar Higgses means that there is not a one-to-one mapping between t_β and a single Higgs mass. t_β gets tangled up with a reconstruction of A_κ and A_λ which are not needed for the chargino/neutralino sector. Therefore, at least three Higgs masses are needed. The most natural Higgs masses for this setup, directly related to A_κ and A_λ, are the two pseudoscalar masses $m_{A_1^0}$, $m_{A_2^0}$. To these one can add the charged Higgs boson, $H^\pm$, or one of the neutral CP even Higgsses. In any case, the addition of two more inputs for a better determination of the whole set of the chargino/neutralino sector means we are dealing with $\mathcal{P}_{8,\text{param}}$. One can also appeal to the Higgs sector for a better determination of λ trading another (second) CP even Higgs for a Higgsino-like neutralino. Summarizing these observations, the variants of the full OS scheme to extract the counterterms for t_β, λ, κ, μ, M_1, M_2, A_λ, A_κ use the masses

$$m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}, \quad m_{\tilde{\chi}^0_{\text{“singlino”}}}, \quad m_{\tilde{\chi}^0_{\text{“bino”}}}, \quad (m_{\tilde{\chi}^0_{\text{“Higgsino”}}} \text{ or } m_{h_i^0}),$$
$$(m_{H^\pm} \text{ or } m_{h_j^0}), \quad m_{A_1^0}, m_{A_2^0}$$

We refrain from giving the complete formulas for this setup since it relies heavily on the details of the implementation of the renormalization of the Higgs sector which will be presented elsewhere [52]. Although using an OS approach with the help of Higgs masses can constrain the singlino parameters we should not expect to have a very good determination of t_β. Indeed, even in the MSSM limit, we have shown [35] that if one takes the heavy CP-even Higgs mass, M_{H^0} as input together with the pseudoscalar from the doublet, M_{A^0}, then when $M_{A^0} \gg M_Z$,

$$\frac{\delta t_\beta}{t_\beta} \sim \frac{1}{M_{H^0}^2/M_{A^0}^2 - 1}(-\delta M_{A^0}^2/M_{A^0}^2 + \delta M_{H^0}^2/M_{H^0}^2). \qquad (40)$$

This could lead to a large finite part when $M_{H^0} \sim M_{A^0}$ as occurs in the decoupling limit.

V. RENORMALIZATION OF THE SFERMIONIC SECTOR

We now deal with the determination of the last set of the parameters listed in Eq. (8) concerning the sfermion sector. Since the implementation of the sfermionic sector in the NMSSM is exactly the same as in the MSSM, we have followed the same approach as the one we developed in [36]. We, therefore, refer to [36] for details and only summarize the setup here.

A. Squarks

For each generation, five parameters, $m_{\tilde{Q}}$, $m_{\tilde{u}_R}$, $m_{\tilde{d}_R}$, A_u, A_d, need to be defined (or renormalized) in the squark sector. Recall that each quark q ($q = u$, d) has two scalar superpartners, one for each chirality, $\tilde{q}_L$ and $\tilde{q}_R$. The squark mass matrix encodes the elements one needs to renormalize. In the ($\tilde{q}_L$, $\tilde{q}_R$) basis, the mass matrix $\mathcal{M}^2_{\tilde{q}}$ takes the form [see also Eq. (5)]

$$\mathcal{M}^2_{\tilde{q}} = \begin{pmatrix} m^2_{\tilde{Q}} + m^2_q + c_{2\beta}(T^3_q - Q_q s^2_W)M^2_Z & m_q(A_q - \mu t_\beta^{-2T^3_q}) \\ m_q(A_q - \mu t_\beta^{-2T^3_q}) & m^2_{\tilde{q}} + m^2_q + c_{2\beta}Q_q s^2_W M^2_Z \end{pmatrix}, \tag{41}$$

where T^3_q is the third component of the isospin for q whose mass is m_q. To define the physical eigenstates, we introduce the diagonalizing matrix R such that

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = R \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}, \qquad R = \begin{pmatrix} c_{\theta_q} & s_{\theta_q} \\ -s_{\theta_q} & c_{\theta_q} \end{pmatrix}, \tag{42}$$

The mass eigenstates will be denoted as $\tilde{q}_{1,2}$ with masses

$$\mathrm{diag}(m^2_{\tilde{q}_1}, m^2_{\tilde{q}_2}) = R\mathcal{M}^2_{\tilde{q}}R^T. \tag{43}$$

We will take $\tilde{q}_1$ to be the lightest eigenstate.

We then follow exactly the same procedure as in the neutralino/chargino sector. Namely we shift the underlying parameters in the mass matrix [Eq. (41)] and introduce wave function renormalization for the fields

$$\mathcal{M}^2_{\tilde{q}} = \mathcal{M}^2_{\tilde{q}} + \delta\mathcal{M}^2_{\tilde{q}}, \tag{44}$$

$$\tilde{q}_i = \left(\delta_{ij} + \frac{1}{2}\delta Z^{\tilde{q}}_{ij}\right)\tilde{q}_j. \tag{45}$$

The ensuing renormalized self-energies for the squarks read

$$\hat{\Sigma}_{\tilde{q}_i\tilde{q}_j}(q^2) = \Sigma_{\tilde{q}_i\tilde{q}_j}(q^2) - \delta m^2_{\tilde{q}_{ij}} + \frac{1}{2}\delta Z^{\tilde{q}}_{ij}(q^2 - m^2_{\tilde{q}_i}) + \frac{1}{2}\delta Z^{\tilde{q}}_{ji}(q^2 - m^2_{\tilde{q}_j}). \tag{46}$$

As was the case in the neutralino/chargino sector, the rotation matrices R, Eq. (42), are not renormalized. This means that the counterterms $\delta m^2_{\tilde{q}_{ij}}$ of the physical mass matrix are given by

$$\delta m^2_{\tilde{q}_{ij}} = (R\delta\mathcal{M}_{\tilde{q}^2}R^T)_{ij}. \tag{47}$$

Keeping with our general strategy we forbid mixing between different fields when they are on their mass shell, $\mathrm{Re}\hat{\Sigma}_{\tilde{q}_i\tilde{q}_j}(m^2_{\tilde{q}_i}) = 0$. Furthermore, we set the residue of the renormalized propagators to unity, $\mathrm{Re}\hat{\Sigma}'_{\tilde{q}_i\tilde{q}_i}(m^2_{\tilde{q}_i}) = 0$.

Because $SU(2)$ symmetry imposes a common mass to two of the four squarks (before mixing), in our scheme we take three physical squark masses as input. In SloopS the selected squark masses are $m_{\tilde{d}_1}$, $m_{\tilde{d}_2}$ and $m_{\tilde{u}_1}$. The definition of the mixings is directly related to physical observables namely the amplitude describing the decays $\tilde{u}_2 \to \tilde{u}_1 Z^0$ and $\tilde{d}_2 \to \tilde{d}_1 Z^0$. At tree level, this amplitude is a substitute for the mixing parameter θ_q, $q = u$, d, $\mathcal{M}^{\tilde{q}_2\tilde{q}_1 Z} = ig_Z T^3_q \sin(2\theta_q)/2$. θ_q defined this way is then promoted to the status of a physical observable ($g_Z = e/s_W c_W$ is extracted from the gauge sector). Therefore. the other two input parameters are $\theta_{u,d}$ for which a counterterm can be defined as

$$\delta m^2_{\tilde{q}_{12}} = -\mathrm{Re}\Sigma_{\tilde{q}_1\tilde{q}_2}\left(\frac{m^2_{\tilde{q}_1} + m^2_{\tilde{q}_2}}{2}\right); \tag{48}$$

see [36] for details. These inputs and conditions allow to construct the counterterms for the five underlying parameters of the squark sector [Eq. (8)]. Among the many predictions is that the mass of the squark $\tilde{u}_2$ receives a one-loop correction. UV finiteness of this correction is another test of our implementation.

B. Sleptons

The renormalization of the slepton sector follows the same methodology and can be considered as a simpler case of the squark system. Indeed, the absence of right-handed neutrinos means that, for each generation, there are only three associated particles: two charged sleptons and one sneutrino. Mixing occurs only in the charged sector. Three parameters, for each family, need to be fixed, $m_{\tilde{L}}$, $m_{\tilde{e}_R}$, A_e; see Eq. (8). The physical masses are $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{\nu}$. A simple OS scheme is to take the physical masses of these three particles as input parameters, $m_{\tilde{e}_1}$, $m_{\tilde{e}_2}$, and $m_{\tilde{\nu}}$. An alternative scheme is to take the (two) charged slepton masses as input with the addition of a constraint on the mixing as we have done for the squark sector, namely $\delta m^2_{\tilde{e}_{12}} = -\mathrm{Re}\Sigma_{\tilde{e}_1\tilde{e}_2}(\frac{m^2_{\tilde{e}_1}+m^2_{\tilde{e}_2}}{2})$ as could be extracted form $\tilde{e}_2 \to \tilde{e}_1 Z$, see [36]. We will stick with the first scheme that requires the three slepton masses. These different implementations for the squarks and sleptons may be useful when comparing the scheme dependence of the results for sfermion decays.

VI. BENCHMARK POINTS AND DEFINITION OF THE SCHEMES

To apply our formalism we obviously need to fully define a model. In particular, our OS renormalization requires physical input parameters and most importantly, as we saw, the use of a set of physical masses among the full spectrum. Since no particle of the NMSSM has been discovered yet it is difficult, even within a particular NMSSM scenario, to pick up the minimal set of input masses. Moreover, even after agreeing on a minimal set to carry the renormalization, the other parameters of the model are still needed in order to perform a complete calculation (some particles and their parameters will only enter indirectly through their loop effects). The reason we insist on this seemingly obvious point is that had a particular manifestation of the NMSSM been discovered experimentally, we would have had to use the physical observables, such as some of the physical masses, to reconstruct the underlying parameters of the model. Such an inversion is notoriously complicated even when performed at tree level, see [55] for instance and the discussion for the counterterms in Sec. IV C. The reconstruction would be easier if information on some decays and cross sections were given [35]. The best we can do is the following. We generate models by supplying all the needed underlying parameters, such as $M_1, M_2, \cdots, A_b, \cdots$. These parameters can be considered as parameters at the electroweak scale. Tree-level formulas are used to calculate the full spectrum. In turn, for one-loop calculations, masses of a subset of this spectrum are used as physical masses. The other masses will receive a loop correction. For example, we can take three neutralino masses as input and predict the one-loop corrections for the remaining two neutralinos of the NMSSM. Another related issue is that these theory generated (physical) masses from a"known" set of underlying parameters introduce a bias in our analysis in the sense that we know what the composition of the neutralino is. In particular, from the mass alone one cannot distinguish the singlinolike or binolike neutral state. What we want to stress here is that despite our OS approach we have some insider's knowledge due to the way we generate the points. This is the reason we will talk about a bino-dominated neutralino for example, an information easily accessed through the underlying parameters but much harder to assess from the mass spectrum. For the same model, we will consider different schemes. These correspond to different choices of the input masses for example.

A. Choice of the benchmark points

We choose five benchmark points in order to cover various hierarchies in the neutralino sector. In particular, the points we selected are classified according to the nature of the lightest supersymmetric particle (LSP) neutralino. The crucial parameter that defines the properties of the singlino component of the neutralino is λ, it ranges from a small value 0.03 (point 4) to moderate values of the order of the weak gauge coupling 0.1–0.4. The other parameter that defines the singlino and controls its mass, $m_{\tilde{S}}$, is κ. It is chosen to cover the range $2\kappa/\lambda = 0.5$–2. $2\kappa/\lambda$ is roughly the ratio between the singlino and Higgsino masses. Sfermions masses of the first two generations as well as the right-handed sbottom are ≈1 TeV for all the 5 benchmark points. While the mixing for the sbottom is always tiny leading to $\tilde{b}_1 = \tilde{b}_R$, we take large mixings for the stops. Three benchmark points have the lightest stop with mass around 0.5 TeV. Two scenarios have rather light $\tilde{\tau}_1$ of about 150 GeV. The LSPs are in the narrow range 110–140 GeV. The values for the underlying parameters for each of the benchmarks are summarized in Table I. The parameters of the NMSSM that do not appear in Table I take a common value for all points, $A_b = A_\tau = m_{L_{1,2}} = m_{\tilde{D}_{1,2}} = m_{\tilde{Q}_{1,2}} = 1000$ GeV while the SM parameters are fixed to $\alpha = 1/137.06$, $M_Z = 91.188$ GeV, $s_W = 0.481$, $\alpha_s(M_Z) = 0.118$. To summarize:

TABLE I. Parameters for the five benchmark points and tree-level masses of the neutralinos, charginos and third generation sfermions. For all points, $m_{D_{1,2}} = m_{Q_{1,2}} = m_{L_{1,2}} = A_b = A_\tau = 1000$ GeV. Parameters with mass dimension are expressed in GeV.

Parameter	Point 1	Point 2	Point 3	Point 4	Point 5
t_β	10	4.5	10	7	3.4
μ	250	250	120	600	550
M_1	1000	230	700	140	400
M_2	150	600	1000	200	150
M_3	2500	1000	1000	1000	1000
λ	0.1	0.2	0.1	0.03	0.4
κ	0.1	0.05	0.1	0.007	0.1
A_λ	150	1250	150	1000	1800
A_κ	0	0	0	0	0
A_t	3000	2200	4000	2300	2400
$m_{\tilde{Q}_3}$	2000	1500	2000	1600	1500
$m_{\tilde{U}_3}$	2000	500	1000	400	500
$m_{\tilde{D}_3}$	1000	1000	1000	1000	1000
$m_{\tilde{L}_3}$	1000	1000	1005	1000	1001.5
$m_{\tilde{R}_3}$	1000	149.5	1000	140	1005
$m_{\tilde{\chi}_1^0}$	125.7	123.4	112.8	138.1	139.4
$m_{\tilde{\chi}_2^0}$	257.3	200.9	123.8	193.1	276.2
$m_{\tilde{\chi}_3^0}$	278.7	255.7	241.6	280.0	392.7
$m_{\tilde{\chi}_4^0}$	500.8	271.7	702.8	603.8	557.3
$m_{\tilde{\chi}_5^0}$	1002.2	614.8	1006.6	612.6	574.1
$m_{\tilde{\chi}_1^+}$	126.8	239.6	118.0	192.9	140.1
$m_{\tilde{\chi}_2^+}$	285.9	614.7	1006.6	612.8	564.0
$m_{\tilde{t}_1}$	1873.0	459.7	935.4	358.3	453.4
$m_{\tilde{t}_2}$	2132.9	1531.7	2045.1	1627.5	1533.7
$\sin\theta_t$	0.707	0.984	0.976	0.988	0.983
$m_{\tilde{b}_1}$	1000.3	1000.3	1000.3	1000.2	1000.3
$m_{\tilde{b}_2}$	2000.9	1501.1	2000.9	1601.1	1501.0
$\sin\theta_b$	1	1	1	1	1
$m_{\tilde{\tau}_1}$	999.7	155.2	1000.9	146.4	1000.8
$m_{\tilde{\tau}_2}$	1002.3	1001.0	1006.1	1001.1	1006
$m_{\tilde{\nu}_\tau}$	998.0	998.1	1003	998.0	998.3
$\sin\theta_\tau$	0.727	1	0.997	1	0.153
m_{h_1} (tree)	88.4	78.0	88.5	85.9	85.7

(i) Point 1 features a winolike neutralino LSP. This point exhibits the largest wino-Higgsino mixing among all five scenarios, note that $\mu - M_2 \sim M_Z$.
(ii) Point 2 has a singlino dominated LSP. It is also the scenario where the singlino mixings to the other components, while still quite small, are the largest of all 5 scenarios. It also features the largest bino-Higgsino mixing, observe that here $\mu - M_1 \sim M_W/4$, a property that will enhance the bino-Higgsino mixing.
(iii) Point 3 features a Higssino-like LSP.
(iv) Point 4 has a binolike LSP. The singlino is practically decoupled with a very small value of λ.
(v) Point 5 also has winolike LSP but differs from point 1 in that the Higgsinos are the heaviest neutralinos. The lightest τ is mxed though dominantly left-handed.

We have also ensured that these benchmarks were phenomenologically viable, that is they possess a Higgs boson in the 122–128 GeV mass range (after including all loop corrections provided by NMSSMTools) and they satisfy theoretical and experimental constraints implemented in NMSSMTools. The SM-like Higgs is always the lightest *CP*-even scalar and for further reference to the benchmark points we give its mass at tree level in Table I. Note that the precise value of the Higgs mass will not play any role in the forthcoming numerical analysis. All these points also feature a light pseudoscalar particle in the range 4–60 GeV. This particle is, however, not directly relevant for the numerical examples that follow. We have also checked that all points satisfy at least the upper bound on the relic density extracted from Planck, $\Omega h^2 < 0.131$ after taking into account a 10% theoretical uncertainty [60]. Points 1, 3, and 5 have a value for the relic density below this range, as is typical of wino and Higgsino DM below the TeV scale, while points 2 and 4 fulfill the Planck condition. To achieve this, we required substantial coannihilation with sfermions, by adjusting $m_{\tilde{R}_3}$ the soft mass term for right-handed sleptons since, typically, scenarios with bino or singlino LSP lead to too much dark matter.

The components (bino, wino, Higgsino, singlino) of the neutralinos are shown in Table II. The neutralinos are labeled from lightest $\tilde{\chi}_1^0$ to heaviest $\tilde{\chi}_5^0$. Since for most points there is not a large mixing between the components, in order to capture the main properties of the benchmark point at a glance, we will refer to the benchmark in terms of its largest components as $(\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0) \sim (\tilde{W}_3^0, \tilde{H}^0, \tilde{H}^0, \tilde{S}^0, \tilde{B}^0)$. Note that in the gaugeless limit, $g \to 0 (M_Z, M_W \to 0)$, mixing occurs only between a singlino and a Higgsino, the strength of the latter being measured by λ. In the MSSM limit, (λ is small) and provided $|M_{1,2} - \mu| > M_Z$, the mixing between the wino and the Higgsino is of the order $M_W/\mathrm{Max}(\mu, M_2)$ and the mixing between the bino and the Higgsino is of order $M_Z s_W/\mathrm{Max}(\mu, M_1)$. In the same limit, the mixing between the bino and wino is vanishingly small, this mixing will first transit via a Higgsino. The t_β dependence is weak, for example in the chargino case the dependence is hidden in the small mixing factor $M_W(\mu + M_2/t_\beta)/\mathrm{Max}(\mu^2, M_2^2)$ and/or $M_W(M_2 + \mu/t_\beta)/\mathrm{Max}(\mu^2, M_2^2)$. These general observations explain the values of the mixing in Table II. In particular, the largest mixings occur for point 1 between the wino and the Higgsinos and for point 2 between the bino and the Higgsinos. Point 2 is also the point where the singlino component may be relevant for some of the states (apart from the LSP singlino, of course). Points 3 and 4 are the ones where the all neutralinos are the "purest."

TABLE II. Components of neutralino mass eigenstates for the five benchmark points. The dominant component is highlighted.

		Point 1	Point 2	Point 3	Point 4	Point 5
$\tilde{\chi}_1^0$	$\tilde{B}^0$	···	0.63%	···	**98.8%**	···
	$\tilde{W}^0$	**78.6%**	···	···	···	**96.2%**
	$\tilde{h}^0$	21.4%	3.88%	**98.4%**	0.85%	3.31%
	$\tilde{S}^0$	···	**95.4%**	0.77%	···	···
$\tilde{\chi}_2^0$	$\tilde{B}^0$	···	**55.8%**		0.49%	···
	$\tilde{W}^0$	1.6%	1.0%	···	**97.0%**	0.67%
	$\tilde{h}^0$	**98.3%**	40.0%	**99.5%**	2.54%	1.69%
	$\tilde{S}^0$	···	3.20%	···	···	**97.4%**
$\tilde{\chi}_3^0$	$\tilde{B}^0$	···	···	···	···	**95.2%**
	$\tilde{W}^0$	19.8%	···	···	···	···
	$\tilde{h}^0$	**79.8%**	**98.9%**	0.9%	···	4.05%
	$\tilde{S}^0$		0.58%	**99.1%**	**99.98%**	0.48%
$\tilde{\chi}_4^0$	$\tilde{B}^0$	···	43.3%	**99.6%**	···	···
	$\tilde{W}^0$	···	2.31%	···	···	···
	$\tilde{h}^0$		**53.6%%**	···	**99.51%**	**99.1%**
	$\tilde{S}^0$	**99.8%**	0.83%	···	···	0.53%
$\tilde{\chi}_5^0$	$\tilde{B}^0$	**99.7%**	···	···	0.54%	4.52%
	$\tilde{W}^0$	···	**96.3%**	**99.3%**	2.36%	2.53%
	$\tilde{h}^0$		3.62%	0.69%	**97.1%**	**91.8%**
	$\tilde{S}^0$	···	···	···	···	1.13%

B. Selecting the renormalization schemes

As we discussed in detail for the neutralinos, the choice of the renormalization scheme is crucial for a most efficient extraction of the counterterms. For instance, we argued that δM_1 will be badly reconstructed if the binolike neutralino was not used as an input parameter in a scenario with little mixing. This is the reason we will adapt the renormalization scheme for each benchmark point. We will compare the predictions for the same observable for different renormalization schemes within the same benchmark. The difference between these schemes lies in how we extract the (six) underlying parameters entering the neutralino/chargino sector. For all OS schemes, the chargino masses are always chosen as input. We consider the following schemes:

(1) Fully OS schemes with four neutralino masses classified as $\mathcal{P}_6$. These schemes will be denoted as
  (a) OS_{1234} when taking the 4 lightest neutralinos
  (b) OS_{2345} when taking the 4 heaviest neutralinos
(2) OS schemes where we take a $\overline{\mathrm{DR}}$ condition for δt_β in addition to three neutralino masses as input. We do so in order to have a better determination of t_β and decouple the system of equations for the neutralinos and charginos. These schemes will be denoted as
  (a) t_{123} when taking the three lightest neutralinos
  (b) t_{345} when taking the three heaviest neutralinos
  (c) t_{134} when taking the first, third, and fourth neutralinos
(3) Fully OS schemes of the $\mathcal{P}_8$ class where some masses from the Higgs sector are used as inputs. To fully determine the system, we need all in all (including the chargino masses) eight input parameters in this case. We resort to these schemes since, as pointed out earlier, schemes based on using solely the masses of the neutralinos are not expected to be good enough in reconstructing neither t_β nor λ. These two parameters will have a strong impact on the couplings of the neutralinos and hence a crucial influence on many of their decays. In this category, we use two types of schemes,
  (a) $OS_{ijkh_2A_1A_2}$ or $OS_{ijkH^+A_1A_2}$ schemes where three neutralinos, both pseudoscalars Higgses and either h_2 or H^+ are chosen as input in addition to the two charginos. The indices i, j, k indicate the relevant neutralinos. For each of these scenarios, we avoid taking the mass of the wino-dominated neutralino as input since M_2 is well extracted form the chargino mass measurements.
  (b) $OS_{ijh_2H^+A_1A_2}$ schemes where two neutralino masses as well as the Higgs singlet, charged Higgs and the two pseudoscalar Higgses are used as inputs.
(4) Full $\overline{\mathrm{DR}}$ scheme is also used for comparison. In this case, we take the renormalization scale $\bar{\mu}$ at the mass of the decaying particle as discussed earlier.

For processes involving sfermion decays, we stick with only one scheme as described in Sec. V.

VII. ONE-LOOP RESULTS FOR NEUTRALINO MASSES AND NEUTRALINO/CHARGINO DECAYS TO GAUGE BOSONS

In the absence of not too large mixings between the different components in the $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector, like in the five points we have chosen, the masses of the physical states are determined essentially by $M_1, M_2, \mu, 2\kappa s$. These parameters must, therefore, be determined accurately for a precise determination of the physical masses. Small contributions to these masses involve a knowledge of λ and t_β, but as argued previously the dependence in these two parameters is expected to be mild. When it comes to the decays, the situation is different since most decays involve transitions between different gauge eigenstates and, therefore, the decays are very often quite sensitive on the parameters that set the mixing. Therefore, in the decays we will be more careful about how λ and t_β are defined.

A. Neutralino masses

The calculation of the one-loop corrected neutralino masses only calls for the computation of two-point functions. Yet, ultraviolet finiteness of the full one-loop corrected neutralino masses is a nontrivial check on the theoretical consistency of our setup and its good implementation in our automated calculator SloopS, since a large number of counterterms is involved. Depending on the schemes we will select, only one or two neutralino masses will receive corrections at one-loop. For all five points, we compare the results of the schemes OS_{1234}, OS_{2345}, t_{123}, t_{234} and $\overline{\mathrm{DR}}$ for the masses. Predictions on the masses based on the schemes that rely on the Higgs sector will be briefly commented upon when we discuss the decays, this is motivated by the fact that these Higgs schemes bring in improvements on the mixings (essentially λ and to a lesser degree t_β) which are not supposed to be very important for the calculations of the masses.

We advocated that a good scheme should include at least one binolike, one singlinolike and one Higgsino-like from the neutralino sector (the winolike being well reconstructed from the chargino masses). The numerical results given in Table III generally follow our expectations.

In the OS_{1234} scheme, the only mass to be predicted is that of the heaviest neutralino, $m_{\tilde{\chi}_5^0}$. For point 1, the latter is dominantly bino. Since M_1 can not be reliably extracted from the four input masses $m_{\tilde{\chi}^0_{1,2,3,4}}$, the corrected mass $m_{\tilde{\chi}_5^0}$ is not trustworthy giving a correction of about 30%. This is in contrast with points 2 and 3 where the heaviest neutralino is dominantly wino. In this case, the chargino masses constrain M_2 very well. In both cases the heaviest neutralino receives a mass correction at the per-mil level. A similar statement can be made for points 4 and 5 for which the heaviest neutralino is a Higgsino whose main parameter, μ, is quite well constrained by the input from the charginos. Note that the correction here, though very modest, is slightly larger than in the case of the wino due to the fact that a full reconstruction still requires a knowledge of the underlying t_β and even λ for point 5.

In the OS_{2345} scheme, the only mass to be predicted is that of the lightest neutralino, $m_{\tilde{\chi}_1^0}$. As before, the masses which get the smallest correction correspond to the

TABLE III. One-loop corrected masses of neutralinos for different schemes and benchmark points. In bold, points for which the masses cannot be computed reliably. All masses are given in GeV. The one-loop corrections for all five neutralino masses in the $\overline{\mathrm{DR}}$ scheme are also given.

Scheme	Masses	Point 1	Point 2	Point 3	Point 4	Point 5
OS_{1234}	$m^{\mathrm{tree}}_{\tilde{\chi}_5}$	1002.17	614.78	1006.64	612.62	574.10
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_5}$	**729.01**	614.81	1006.56	608.83	573.22
OS_{2345}	$m^{\mathrm{tree}}_{\tilde{\chi}_1}$	125.67	123.42	112.77	138.09	139.37
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_1}$	125.56	**−89.66**	**147.38**	**205.31**	139.36
t_{123}	$m^{\mathrm{tree}}_{\tilde{\chi}_4}$	500.78	271.67	702.82	603.84	557.31
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_4}$	**−515.19**	275.13	**3802.01**	601.19	556.98
	$m^{\mathrm{tree}}_{\tilde{\chi}_5}$	1002.17	614.78	1006.64	612.62	574.10
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_5}$	**1426.14**	614.84	1006.99	613.34	577.17
t_{345}	$m^{\mathrm{tree}}_{\tilde{\chi}_1}$	125.67	123.42	112.77	138.09	139.37
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_1}$	125.61	**−1808.4**	**−2151.8**	**−479.9**	138.54
	$m^{\mathrm{tree}}_{\tilde{\chi}_2}$	257.30	200.86	123.80	193.12	276.19
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_2}$	257.83	**146.03**	**1236.66**	189.51	**74.11**
$\overline{\mathrm{DR}}$	$m^{1-\mathrm{loop}}_{\tilde{\chi}_1}$	136.00	124.10	120.38	140.90	147.84
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_2}$	265.61	204.60	129.52	204.23	278.56
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_3}$	286.68	259.56	241.533	280.01	395.25
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_4}$	500.72	278.72	703.09	601.75	557.64
	$m^{1-\mathrm{loop}}_{\tilde{\chi}_5}$	995.41	625.50	1009.26	613.84	577.47

winolike LSP, points 1 and 5. This is in sharp contrast to the singlino in point 2 and the bino in point 4 whose masses receive very large corrections. The LSP in point 3, although Higgsino-like, gets a non-negligible correction. This means, a point we hinted at previously, that the chargino system does not fully define the Higgsino-like neutralino due to the reconstruction of t_β. We expect the scheme t_{123} to fare better than OS_{1234}. Indeed, as compared to OS_{1234} the masses of the heaviest neutralino are changed very little for points 2, 3, and 5 and are predicted with a smaller correction for point 4. The t_{123} scheme, however, does not improve the situation for point 1 where the mass of the heaviest bino is, as always, badly reconstructed. The same problem afflicts the prediction of the mass of the binolike χ^0_4 for point 3 and of the singlino for point 1. Otherwise, the corrections for $\tilde{\chi}^0_4$ are negligible since these neutralinos are either winos or Higgsinos. Similar arguments explain the results for the one-loop corrected neutralino masses in the scheme t_{345}. This scheme works well for point 1 as the three heaviest neutralinos correspond to the bino, singlino and dominantly Higgsino. For point 2, the singlino component is not accessed which explains why the mass of the dominantly singlino $\tilde{\chi}^0_1$ cannot be predicted reliably. The correction to the mass of $\tilde{\chi}^0_2$, a dominantly bino neutralino with a large Higgsino admixture, is also large, around 30%. For point 3, the two lightest neutralinos are Higgsino-like and receive very large corrections, this illustrates the futility in using the mass of the neutral wino as input at the expense of one of the other neutralino masses. For point 4, the wino dominated neutralino receives small corrections while, as expected, the mass of the dominantly bino $\tilde{\chi}^0_1$ is unreliable. Similarly for point 5 where the singlino dominated $\tilde{\chi}^0_2$ is unreliably predicted. Note that a scheme which does not allow a good reconstruction of some of the parameters, for example the singlino mass term κs, can be nevertheless appropriate for observables where the singlino component does not play a role.

It is also interesting to look at the predictions given by a $\overline{\mathrm{DR}}$ scheme, the renormalization scale $\bar{\mu}$ is taken at the (tree-level) mass of the particle. Here corrections to all masses are calculated. For all masses and for all points, the corrections are small and never exceed 10%, compare Table I and Table III. However, note that when the underlying parameters for the OS scheme are reconstructed efficiently, the OS scheme for that particular mass gives smaller corrections than the $\overline{\mathrm{DR}}$ scheme.

To summarize, in order to compute radiative corrections to the masses reliably, one then has to be careful about the choice of the renormalization scheme. A good scheme should be chosen according to the characteristics of the point considered, such that the input parameters should reconstruct the main ingredients that define the nature of the particle whose mass is to be corrected at one-loop. The $\overline{\mathrm{DR}}$ scheme is versatile and reliable but a good OS scheme fares better, in the sense of leading to smaller corrections, as far as masses are concerned.

B. Two-body neutralino/chargino decays to a gauge boson

We now study the one-loop corrections to decays of the type $\chi_i \to \chi'_j V$, $V = W^\pm$, Z. If the charginos and neutralinos did not mix these transitions would not be possible at all. This of course applies to a (pure) singlino state. It also applies to transitions between two neutralinos through a Z for the case of a wino and bino. This transition is only possible among Higgsinos but there is generally little mass difference between these Higgsinos for these decays to occur on-shell. Other transitions are possible among winos and separately among Higgsinos, but again phase-space is restrictive. These observations together with those we made about mixing in Sec. VI A explain the main features of the decays. We will study some of the schemes we used for the calculations of the masses. For each of the five points, we will add another scheme of the category $\mathcal{P}_8$ that requires inputs from the Higgs sector. We restrict ourselves to processes that have a branching ratio at tree level of at least 1% since they are the only ones of any physical relevance. In the $\overline{\mathrm{DR}}$ scheme, we take the scale $\bar{\mu}$ at the mass of the decaying particle.

1. Point 1

This benchmark point (see Table II) can be characterized as $(\tilde{W}^0_3, \tilde{H}^0, \tilde{H}^0, \tilde{S}^0, \tilde{B}^0)$ and $(\tilde{W}^+, \tilde{H}^+)$ according to our discussion in Sec. VI A. The LSP neutralino is very much winolike. Choosing the LSP mass as an input is redundant since, as we saw, the extraction of M_2, which sets the characteristics of the wino, from the chargino mass is sufficient. Therefore, we take here the t_{345} and OS_{2345} schemes to get access to a maximum of information on the neutralino sector. We did expose at some length the shortcomings of these schemes when it comes to a good reconstruction of the singlino and Higgsino characteristics and for the need to revert to mass measurements from the Higgs sector. The latter should constrain λ much better and contribute to improve the determination of t_β. We, therefore, study the predictions of the scheme $OS_{245h_2A_1A_2}$. Note that we advocate the use of the mass of the binolike and the singlinolike neutralino in conjunction with the Higgs masses. We first observe that in the scheme $OS_{245h_2A_1A_2}$ the one-loop corrected neutralino masses are $m_{\tilde{\chi}^0_1} = 125.66$ GeV and $m_{\tilde{\chi}^0_3} = 278.97$ GeV. These are per-mil level corrections. Therefore, *a priori*, this scheme seems to be indeed a very good scheme. This statement is confirmed if one looks at the one-loop corrections to all the decays listed in Table IV. For all decays not involving the singlino-dominated neutralino χ^0_4, the corrections in the scheme $OS_{245h_2A_1A_2}$ are below 5%. Even for singlino decays, the corrections are not larger than 10%, the largest correction is reached for the smallest branching fraction. The corrections in this scheme are smaller than in the $\overline{\rm DR}$ scheme for all the considered channels but $\chi^0_4 \to \chi^0_3 Z$ which is the smallest branching ratio for the singlino dominated χ^0_4.

Both the t_{345} and OS_{2345} schemes give small corrections (typically less than 10%) for all channels apart those involving the decay of the singlino. For the case of the singlino decays, the corrections in the mixed $\overline{\rm DR}$-OS scheme t_{345} are under control (below 20%) but they should not be trusted in the scheme OS_{2345}. To summarize, a reconstruction of λ is essential to compute the decays of the singlino. As expected neutralino/chargino mass measurements do not allow a good reconstruction in the OS_{2345} scheme while it is perfectly fine when it comes to the decays of the other particles. We would have expected the scheme t_{123} to do as badly as the OS_{2345} scheme for singlino-dominated decays since a direct access to λ is not possible here also. However, in the t_{123} scheme, t_β is solved independently thus permitting a better access to λ, even if not as good as in $OS_{245h_2A_1A_2}$.

We argued earlier that the mixings in this sector are not very sensitive to t_β. To quantify this statement we looked precisely at the determination of the finite part of each of the counterterms for (μ, t_β, λ) for the three schemes. We have, respectively for the schemes t_{123}; OS_{2345}; $OS_{245h_2A_1A_2}$,

TABLE IV. Point 1: Partial widths (in MeV) for decays of neutralinos and charginos into one gauge boson at tree-level (tree) and at one-loop (tree + one-loop) with four different renormalization schemes. The relative correction to the partial decay widths is also indicated in parentheses. The schemes for the one-loop results (tree + one-loop), here t_{234}, OS_{2345}, $OS_{245h_2A_1A_2}$ and $\overline{\rm DR}$, are defined in the text.

	Tree	t_{345}	OS_{2345}	$OS_{245h_2A_1A_2}$	$\overline{\rm DR}$
$\tilde{\chi}^+_2 \to W^+\tilde{\chi}^0_1$	406	412 (1%)	419 (3%)	420 (3%)	417 (3%)
$\tilde{\chi}^+_2 \to Z\tilde{\chi}^+_1$	341	349 (2%)	357 (5%)	355 (4%)	354 (4%)
$\tilde{\chi}^0_2 \to W^-\tilde{\chi}^+_1$	271	274 (1%)	280 (3%)	280 (3%)	276 (2%)
$\tilde{\chi}^0_2 \to Z\tilde{\chi}^0_1$	183	184 (0.8%)	192 (5%)	190 (4%)	190 (4%)
$\tilde{\chi}^0_3 \to W^-\tilde{\chi}^+_1$	452	456 (0.9%)	467 (3%)	461 (2%)	458 (1%)
$\tilde{\chi}^0_3 \to Z\tilde{\chi}^0_1$	33.5	37.2 (11%)	33.8 (1%)	35.1 (5%)	30.2 (−10%)
$\tilde{\chi}^0_4 \to W^-\tilde{\chi}^+_1$	10.4	10.6 (2%)	18.2 (75%)	9.56 (−8%)	9.54 (−8%)
$\tilde{\chi}^0_4 \to W^-\tilde{\chi}^+_2$	22.9	26.3 (15%)	42.1 (84%)	23.2 (1%)	24.6 (7%)
$\tilde{\chi}^0_4 \to Z\tilde{\chi}^0_1$	6.26	6.44 (3%)	11.0 (76%)	5.83 (−7%)	5.70 (−9%)
$\tilde{\chi}^0_4 \to Z\tilde{\chi}^0_2$	26.2	29.9 (14%)	47.7 (82%)	26.1 (−0.7%)	28.1 (7%)
$\tilde{\chi}^0_4 \to Z\tilde{\chi}^0_3$	3.12	3.64 (17%)	6.02 (93%)	3.44 (10%)	3.16 (1%)
$\tilde{\chi}^0_5 \to W^-\tilde{\chi}^+_1$	26.8	22.4 (−14%)	26.8(0.1%)	26.1 (−2.5%)	27.3 (2%)
$\tilde{\chi}^0_5 \to W^-\tilde{\chi}^+_2$	611	618 (1%)	625 (2%)	624 (2%)	630 (3%)
$\tilde{\chi}^0_5 \to Z\tilde{\chi}^0_2$	515	517 (0.4%)	533 (4%)	531 (3%)	531 (3%)
$\tilde{\chi}^0_5 \to Z\tilde{\chi}^0_3$	118	122 (3%)	116 (−2%)	117 (−1%)	121 (3%)

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\text{finite}} = \overbrace{(-2.3\%, 0, +6\%)}^{t_{123}}; \overbrace{(-2.6\%, -20\%, +43\%)}^{OS_{2345}}; \overbrace{(-2.6\%, -17\%, +0.9\%)}^{OS_{245h_2A_1A_2}}$$

which shows first of all that μ is well reconstructed, independently of the scheme. This is easy to understand since all schemes rely on the chargino masses and confirm that the t_β dependence in the masses is very weak. Note that both OS_{2345} and $OS_{245h_2A_1A_2}$ do not determine t_β well. This was to be expected from the discussion in Sec. VI, i.e., masses of the neutralinos and Higgses are not much dependent on t_β. Using extra measurements with the Higgs masses improves the extraction of t_β only marginally. Yet, for these decays, the t_β dependence is generally very weak, and so is the t_β scheme dependence. What makes a huge difference is the extraction of λ. We see that the OS_{2345} is ineffective, the contribution of the counterterm alone would contribute about $2 \times 43\% \sim 86\%$ in decays directly proportional to λ like those in transitions involving χ_4^0. Indeed, for χ_4^0 decays the difference between the OS_{2345} and $OS_{245h_2A_1A_2}$ can be, to a large extent, accounted for by the difference in the value of δt_β for all channels listed in the decays of χ_4^0.

Two final remarks concerning this point. Leaving aside the decays of the problematic singlino, we see that the differences between the schemes is quite small, in fact the largest discrepancies occur when the branching fraction is smallest among all the decays of a given neutralino. Again, the small branching fraction is an indication of the smallness of the coupling which is most sensitive to mixing and hence would be most dependent on the scheme. The corrections though generally small can not be accounted simply by taking an effective running of the gauge coupling (which would give a correction of about 7%), there are, therefore, genuine electroweak corrections. These observations should be kept in mind for the other points we will study.

2. Point 2

This benchmark point features a singlino LSP where the neutralinos can be characterized as $(\tilde{S}^0, \tilde{B}^0, \tilde{H}^0, \tilde{H}^0, \tilde{W}_3^0)$ and the charginos $(\tilde{\chi}_1^+, \tilde{\chi}_2^+) = (\tilde{H}^+, \tilde{W}^+)$. $\tilde{\chi}_2^0$ and $\tilde{\chi}_4^0$ are almost equal mixture of bino and Higgsino. Note that phase space does not allow the decay $\tilde{\chi}_2^0 \to Z\tilde{\chi}_1^0$. Due to the singlino nature of the LSP, decays to the LSP $\tilde{\chi}_1^0$ have a very small partial width. The only exception is $\tilde{\chi}_5^0 \to Z\tilde{\chi}_4^0$; however, this is due to the very small Higgsino component in $\tilde{\chi}_4^0$ compared for example to $\tilde{\chi}_3^0$ which is almost pure Higgsino (recall that the coupling of the Z to neutralinos requires both neutralinos to be Higgsino-like). The need to access the singlino and bino component leads us to take the masses of their corresponding neutralinos as input. We, therefore, consider the schemes t_{123} and OS_{1234}. As argued previously this does not guarantee a good reconstruction of the mixing λ which is crucial in calculating the decays where the singlino dominated state, $\tilde{\chi}_1^0$, is involved.

For the OS scheme that uses the Higgs mass measurements, we advocate $OS_{12h_2A_1A_2H^+}$ where only the singlino-like and binolike are used from the neutralino sector (we still of course use the masses of the charginos) as well as four Higgs masses, the next-to-lightest CP-even neutral Higgs, the two CP-odd neutrals and the charged Higgs. The aim here is a better determination of λ. We recall first that this scheme does a very good job in predicting the masses of the three heaviest neutralinos with corrections below 1% for all three masses (we find for the corrected masses, $m_{\chi^0_{2,3,4}} = 253.12,\ 273.63,\ 614.82$ GeV). Table V shows clearly that the $OS_{12h_2A_1A_2H^+}$ and $\overline{\text{DR}}$ schemes are the ones that give the smallest corrections to the decays especially

TABLE V. Point 2: same as Table IV.

	Tree	t_{123}	OS_{1234}	$OS_{12h_2A_1A_2H^+}$	$\overline{\text{DR}}$
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0$	79.0	155 (96%)	182 (130%)	84.4 (7%)	58.5 (−26%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_2^0$	368	216 (−41%)	134 (−64%)	293 (−20%)	266 (−28%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_3^0$	1370	1200 (−12%)	1140 (−17%)	1180 (−14%)	1170 (−15%)
$\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+$	1400	1270(−9%)	1210 (−14%)	1240 (−12%)	1221 (−13%)
$\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0$	34.5	70.9 (106%)	86.8 (152%)	42.7 (24%)	29.6 (−14%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_1^0$	11.3	23.9 (110%)	26.8 (137%)	13.3 (18%)	11.9 (5%)
$\tilde{\chi}_5^0 \to W^-\tilde{\chi}_1^+$	1430	1270 (−12%)	1210 (−16%)	1240 (−14%)	1220 (−15%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_3^0$	1250	1120 (−11%)	1040 (−17%)	1090 (−13%)	1080 (−14%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_4^0$	58.8	55.1 (−6%)	65.1 (11%)	60.4 (3%)	57.3 (−2.5%)

when the singlino is involved. In many channels, the predictions of the full $\overline{\rm DR}$ are within a few per-cent of what is obtained with the $OS_{12h_2A_1A_2H^+}$ scheme, for these decays the impact of singlino mixing is marginal. It is, therefore, not surprising that for these same channels the prediction of the other two schemes agree within 5%. The channels where the difference between the $\overline{\rm DR}$ and $OS_{12h_2A_1A_2H^+}$ scheme is above 5% are those where the t_{123} and the OS_{1234} become totally unreliable with corrections of order 100% or worse. Not surprisingly the worst cases involve decays into a singlino, channels where the partial widths are very small. These channels require an excellent knowledge of the full mixing structure of the singlino. To confirm these findings we have extracted, as for point 1, the finite part of each of the counterterms for (μ, t_β, λ) for the three schemes $(t_{123}; OS_{1234}; OS_{12h_2A_1A_2H^+})$, we find

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\rm finite} = \overbrace{(-2.6\%, 0, 57\%)}^{t_{123}}; \overbrace{(-3.0\%, 23\%, +73\%)}^{OS_{1234}}; \overbrace{(-2.8\%, 12\%, +10\%)}^{OS_{12h_2A_1A_2H^+}}.$$

These values confirm the observation that we made for point 1, t_{123} and OS_{2345} schemes entail large corrections for $\delta\lambda/\lambda$. We note that for decays into singlinos the difference between the relative corrections given by the schemes can be approximated by $\Delta(\delta\Gamma/\Gamma) \sim \Delta(\delta\lambda/\lambda)$. It should also be observed that although t_β is not an issue for these decays, the finite part of t_β in the $OS_{12h_2A_1A_2H^+}$ is not so small, it amounts to about 12%. We have also computed one-loop corrections in a scheme where we rather chose the lightest CP-even Higgs $OS_{245h_1A_1A_2}$ and we found similar corrections for the MSSM-like transitions but large corrections for singlino processes. Once again using the singlet Higgs h_2 allows for a better reconstruction of $\delta\lambda$.

3. Point 3

Once the identification of the neutralinos has been made we realize that this point is somehow a reshuffling of point 2. The rôle played by $\tilde\chi_1^0$ is replaced by $\tilde\chi_3^0$ and the neutral Higgsino-like have become $\tilde\chi_1^0$, $\tilde\chi_2^0$. Therefore, we expect similar conclusions to emerge, especially as regards singlinolike decays, even though all the states are almost pure here. Following our characterization, the point is identified as $(\tilde H^0, \tilde H^0, \tilde S^0, \tilde B^0, \tilde W_3^0)$ and $(\tilde H^+, \tilde W^+)$. The hierarchy in the charginos has not changed as compared to point 2. We, therefore, advocate the use of the scheme $OS_{34h_2A_1A_2H^+}$. A careful look at the predictions of the different decays, taking into account the identity of the particles involved in the decay, shows similar corrections as those for point 2, see Table VI. Whenever large discrepancies occur they can be explained along the same arguments as those we have just put forward for point 2. In particular the schemes t_{134} and OS_{1234} give unreliable predictions when the decay involves the singlinolike $\tilde\chi_3^0$. Results in the $\overline{\rm DR}$ scheme are very similar to the $OS_{34h_2A_1A_2H^+}$ scheme, the difference between the two never exceeds more than 7%.

We should point at another issue not directly related to the singlino. For the decays $\tilde\chi_4^0, \tilde\chi_5^0 \to \tilde\chi_2^0 Z$, the predictions in the scheme OS_{1234} differ by +27% with those in the scheme t_{134}, while the difference between $OS_{34h_2A_1A_2H^+}$ and t_{134} is just +1%. Similarly for $\tilde\chi_4^0, \tilde\chi_5^0 \to \tilde\chi_1^0 Z$ the difference is −42% (between the OS_{1234} and t_{134} schemes) and about −2% (between $OS_{34h_2A_1A_2H^+}$ and t_{134}). This has to do with the t_β dependence. Although these decays are mildly sensitive to t_β, it remains that t_β is so badly reconstructed in the scheme OS_{1234} that it leads to noticeable differences with the prediction for the other schemes. To wit, if we again look into the finite part of the counterterms, we find

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\rm finite} = \overbrace{(-3.3\%, 0, 42\%)}^{t_{134}}; \overbrace{(-1.5\%, -169\%, 80\%)}^{OS_{1234}}; \overbrace{(-3.2\%, -5.39\%, +4\%)}^{OS_{34h_2A_1A_2H^+}}.$$

We see that the scheme OS_{1234} fares quite badly $(\delta t_\beta/t_\beta|_{\rm finite} \sim -170\%)$. These values also confirm the observation that we made for point 1, t_{134} and OS_{1234} schemes entail large corrections for $\delta\lambda/\lambda$. Note also that here $OS_{34h_2A_1A_2H^+}$ proves to be a quite good scheme, in particular for both t_β and λ. The fact that the extraction of t_β proves very uncertain for this point in the OS_{1234} scheme is easy to understand. Remember that all charginos and neutralinos are to a very good approximation almost in a pure state; therefore, from their masses the small t_β dependence hidden in the mixing is reconstructed badly (inversely proportional to the very small mixing).

4. Point 4

The most important feature of this scenario is that the singlino is for all purposes totally decoupled, here λ is extremely small. In particular, the near-pure singlino nature of $\tilde{\chi}_3^0$ explains why its partial decay width into gauge bosons is strongly suppressed, the largest two-body decay involving a gauge boson is into $W\tilde{\chi}_1^+$ with a partial width $\Gamma = 2.09 \times 10^{-5}$ GeV. The preferred decays of $\tilde{\chi}_3^0$ involve Higgses which we do not study here. We should, therefore, not be surprised that in Table VII $\tilde{\chi}_3^0$ is not present. This benchmark point is characterized as $(\tilde{B}^0, \tilde{W}_3^0, \tilde{S}^0, \tilde{H}^0, \tilde{H}^0)$; $(\tilde{W}^+, \tilde{H}^+)$ where the LSP is binolike. Therefore, the mass of the LSP features in all the OS schemes. For the scheme that relies on inputs from the Higgs sector, we propose $OS_{134A_1A_2H^+}$. Since we are looking at decays in what is essentially the MSSM where $M_{1,2}, \mu$ are well reconstructed, any discrepancy between the schemes has to do with t_β. We first observe that the fully $\overline{\rm DR}$ scheme *and* the t_{123} scheme give predictions which for all decays shown in Table VII are within 5%, the only exception is $\tilde{\chi}_4^0 \to Z\tilde{\chi}_2^0$ where the difference is 8%. The $OS_{134A_1A_2H^+}$ scheme also agrees with the fully $\overline{\rm DR}$ scheme within 10%. For a few decays, the OS_{1234} scheme gives corrections of order 70% and −40% that are quite different from the results in the other schemes. These observations lead to suspect that once again the determination of t_β is in question especially in the OS_{1234} scheme. Indeed if we look at the finite part of the counterterms we find

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\rm finite} = \overbrace{(0.03\%, 0, -1430\%)}^{t_{123}}; \overbrace{(0.3\%, +150\%, -4500\%)}^{OS_{1234}}; \overbrace{(0.08\%, 27.4\%, 2841\%)}^{OS_{134A_1A_2H^+}}$$

showing the very poor reconstruction of t_β in OS_{1234} and to a lesser extent in $OS_{134A_1A_2H^+}$.

Observe that a good reconstruction of t_β has also an impact on the reconstruction of μ, although all three schemes perform well for μ, t_{123} does ten times better than OS_{1234}. Since many decays into gauge bosons are triggered from Higgsino to Higgsino transitions a very precise determination of μ is important. Note in passing that $\delta\lambda$ is totally unreliable as expected since the resolution of the system leads to a division by λ, i.e., a division by a very small number. This has no direct impact on the corrections computed since we did not consider decays involving singlinos for this point. To summarize, for this point all schemes, apart from OS_{1234}, do a good job giving moderate corrections. The largest correction of order 20% occurs for $\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0$. This is a genuine correction which is fairly independent of the scheme.

5. Point 5

While point 4 had the smallest λ and featured the most decoupled singlino of our benchmarks, point 5 has the largest λ while the μ parameter has slightly changed. λ is rather small but it is large enough to allow for a few percent mixing of the singlino with the Higgsino (see Table II) and subsequently with the other neutralinos, leading for example to a partial width of 10^{-2} GeV for the decay of the singlinolike neutralino, $\tilde{\chi}_2^0 \to W\tilde{\chi}_1^+$. The LSP is winolike

TABLE VI. Point 3: same as Table IV.

	Tree	t_{134}	OS_{1234}	$OS_{34h_2A_1A_2H^+}$	$\overline{\rm DR}$
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0$	1960	1670(−15%)	1830 (−7%)	1680 (−15%)	1670 (−15%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_2^0$	2110	1730 (−18%)	1870 (−11%)	1740 (−17%)	1730 (−18%)
$\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+$	2050	1710 (−16%)	1860 (−9%)	1720 (−16%)	1730 (−16%)
$\tilde{\chi}_3^0 \to W^-\tilde{\chi}_1^+$	11.2	20.5 (84%)	27.0 (141%)	12.0 (7%)	12.0 (7%)
$\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0$	4.32	7.68 (78%)	11.7 (171%)	4.51 (4%)	4.20 (−3%)
$\tilde{\chi}_3^0 \to Z\tilde{\chi}_2^0$	3.78	7.00 (85%)	8.34 (120%)	4.08 (9%)	4.28 (13%)
$\tilde{\chi}_4^0 \to W^-\tilde{\chi}_1^+$	445	479 (8%)	525 (18%)	480 (8%)	460 (3%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_1^0$	99.4	113 (13%)	70.3 (−29%)	110 (11%)	105 (6%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_2^0$	306	328 (7%)	410 (34%)	332 (8%)	323 (6%)
$\tilde{\chi}_5^0 \to W^-\tilde{\chi}_1^+$	2060	1720 (−16%)	1860 (−10%)	1720 (−16%)	1710 (−17%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_1^0$	578	500 (−14%)	256 (−56%)	487(−16%)	502 (−13%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_2^0$	1480	1220 (−17%)	1620 (9%)	1240 (−16%)	1270 (−13%)

and the point is characterized as $(\tilde{W}_3^0, \tilde{S}^0, \tilde{B}^0, \tilde{H}^0, \tilde{H}^0)$ and $(\tilde{W}^+, \tilde{H}^+)$. This hierarchy suggests to choose the masses of $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$ as inputs for all OS schemes. With t_{123} and OS_{2345} we also consider the $OS_{234A_1A_2H^+}$ scheme where the Higgs masses are used for a better extraction of δt_β and $\delta\lambda$. In this scheme, the corrections to the masses of $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_4^0}$ are totally negligible, not exceeding 0.5 per mil. Moreover the corrections to all the decays we have considered are quite moderate, below 10%, see Table VIII. Results in the $\overline{\rm DR}$ are very similar, the difference between the two never exceeds more than 5%. The differences between the $OS_{234A_1A_2H^+}$ and t_{123} are also quite small, within a margin of 7% even for the decay of the singlino. One can already guess that the presence of a non-negligible λ can help not only in better reconstructing this parameter from the Higgs masses but also in better reconstructing t_β. This is in contrast with the results obtained with the OS_{2345} scheme. The latter is totally unreliable essentially due to a failed reconstruction of both λ, see the large corrections involving the singlino $\tilde{\chi}_2^0$, but also due to a quite bad reconstruction of t_β (see the decays of the other neutralinos in particular the decays of the binolike $\tilde{\chi}_3^0$). These observations are borne out by the values of the finite part of the key counterterms in the schemes t_{123}, OS_{2345} and $OS_{234A_1A_2H^+}$ with

$$(\delta\mu/\mu, \delta t_\beta/t_\beta, \delta\lambda/\lambda)_{\rm finite} = \overbrace{(-0.1\%, 0, -1.2\%)}^{t_{123}}; \overbrace{(0.2\%, +115\%, -30\%)}^{OS_{2345}}; \overbrace{(-0.1\%, 1.9\%, 1.6\%)}^{OS_{234A_1A_2H^+}}.$$

The numerical results presented here concerning the decays of the neutralinos and charginos to gauge bosons confirm the general expectation and arguments we have given outlined in Secs. II–IV. Indeed, the choice of neutralino masses to be used as input is crucial to allow a precise reconstruction of the parameters of the neutralino sector. Moreover a good reconstruction of t_β and λ is important in order to make accurate predictions for partial decay widths. In this regard, renormalization schemes using some Higgs masses as input fare better than those using only masses from the neutralino sector. This is especially relevant for all decays involving singlinos where the parameter λ plays an important role and, therefore, must be reconstructed precisely.

VIII. ONE-LOOP CORRECTIONS TO TWO-BODY SFERMION DECAYS TO FERMIONS

We now compute the one-loop corrections to the decays of third generation sfermions into a fermion and a neutralino/chargino. These processes are often the preferred decay modes of sfermions and are the main channels used for third generation squark searches at the LHC [61–63]. Other decay channels involving Higgses will be considered in a separate publication [52]. For squarks, we compute both QCD and EW corrections. As before we include only the decays for which the tree-level branching ratio is above a few percent as they are the only physically relevant ones. For the definition of the parameters of the sfermions, we will consider the scheme presented in Sec. V. Namely, for the squarks the input parameters for the third generation will be $m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{t}_1}$ θ_b and θ_t for the squarks and $m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}$ and $m_{\tilde{\nu}_\tau}$ for the staus. For the QCD corrections, we take $\alpha_s(1\ \text{TeV}) = 0.0894$, this scale of α_s corresponds to the mass of $\tilde{b}_1$ in all of our benchmarks. As in the previous section we will test different schemes for the neutralino sector. The difference between the latter schemes will impact the predictions for the electroweak corrections. QCD corrections do not impact these schemes but only the squark sector. Since all the OS schemes adopt the same definition for the input parameters for the squark sector there will be no difference between the OS schemes, including the t_{123}-type schemes. There may be differences in the QCD corrections between the OS schemes and the full $\overline{\rm DR}$ scheme. As we will see, the QCD scheme dependence is very small and generally hardly noticeable.

The couplings of the type $\tilde{f}f'\tilde{\chi}$ (for both charged and neutral $\tilde{\chi}$) responsible for these decays originate from two sources. First, gauge type coulings ($\propto g$, g') occur with wino and binolike $\tilde{\chi}$. A right-handed $\tilde{f}_R$ will only couple to the bino component. Second, Yukawa type couplings

$$y_{d,u} = \left(\frac{g\sqrt{2}}{M_W}\right)\left(\frac{m_d}{c_\beta}, \frac{m_u}{s_\beta}\right) \sim \frac{g\sqrt{2}}{M_W}(m_d t_\beta, m_u) \quad \text{for } t_\beta > 3$$

are important only for third family sfermions in particular the stops and sbottoms. For sbottoms, this coupling is enhanced by t_β. Therefore, if the phase space allows, the main decay of the $\tilde{\tau}_1$ is into gauginos in particular into the binolike neutralino for $\tilde{\tau}_1 \sim \tilde{\tau}_R$. For all our benchmarks, the $\tilde{b}_1$ is right-handed, $\tilde{b}_1$ will, therefore, also decay preferably into a bino if the latter is lighter, otherwise decays into the Higgsino are preferred. For such decays, it is crucial to specify the exact value of the sbottom mass. Our tree-level calculation is done with a pole mass for the bottom, $m_b = 4.7$ GeV. If the decay is indeed dominated by the Yukawa coupling, we should note that the use of a running $\tilde{q}$ mass at the scale of the decaying particle, i.e., the sbottom mass of around 1 TeV, would be more appropriate in order to take into account the bulk of the QCD corrections. Using the running bottom mass brings in a relative correction of order $2\delta m_b/m_b \sim -72\%$. Indeed, at one-loop, we have $m_b^{\overline{\rm DR}}(\bar{\mu} = 1\ \text{TeV}) \sim m_b^{\rm pole}(1 + a_s(\ln(m_b^2/\bar{\mu}^2) - 5/3))$,

TABLE VII. Point 4: same as Table IV for point 4.

	Tree	t_{123}	OS_{1234}	$OS_{134A_1A_2H^+}$	$\overline{\text{DR}}$
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0$	307	364 (19%)	388 (26%)	371 (21%)	379 (23%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_2^0$	1420	1340 (−6%)	1040 (−27%)	1220 (−14%)	1360 (−4%)
$\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+$	1300	1210 (−7%)	950 (−27%)	1160 (−10%)	1260(−3%)
$\tilde{\chi}_4^0 \to W^-\tilde{\chi}_1^+$	1310	1210 (−7%)	960 (−27%)	1160 (−11%)	1260 (−3%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_1^0$	383	425 (11%)	232 (−40%)	413 (8%)	417 (9%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_2^0$	1020	916 (−10%)	605 (−41%)	927 (−9%)	1000 (−2%)
$\tilde{\chi}_5^0 \to W^-\tilde{\chi}_1^+$	1340	1240 (−8%)	1050 (−22%)	1150 (−14%)	1230 (−8%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_1^0$	89.5	109 (21%)	152 (70%)	110 (23%)	103 (16%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_2^0$	165	169 (2%)	293 (77%)	165(−0.6%)	158(−4%)

$a_s = \alpha_s(\bar{\mu})/\pi$. From these observations we should expect a strong t_β (and scheme) dependence whenever the QCD corrections are large for sbottom decays and could be accounted for by the running of m_b. For the stop, the Higgsino coupling is large due to the large m_t and, therefore, decays into a Higgsino will generally dominate. Likewise the correction driven by the running of the top mass is $2\delta m_t/m_t \sim -30\%$ at a scale of 1 TeV (and about -37% for a scale at 2 TeV). In all cases, decays into singlinos are strongly disfavored unless the singlino is the only kinematically accessible mode.

A. Point 1

For this point, all sfermions are at the TeV scale, in fact the stops are even heavier with a mass around 2 TeV. $\tilde{\tau}_1$ and $\tilde{t}_1$ are heavily mixed (between LH and RH) while the $\tilde{b}_1$ is dominantly RH. t_β is rather large. Recall that this point is characterized as $(\tilde{W}_3^0, \tilde{H}^0, \tilde{H}^0, \tilde{S}^0, \tilde{B}^0)$ and $(\tilde{W}^+, \tilde{H}^+)$ with rather large mixing between the winos and the Higgsinos; see Table II.

The decays of the $\tilde{b}_1$ are easy to understand. For this RH state, the gauge decay would have been into the binolike neutralino, but this channel is kinematically closed. Decays are, therefore, totally triggered by the t_β enhanced Yukawa coupling into Higgsino states which seep into $\tilde{\chi}_1^0$ through $\tilde{H} - \tilde{W}$ mixing. Table IX shows large (negative) corrections for sbottom decays with essentially the same corrections for all channels. The bulk of the corrections comes from the running of the bottom mass, remember the −72% QCD correction, which our full calculations reproduces rather

TABLE VIII. Point 5: same as Table IV.

	Tree	t_{123}	OS_{2345}	$OS_{234A_1A_2H^+}$	$\overline{\text{DR}}$
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0$	1250	1160 (−7%)	909 (−27%)	1150 (−8%)	1190 (−5%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_2^0$	531	483 (−9%)	545 (3%)	518 (−2%)	501 (−6%)
$\tilde{\chi}_2^+ \to W^+\tilde{\chi}_3^0$	250	265 (6%)	172 (−31%)	261 (5%)	264 (6%)
$\tilde{\chi}_2^+ \to Z\tilde{\chi}_1^+$	1310	1220 (−7%)	945 (−28%)	1210 (−7%)	1240 (−5%)
$\tilde{\chi}_2^0 \to W^-\tilde{\chi}_1^+$	34.3	30.6 (−11%)	37.1 (8%)	33.0 (−4%)	34.1 (−1%)
$\tilde{\chi}_3^0 \to W^-\tilde{\chi}_1^+$	58.8	55.7 (−5%)	−17.0 (−128%)	53.7 (−9%)	57.8 (−2%)
$\tilde{\chi}_3^0 \to Z\tilde{\chi}_1^0$	2.75	2.94 (7%)	5.41 (96%)	2.97 (8%)	2.83 (3%)
$\tilde{\chi}_4^0 \to W^-\tilde{\chi}_1^+$	1320	1220 (−8%)	953 (−28%)	1201 (−9%)	1210(−8%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_1^0$	1280	1160 (−9%)	746 (−42%)	1150 (−10%)	1210 (−5%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_2^0$	233	223 (−4%)	377 (62%)	240 (3%)	230(−3%)
$\tilde{\chi}_4^0 \to Z\tilde{\chi}_3^0$	157	168 (7%)	116 (−26%)	165 (5%)	166 (6%)
$\tilde{\chi}_5^0 \to W^-\tilde{\chi}_1^+$	1230	1120 (−9%)	940 (−23%)	1110 (−10%)	1150 (−7%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_1^0$	108	106 (−3%)	254 (133%)	107 (−2%)	101 (−7%)
$\tilde{\chi}_5^0 \to Z\tilde{\chi}_2^0$	166	147(−11%)	13.0 (−92%)	155 (−7%)	151 (−9%)

TABLE IX. Point 1: Partial decay widths (in GeV) of third generation sfermions into a fermion and a neutralino/chargino at tree-level (tree) and at one-loop in three schemes (see text for their definition) including for the squarks both the electroweak and QCD effects. The total (electroweak and QCD) relative correction is indicated between round parentheses (). The relative QCD correction is given in squared parentheses []. The relative QCD correction is the same in both the t_{345} and $OS_{245h_2A_1A_2}$ scheme, see text. It is, therefore, not listed.

	Tree	t_{345}	$OS_{245h_2A_1A_2}$	$\overline{\text{DR}}$
$\tilde{b}_1 \to b\tilde{\chi}_1^0$	0.210	0.058 (−72%)	−0.013 (−106%) [−68%]	0.065 (−69%) [−68%]
$\tilde{b}_1 \to b\tilde{\chi}_2^0$	0.551	0.164 (−70%)	−0.034(−106%) [−75%]	0.165 (−70%) [−75%]
$\tilde{b}_1 \to b\tilde{\chi}_3^0$	0.408	0.133 (−67%)	−0.018 (−104%) [−75%]	0.126 (−69%)[−75%]
$\tilde{b}_1 \to t\tilde{\chi}_1^-$	0.357	0.077 (−78%)	−0.044 (−112%) [−74%]	0.088 (−75%)[−74%]
$\tilde{b}_1 \to t\tilde{\chi}_2^-$	0.732	0.231 (−68%)	−0.040 (−105%) [−75%]	0.222 (−70%) [−75%]
$\tilde{t}_1 \to t\tilde{\chi}_2^0$	15.3	10.5 (−31%)	10.6 (−31%) [−34%]	10.5 (−31%)[−34%]
$\tilde{t}_1 \to t\tilde{\chi}_3^0$	20.1	14.5 (−28%)	14.7 (−27%) [−28%]	14.3 (−29%)[−28%]
$\tilde{t}_1 \to b\tilde{\chi}_2^+$	23.4	16.8 (−28%)	16.7(−29%) [−29%]	17.1 (−27%)[−29%]
$\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$	1.73	1.65 (−4%)	1.60 (−7%)	1.62 (−6%)
$\tilde{\tau}_1 \to \nu_\tau\tilde{\chi}_1^+$	3.13	3.01 (−4%)	2.93 (−6%)	2.98 (−5%)

well. Once this correction is taken into account the remaining QCD correction is less than 5% for all the channels and no scheme dependence is to be noticed for the QCD part of the corrections. As for the electroweak corrections, these are very small in both the t_{345} and $\overline{\text{DR}}$ scheme, they do not exceed 7%. The electroweak corrections in the $OS_{245h_2A_1A_2}$ scheme are about −34% off compared to any of the other two schemes. This difference is due to the large finite term induced by the counterterm $\delta t_\beta/t_\beta$, recall that compared to $\overline{\text{DR}}$ we had found a difference of about −17%; see Sec. VII A. This is exactly what is needed to account for the difference between the predictions of the two schemes. $\Delta(\delta\Gamma/\Gamma) \simeq 2\Delta(\delta t_\beta/t_\beta)$. We are referring to Δ as the difference between two schemes and δ as the loop correction. This calculation shows that for such decays a very good reconstruction (scheme) for t_β is crucial.

The decays of the $\tilde{t}_1$ proceed dominantly through the Yukawa coupling, again the bulk of the correction is from QCD and can be accounted for by the running of the top mass, as expected. Unlike the case with $\tilde{b}_1$ the dependence on t_β and, therefore, the scheme is hardly noticeable. The electroweak corrections here are not larger than 3%. For $\tilde{\tau}_1$, the largest decays process through the gauge $SU(2)$ component, since the mixing with the very heavy binos are unreachable and the Yukawa couplings are too small for a transition through the Higgsino. This also explain the very weak scheme dependence.

1. Point 2

As compared to point 1, the bino is much lighter and at the same time the Yukawa coupling is smaller due to a smaller t_β (4.5 instead of 10). The decay of $\tilde{b}_1 \simeq \tilde{b}_R$ is, therefore, dominated by the bino when the neutralino has a fair amount of bino. This is the case for $\tilde{b}_1 \to b\tilde{\chi}_2^0$ that is triggered by the (hypercharge) gauge coupling. Expectedly, this decay which is not sensitive to the Yukawa of the bottom shows no scheme dependence, for both the electroweak and the QCD corrections. The −16% QCD correction is counterbalanced by a +12% electroweak correction. When the decay is into Higgsino dominated states ($\tilde{b}_1 \to t\tilde{\chi}_1^-$ and $\tilde{b}_1 \to b\tilde{\chi}_3^0$) we reach similar conclusions as for point 1, namely the bulk of the correction is from QCD and can be accounted for by a running of m_b. For decays into Higgsinos, the discrepancy between the $\overline{\text{DR}}$ and t_{123} schemes on the one hand and the $OS_{12h_2A_1A_2H^+}$ scheme on the other is due to the finite part of the δt_β contribution; see Sec. VII B 2. The decay into $\tilde{\chi}_4^0$ involves both the bino (gauge) and the Higgsino (Yukawa) couplings, the bulk of the correction is due to the running b mass, while the 10% discrepancy in the electroweak corrections found for the $OS_{12h_2A_1A_2H^+}$ is due to the t_β reconstruction.

Here the lightest stop is mainly $\tilde{t}_R$ and has a mass of about 460 GeV. Normally, the dominant decays would be to the Higgsino rich states and eventually to the bino rich $\tilde{\chi}_2^0$; however, phase space penalizes the decays into $\tilde{t}_1 \to b\tilde{\chi}_1^+$, $t\tilde{\chi}_3^0$, $t\tilde{\chi}_4^0$. The corrections for these three decays are quite moderate, in part because in the QCD corrections the running top mass should be evaluated at lower scale and as is the case for $\tilde{t}_1 \to t\tilde{\chi}_4^0$ the bino (gauge decay contribution) is competitive. In any case, contrary to the sbottom the t_β dependence is weak as is reflected in Table X. Stop decays into the LSP singlino, $\tilde{\chi}_1^0$, is fraught with uncertainties. First, these decays are possible because of the small Higgsino component which through mixing allows decays

TABLE X. Point 2: Same as in Table IX but for point 2.

	Tree	t_{123}	$OS_{12h_2A_1A_2H^+}$	$\overline{\mathrm{DR}}$
$\tilde{b}_1 \to b\tilde{\chi}_2^0$	0.332	0.318 (−4%)	0.318 (−4%)[−16%]	0.320 (−4%)[−16%]
$\tilde{b}_1 \to b\tilde{\chi}_3^0$	0.120	0.037 (−69%)	0.059 (−51%)[−72%]	0.038 (−69%)[-−72%]
$\tilde{b}_1 \to b\tilde{\chi}_4^0$	0.258	0.208 (−19%)	0.234 (−9%)[−20%]	0.213 (−18%)[−20%]
$\tilde{b}_1 \to t\tilde{\chi}_1^-$	0.228	0.066 (−71%)	0.107 (−53%)[−72%]	0.066 (−71%)[−72%]
$\tilde{t}_1 \to t\tilde{\chi}_1^0$	0.178	0.346 (94%)	0.185 (4%)[−20%]	0.133 (−25%)[−20%]
$\tilde{t}_1 \to t\tilde{\chi}_2^0$	0.414	0.241 (−42%)	0.334 (−19%) [−19%]	0.328 (−21%)[−19%]
$\tilde{t}_1 \to t\tilde{\chi}_3^0$	0.639	0.572 (−11%)	0.574 (−10%)[−17%	0.567 (−11%)[−16%]
$\tilde{t}_1 \to t\tilde{\chi}_4^0$	0.648	0.631 (−2%)	0.624 (−4%)[−11%]	0.648 (0%)[−12%]
$\tilde{t}_1 \to b\tilde{\chi}_1^+$	4.19	3.76 (−10%)	3.73 (−11%) [−22%]	3.75 (−10%)[−21%]
$10^4 \times (\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0)$	6.16	15.9 (141%)	9.27 (40%)	6.99 (6%)

to an almost singlino state. The bulk of the corrections in the fully $\overline{\mathrm{DR}}$ scheme is in line with a running of m_t which provides about −20% corrections. The discrepancies in the other two schemes are rendered by a large correction in the finite part of $\delta\lambda$, see the values in Sec. VII B 2.

Due to phase space the light $\tilde{\tau}_1$ which is mostly $\tilde{\tau}_R$ can only decay to the LSP singlino. Not surprisingly the rate is ridiculously small. Since the only nonsinglino component of $\tilde{\chi}_1^0$ is the Higgsino, the decay is sensitive to λt_β. The difference between large corrections in the schemes t_{123} and $OS_{12h_2A_1A_2H^+}$ on the one hand and the fully $\overline{\mathrm{DR}}$ on the other hand can be explained by the finite part of the $\delta\lambda$ and δt_β which can be found in our discussion in Sec. VII B 2.

2. *Point 3*

For this point, all third generation sfermions are around the TeV scale. $\tilde{\tau}_1$ is almost $\tilde{\tau}_R$ and a similar statement can be made for $\tilde{t}_1 \sim \tilde{t}_R$. Apart from the heaviest neutralino and chargino which are winolike, the other neutralinos and the chargino are kinematically accessible to all 3 sfermions studied. $\tilde{\chi}_3^0$ being dominantly singlino does not show up in our list of decays, it couples to sfermions far too feebly. Decays to $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_1^+$ which are all Higgsino-like are dominant for $\tilde{t}_1$ and $\tilde{b}_1$ and small for $\tilde{\tau}_1$, since these couplings are proportional to the Yukawa coupling. As for other points, large (negative) radiative corrections are found for these decays for both stops and especially sbottoms for all the schemes, see Table XI, these corrections can be incorporated in the running of m_t and m_b. For both $\tilde{\tau}_1$ and $\tilde{b}_1$, we notice again a non-negligible scheme dependence due to the implementation of δt_β. The difference between the t_{123} and the $OS_{34h_2A_1A_2H^+}$ is very well accounted for by the finite value of $\delta t_\beta / t_\beta$ of the scheme. While for the squarks the difference between the full $\overline{\mathrm{DR}}$

TABLE XI. Point 3: Same as in Table IX but for point 3.

	Tree	t_{134}	$OS_{34h_2A_1A_2H^+}$	$\overline{\mathrm{DR}}$
$\tilde{b}_1 \to b\tilde{\chi}_1^0$	0.660	0.180 (−73%)	0.112 (−83%)[−75%]	0.190 (−71%)[−75%]
$\tilde{b}_1 \to b\tilde{\chi}_2^0$	0.624	0.192 (−69%)	0.118 (−81%) [−75%]	0.192 (−69%) [−75%]
$\tilde{b}_1 \to b\tilde{\chi}_4^0$	0.135	0.146 (8%)	0.146 (8%) [−1.5%]	0.146 (8%) [−1.5%]
$\tilde{b}_1 \to t\tilde{\chi}_1^-$	1.21	0.350 (−71%)	0.207 (−83%) [−74%]	0.350 (−71%)[−74%]
$\tilde{t}_1 \to t\tilde{\chi}_1^0$	7.59	5.89 (−22%)	5.89 (−22%) [−27%]	5.88 (−23%)[−27%]
$\tilde{t}_1 \to t\tilde{\chi}_2^0$	7.89	5.91 (−25%)	5.95 (−25%) [−26%]	5.93 (−25%)[−26%]
$\tilde{t}_1 \to t\tilde{\chi}_4^0$	0.276	0.280 (2%)	0.281 (2%) [−0.3%]	0.281 (2%)[−0.01%]
$\tilde{t}_1 \to b\tilde{\chi}_1^+$	15.8	12.5 (−21%)	12.5 (−21%) [−28%]	12.5 (−21%)[−28%]
$\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$	0.116	0.150 (29%)	0.143 (23%)	0.120(3%)
$\tilde{\tau}_1 \to \tau\tilde{\chi}_2^0$	0.0950	0.0820 (−14%)	0.0695 (−27%)	0.1024 (8%)
$\tilde{\tau}_1 \to \tau\tilde{\chi}_4^0$	1.214	1.312 (8%)	1.311 (8%)	1.328 (9%)
$\tilde{\tau}_1 \to \nu_\tau\tilde{\chi}_1^+$	0.193	0.213 (10%)	0.191 (−1%)	0.198 (3%)

scheme and the t_{123} scheme is not noticeable, it is not the case for the $\tilde{\tau}_1$. We have traced this difference to the implementation of the $\tilde{\tau}$ mixing angle, where we applied different definitions for the squark and the slepton sector; see Sec. V. Despite the fact that $\tilde{\chi}_4^0$ is far heavier than $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$, $\tilde{\tau}_1 \to \tau\tilde{\chi}_4^0$ has the largest partial width for $\tilde{\tau}_1$. Decays of $\tilde{t}_1$ and especially $\tilde{b}_1$ into $\tilde{\chi}_4^0$ are also not negligible. This is normal, $\tilde{\chi}_4^0$ is essentially binolike with a relatively large coupling to f_R states, in particular sleptons. In this case, the radiative corrections are modest and scheme independent. This is also not surprising since the decays are driven essentially by the $U(1)$ gauge coupling.

3. Point 4

This point is the most MSSM-like with a very small λ so that the singlino is practically decoupled. It is, therefore, normal that $\tilde{\chi}_3^0$ does not show up in the table of decays, Table XII. $\tilde{t}_1$ and $\tilde{\tau}_1$ are quite light here and are, like $\tilde{b}_1$, mainly RH. Decays to the wino dominated states $\tilde{\chi}_2^0, \chi_1^+$ are, therefore, suppressed. Because the Higgsino-like state are too heavy for $\tilde{t}_1$ and $\tilde{\tau}_1$ to decay into, the only channel left for $\tilde{t}_1$ and $\tilde{\tau}_1$ is into the LSP which is almost binolike. Radiative corrections to the decays into the LSP (bino dominated) for all three sfermions are relatively small and most importantly the scheme dependence is hardly noticeable. This is as expected since these transitions are triggered by the $U(1)$ gauge coupling. For the sbottom, the other decays heavily involve the Higgsino component. Again it is the same story, the large negative QCD corrections are accounted for by the running of m_b and the discrepancy between the full $\overline{\mathrm{DR}}$, the t_{123} and the $OS_{134A_1A_2H^+}$ scheme are accounted for by the large contribution from the finite part of the δt_β counterterm derived in the latter scheme.

4. Point 5

Compared to the previous point, point 4, the nature of the stop and sbottom has not changed. $\tilde{t}_1$ which is mainly $\tilde{t}_R$ is not very heavy and the would-be preferred decays into the bino-like, $\tilde{\chi}_3^0$ and Higgsino-like states, $\tilde{\chi}_4^0$, $\tilde{\chi}_5^0$, $\tilde{\chi}_2^+$ are kinematically not possible. Decays into the remaining winolike, $\tilde{\chi}_1^0$, and singlinolike, $\tilde{\chi}_2^0$, state are extremely suppressed as Table XIII shows. Note that point 5 has the largest value of λ of all the benchmarks we proposed. Although the amount of singlino mixing remains small, $\tilde{t}_1$ decays into the singlino as it is the only kinematically accessible state in this category. The decay is inherited from the Yukawa Higgsino coupling and transmitted then to the singlino-rich $\tilde{\chi}_2^0$. With stops, the t_β dependence is small but the singlino parameter λ controls this decay. It turns out that the difference between the three schemes is quite small and follows from the fact the finite part of $\delta\lambda$ is quite small (contrary to what is found for many of the points we studied); see Sec. VII B 5. It follows also that a large part of the correction is due to QCD and is encoded in the running of the top mass, while the electroweak corrections amount to less than $+10\%$.

$\tilde{b}_1$ is much heavier than $\tilde{t}_1$, in particular the channel into the bino-rich $\tilde{\chi}_3^0$ is open. This constitutes the largest partial width for $\tilde{b}_1$. Again, since this is mediated by the hypercharge gauge coupling, the corrections are modest and scheme independent with very small corrections for both the QCD and the electroweak part. The decays of $\tilde{b}_1$ to the heavier Higgsino-dominated neutralinos and charginos are Yukawa induced especially for the almost pure $\tilde{\chi}_4^0$, $\tilde{\chi}_2^+$ states. Note also that $\tilde{\chi}_5^0$ has a non-negligible bino component that seeps in also; see Table II. The large negative corrections are essentially QCD corrections that can be accounted for by a running of the b mass as we explained for similar cases before. Note that this time the $OS_{234A_1A_2H^+}$ does not differ by more that 2% from the other schemes for these type of decays. This again is easily understood on the basis of the finite part for $\delta t_\beta/t_\beta$ that we calculated for this point; see Sec. VII B 5.

For the $\tilde{\tau}_1$ which has a large $\tilde{\tau}_L$ component, decays to the winolike states $\tilde{\chi}_1^0$, $\tilde{\chi}_1^+$ dominate, note the (almost) factor 2 ratio between the charged and neutral channels (due to isospin). Decays into the bino-rich $\tilde{\chi}_3^0$ are not small, of order s_W^2/c_W^2 compared to the decays into the wino dominated $\tilde{\chi}_1^0$.

TABLE XII. Point 4: Same as in Table IX but for point 4.

	Tree	t_{123}	$OS_{134A_1A_2H^+}$	$\overline{\mathrm{DR}}$
$\tilde{b}_1 \to b\tilde{\chi}_1^0$	0.508	0.563 (11%)	0.538 (6%) [−4%]	0.536 (5%)[−4%]
$\tilde{b}_1 \to b\tilde{\chi}_4^0$	0.131	0.030 (−75%)	0.101 (−23%) [−78%]	0.033 (−75%) [−78%]
$\tilde{b}_1 \to b\tilde{\chi}_5^0$	0.123	0.016 (−87%)	0.094 (−23%) [−77%]	0.030 (−76%)[−77%]
$10^2 \times (\tilde{b}_1 \to t\tilde{\chi}_1^-)$	3.41	0.53 (−84%)	2.04 (−40%) [−67%]	0.65 (−81%) [−67%]
$\tilde{b}_1 \to t\tilde{\chi}_2^-$	0.197	0.046 (−77%)	0.151 (−23%)[−77%]	0.0451 (−77%) [−77%]
$\tilde{t}_1 \to t\tilde{\chi}_1^0$	0.181	0.210 (16%)	0.211 (17%) [10%]	0.21 (16%) [10%]
$10^3 \times (\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0)$	8.01	8.78 (10%)	8.75 (9%)	8.79 (10%)

TABLE XIII. Point 5: Same as in Table IX but for point 5.

$10^3 \times (\tilde{t}_1 \to t\tilde{\chi}_2^0)$	Tree 9.17	t_{123} 7.71 (−16%)	$OS_{234A_1A_2H^+}$ 8.28 (−10%)[−17%]	$\overline{\rm DR}$ 8.18 (−11%)[−19%]
$\tilde{b}_1 \to b\tilde{\chi}_3^0$	0.362	0.383 (6%)	0.383 (6%) [−3%]	0.383 (6%)[−3%]
$10^2 \times (\tilde{b}_1 \to b\tilde{\chi}_4^0)$	3.88	1.12 (−71%)	1.17 (−70%) [−74%]	1.12 (−71%)[−74%]
$10^2 \times (\tilde{b}_1 \to b\tilde{\chi}_5^0)$	4.45	2.16 (−52%)	2.21 (−50%) [−57%]	2.15 (−52%) [−57%]
$10^2 \times (\tilde{b}_1 \to t\tilde{\chi}_2^+)$	6.17	1.77 (−71%)	1.86(−70%)[−74%]	1.75 (−72%) [−74%]
$\tilde{\tau}_1 \to \nu_\tau\tilde{\chi}_1^+$	7.07	6.912 (−2%)	6.98 (−1%)	6.872 (−3%)
$\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$	3.49	3.409 (−2%)	3.441 (−1%)	3.381 (−3%)
$\tilde{\tau}_1 \to \tau\tilde{\chi}_3^0$	1.04	1.129 (9%)	1.104 (6%)	1.145 (10%)

The $\tilde{\tau}_R$ component is not large enough to participate in the coupling. Since these decays are driven by couplings of a gauge origin there is very little scheme dependence.

In summary, we have found that, as expected, the size of the corrections for the decays of sfermions into neutralinos/charginos strongly depend one the nature and composition (L or R content, gaugino/Higgsino fractions) of the particles involved. Large corrections are found for decays into Higgsinos; however, the bulk of these corrections originate from QCD and are easily accounted for by using the running quark mass.A scheme dependence remains for purely electroweak corrections, we advocate the use of a scheme where t_β and/or λ are reconstructed precisely.

IX. CONCLUSIONS

The present paper is the first in a series that addresses the renormalization, at one-loop, of the NMSSM, paying particular attention to the implementation of on-shell schemes. We have concentrated here on the neutralino/chargino system and exposed the sfermion sector. We also appealed to some issues and features that reside within the Higgs sector and which help in better defining some key parameters which are also of importance when studying observables that only involve the neutralinos, charginos, and sfermions. Details of the Higgs sector are left for a follow-up paper. After presenting the theoretical setup, in particular the different schemes that allow us to define the necessary counterterms for a complete renormalization of the chargino/neutralino and sfermion systems, we turn to two classes of applications. For this, we have first defined a set of five benchmark points which select different hierarchies of neutralinos and charginos depending on the nature of these particles (winolike, singlinolike, binolike, Higgsino-like, and mixed). In the first class of applications, we studied the electroweak radiative correction for the decays of the type $\tilde{\chi} \to \tilde{\chi}' V$, $V = W^\pm$, Z, $\tilde{\chi}^{(\prime)} = \tilde{\chi}^0$, $\tilde{\chi}^+$. In the second class, we considered sfermion decays to a fermion and a chargino/neutralino ($\tilde{f} \to f'\chi$); in particular, we calculated the one-loop QCD and electroweak corrections to the lightest stop and sbottom and the electroweak corrections to the lightest stau. The results are presented for different on-shell renormalization schemes and compared to a full $\overline{\rm DR}$ scheme. Considering the importance of the ubiquitous t_β, we also study a mixed scheme which is essentially OS apart from a $\overline{\rm DR}$ implementation of t_β. All these calculations are obtained with `SloopS` a code for the automatic generation of counterterms and the calculation of corrected masses, decays, and cross sections. The theoretical setup that we have detailed in this paper is now fully implemented in `SloopS`.

The OS schemes we have presented in this study are based on the use of a minimal set of physical masses with the view of reconstructing the totality of the underlying parameters of the NMSSM in order that any observable can be predicted at the one-loop order. Obviously there are different choices for the minimal set of physical masses. One would have thought that if one is interested in the chargino/neutralino system, providing the masses for a subset of these particles should have been sufficient to determine all the needed counterterms. Algebraically, this is indeed the case; however, masses of the neutralinos and charginos are not very sensitive to some key parameters such as t_β and λ. The latter sets the amount of mixing between the singlet and the other (MSSM-like) components. As a consequence, when we study decays which are much more sensitive to some of these parameters, we may end up with large radiative corrections due to ill-reconstructed mixing parameters. We have studied how one can improve the predictions by trading off some of the neutralino masses by some Higgs masses since the Higgs sector also experiences mixing that are driven by the same parameters. Our results indicate that a judicious choice of Higgs masses leads to significant improvement in the reconstruction of λ while issues remain with t_β, even though some improvement on t_β is always found. We, therefore, recommend to use an OS renormalization scheme with at least three Higgs masses as input for a reliable estimate of the partial decay widths of neutralino/charginos. The issue of the best choice of scheme for the decays involving Higgsinos and singlinos which are still dependent on a precise reconstruction of t_β is left for further investigation, one possiblity would be to use as input parameter an observable other than a mass, say a decay such as the decay of one of the neutral pseudoscalar Higgses to $b\bar{b}$ as was suggested for the MSSM, see [35] or the two-body decay of a neutralino into a gauge boson.

The electroweak corrections to the decays $\chi \to \chi' V$ are generally modest, within 20% and often much less. Larger corrections do show up in some schemes but these are due to a large contribution for the finite part of the counterterm of t_β and/or λ when those are extracted from a system of masses which is marginally affected by t_β thus explaining the large finite part of the counterterm. For sbottom decays, the QCD one-loop calculation reveals corrections of order -70% and about -20% for some stop decays. These large corrections happen when the decays are triggered through the Higgsino coupling. The large QCD corrections can be absorbed, almost entirely, in the running of the respective quark masses, set at the scale of the sbottom/stop mass. The different renormalization schemes described here and the extension of `SloopS` to include the NMSSM can now be used to compute any scattering process, in particular processes involving dark matter particles relevant for computing the relic density or processes for sparticle production and decays needed for collider physics.

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