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Supernatural supersymmetry and its classic example: M-theory inspired NMSSM

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We briefly review the supernatural supersymmetry (SUSY), which provides a most promising solution to the SUSY electroweak fine-tuning problem. In particular, we address its subtle issues as well. Unlike the minimal supersymmetric standard model (MSSM), the next to MSSM (NMSSM) can be scale invariant and has no mass parameter in its Lagrangian before SUSY and gauge symmetry breakings. Therefore, the NMSSM is a perfect framework for supernatural SUSY. To give the SUSY breaking soft mass to the singlet, we consider the moduli and dilaton dominant SUSY breaking scenarios in M-theory on S^1/Z_2. In these scenarios, SUSY is broken by one and only one F term of moduli or dilaton, and the SUSY breaking soft terms can be determined via the Kähler potential and superpotential from Calabi-Yau compactification of M-theory on S^1/Z_2. Thus, as predicted by supernatural SUSY, the SUSY electroweak fine-tuning measure is of unity order. In the moduli dominant SUSY breaking scenario, the right-handed sleptons are relatively light around 1 TeV, stau can even be as light as 580 GeV and degenerate with the lightest neutralino, chargino masses are larger than 1 TeV, the light stop masses are around 2 TeV or larger, the first two-generation squark masses are about 3 TeV or larger, and gluinos are heavier tha.n squarks. In the dilaton dominant SUSY breaking scenario, the qualitative picture remains the same but we have heavier spectra as compared to the moduli dominant SUSY breaking scenario. In addition to it, we have Higgs H_2/A_1-resonance solutions for dark matter (DM). In both scenarios, the minimal value of DM relic density is about 0.2. To obtain the observed DM relic density, we can consider the dilution effect from supercritical string cosmology or introduce the axino as the lightest supersymmetric particle.

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I. INTRODUCTION

Supersymmetry (SUSY) provides a natural solution to the gauge hierarchy problem in the Standard Model (SM). In the supersymmetric SMs (SSMs) with R parity, gauge coupling unification can be obtained, the lightest supersymmetric particle (LSP) such as neutralino can be a dark matter (DM) candidate, and the electroweak (EW) gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling, etc. Moreover, gauge coupling unification strongly implies the grand unified theories (GUTs), and the SUSY GUTs can be constructed from superstring theory, which is the most competitive candidate for quantum gravity. Therefore, supersymmetry is not only the most promising new physics beyond the SM, but also a bridge between the low energy phenomenology and high energy fundamental physics.

It is well known that a SM-like Higgs boson with mass m_H around 125 GeV was discovered during the first run of the LHC [1,2]. In the minimal supersymmetric standard model (MSSM), to realize such a Higgs boson mass, we need the multi-TeV top squarks with small mixing or TeV-scale top squarks with large mixing [3]. There also exists strong constraints on the parameter space in the SSMs from the LHC SUSY searches. For example, the gluino mass $m_{\tilde{g}}$ and first two-generation squark mass $m_{\tilde{q}}$ should be heavier than about 1.7 TeV if they are roughly degenerate $m_{\tilde{q}} \sim m_{\tilde{g}}$, and the gluino mass is heavier than about 1.3 TeV for $m_{\tilde{q}} \gg m_{\tilde{g}}$ [4,5]. Thus, the naturalness in the SSMs is challenged from both the Higgs boson mass and the LHC SUSY searches.

To quantize the size of fine-tuning in the SSMs, we need to define the measure. There are two kinds of definitions for fine-tuning measures: the low energy definition [6–8] and the high energy definition [9,10]. We emphasize that the naturalness conditions from the low energy definition can still be satisfied in principle, but the naturalness condition from the high energy definition is indeed a big challenge. However, because SUSY is the connection between the low and high energy physics, we do need to consider seriously the fine-tuning problem via the high energy definition. To solve this problem, we proposed the supernatural SUSY, which provides a most promising solution to the SUSY EW fine-tuning problem. It was shown in Refs. [11–13] that the high energy fine-tuning measure will automatically be at the order one $\mathcal{O}(1)$ in the $\mathcal{F}$-$SU(5)$ models [14–17] and the MSSM with no-scale supergravity (SUGRA) [18] and Giudice-Masiero (GM) mechanism [19]. We will briefly review the supernatural SUSY in Sec. III and for the first time address its subtle issues publicly. Especially, the major challenge to the previous studies is the μ term, which is generated by the GM mechanism and then is proportional to the universal gaugino mass $M_{1/2}$. The ratio $\mu/M_{1/2}$ is of order one but cannot be determined as an exact number. We have studied it carefully before and did not find any loophole [11–13].

On the other hand, the MSSM suffers from the so-called μ problem [20]. In the next to MSSM (NMSSM) which is the simplest extension of the MSSM [21–23], due to the presence of an extra singlet superfield $\hat{S}$, the effective $\mu_{\rm eff} \equiv \lambda\langle\hat{S}\rangle$ term can be generated via the superpotential term $\lambda\hat{S}\hat{H}_d\hat{H}_u$ after $\hat{S}$ acquires a vacuum expectation value (VEV), where λ is the Yukawa coupling while $\hat{H}_d$ and $\hat{H}_u$ are one pair of Higgs doublets in the MSSM. Moreover, the SUSY breaking scale is the only scale in the Lagrangian, since it allows for a scale invariant superpotential [24]. The SM-like Higgs, due to the above superpotentional term, gets additional contributions at tree level, and its mass can be pushed up by the mixing effects in diagonalizing the mass matrix of CP-even Higgs fields [25–27]. This results in a SM-like Higgs boson with mass around 125 GeV without large loop contributions, and then the SUSY EW fine-tuning problem can be ameliorated [28]. Similar to the constrained MSSM (CMSSM)/minimal supergravity (mSUGRA) [29], one can also define the constrained NMSSM (CNMSSM) [30–42], where the SUSY breaking (SSB) soft terms are the universal scalar mass m_0, the universal gaugino mass $M_{1/2}$, and the universal trilinear coupling term A_0 at the GUT scale $M_{\rm GUT}$. In the CNMSSM, in contrast to the unconstrained NMSSM, one needs a small value of λ but a large value of $\tan\beta \equiv \frac{\langle\hat{H}_u\rangle}{\langle\hat{H}_d\rangle}$ to get the SM-like Higgs mass around 125 GeV (for example, see [30]).

In this paper, we point out that the NMSSM provides an excellent framework for supernatural SUSY since its superpotential can be scale invariant [24]. In particular, we do not have the μ term issue any more. To satisfy three conditions of supernatural SUSY (see Sec. III) and give a soft mass to the singlet, we shall consider the moduli dominant SUSY breaking (MDSB) and dilaton dominant SUSY breaking (DDSB) scenarios in M-theory on S^1/Z_2 [43–47], and propose the M-theory inspired CNMSSM (MCNMSSM). In the MCNMSSM, SUSY is broken by one and only one F term of moduli or dilaton. The SUSY breaking soft terms, such as m_0, $M_{1/2}$, and A_0, can be calculated explicitly via the Kähler potential and superpotential from Calabi-Yau compactification of M-theory on S^1/Z_2, and they are functions of the gravitino mass ($M_{3/2}$) and hidden/observable sector gauge couplings at the GUT or string scale [47] which should be determined after moduli stabilization. Therefore, according to the supernatural SUSY, the fine-tuning measure is order of unity. In other words, there will be no EW fine-tuning problem at all in the MCNMSSM. In the MDSB scenario, we find that the minimal values for m_0 and $M_{1/2}$ consistent with sparticle mass bounds, B-physics bounds, and the light CP-even Higgs mass bound of 125 ± 2 GeV are about 0.6 TeV and 1.4 TeV, respectively, and the corresponding A_0 range is $[-4, -2]$ TeV. We also find that the range of parameter λ is [0, 0.1], and $\tan\beta$ is from 5 to 28. Moreover, we notice $m_{H_2} \approx m_{H^\pm} \approx m_{A_1}$ in most of the parameter space, while we have $m_{H_3} \approx m_{H^\pm}$ in the mass range [1.8,2.7] TeV. The gluino mass $m_{\tilde{g}}$ is found to be relatively heavy $\gtrsim 3$ TeV, and the light stop is the lightest colored sparticle ($\gtrsim 2$ TeV). The first two-generation squarks are about 3 TeV but they are lighter than the gluinos. In the slepton sector, the first two-generation sleptons have masses around 1 TeV or larger, while the light stau, which is mainly the right-handed stau, can be as light as 560 GeV. The LSP neutralino (which is bino in both the models) are in the mass range [0.55, 1.1] TeV while charginos are heavier than 1 TeV. We notice that despite the fact that the LSP neutralino and light stau are almost degenerate, the minimal value of DM relic density we get is about 0.2. In the DDSB scenario, the minimal values for m_0 and $M_{1/2}$ consistent with various constraints are about 0.8 TeV and 1.6 TeV, respectively. The ranges for A_0, λ, and $\tan\beta$ are, respectively, $[-8.8, -2]$ TeV, [0, 0.15], and [2, 41]. Because of this slightly larger range of λ, the low mass values of the CP-even Higgs $m_{H_{2,3}}$ and CP-odd Higgs $m_{A_{1,2}}$ are somewhat smaller than the MDSB. So these Higgs particles can come closer in mass with the LSP neutralino which can have mass in the range [0.6,4] TeV. It is also observed that $m_{A_1} = m_{H^\pm}$, while $m_{A_1} \approx m_{A_{A_2}} \approx m_{H_2} \approx m_{H_3}$ in some portions of parameter space. The light stop is still the lightest colored sparticle with mass $\gtrsim 2$ TeV, the first two-generation squark masses are $\gtrsim 3.4$ TeV, while the gluino mass is $\gtrsim 3.5$ TeV. The first two-generation sleptons are heavier than 1 TeV while the light stau can be as light as 600 GeV. The chargino masses are $\gtrsim 1.2$ TeV. Even though we have the resonance conditions such as $2m_{\tilde{\chi}_1^0} \approx m_{H_2,A_1}$ as well as the neutralino-stau coannihilation scenario, the minimal relic density we get is still around 0.2. We also present a couple of tables for benchmark points as examples of our findings. Furthermore, the minimal DM relic density is about 0.2 in both scenarios. To obtain the correct DM relic density, we can consider the lightest neutralino as the next to the LSP (NLSP) which decays to the LSP axino ($\tilde{a}$) and gamma (γ) final state. Moreover, in the supercritical string cosmology, the DM relic density can be diluted by a factor of 10 [$\mathcal{O}(10)$] [48], but it is model dependent.

This paper is organized as follows. In Sec. II, we introduce the CNMSSM as well as its SUSY breaking soft terms. In Sec. III, we briefly review the supernatural SUSY and address its subtle issues. We give the SUSY breaking soft terms from M-theory on S^1/Z_2 as well. We outline the detailed scanning procedure and the relevant experimental constraints in Sec. IV. We present in detail results of our scans in Sec. V. A summary and conclusion are given in Sec. VI.

II. THE CONSTRAINED NEXT TO MINIMAL SUPERSYMMETRIC STANDARD MODEL

The NMSSM is the simplest extension of the MSSM. In the NMSSM, we introduce an SM singlet superfield $\hat{S}$, as well as a Z_3 symmetry that forbids the μ term in the MSSM. The scale-invariant superpotential is

$$W_{\mathrm{NMSSSM}} = (\text{MSSM Yukawa terms}) + \lambda \hat{S}\hat{H}_u\hat{H}_d + \frac{\kappa}{3}\hat{S}^3, \tag{1}$$

where λ and κ are Yukawa couplings. The above two terms substitute the $\mu\hat{H}_u\hat{H}_d$ term in the MSSM superpotential. After spontaneous EW gauge symmetry breaking, a non-vanishing VEV v_S of $\hat{S}$ at the minimum of the Higgs potential generates an effective μ_{eff} term in the MSSM, i.e., $\mu_{\mathrm{eff}} \equiv \lambda v_S$. The SUSY breaking soft terms in the Higgs sector are then given by

$$V_{\mathrm{soft}} = m_{H_u}^2|H_u|^2 + m_{H_d}^2|H_d|^2 + m_S^2|S|^2 + \left(\lambda A_\lambda S H_u H_d + \frac{1}{3}\kappa A_\kappa S^3 + \text{H.c.}\right), \tag{2}$$

where A_λ and A_κ are soft trilinear terms associated with the λ and κ terms in the superpotential. The VEV v_S of $\hat{S}$, determined by the minimization conditions of the Higgs potential, is effectively induced by the SUSY breaking soft terms in Eq. (2) and is naturally set by M_{SUSY}. Thus, the μ problem in the MSSM is solved.

Just like the CMSSM/mSUGRA, we can assume that the SSB terms are universal at the GUT scale and the SUSY breaks spontaneously in a hidden sector. The gravitational interactions then mediate SUSY breaking effects from the hidden sector to the observable sector. This Z_3-invariant NMSSM is denoted as CNMSSM. In the CNMSSM, we assume universality for all the soft scalar masses squared, gaugino masses, and trilinear scalar couplings denoted as m_0^2, $M_{1/2}$, and A_0 at the GUT scale. In addition to these parameters, we also have new Yukawa couplings λ and κ. Through the minimization of the Higgs potential, m_S^2 can be traded for $\tan\beta$, and κ can be determined in terms of the other parameters for a correct value of M_Z [32,40]. Moreover, one can also chose either κ or $\mathrm{sgn}(\mu_{\mathrm{eff}})$. For conventional reasons we chose $\mathrm{sgn}(\mu_{\mathrm{eff}})$. In short, the CNMSSM can be defined in terms of five continuous input parameters and one sign as follows:

$$m_0, M_{1/2}, A_0, \tan\beta, \lambda, \mathrm{sgn}(\mu_{\mathrm{eff}}). \tag{3}$$

It is important to note that the addition of a gauge singlet superfield in the NMSSM modifies the neutralino and Higgs sectors of the MSSM with two on-shell fermionic and two on-shell scalar degrees of freedom. The two new scalar degrees of freedom yield two extra Higgs bosons and alter the mixing angles between the physical Higgs bosons and the gauge eigenstates. There are now three CP-even neutral Higgs bosons, H_1, H_2, and H_3; two CP-odd neutral Higgs bosons, A_1 and A_2; and one charged Higgs boson, $H^\pm$. The observed Higgs boson need not be the lightest CP-even neutral Higgs boson in the CNMSSM as compared to the CMSSM scenario where the SM-like Higgs boson has to be the lightest CP-even Higgs boson.

The two on-shell fermionic degrees of freedom yield a new fermionic state "singlino," which after EW symmetry breaking mix with the two neutral Higgsinos (which are mixed with the two neutral gauginos), resulting in five neutralinos. The Higgsino-singlino mixing is proportional to λ. The singlino decouples, if λ is small, and as a consequence we get four MSSM-like neutralinos and a singlino. It might be difficult to distinguish the MSSM and NMSSM neutralino sectors in a scenario where the singlino soft-breaking mass, m_S, is substantial and the singlino is decoupled.

An important point to remember is that larger values of λ do not necessarily imply an increase of the mass of the SM-like CP-even Higgs scalar. It is indicated in [36] that the mixing of the SM-like Higgs with a heavy singletlike Higgs (which is also proportional to λ) leads to a decrease of the SM-like Higgs mass. In the CNMSSM, the off-diagonal matrix elements cannot be fine-tuned to 0. In this way, the Higgs mass constraint puts an upper bound on λ. Moreover, Since $\mu_{\mathrm{eff}} = \lambda v_S$, if we know the μ_{eff} which is approximately equal to the chargino mass $m_{\tilde{\chi}_1^\pm}$, we can estimate v_S.

III. THE SUPERNATURAL SUSY AND THE M-THEORY INSPIRED SUSY BREAKING SOFT TERMS

To study the fine-tuning issue in the supersymmetric SMs, we need to define the fine-tuning measures first. There are two kinds of definitions: the low energy definition [6–8], and the high energy definition [9,10]. The low energy definition of the fine-tuning measure does not give strong constraints on the SSMs. In particular, if we allow a few percent fine-tuning, we can still have the viable parameter spaces in the MSSM and NMSSM, which satisfy all the current experimental constraints including the low bounds on the masses of the gluino, first/second generation squarks, and sleptons, from the LHC SUSY searches [4,5,49–51]. However, the high energy definition of fine-tuning measure is still a big challenge. For example, it is shown in Table 1 of Ref. [52] that the benchmark points 1 and 2 have the low energy fine-tuning measure Δ_{EW} around 20 while the high energy fine-tuning measure Δ_{EENZ} is around 1500. Since the fine-tuning measures for the high energy definition in the viable SSMs are very large at the order of 10^3 [$\mathcal{O}(10^3)$], we shall concentrate on it in the following discussions. The typical quantitative measure $\Delta_{\rm EENZ}$ of SUSY EW fine-tuning is defined by the maximum of the logarithmic derivative of M_Z with respect to all fundamental parameters a_i at the GUT scale [9,10]

$$\Delta_{\rm EENZ} = {\rm Max}\{\Delta_i^{\rm GUT}\}, \qquad \Delta_i^{\rm GUT} = \left|\frac{\partial \ln(M_Z)}{\partial \ln(a_i^{\rm GUT})}\right|. \quad (4)$$

The fine-tuning measure in Eq. (4) is exactly one in the dream case. So we would like to explore the supersymmetry breaking scenario whose fine-tuning measure for the high energy definition is automatically at the order one [$\mathcal{O}(1)$]. Assuming that there is one and only one mass parameter M_* in the SSMs, to be concrete, we shall take M_* as the universal gaugino mass $M_{1/2}$ for no-scale supergravity and gravitino mass $M_{3/2}$ for all the other supergravity including the M-theory supergravity. Thus, M_Z will be a trivial function of M_*, and we have the following approximate scale relation:

$$M_Z^n = f_n(c_i) M_*^n, \quad (5)$$

where f_n is a dimensionless parameter, and c_i denote the dimensionless coupling parameters, such as gauge and Yukawa couplings, as well as the ratio between μ and $M_{1/2}$ for the MSSM with the GM mechanism (for more details see [13]). For the nearly constant f_n of Eq. (5), we have

$$\frac{\partial M_Z^n}{\partial M_*^n} \simeq f_n, \quad (6)$$

and therefore we obtain

$$\frac{\partial \ln(M_Z^n)}{\partial \ln(M_*^n)} \simeq \frac{M_*^n}{M_Z^n}\frac{\partial M_Z^n}{\partial M_*^n} \simeq \frac{M_*^n}{M_Z^n}\frac{\delta M_Z^n}{\delta M_*^n} \simeq \frac{1}{f_n} f_n. \quad (7)$$

Consequently, the fine-tuning measure is an order one constant

$$\left|\frac{\partial \ln(M_Z^n)}{\partial \ln(M_*^n)}\right| \simeq \mathcal{O}(1). \quad (8)$$

For the first time, we would like to address a few subtle issues publicly in the supernatural SUSY as follows:

(i) The EW symmetry breaking and determination of M_* from the Z boson mass.

Assuming that the SSMs arise from string theories with suitable compactification and moduli stabilization, and there is one and only one F term of moduli or dilaton whose F term breaks SUSY, we can calculate the corresponding Kähler potential and superpotential, and then all the SUSY breaking soft terms can be determined in terms of M_*. Also, we can calculate the corresponding gauge couplings and Yukawa couplings at the GUT or string scale in principle, which should be required to be consistent with the low energy experimental values via renormalization group equation (RGE) running. For any set of the gauge couplings, Yukawa couplings, and SUSY breaking soft terms at the GUT or string scale, because the only free parameter is M_*, we might have three cases: (1) No RGE solution. (2) No EW gauge symmetry breaking; for example, stau is tachyonic. (3) The EW gauge symmetry breaking. In particular, for case (3), the observed Z boson mass M_Z as a low energy input will determine the corresponding M_* since it is the only dimensionful free parameter. Of course, if the RGEs have several solutions, we may have a few corresponding M_* values.

(ii) New μ problem in the MSSM and $\mathcal{F}$-$SU(5)$ model.

In the MSSM and $\mathcal{F}$-$SU(5)$ model with no-scale supergravity, to solve the μ problem, we employ the GM mechanism [19]. Thus, we have $\mu \propto M_{1/2} \propto M_{3/2}$, and M_* is assumed to be $M_{1/2}$. The ratio $c \equiv \mu/M_{1/2}$ is an order one constant, but we cannot determine the exact value of c from the GM mechanism since we cannot determine the coefficient of the high-dimensional operator up to order one, which generates the μ term. This new μ problem was pointed out to us not only by referees but also by audiences.

We have considered it in detail and confirmed that there is no gap in our previous studies [11–13]. From a top-down approach, c is a fixed real number at the order one, and it can be determined from our above string model assumptions in principle. So the low energy Z boson mass M_Z is predicted from the high energy fundamental theory. From the phenomenological point of view, the observed value of Z boson mass M_Z determines the gaugino mass $M_{1/2}$ at the GUT scale for some narrow range of c. By the way, for the other numerical values of c, we do not have the correct M_Z value, or the EW gauge symmetry breaking, or the RGE solution for no-scale boundary conditions. To be concrete, from Fig. 2 of Ref. [12], we found that for a fixed c, there is clearly one-to-one correspondence between M_Z and $M_{1/2}$.

On the other hand, if people are still not satisfied with our above explanation to the new μ problem, we can study the supernatural supersymmetry in the NMSSM, which solves the μ problem. This is one of the main motivations of our current paper.

(iii) Symmetry for supernatural supersymmetry.

In the supernatural supersymmetry, the fine-tuning measure is exactly one for the perfect scenario. So it is naive to think that there may exist a symmetry behind it. This symmetry acts like the scale invariance: for the fixed dimensionless coefficients at the unification scale from the top-down approach, we define the mass ratio $r_\phi \equiv M_\phi/M_*$, where ϕ is a supersymmetric particle (sparticle) and

M_ϕ is its mass at low energy. We found r_ϕ is scale invariant; i.e., r_ϕ does not depend on M_*. This has been confirmed numerically by the previous studies in the MSSM and $\mathcal{F}$-$SU(5)$ model with no-scale supergravity and GM mechanism [11–13]. In other words, the sparticle mass spectra for different M_* are correlated by an overall rescale.

Similar to the low energy definition of the fine-tuning, we may require extra naturalness conditions at the GUT or string scale. In the MSSM, with the one-loop effective potential contributions to the tree-level Higgs potential, we get the Z-bosom mass M_Z[1]

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \quad (9)$$

where Σ_u^u and Σ_d^d denote the corrections to the scalar potential coming from the one-loop effective potential defined in [8] while m_{H_u} and m_{H_d} are the Higgs soft masses. $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$ is the ratio of the Higgs VEVs. For moderate large $\tan\beta$, we have

$$\frac{M_Z^2}{2} \simeq -m_{H_u}^2 - \mu^2. \quad (10)$$

At the GUT or string scale, although we do not have the EW gauge symmetry breaking, i.e., $M_Z = 0$, to be natural, one might still require

$$\frac{m_{H_u}^2}{\mu^2} \sim \frac{|m_{H_u}^2 - \mu^2|}{m_{H_u}^2 + \mu^2} \sim \mathcal{O}(1). \quad (11)$$

In the no-scale supergravity, the above requirement cannot be satisfied since the universal scalar mass m_0 vanishes, i.e., $m_0 = 0$. However, our models, such as the MSSM and $\mathcal{F}$-$SU(5)$ model with no-scale supergravity and GM mechanism, are indeed technically natural since $m_0 = 0$ arises from the $SU(N,1)/SU(N)\times U(1)$ symmetry or a Heisenberg symmetry in the Kähler potential [53].

(iv) Multi F-term SUSY breakings.

If SUSY is broken by two or more F terms of moduli and/or dilaton, we should define the corresponding fundamental mass parameters as $M_*^i \equiv F_i/\sqrt{3}M_{\rm Pl}$, and calculate the corresponding fine-tuning measures $\Delta^{\rm GUT}_{M_*^i}$. In other words, M_* cannot be the gravitino mass. This kind of scenarios should be studied in detail as well since there might exist the corresponding supernatural SUSY, which is different from our current study. For example, if the Kähler potential and superpotential are determined from string constructions and the moduli and dilaton are stabilized properly, the supernatural SUSY can still be valid.

(v) Effective supernatural SUSY.

The above definition for supernatural SUSY is very strong, so we can relax the conditions, in addition to the above multi F-term SUSY breakings. In fact, to solve the SUSY EW fine-tuning problem, we only require that the dimensionful parameters at the GUT scale, which have large fine-tuning measures $\Delta_{\rm EENZ}$, are related to the fundamental mass parameter M_* [54]. Similar to the natural SUSY or more effective SUSY where only the third generation sfermions like stops need to be light while the first two-generation sfermions can be very heavy, we shall call it the effective supernatural SUSY [54]. Furthermore, for the supernatural SUSY, we can make small perturbations to the leading order SUSY breaking soft terms. Obviously, the solution to the SUSY EW fine-tuning problem is still valid. Interestingly, although it might only change the particle spectra a little bit, it will have big effects on the DM candidate and DM relic density, which will be studied elsewhere.

In this paper, we shall study the scale invariant NMSSM. Because $\hat{S}$ is an SM singlet, its scalar mass can only be generated via two-loop effects via RGE running, and then it is too small for no-scale supergravity. To solve this problem, we consider the SUSY breaking soft terms from M-theory on S^1/Z_2 [43–47]. As we know, in the weakly coupled heterotic string theory, there exist two simplified scenarios: (1) The moduli dominant SUSY breaking scenario or, say, no-scale scenario [18,55] with $m_0 = A = 0$; (2) the dilaton dominant SUSY breaking scenario [56,57] with $M_{1/2} = -A = \sqrt{3}m_0$, which can also escape the above problem. Generically speaking, the M-theory on S^1/Z_2 seems to be a better candidate than the weakly coupled heterotic string theory to explain the low energy phenomenology and high energy unification of all the fundamental interactions. In particular, we can have the next-to-leading order corrections to the Kähler potential and gauge kinetic functions and then to the SUSY breaking soft terms as well [43–47]. To parametrize the next-to-leading order corrections to the SUSY breaking soft terms, we define [47]

$$x \equiv \frac{\alpha(T+\overline{T})}{S+\overline{S}} = \frac{\alpha_{\rm GUT}^{-1}\alpha_H - 1}{\alpha_{\rm GUT}^{-1}\alpha_H + 1}, \quad (12)$$

where α is related to the extra space dimensions and defined in Refs. [45–47], S and T are dilaton and moduli fields, and $\alpha_{\rm GUT}$ and α_H are the gauge couplings at the GUT scale in the observable and hidden sectors, respectively. With the assumption $\alpha_H \geq \alpha_{\rm GUT}$ and to avoid α_H to be infinity, we obtain

[1]The following comment is based on the private discussions with Daniel Chung.

$$0 \le x \le 1. \tag{13}$$

In the supernatural SUSY, there exists one and only one moduli or dilaton field whose F term breaks the SUSY. Thus, we will consider the MDSB and DDSB scenarios as follows [47]:

(I) Moduli dominant SUSY breaking scenario. The SUSY breaking soft terms are

$$m_0 = \frac{x}{3+x} M_{3/2}, \tag{14}$$

$$M_{1/2} = \frac{x}{1+x} M_{3/2}, \tag{15}$$

$$A = -\frac{3x}{3+x} M_{3/2}. \tag{16}$$

(II) Dilaton dominant SUSY breaking scenario. The SUSY breaking soft terms are

$$m_0^2 = M_{3/2}^2 - \frac{3M_{3/2}^2}{(3+x)^2} x(6+x), \tag{17}$$

$$M_{1/2} = \frac{\sqrt{3}M_{3/2}}{1+x}, \tag{18}$$

$$A = -\frac{\sqrt{3}M_{3/2}}{3+x}(3-2x). \tag{19}$$

From the requirement $m_0^2 > 0$, we obtain that x is smaller than about 0.67423. Choosing $x = 0$, we obtain the relation $M_{1/2} = -A = \sqrt{3}m_0$ in the weakly coupled heterotic string theory.

In short, in the M-theory motivated CNMSSM with MDSB and DDSB scenarios, all the SUSY breaking soft mass parameters have fixed relations with gravitino mass $M_{3/2}$ after the moduli stabilization which determine α_{GUT} and α_H as well. According to the supernatural SUSY, the fine-tuning measure is automatically of order one. In other words, such kinds of models are supernatural, even though their particle spectra are heavy.

IV. PHENOMENOLOGICAL CONSTRAINTS

We use the publicly available code MicrOmegas3.5.5 [58] for random scans on the following parameter space:

$$\begin{aligned} &0 \le x \le 1, \\ &0 \le M_{3/2} \le 5\ \mathrm{TeV}, \\ &2 \le \tan\beta \le 60, \\ &0 \le \lambda \le 0.7. \end{aligned} \tag{20}$$

We consider $\mu > 0$, $m_t = 173.3$ GeV [59] and $m_b^{\overline{\mathrm{DR}}}(M_Z) = 2.83$ GeV.

After collecting the data, we require the following bounds on sparticle masses from the LEP2 experiment

$$m_{\tilde{t}_1}, m_{\tilde{b}_1} \gtrsim 100\ \mathrm{GeV}, \tag{21}$$

$$m_{\tilde{\tau}_1} \gtrsim 105\ \mathrm{GeV}, \tag{22}$$

$$m_{\tilde{\chi}_1^\pm} \gtrsim 103\ \mathrm{GeV}. \tag{23}$$

We use the LEP2 bounds since they are somewhat model independent. On the other hand, because of the LHC ongoing searches, the above-mentioned SUSY particle masses can be in several hundred GeVs ranges depending on the decay channels. As we will also show later in this paper, our sparticle spectra are heavy enough to escape such LHC mass constraints. We implement the following B-physics constraints [60,61]

$$1.6 \times 10^{-9} \le \mathrm{BR}(B_s \to \mu^+\mu^-) \le 4.2 \times 10^{-9} (2\sigma), \tag{24}$$

$$2.99 \times 10^{-4} \le \mathrm{BR}(b \to s\gamma) \le 3.87 \times 10^{-4} (2\sigma), \tag{25}$$

$$0.70 \times 10^{-4} \le \mathrm{BR}(B_u \to \tau\nu_\tau) \le 1.5 \times 10^{-4} (2\sigma). \tag{26}$$

In addition, we impose the following bounds from the LHC SUSY searches as well [1,2,4,5]:

$$m_{H_1} = 123\text{–}127\ \mathrm{GeV}, \tag{27}$$

$$m_{\tilde{g}} \gtrsim 1.7\ \mathrm{TeV} \quad (\text{for } m_{\tilde{g}} \sim m_{\tilde{q}}), \tag{28}$$

$$m_{\tilde{g}} \gtrsim 1.3\ \mathrm{TeV} \quad (\text{for } m_{\tilde{g}} \ll m_{\tilde{q}}). \tag{29}$$

For the muon anomalous magnetic moment a_μ, we require that the benchmark points be at least as consistent with the data as the SM.

V. NUMERICAL RESULTS

In the following, we will present our results for MDSB and DDSB scenarios.

A. Moduli dominant SUSY breaking scenario

In Fig. 1, we present our graphs in the $\Omega h^2 - m_{H_1}$ plane. The ranges of input parameters given in Eq. (20) are also displayed in vertical bars, where red to blue colors represent the lowest to highest values of input parameters. Note that in Fig. 1, we just display data from our scans without applying any constraints. In Fig. 1, we

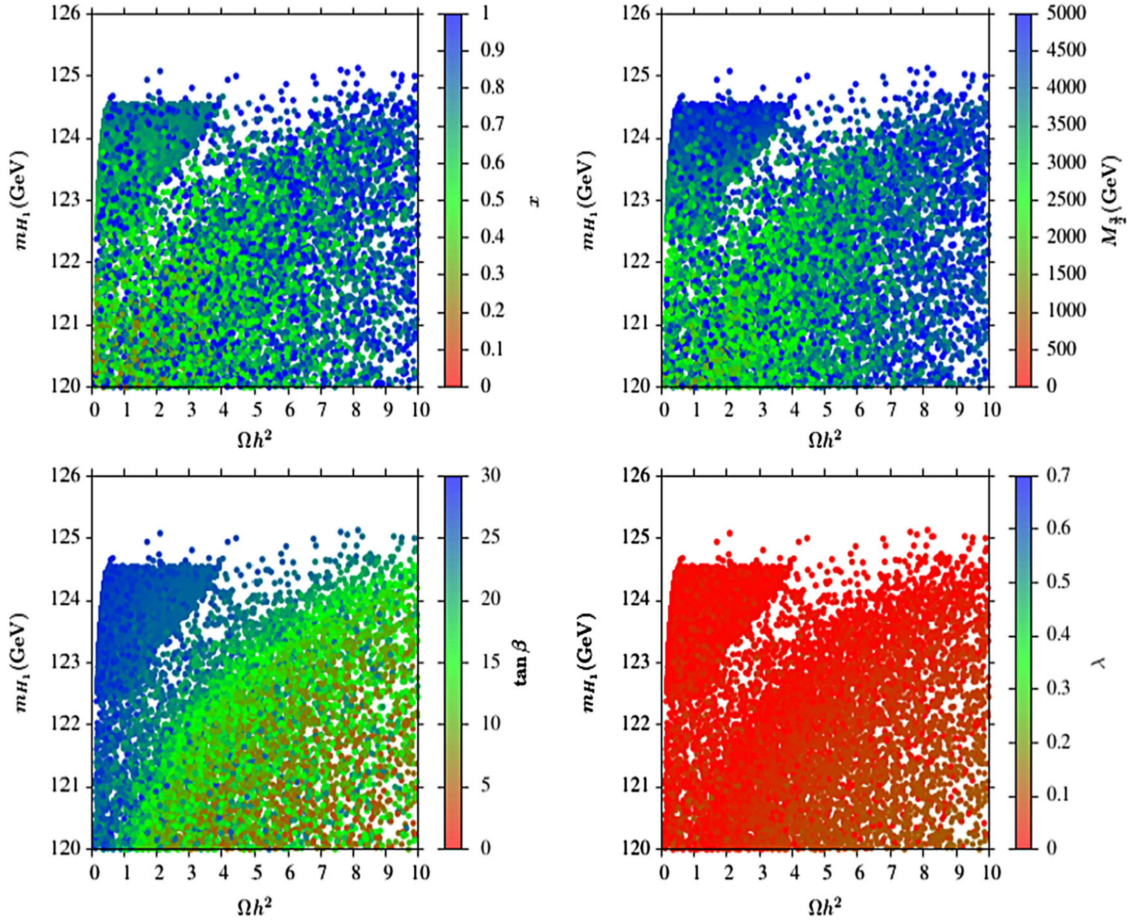

FIG. 1. Plots in the $\Omega h^2 - m_{H_1}$ plane for the moduli dominant SUSY breaking scenario. The ranges of input parameters given in Eq. (20) are shown in vertical bars where red to blue colors represent lowest to largest values.

show the CP-even Higgs mass m_{H_1} larger than 120 GeV and the neutralino dark matter relic density Ωh^2 between 0 and 10 to give a broader picture. These plots show that in order to have m_{H_1} in the range [123, 127] GeV, the input parameters x should be greater than 0.2 and gravitino mass $M_{3/2}$ should be greater than 2 TeV. From the left bottom panel, we see that the Higgs mass constraint pushes $\tan\beta \gtrsim 5$, and if we demand $\Omega h^2 \lesssim 1$ to further constrain the parameter space, it further pushes $\tan\beta$ values above 20. In the right bottom panel, we see that all the points have $\lambda \lesssim 0.2$. Moreover, these plots show that the Higgs mass constraint alone severely restricts the input parameter space, and we will study this scenario in more detail below.

Since the SUSY breaking soft terms m_0, $M_{1/2}$, and A_0 are functions of input parameters x and $M_{3/2}$, we calculate them using Eqs. (14)–(16) and show our results in Fig. 2. In these plots, gray points satisfy successful radiative electroweak symmetry breaking. Blue points, which form a subset of gray points, satisfy particle mass bounds, B-physics bounds, and Higgs mass bounds. Although we do not have the points which satisfy the dark matter relic density bounds, for instance, WMAP9 5σ bounds, we will comment on the possible mechanism to dilute relic density later. Here, we want to see how much we can restrict the parameter space if we insist on $\Omega h^2 \lesssim 1$ besides the above-mentioned constraints. Red color points satisfy $\Omega h^2 \lesssim 1$. As we have already observed, the light CP-even Higgs mass ranges [123, 127] GeV constrain the input parameter space a lot. This constraint already makes the spectra too heavy so that the viable points satisfy various above-mentioned bounds. From the first row of Fig. 2, we see that the minimum value of m_0 consistent with all the constraints is about 0.6 TeV, corresponding to $x \approx 0.4$ and $M_{3/2} \approx 2.5$ TeV while the maximum allowed values of m_0 is about 1.3 TeV. The plots in the second row display dependence of $M_{1/2}$ on x and $M_{3/2}$. Here, we see that the minimum and maximum values of $M_{1/2}$ consistent with all the above-mentioned constraints are about 1.4 TeV and 2.5 TeV, respectively. Because Eqs. (14) and (16) show $A_0 = -3m_0$, we do not present plots for A_0. But one can estimate the allowed range for A_0 from m_0 plots in the first row. It can be seen that the allowed ranges of universal trilinear scalar coupling A_0 are about $[-4, -2]$ TeV. This indicates that the scalar top quarks are not highly mixed. So

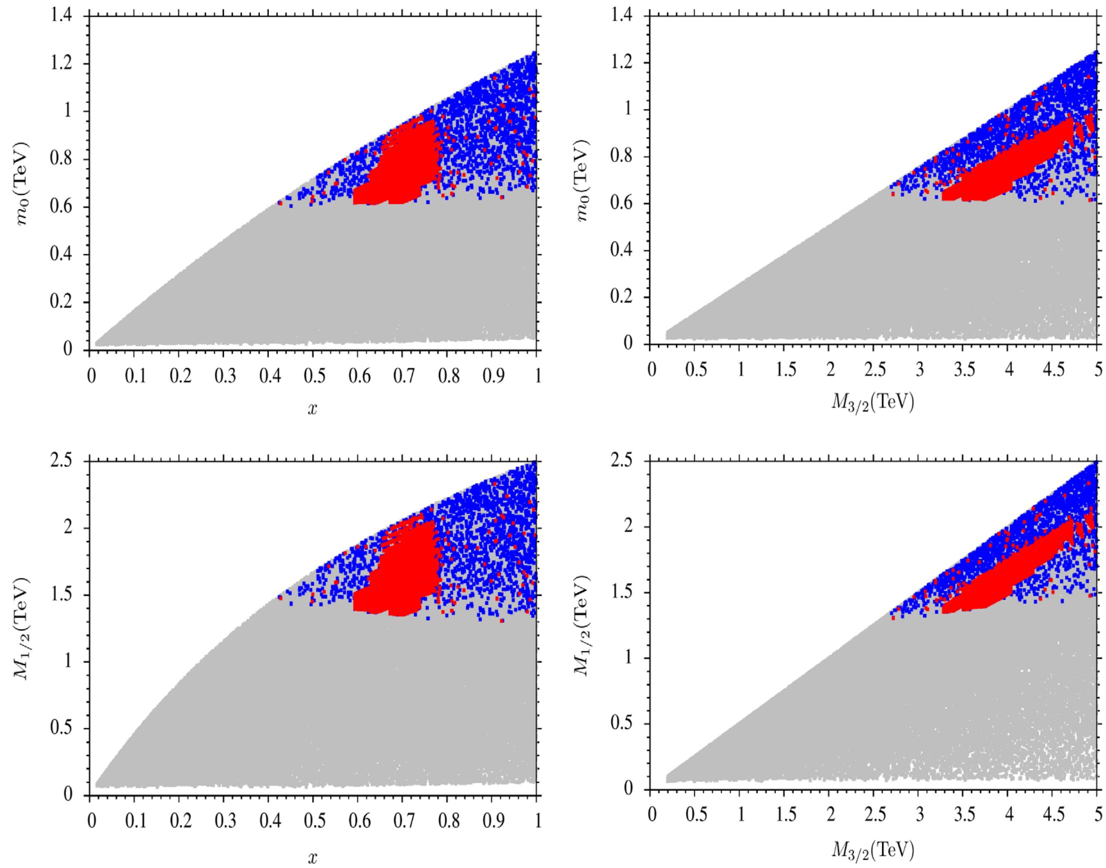

FIG. 2. Plots of m_0 and $M_{1/2}$ as functions of x and $M_{3/2}$ for the moduli dominant SUSY breaking scenario. Gray points satisfy successful radiative electroweak symmetry breaking. Blue points form a subset of gray points and satisfy particle mass bounds, B-physics bounds, and Higgs mass bounds. Red points further form a subset of blue points and satisfy $\Omega h^2 \lesssim 1$.

in order to have the CP-even SM-like Higgs boson mass around 125 GeV, we need heavy squarks/stops. The large values of m_0 and $M_{1/2}$ indicate heavy spectra at low energy. One can write the EW-scale masses of squarks and sleptons in terms of m_0 and $M_{1/2}$ as follows [62]:

$$m_{\tilde{q}}^2 \simeq m_0^2 + (5-6)M_{1/2}^2, \quad (30)$$

$$m_{\tilde{e}_L}^2 \simeq m_0^2 + 0.5M_{1/2}^2, \quad (31)$$

$$m_{\tilde{e}_R}^2 \simeq m_0^2 + 0.15M_{1/2}^2. \quad (32)$$

By plugging in the minimum values of m_0 and $M_{1/2}$ in the above semianalytical expressions we see that the squarks may be around 3 TeV, the left-handed sleptons can be around 1 TeV, while the right-handed slepton can be relatively light around 650 GeV. We will see that this indeed is the case and the right-handed staus are light. One can also observe this trend in Table I.

To have the SM-like Higgs mass around 125 GeV in the NMSSN, the Yukawa coupling λ also plays a very crucial role. In the unconstrained NMSSM, one needs large λ values (but less than 0.7 to avoid the Landau pole problem in GUT models) and small $\tan\beta \lesssim 10$. However, in the CNMSSM, the requirement is almost reversed. One usually needs small values of λ and large values of $\tan\beta$ [30]. This can be seen in Fig. 3 where we display plots in the $\lambda - m_{H_1}$ plane (left panel) and the $\tan\beta - m_{H_1}$ plane (right panel). In these plots, gray points satisfy the successful radiative electroweak symmetry breaking, orange points satisfy all the above mentioned constraints except the Higgs mass constraints, and red points further form a subset of orange points and satisfy $\Omega h^2 \lesssim 1$. The horizontal black line indicates the lower bounds on the Higgs mass of 123 GeV. Here, we see that orange points with $m_{H_1} \approx$ 123 GeV have maximum value of λ about 0.1. The maximum value of λ further shrinks to about 0.08 when we demand $\Omega h^2 \lesssim 1$. Also, the plot in the $\tan\beta - m_{H_1}$ plane shows that the allowed range of $\tan\beta$ is [5, 28].

In the NMSSM, because of the presence of an additional gauge singlet $\hat{S}$, we have an extra CP-even Higgs H_3 and a pseudoscalar A_2 as compared to MSSM. The approximate tree-level Higgs boson masses in the NMSSM are given in Ref. [63]. From there we see that these masses are proportional to v_S. From Fig. 3, we see that the minimum value of λ consistent with all constraints is about 0.1. Note that $\mu_{\rm eff} \equiv \lambda v_S$ and $\mu_{\rm eff} \approx \chi_1^{\pm} \gtrsim 100$ GeV from the LEP

TABLE I. Sparticle and Higgs masses are in GeV units and $\mu > 0$. All of these points satisfy the sparticle mass, B-physics constraints described in Sec. IV. Points 1 and 2 display the neutralino-stau coannihilation scenario when $m_{H_3} \approx m_{A_1} \approx m_{H^\pm}$ and $m_{H_2} \approx m_{A_1} \approx m_{H^\pm}$, respectively.

	Point 1	Point 2
x	0.67683	0.92663
$M_{3/2}$	3396	4329.3
$\tan\beta$	26.354	27.518
λ	9.9924×10^{-3}	1.5796×10^{-2}
m_0	625.13	1021.7
$M_{1/2}$	1370.7	2082.2
A_0	−1875.4	−3065
m_{H_1}	123	124.7
m_{H_2}	1767	2731
m_{H_3}	1840	2979
m_{A_1}	1840	2731
m_{A_2}	2542	4201
$m_{H^\pm}$	1842	2732
$m_{\tilde{\chi}^0_{1,2}}$	596, 1119	923, 1716
$m_{\tilde{\chi}^0_{3,4,5}}$	1891, 1895, 2295	2810, 2813, 3837
$m_{\tilde{\chi}^\pm_{1,2}}$	1119, 1895	1716, 2813
$m_{\tilde{g}}$	2957	4369
$m_{\tilde{u}_{L,R}}$	2722, 2618	4025, 3865
$m_{\tilde{t}_{1,2}}$	1923, 2377	2809, 3466
$m_{\tilde{d}_{L,R}}$	2723, 2605	4026, 3845
$m_{\tilde{b}_{1,2}}$	2344, 2344	3443, 3443
$m_{\tilde{\nu}_{1,2}}$	1084	1680
$m_{\tilde{\nu}_3}$	1019	1567
$m_{\tilde{e}_{L,R}}$	1086, 801	1682, 1271
$m_{\tilde{\tau}_{1,2}}$	596, 1028	928, 1573
σ_{SI} (pb)	9.88×10^{-12}	3.91×10^{-12}
σ_{SD} (pb)	3.45×10^{-9}	6.98×10^{-10}
$\Omega_{\rm CDM} h^2$	0.2085	0.6458

bound on chargino mass, and we obtain $v_S \gtrsim 1$ TeV. Such a large value of v_S in turn implies heavy masses of Higgs bosons as can be seen in Table I. Note that if we use the chargino mass value greater than the LEP2 bound, it will simply increase v_S as shown above, and thus increase the heavy Higgs masses.

The addition of a gauge singlet also affects the neutralino sector of the NMSSM. Now we have the singlino-type neutralino in addition to the bino-type, wino-type, and Higgsino-type neutralinos. For dark matter relic density, one can try to have $m_{H_{1,2,3}/A_{1,2}} \simeq 2m_{\tilde{\chi}^0_1}$ resonance solutions. Moreover, in the CMSSM, small values of m_0 give rise to a stable charged slepton LSP. While in the CNMSSM, this problem can be evaded due to the presence of the extra singlinolike neutralino [41]. We would like to remind readers that in a good approximation one can show that $m_{\tilde{\chi}^0_{1,2}}$ are proportional to gaugino masses $M_{1,2}$, $m_{\tilde{\chi}^0_{3,4}}$ are proportional to $\mu_{\rm eff}$, and singlino mass $m_{\tilde{\chi}^0_5}$ is directly proportional to κ and $\mu_{\rm eff}$ but inversely proportional to λ [64]. From Fig. 2, we see that the minimum allowed value of $M_{1/2}$ is about 1.4 TeV. Since $m_{\tilde{\chi}^0_1} \approx 0.44 M_{1/2}$, the lightest neutralino should be much heavier than the SM-like Higgs boson. We also find that $m_{H_{2,3}}$ and $m_{A_{1,2}}$ are heavier than 1.5 TeV. So no resonance solutions can be realized here. As we have already discussed in this case, λ is small and $\mu_{\rm eff}$ is about 1 TeV. Thus, the singlino is also heavy (as $m_{\tilde{\chi}^0_5} \propto \kappa, \mu_{\rm eff}/\lambda$), and the LSP neutralino in the MDSB scenario is binolike. On the other hand, because $|A_0|$ is not large enough, top squark masses must be heavy to achieve $m_{H_1} \sim 125$ GeV. And thus we do not have the LSP neutralino-stop coannihilation channel. The focus point SUSY or hyperbolic SUSY cannot be realized as well due to $m_0 < M_{1/2}$ from Fig. 2. We have mentioned earlier in Eq. (32) that the right-handed slepton can be relatively light for relatively small values of m_0; thus we can expect the LSP neutralino-stau coannihilation. From Fig. 4 it is evident that we do have a neutralino-stau coannihilation region. The color coding for this figure is the same as in Fig. 2. Here, for the red points, the minimum masses for the light stau and LSP neutralino are, respectively, 580 GeV and 570 GeV while the light stau and LSP neutralino can be as heavy as ≈1400 GeV. We notice here

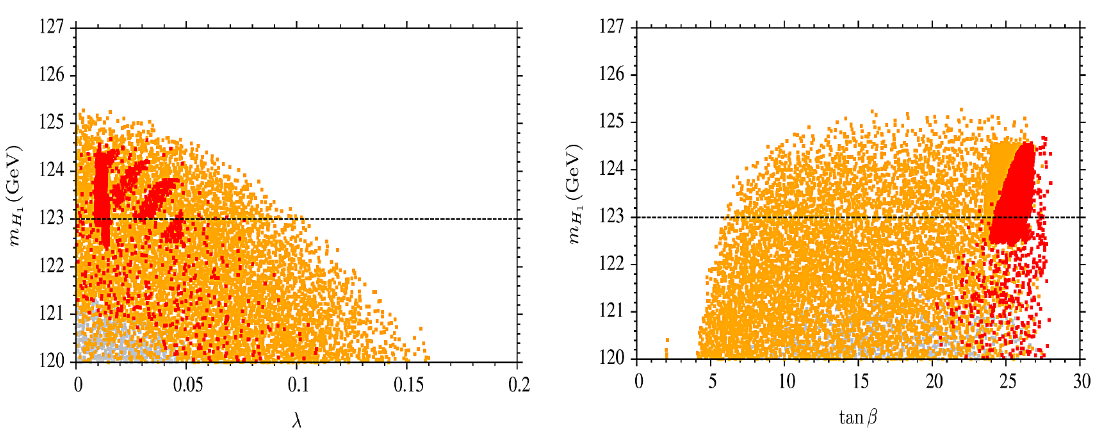

FIG. 3. Plots in the $\lambda - m_{H_1}$ and $\tan\beta - m_{H_1}$ planes for the moduli dominant SUSY breaking scenario. Gray points satisfy successful radiative electroweak symmetry breaking. Orange points, which form a subset of gray points, satisfy particle mass bounds and B-physics bounds. We do not apply Higgs mass bounds here. Red points further form a subset of orange points and satisfy $\Omega h^2 \lesssim 1$.

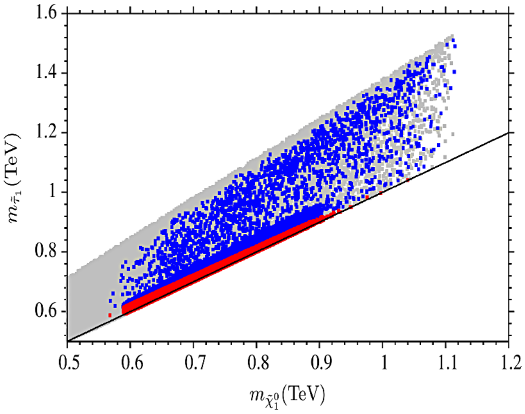

FIG. 4. Plots in the $m_{\tilde{\chi}_1^0} - m_{H_2}$, $m_{\tilde{\chi}_1^0} - m_{A_1}$, and $m_{\tilde{\chi}_1^0} - m_{\tilde{\tau}_1}$ planes for the moduli dominant SUSY breaking scenario. The color coding is the same as in Fig. 2.

that the best point we have here in this plot has $\Omega h^2 \approx 0.2$. However, we are not able to get the points with relic density within 5σ of WMAP9 bounds [65].

We comment here that since relic density calculations are highly sensitive to sparticle spectra, a slight change in sparticle masses may change the relic density a lot. Therefore, $\Omega h^2 \approx 0.2$ is not that bad value. On the other hand, we can employ some relic density dilution mechanisms. For instance, one can assume that the lightest neutralino is the NLSP and then decays into $\tilde{a}\gamma$, where the axino $\tilde{a}$ acts as the LSP [66]. Besides this, one can consider the dilution effect from supercritical string cosmology (SSC). In such a scenario, it was shown that in the presence of a time dependent dilaton, a smoothly evolving dark energy can modify the dark matter allowed region with standard cosmology. The dilaton field dilutes the neutralino relic density by a factor $[\mathcal{O}(10)]$, and consequently the regions with the larger dark matter relic density in the standard scenario are allowed in the SSC [48] but it is model dependent.

B. Dilaton dominant SUSY breaking scenario

In Fig. 5, we present plots for the DDSB scenario in the $\Omega h^2 - m_{H_1}$ plane. We also display the ranges of input parameters given in Eq. (20) in vertical bars. In the top left panel we see that x should be in the range around [0.1, 0.2] to have $m_{H_1} \gtrsim 123$ GeV, while the DM relic density can be anywhere between 0 and 10. In Fig. 5 we see that the viable points with Higgs mass above 123 GeV tend to have more or less one particular color and hence show the narrow ranges of input parameters. This is because of our dedicated searches: if we generated more data around some good points, the corresponding ranges of those input parameter's color dominate (this is very much true for x and $\tan\beta$). These dedicated search effects will also appear in Fig. 6. In the top right panel of Fig. 5, we see that the Higgs mass

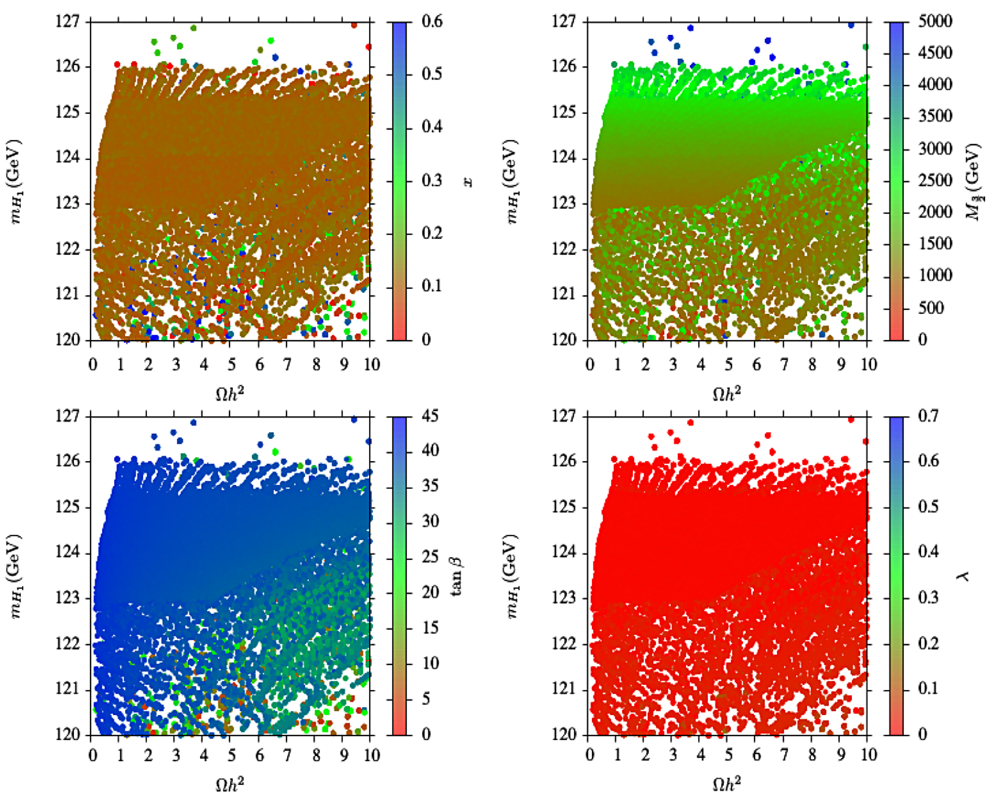

FIG. 5. Plots in the $\Omega h^2 - m_{H_1}$ plane for the dilaton dominant SUSY breaking scenario. The ranges of input parameters given in Eq. (20) are shown in vertical bars.

larger than 123 GeV requires $M_{3/2} \gtrsim 1$ TeV. In the bottom left panel, for $m_{H_1} \gtrsim 123$ GeV, we need $\tan\beta \gtrsim 35$, but we can see some green points at the top of the figure which shows that the low bound on $\tan\beta$ can be relaxed. The appearance of only blue points is just an artifact of dedicated searches. In Fig. 7 one will see that the actual $\tan\beta$ lower limit consistent with 123 GeV Higgs mass is about 5.

We use Eqs. (17)–(19) to calculate m_0, $M_{1/2}$, and A_0 as functions of input parameters x and $M_{3/2}$. We show our results in Fig. 6. The color coding is the same as in Fig. 2. As compared to the MDSB scenario, in the DDSB scenario the input parameters x and $M_{3/2}$ are less constrained under various bounds we mentioned earlier. The patches of points correspond to our dedicated searches. From the plots in the first row of Fig. 6, we observe that the minimum value of m_0 consistent with all the constraints is about 0.7 TeV, corresponding to $x \approx 0.6$, but the maximum value of $m_0 \sim 5$ TeV occurs at very small values of x. On the other hand, the minimum value of m_0 is correlated to $M_{3/2}$ and increases linearly with $M_{3/2}$ up to 5 TeV. We also notice here that the red points have $m_0 \lesssim 3.5$ TeV. The plots in the

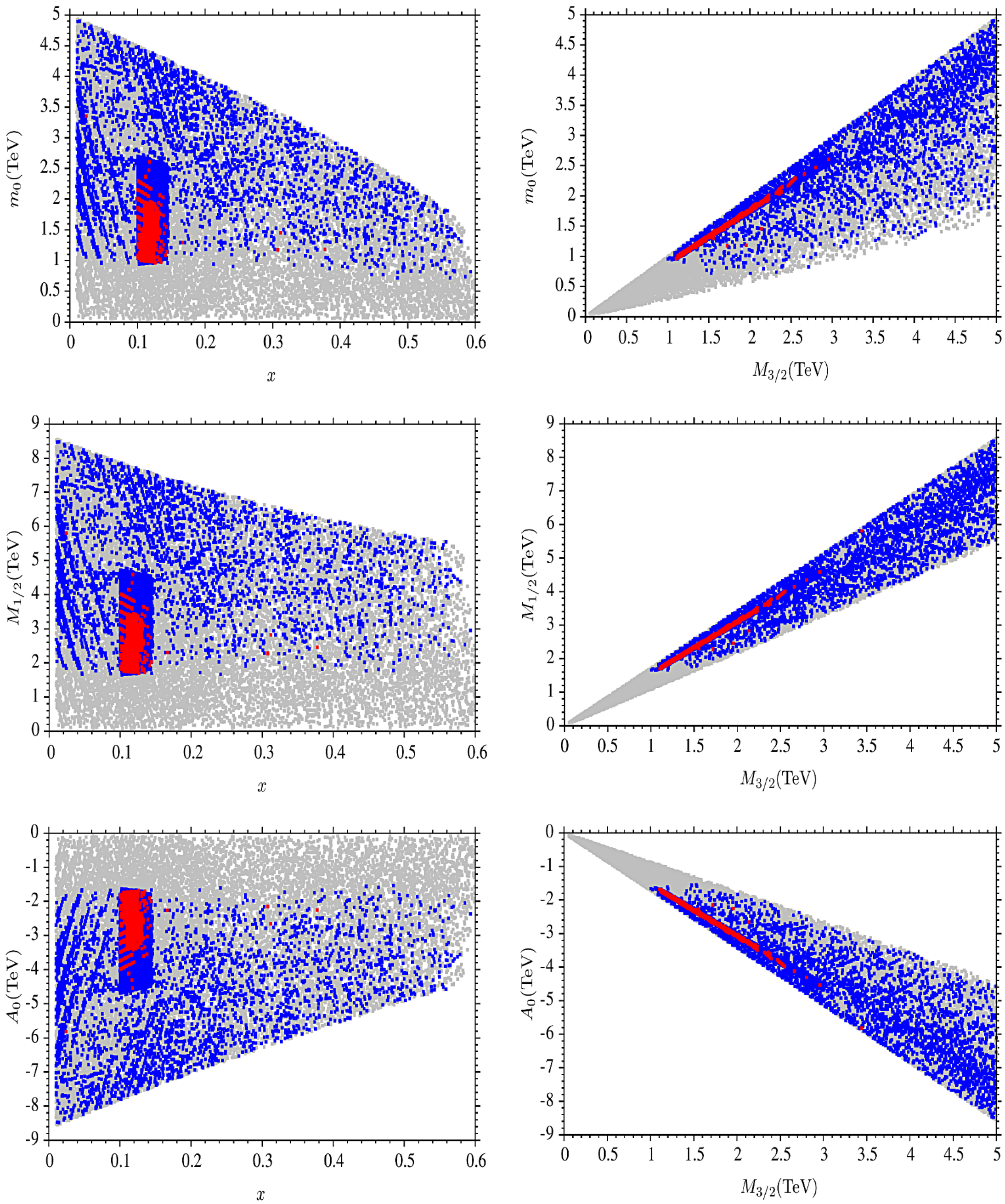

FIG. 6. Plots of m_0, $M_{1/2}$, and A_0 as functions of x and $M_{3/2}$ for the dilaton dominant SUSY breaking scenario. The color coding is the same as in Fig. 2.

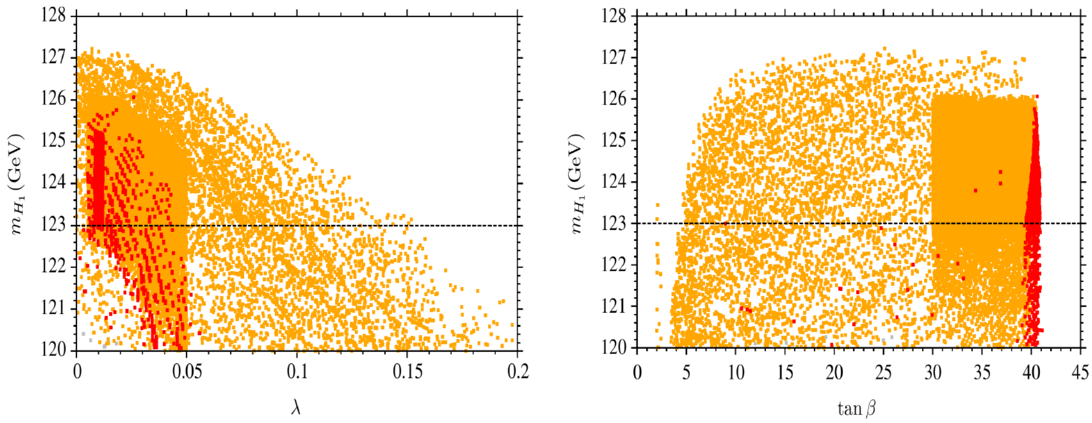

FIG. 7. Plots in the $\lambda - m_{H_1}$ and $\tan\beta - m_{H_1}$ planes for the dilaton dominant SUSY breaking scenario. The color coding is the same as in Fig. 3.

second row display dependence of $M_{1/2}$ on x and $M_{3/2}$. The minimum and maximum values of $M_{1/2}$ consistent with all the above mentioned constraints is about 2 TeV and 8.5 TeV, respectively. Finally, the plots in the third row display that the allowed range of universal trilinear soft term A_0 is $[-8.5, -2]$ GeV. Such relative large values of $|A_0|$ show the top squarks have larger mixing in the DDSB scenario than the MDSB scenario. This implies that now A_0 will share the burden of achieving the SM-like Higgs mass around 125 GeV with top squarks. Using Eqs. (30)–(32), we see that in the DDSB scenario, we have heavier spectra as compared to the MDSB scenario.

In Fig. 7, we depict our results in the $\lambda - m_{H_1}$ (left panel) plane and the $\tan\beta - m_{H_1}$ (right panel) plane. In the left panel we immediately see that the allowed values of λ consistent with Higgs mass bounds as well is about 0.15, which is slightly larger than what we got in the MDSB scenario ($\lambda \sim 0.1$). This slightly larger value of λ has very important consequences on the Higgs sector. Similar to the above discussions, with $\mu_{\rm eff} = \lambda v_S$, v_S should be larger than 666 GeV (taking $\mu = 100$ GeV). Thus, we have relatively small v_S, and then the masses of the CP-even Higgs H_2, H_3 and CP-odd Higgs A_1 and A_2 can have relatively smaller values as compared to the MDSB scenario. In the right panel of Fig. 7, we see that $\tan\beta$ can have any value between 2 to 41.

In Fig. 8, we display plots in $m_{\tilde{\chi}_1^0} - m_{H_2}$, $m_{\tilde{\chi}_1^0} - m_{A_1}$, and $m_{\tilde{\chi}_1^0} - m_{\tilde{\tau}_1}$. The color coding of Fig. 8 is the same as in Fig. 2. The black solid lines indicate $2m_{\tilde{\chi}_1^0} = m_{H_2,A_1}$ in the first row and $m_{\tilde{\chi}_1^0} = m_{\tilde{\tau}_1}$ in the second row. Here, we see that for red points, the neutralino mass range is about [0.75, 2.7] TeV while $\sim[1.5, 5.5]$ TeV is the corresponding mass range of m_{A_1}. In the bottom panel, we present the LSP neutralino-stau coannihilation scenario. We see that for red points $m_{\tilde{\tau}_1}$ is in the mass range $\sim[0.75, 2.7]$ TeV while without demanding $\Omega h^2 \lesssim 1$ (blue points), $m_{\tilde{\tau}_1}$ can be as heavy as 5.8 TeV. It is very clear that in such a parameter space the gluino and the first two-generation squarks/sleptons cannot be probed at the 14 TeV LHC, which will provide a strong motivation for 33 TeV and 100 TeV proton-proton colliders. It is shown in Ref. [67] that the squarks/gluinos of 2.5 TeV, 3 TeV, and 6 TeV may be probed by the LHC14, high luminosity LHC14 and high energy LHC33, respectively. Thus, our models have testable predictions. If we have the collider facility with even higher energy in the future, we will be able to probe over even larger values of sparticle masses.

C. The benchmark points for the MDSB and DDSB scenarios

In Table I, we display two benchmark points for the MDSB scenario. The first point is an example of a relatively light sparticle spectrum. Here, $\lambda \sim 9.9 \times 10^{-3}$ and $\tan\beta \sim 26$ while the light CP-even Higgs $m_{H_1} \sim 123$ GeV. This point is also an example of solutions where $m_{H_3} \approx m_{A_1} \approx m_{H^\pm}$. The first two generation squarks are about 2.5 TeV or heavier. Since gluino is about 3 TeV, the light stop $\tilde{t}_1$ with mass around 2 TeV is the lightest colored sparticle. The $\tilde{t}_2$ and $\tilde{b}_{1,2}$ have comparable masses of about 2.3 TeV. The slepton masses are $\lesssim$1 TeV. We also notice that the LSP neutralino and NLSP stau are almost degenerate $\approx$596 GeV. The neutralino-proton spin independent and spin dependent cross sections are very small for this point that is $\sim 10^{-12}$ and 10^{-9}, respectively. The dark matter relic density is about 0.2. For the second benchmark point, the input parameters have relatively large values and then imply a heavier sparticle spectrum. For example, $x \approx 0.93$, $M_{3/2} \approx 4329$ GeV, $\tan\beta \approx 28$, and $\lambda \approx 1.6 \times 10^{-2}$. The light CP-even Higgs $m_{H_1} \approx 125$ GeV. This point represents the part of the parameter space with $m_{H_2} \approx m_{A_1} \approx m_{H^\pm}$. Here, the light stop is also the lightest colored sparticle with mass around 2.8 TeV while the gluino mass is around 4.3 TeV. The first two generations of squarks have masses about 4 TeV, while $\tilde{t}_2$ and $\tilde{b}_{1,2}$ have comparable masses about 3.4 TeV. The slepton masses are about 1 TeV or heavier. Although the LSP neutralino and the lighter stau are almost degenerate and, respectively, have masses 923 GeV and 928 GeV, the dark matter relic density is still about 0.65.

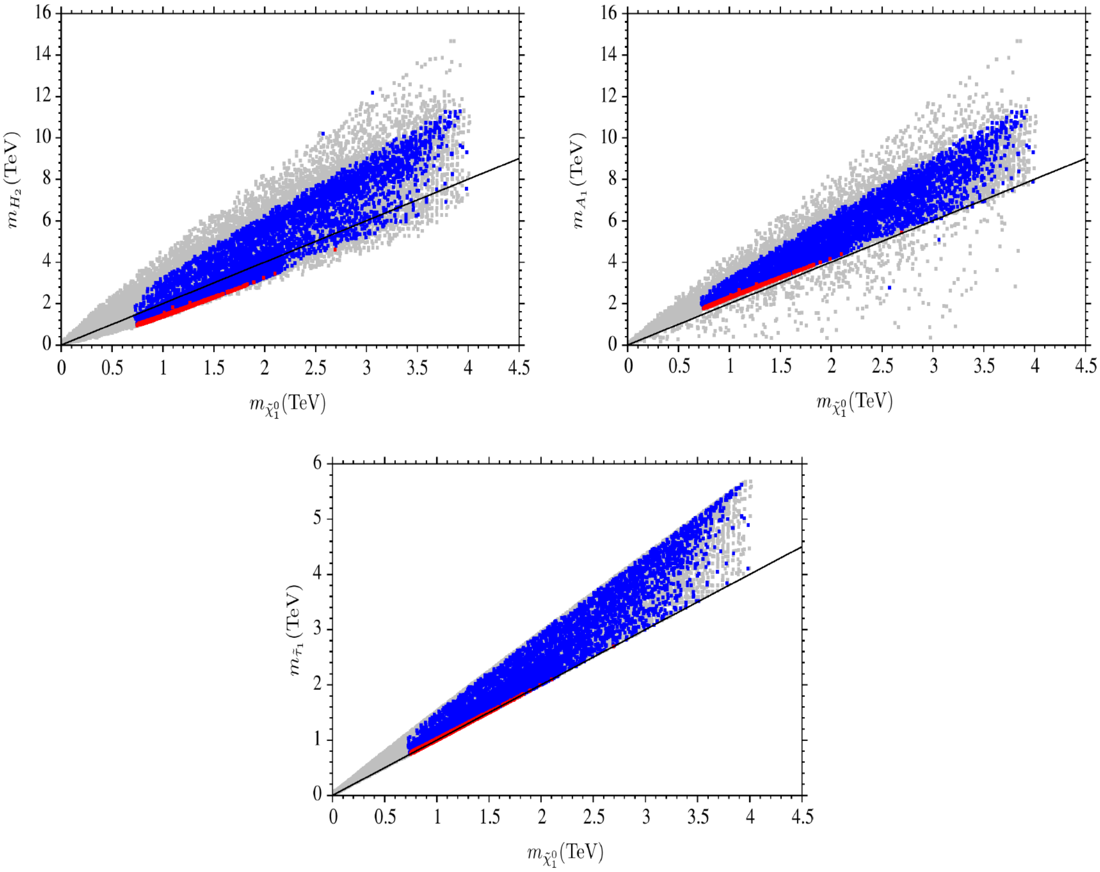

FIG. 8. Plots in the $m_{\tilde{\chi}_1^0} - m_{H_2}$, $m_{\tilde{\chi}_1^0} - m_{A_1}$, and $m_{\tilde{\chi}_1^0} - m_{\tilde{\tau}_1}$ planes for the dilaton dominant SUSY breaking scenario. The color coding is the same as in Fig. 2.

In Table II, we display three benchmark points for the DDSB scenario. Because we have already seen in Fig. 6 that the minimum required values for m_0 and $M_{1/2}$ are large as compared to the MDSB scenario, all these three points have heavier spectra. Point 1 is relatively light as compared to point 2 and point 3. For point 1, since x and $M_{3/2}$ have smaller values, this translates into relatively small values of m_0, $M_{1/2}$, and A_0 as 998.06 GeV, 1755.4 GeV, and −1738.8 GeV, respectively. The light CP-even Higgs boson is about 123 GeV. In these three points, we have $m_{H_3} \approx m_{A_1} \approx m_{H^\pm}$. As in the MDSB scenario, the gluino is heavier than the light stop, and they are about 3.7 TeV and 2.6 TeV, respectively. The first two family squark masses are heavier than 3 TeV while $\tilde{t}_2$ and $\tilde{b}_{1,2}$ are about 2.9 TeV. The first two generation slepton masses and $\tilde{\tau}_2$ are heavier than 1 TeV. The LSP neutralino and light stau masses are degenerate and about 773 GeV. Here, we also notice that apart from representing the neutralino-stau coannihilation scenario, this point also satisfies the A-resonance condition $|2m_{\tilde{\chi}_1^1} - m_{A_1}|/m_{A_1} \lesssim 0.3$. But it still has relatively large dark matter relic density 0.23569 and small neutralino-proton spin independent and spin dependent cross sections. Moreover, point 2 and point 3 share similar properties but have relatively heavier spectra.

VI. DISCUSSIONS AND CONCLUSION

We briefly reviewed the supernatural SUSY and addressed its subtle issues. We pointed out that the NMSSM is a perfect framework for supernatural SUSY since unlike the MSSM it can be scale invariant and then has no mass parameter in its Lagrangian before SUSY and gauge symmetry breakings. To generate the SUSY breaking soft mass to singlet, we studied the moduli and dilaton dominant supersymmetry breaking scenarios in M-theory on S^1/Z_2. In these scenarios, SUSY is broken by one and only one F term of moduli or dilaton superfield, and the SUSY breaking soft terms can be determined via the Kähler potential and superpotential from Calabi-Yau compactification of M-theory on S^1/Z_2. Thus, according to the supernatural SUSY, the SUSY EW fine-tuning measure is predicted to be of unity order. In the moduli dominant SUSY breaking scenario, we found that the right-handed sleptons are relatively light around 1 TeV, and stau can even be as light as 580 GeV and degenerate with the LSP neutralino. Moreover, charginos are $\gtrsim 1$ TeV, the light stop masses are around 2 TeV or larger, the first two-generation squark masses are about 3 TeV or larger, and gluinos are heavier than squarks as well. In the dilaton dominant SUSY breaking scenario, the above qualitative picture is preserved

TABLE II. Sparticle and Higgs masses are in GeV units and $\mu > 0$. All of these points satisfy the sparticle mass, B-physics constraints described in Sec. IV. Point 1 displays neutralino-stau coannihilation, and point 2 and point 3 represent m_{A_1}-resonance solutions though stau and is also very near to neutralino in mass. For these points $m_{H_3} \approx m_{A_1} \approx m_{H^\pm}$.

	Point 1	Point 2	Point 3
x	0.12148	0.11913	0.10126
$M_{3/2}$	1136.6	2145.5	2551.7
$\tan\beta$	40.695	40.554	40.491
λ	9.0562×10^{-3}	1.2251×10^{-2}	2.4202×10^{-2}
m_0	998.06	1889.1	2292.7
$M_{1/2}$	1755.4	3320.6	4013.3
A_0	−1738.8	−3290.4	−3986.7
m_{H_1}	123	125	125
m_{H_2}	993	2299	2924
m_{H_3}	1811	3252	3874
m_{A_1}	1811	3252	3874
m_{A_2}	1990	4013	4938
$m_{H^\pm}$	1812	3553	3874
$m_{\tilde{\chi}^0_{1,2}}$	773, 1442	1502, 2760	1830, 3346
$m_{\tilde{\chi}^0_{3,4,5}}$	1516, 2184, 2188	3263, 3864, 3867	4083, 4579, 4581
$m_{\tilde{\chi}^\pm_{1,2}}$	1442, 2188	2760, 3867	3346, 4582
$m_{\tilde{g}}$	3732	6770	8085
$m_{\tilde{u}_{L,R}}$	3472, 3339	6277, 6021	7491, 7180
$m_{\tilde{t}_{1,2}}$	2556, 2998	4625, 5403	5515, 6446
$m_{\tilde{d}_{L,R}}$	3473, 3322	6277, 5987	7491, 7139
$m_{\tilde{b}_{1,2}}$	2950, 2950	5369, 5369	6416, 6416
$m_{\tilde{\nu}_{1,2}}$	1506	2827	3415
$m_{\tilde{\nu}_3}$	1372	2580	3117
$m_{\tilde{e}_{L,R}}$	1508, 1184	2828, 2237	3416, 2710
$m_{\tilde{\tau}_{1,2}}$	774, 1381	1503, 2584	1834, 3121
σ_{SI}(pb)	8.96×10^{-12}	2.62×10^{-12}	1.92×10^{-12}
σ_{SD}(pb)	2.95×10^{-9}	3.40×10^{-10}	1.79×10^{-10}
$\Omega_{\rm CDM}h^2$	0.23569	0.74549	0.99767

but the particle spectra are heavier as compared to the moduli dominant SUSY breaking scenario. In addition to it, we have Higgs H_2/A_1-resonance solutions. In both scenarios, the minimum value of DM relic density is about 0.2. To realize the correct DM relic density, we can employ the dilution effect from supercritical string cosmology or introduce the axino as the lightest supersymmetric particle.

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