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A visible QCD axion from an enlarged color group

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We consider the possibility of an enlarged QCD color group, SU(3+N') spontaneously broken to $SU(3)_c \times SU(N')$ with extra vectorlike quarks transforming in the fundamental representation. When the heavy quarks are integrated out below the PQ-breaking scale, they generate an axion coupling which simultaneously solves the strong CP problem for both gauge groups. However, the axion mass now receives a new nonperturbative contribution from the SU(N') confinement scale, which can be substantially larger than the QCD scale. This can increase the axion mass to be at or above the electroweak scale. This visible axion can then decay into gluons and photons giving rise to observable signals at run II of the LHC. In particular, if the mass is identified with the 750 GeV diphoton resonance, then the new confinement scale is \sim TeV and the PQ-breaking scale is \sim 10 TeV. This predicts vectorlike quarks and a PQ scalar resonance in the multi-TeV range, with the possibility that dark matter is an SU(N') baryon.

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I. INTRODUCTION

It has long been known that a nonzero θ angle in OCD leads to large CP-violating effects which are not observed, such as a neutron electric dipole moment [1,2]. A simple way to address this strong CP problem is to introduce a global Peccei-Quinn (PQ) symmetry [3,4] which is spontaneously broken at a scale f_a and gives rise to a Nambu-Goldstone (NG) boson, the axion [5,6]. Nonperturbative effects then generate an axion potential with a minimum that occurs at an axion vacuum expectation value (VEV) that cancels a nonzero θ angle, thereby dynamically solving the strong CP problem. The axion can be considered to be part of a complex scalar field Φ , which couples to vectorlike quarks in the fundamental representation of the QCD color group $SU(3)_c$, and is charged under the PQ symmetry [7,8]. When Φ obtains a VEV, $\langle \Phi \rangle = f_a/\sqrt{2}$, the PQ symmetry is spontaneously broken and the vectorlike quarks obtain a mass. After these quarks are integrated out, they generate an axion coupling to the gluon field strength, giving a simple realization of the PQ mechanism.

The QCD axion solution relates the axion mass m_a to the PQ-breaking scale f_a . For example, in the KSVZ model [7,8], the relation, assuming two light quarks, is given by

$$m_a^2 f_a^2 = \frac{1}{8} f_\pi^2 m_\pi^2 \frac{4m_u m_d}{(m_u + m_d)^2},\tag{1}$$

implying that

$$m_a^2 f_a^2 \sim \frac{1}{8} \Lambda_c^4. \tag{2}$$

Here Λ_c is the QCD confinement scale, and we have used the experimental values of the quark masses, the pion decay

constant $f_{\pi} \simeq 130 \,\text{MeV}$ and the pion mass $m_{\pi} \simeq 135 \,\text{MeV}$. Note that the right-hand side of Eq. (1), which is given by the topological susceptibility [8]

$$\mathcal{T} \equiv -i \int d^4 x \langle 0 | \mathbf{T} \left[\frac{1}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}(x), \frac{1}{32\pi^2} G^b_{\rho\sigma} \tilde{G}^{b\rho\sigma}(0) \right] | 0 \rangle, \tag{3}$$

tends to zero in the chiral limit, where $G_{\mu\nu}^a$ is the gluon field-strength tensor and $\tilde{G}_{\mu\nu}^a \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma}$ with $\epsilon^{\mu\nu\rho\sigma}$ the totally antisymmetric tensor ($\epsilon^{0123} = +1$). The fact that $f_{\pi}m_{\pi} \sim \Lambda_c^2$ is a numerical coincidence. In the absence of light quarks, the topological susceptibility is of order Λ_c^4 [8] and, therefore, $m_a^2 f_a^2 \sim \Lambda_c^4$.

The electroweak scale would be a natural choice for the value of f_a , as was first considered in Refs. [5,6]. However, the Weinberg-Wilczek axion was ruled out almost immediately, while the current astrophysical and cosmological constraints on invisible axions [7,8] restrict f_a to lie in the narrow range $10^9~{\rm GeV} \lesssim f_a \lesssim 10^{12}~{\rm GeV}$ (although the upper bound, due to dark matter overclosure, can be relaxed if the initial misalignment angle is tuned). These bounds result from the fact that using Eq. (1) with $\Lambda_c \sim$ 250 MeV makes the axion sufficiently light (10^{-5} eV \lesssim $m_a \lesssim 10^{-2} \text{ eV}$) that it can be produced in stars. For instance, a stringent constraint comes from the observation of supernova (SN)1987A. The axion emission must not shorten the burst duration implying $f_a \gtrsim 4 \times 10^8 \text{ GeV}$ (see, e.g., Refs. [9-11]). Moreover, in the center of the Sun, keV axions (which were originally predicted with $f_a \simeq$ electroweak scale) can be produced through the axion-photon conversion in the presence of the solar magnetic field. Negative results from searches for such axions lead to a similar albeit less stringent bound. Clearly to invalidate current astrophysical and cosmological constraints and allow heavier axion masses with electroweak values of f_a , the relation (1) must therefore be modified.

To untie the relation (2) between m_a and Λ_c , we will entertain the possibility that above some ultraviolet (UV) unification scale, M_U , there is an enlarged QCD gauge group SU(3 + N'), which is then spontaneously broken as

$$SU(3 + N') \rightarrow SU(3)_c \times SU(N').$$
 (4)

The θ angle from the SU(3 + N') group descends down to the SU(3)_c and SU(N') subgroups intact. The quark fields at short distances belong to the fundamental representation of SU(3 + N'), and can be decomposed with regards to SU(3)_c and SU(N'), according to Eq. (4).

As in the KSVZ model, extra vectorlike quarks are charged under a PQ symmetry but now they transform in the fundamental representation of both $SU(3)_c$ and SU(N'). The PQ symmetry is spontaneously broken by a complex scalar field $\boldsymbol{\Phi}$ with the axion identified as the NG boson. The extra vectorlike quarks Ψ obtain a mass, hf_a where h is a Yukawa coupling. When they are integrated out, they generate an axion coupling to both gauge field strengths. Since both $SU(3)_c$ and SU(N') originate from a unified color group SU(3 + N'), they have the same θ angle, which is not renormalized at low energies. The axion coupling to the topological charge in these subgroups will be the same too. In addition, since the physical theta parameter is $\bar{\theta} = \theta + \arg(\det \mathcal{M})$, where \mathcal{M} is a complex mass matrix, unification guarantees the same Yukawa terms and, therefore, the same phase $arg(\det \mathcal{M})$ in the two sectors. This assumes that no new phases are introduced when the unified partners of the Standard Model quarks are decoupled, and a possible UV framework which sequesters the SU(3 + N')-preserving CP violation from the symmetry breaking is given in Appendix A. Thus, when nonperturbative effects generate an axion potential, the axion VEV will simultaneously solve both strong CP problems.

Since the colored matter content of the two groups is not necessarily the same (and N' is not necessarily equal to 3), the SU(N') group can confine at a scale $\Lambda' \gtrsim \Lambda_c$. This gives a new contribution to the axion mass relation which now becomes

$$m_a^2 f_a^2 \sim \frac{1}{8} \Lambda_c^4 + \Lambda'^4,$$
 (5)

where we have assumed that there are no light quarks in the SU(N') sector. A dramatic consequence of the modification (5) is that the axion can now have an electroweak scale mass.

An electroweak scale axion can be searched for at the LHC and future colliders since the generic signal decays to photons, gluons, and possibly W and Z bosons and Standard Model quarks and leptons. Not only is this experimentally accessible but it is also theoretically appealing because the global PQ symmetry is known to be explicitly violated by gravitational effects. In order not to spoil the PQ mechanism, these gravitational effects must also be suppressed to a very high order in the case of invisible axion models [12–14]; this difficulty results from the fact that the PQ-symmetry-breaking scale is very high in these models, and thus the effects of Planck-suppressed operators are sizable compared to the QCD effects on the generation of the axion potential. An axion at the electroweak scale helps to suppress the gravitational violations, without any need for further sequestering mechanisms.

In particular, the electroweak axion can be identified with the recent 750 GeV diphoton resonance [15–18]. This requires a confinement scale $\Lambda' \sim 1$ TeV and a PQ-breaking scale $f_a \sim 10$ TeV. With these values, the PQ scalar radial mode and vectorlike quarks have masses in the multi-TeV range. Furthermore, the required cross section for the diphoton excess can be fit if the vectorlike quarks have $\mathcal{O}(1)$ hypercharges. Thus, an electroweak axion gives a simple picture of the putative signal.

The idea of extending the color group to raise the axion mass was first considered in Refs. [19,20], where unlike in our case, the unified quark partners remain below the symmetry-breaking scale. A modified axion mass relation (5) was also proposed by Rubakov [21], who considered a mirror copy of the Standard Model with gauge group $SU(5) \times SU(5)$. For subsequent work, see Refs. [22–26]. More recently this mirror version was studied in Ref. [27] in order to obtain a visible QCD axion, which was then used to explain the recent diphoton excess where the PO scalar radial mode was identified with the 750 GeV resonance. The difference with our approach is that we do not require a mirror copy of the Standard Model. Instead, in our model, the two colored sectors are related by a unified gauge group with a minimal particle content. This means that we do not have mirror copies of Standard Model quarks and leptons which leads to extra collider and cosmological constraints on the axion sector that results from the more complicated phenomenology. Furthermore, we identify the 750 GeV resonance with an axion which directly decays to two photons, as opposed to the PQ scalar radial mode whose decay via a pair of axions produces a four-photon signal [27].

II. ENLARGING QCD COLOR

A. Gauge couplings and vacuum angles

We will assume that the QCD color group $SU(3)_c$ is a subgroup of SU(3 + N'). In the UV, the Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4q^2} F^{A}_{\mu\nu} F^{A\mu\nu} + \frac{\theta}{32\pi^2} F^{A}_{\mu\nu} \tilde{F}^{A\mu\nu}, \tag{6}$$

where $F_{\mu\nu}^A$, g, θ are the field strength tensor, gauge coupling, and θ parameter of the SU(3 + N') gauge theory, respectively, and A is the adjoint color index of SU(3 + N'), $A = 1, 2, ..., (3 + N')^2 - 1$. At a scale M_U , this group is spontaneously broken down to SU(3) $_c \times$ SU(N') where SU(N') is the hidden color gauge group. This occurs via the VEV of an adjoint Higgs field Σ :

$$\langle \Sigma \rangle = V \operatorname{diag}\{N', N', N', \underbrace{-3, -3, \dots, -3}_{N'}\}. \tag{7}$$

In addition, we require that the U(1) subgroup of SU(3+N') is broken at approximately the same scale V, by the $SU(3)_c \times SU(N')$ singlet component VEV of a scalar field transforming as a three-index antisymmetric tensor of SU(3+N') with zero hypercharge. After this combined symmetry breaking, the gauge bosons not belonging to $SU(3)_c \times SU(N')$ acquire masses proportional to gV and can be dropped from the sum in Eq. (6). Thus, below the scale gV, the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4g^2} \left[\sum_{a=1}^8 G^a_{\mu\nu} G^{a\mu\nu} + \sum_{\alpha=1}^{N^2 - 1} G'^{\alpha}_{\mu\nu} G'^{\alpha\mu\nu} \right] + \frac{\theta}{32\pi^2} \left[\sum_{a=1}^8 G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \sum_{\alpha=1}^{N^2 - 1} G'^{\alpha}_{\mu\nu} \tilde{G}'^{\alpha\mu\nu} \right], \quad (8)$$

where $G^a_{\mu\nu}$ and $G'^\alpha_{\mu\nu}$ denote the field strength tensors of the $SU(3)_c$ and SU(N') gauge theories, respectively; a=1,...8 is the $SU(3)_c$ color index while $\alpha=1,...N'^2-1$ is the SU(N') hidden color index. Consequently, at the scale M_U , the gauge couplings g_s,g_s' and the theta parameters θ_s,θ_s' of the two gauge groups satisfy

$$g = g_s = g'_s, \qquad \theta = \theta_s = \theta'_s.$$
 (9)

In order to generate axion couplings at a lower scale compatible with assuming that the PQ symmetry is broken at 10 TeV, we require that the strong coupling scale Λ' of the hidden gauge group satisfy 1 TeV $\lesssim \Lambda' \lesssim 10$ TeV. This requirement gives a strong constraint on the numbers of hidden colors N' and hidden quark flavors n'_F . To see this, we first note that at the one-loop level the strong coupling constant at the scale M_U is given by

$$\frac{1}{\alpha_s(M_U)} = \frac{1}{\alpha_s(m_Z)} - \frac{b}{2\pi} \ln\left(\frac{M_U}{m_Z}\right),\tag{10}$$

where $\alpha_s \equiv g_s^2/(4\pi)$, m_Z is the Z-boson mass, $\alpha_s(m_Z) = 0.1185(6)$ [28], and $b = -7 + \frac{2}{3}n_F'$ with six quark flavors assumed. As we will see in more detail in Sec. II B, the number of extra quark flavors is equal to that of the hidden

TABLE I. The values of M_U (in GeV) for various N', n_F' , and Λ' .

$\overline{N'}$	n_F'	$\Lambda' = 1 \text{ TeV}$	$\Lambda' = 10 \text{ TeV}$
	1	2.5×10^{10}	9.7×10^{12}
	2	1.7×10^{10}	4.4×10^{12}
3	3	1.1×10^{10}	2.0×10^{12}
	4	7.6×10^{9}	9.3×10^{11}
	5	5.1×10^9	4.2×10^{11}
4	1	7.3×10^{6}	4.9×10^{8}
	2	5.9×10^{6}	3.2×10^{8}
	3	4.8×10^{6}	2.2×10^{8}
	4	3.9×10^{6}	1.4×10^{8}
	5	3.2×10^{6}	9.5×10^{7}

quark flavor n'_F in our setup since they originate from fundamental representations of SU(3 + N'). On the other hand, the hidden coupling at M_U is given by

$$\frac{1}{\alpha_s'(M_U)} = -\frac{b'}{2\pi} \ln\left(\frac{M_U}{\Lambda'}\right),\tag{11}$$

with $b' = -\frac{11}{3}N' + \frac{2}{3}n'_F$. Here we assume that there are no mirror Standard Model quarks and leptons at low energies. By requiring $\alpha_s(M_U) = \alpha'_s(M_U)$, we can express M_U as a function of n'_F , N', and Λ' .

In Table I, we summarize the values of M_U (in GeV) for various N', n'_F , and Λ' . It turns out that the N'=2 cases do not yield any reasonable value for M_U . For a larger N', we obtain a lower M_U . From this table, we find that this setup accommodates multiflavors for extra quarks while keeping M_U sufficiently high. The more vectorlike quarks we add to the theory, the larger the beta function of the hidden strong interaction becomes, which results in a smaller coupling constant at low energies. On the other hand, these extra quarks make the strong coupling constant larger at high scales, and thus the unified coupling g also becomes large. As these two effects compensate each other, the resultant M_U is rather insensitive to the number of extra quarks. This feature is actually desirable for the explanation of the 750 GeV diphoton anomaly in our model, as we discuss in Sec. III B.

In Fig. 1, we show the running of α_s and α_s' with orange and blue lines for representative values, N'=3 and $M_U=3\times 10^{10}$ GeV. The solid and dashed lines correspond to the cases of $n_F'=1$ and 5, respectively. Here, we have used the two-loop renormalization group equations, and neglected threshold corrections at M_U . The masses of the vectorlike quarks are set to be 1 TeV. As can be seen, Λ' is less sensitive to n_F' , which allows us to introduce a

¹The uncertainty in the input value of $\alpha_s(m_Z)$ causes less than 10% errors for the resultant values of M_U given in Table I.

²Strictly speaking, the coefficients b and b' should be modified below each extra-quark mass threshold. However, since the extra quark masses (1–10 TeV) are not far from the electroweak scale, we expect that one-step matching adopted here does not cause significant uncertainty in this estimation.

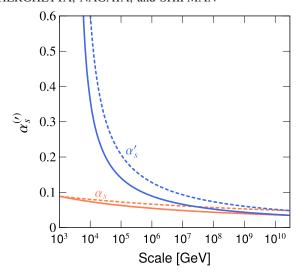


FIG. 1. The running of α_s and α'_s , where N'=3 and $M_U=3\times 10^{10}$ GeV. The solid and dashed lines correspond to the cases of $n'_F=1$ and 5, respectively.

number of vectorlike quarks at low energies. We note in passing that our model does not suffer from a domain wall problem [29,30] even though $n_F' \ge 2$. As we will see in Sec. II D, we can introduce the PQ-symmetry-violating Planck-suppressed operators without spoiling the PQ mechanism. These operators explicitly break a discrete symmetry, and thus destabilize domain walls.

B. Axion couplings and mass

We will assume that there are new Dirac quarks, Ψ in the fundamental representation of the unified color group SU(3+N'). After this group is spontaneously broken at the scale M_U , these quarks split into a fundamental representation of $SU(3)_c$, denoted ψ , and a fundamental representation of the hidden color group SU(N'), denoted ψ' . In addition, we assume that there is a complex scalar field Φ that couples to the new Dirac fermions. As in the KSVZ model we assume that these fields are charged under a Peccei-Quinn U(1) global symmetry,

$$\Psi \to e^{iq_{\Psi}\alpha\gamma_5}\Psi, \qquad \Phi \to e^{iq_{\Phi}\alpha}\Phi,$$
 (12)

where α is an arbitrary parameter and $q_{\Psi,\Phi}$ are the PQ charges. We will assume $q_{\Phi}=1$ and $q_{\Psi}=\frac{1}{2}$ for simplicity. This symmetry forbids a Dirac mass term but allows the Yukawa couplings

$$\Delta \mathcal{L} = h_{ij} \Phi \bar{\Psi}_{Ri} \Psi_{Lj} + \text{H.c.} \rightarrow h_{ij} \Phi (\bar{\psi}_{Ri} \psi_{Lj} + \bar{\psi}'_{Ri} \psi'_{Lj}) + \text{H.c.},$$
(13)

where h_{ij} are dimensionless couplings and $i, j = 1, ... n'_F$ denotes the flavor index. As one can see, the number of extra quarks is equal to that of extra hidden quarks. The

spontaneous breaking of the PQ symmetry then occurs when the scalar field obtains a VEV, which is parametrized as

$$\Phi = \frac{1}{\sqrt{2}} (f_a + \rho) e^{i\frac{a}{f_a}},\tag{14}$$

where f_a is the PQ breaking scale, ρ is the radial mode and a is the axion field. The radial mode obtains a mass of order $\sqrt{\lambda_{\Phi}}f_a$, where λ_{Φ} is the quartic coupling in the scalar potential. The PQ current becomes

$$j_{\mu}^{\text{PQ}} = iq_{\Phi}(\Phi^*\partial_{\mu}\Phi - \Phi\partial_{\mu}\Phi^*) + q_{\Psi}\bar{\psi}\gamma_{\mu}\gamma_{5}\psi + q_{\Psi}\bar{\psi}'\gamma_{\mu}\gamma_{5}\psi',$$

$$\rightarrow -f_{a}\partial_{\mu}a + \frac{1}{2}\bar{\psi}\gamma_{\mu}\gamma_{5}\psi + \frac{1}{2}\bar{\psi}'\gamma_{\mu}\gamma_{5}\psi'. \tag{15}$$

Under a PQ transformation the axion will shift as $a \to a + f_a \alpha$, giving rise to an anomalous term that matches the axial anomaly from (12). Since the axion couples to the divergence of the PQ current, we see from Eq. (15) that the axion couples to the new quarks ψ, ψ' , which obtain a mass of order $m_{\Psi} \sim h f_a$ after the PQ symmetry is broken.

At low scales, these heavy quarks are integrated out (assuming $m_{\Psi} \gtrsim \Lambda'$) and generate a coupling of the axion field (and the radial field ρ) to the QCD gluons, the hidden sector gluons, and possibly photons (provided the heavy fermions also carry hypercharge). In particular,

$$\mathcal{L}_{a} = \frac{1}{32\pi^{2}} \left(\frac{a}{f_{a}} + \theta \right) G^{a}_{\mu\nu} \tilde{G}^{a\mu\nu},$$

$$\mathcal{L}_{a'} = \frac{1}{32\pi^{2}} \left(\frac{a}{f_{a}} + \theta \right) G^{\prime\alpha}_{\mu\nu} \tilde{G}^{\prime\alpha\mu\nu}, \tag{16}$$

where we have used (9) and θ nonrenormalization. Note that the triangle graphs which generate (16) are saturated at virtual momenta $m_{\Psi} \sim h \langle \Phi \rangle \sim h f_a$.

The axion mass-squared is determined by the two-point function

$$i \int d^{4}x \langle F_{\mu\nu}^{A} \tilde{F}^{A\mu\nu}(x), F_{\rho\sigma}^{B} \tilde{F}^{B\rho\sigma}(0) \rangle$$

$$\rightarrow i \int d^{4}x \langle G_{\mu\nu}^{a} \tilde{G}^{a\mu\nu}(x), G_{\rho\sigma}^{b} \tilde{G}^{b\rho\sigma}(0) \rangle$$

$$+ i \int d^{4}x \langle G_{\mu\nu}^{\prime a} \tilde{G}^{\prime a\mu\nu}(x), G_{\rho\sigma}^{\prime \beta} \tilde{G}^{\prime \beta\rho\sigma}(0) \rangle, \qquad (17)$$

where the latter correlation function is saturated in the IR and reduces to $\sim \frac{1}{8}\Lambda_c^4 + \Lambda'^4$. Since we deal with a single combination $a + \theta$, the axion Lagrangian takes the form

$$\mathcal{L}_{a} = \frac{1}{2} \partial_{\mu} a \partial^{\mu} a - \frac{1}{2} \mathcal{T} \left(\frac{a}{f_{a}} + \theta \right)^{2}, \tag{18}$$

and thus the θ term is eliminated in the vacuum. Here, $\mathcal T$ is the sum of the topological susceptibilities for the two Yang-Mills theories, QCD and the hidden color group. It is given by

$$\mathcal{T} = \frac{1}{8}\Lambda_c^4 + (\Lambda')^4; \tag{19}$$

see Eq. (1) for the first term and the subsequent discussion for the second. A dual interpretation of this mechanism is given in Appendix B.

The axion mass relation then becomes

$$f_a^2 m_a^2 = \mathcal{T} \sim \frac{1}{8} \Lambda_c^4 + (\Lambda')^4.$$
 (20)

The second term on the right-hand side of (20) can be arbitrarily large compared to the first term from QCD and, therefore, can give the dominant contribution to the axion mass. This destroys the standard KSVZ relation between m_a and Λ_c allowing for much larger values of the axion mass. For example, for $\Lambda' \sim 1$ TeV and $f_a \sim 10$ TeV, the axion mass m_a can be as large as $\mathcal{O}(100)$ GeV. This then invalidates the standard axion limits from astrophysics.

C. Unified symmetry-breaking effects

After the spontaneous breaking of SU(3 + N'), there could be possible sources of CP violation that spoil the equation $\theta_s = \theta'_s$ at low energies, since the physical theta parameter is given by

$$\bar{\theta} = \theta + \arg(\det \mathcal{M}),$$
 (21)

where \mathcal{M} is a complex mass matrix for quarks. These include threshold effects and renormalization group effects caused by visible and hidden quarks, and higher-dimensional operators that contain the SU(3+N') breaking field Σ .

Firstly, we consider the effects of vectorlike quarks on the vacuum angles. Above M_U , the vectorlike quarks form the fundamental representation of SU(3+N'), and they have Yukawa couplings with the scalar field Φ as in Eq. (13). Below M_U , the Yukawa interaction splits into two parts as shown in the right-hand side of Eq. (13), but the coefficients of the two parts, h_{ij} , are identical. For this reason, after Φ develops a VEV, the resultant mass matrices for ψ and ψ' also become identical, $hf_a/\sqrt{2}$. Therefore, these mass terms contribute to the θ angles with the same amount, $\arg\{\det(hf_a/\sqrt{2})\}$, and do not spoil the relation $\theta_s = \theta_s'$.

Secondly, we consider the contribution of the Standard Model quarks, Q_L , u_R^c , and d_R^c , and their SU(3+N') partners, Q_L' , $u_R'^c$, and $d_R'^c$, respectively. These fields form fundamental representations of SU(3+N'), Ψ_Q , $\Psi_{\bar{u}}$, and $\Psi_{\bar{d}}$. The leptons are irrelevant for the present discussion

and thus we will neglect them in what follows. As we will see, there are subtleties in this case since the low-energy spectrum of our model does not contain the partner quarks, and thus the SU(3 + N') symmetry is explicitly broken in this sector.

These fields have Yukawa interactions with the Standard Model Higgs boson in order to reproduce the ordinary Standard Model Yukawa couplings. In the SU(3 + N') gauge theory, these Yukawa interactions are written as

$$\mathcal{L}_{\text{Yukawa}} = -\Psi_{Qi}(\mathcal{Y}_u)_{ij}\Psi_{\bar{u}j}H - H^{\dagger}\Psi_{Qi}(\mathcal{Y}_d)_{ij}\Psi_{\bar{d}j} + \text{H.c.},$$
(22)

where i, j = 1, 2, 3 is the generation index, \mathcal{Y}_u and \mathcal{Y}_d are 3×3 matrices, and H is the Standard Model Higgs field. Since the values of the theta terms are basis dependent, we first specify the basis for the following discussion. Of course, the derived consequences do not depend on the choice of the basis.

Using the possible field redefinitions, the Yukawa matrices can be transformed to the following form,

$$\mathcal{Y}_{u} = \operatorname{diag}(y_{u}, y_{c}, y_{t}),$$

$$\mathcal{Y}_{d} = V_{\text{CKM}}^{*} \cdot \operatorname{diag}(y_{d}, y_{s}, y_{b}),$$
(23)

where $V_{\rm CKM}$ is the ordinary CKM matrix. As discussed in Sec. II A, we have $\theta_s = \theta_s'$ below the SU(3 + N') symmetry-breaking scale. On the other hand, the Yukawa interactions lead to

$$\mathcal{L}_{\text{Yukawa}} = -Q_L \mathcal{Y}_u u_R^c H - H^{\dagger} Q_L \mathcal{Y}_d d_R^c - Q_L' \mathcal{Y}_u u_R'^c H - H^{\dagger} Q_L' \mathcal{Y}_d d_R'^c + \text{H.c.}$$
(24)

Now let us examine the physical θ terms of both sectors. In the SU(3)_c sector,

$$\bar{\theta} \equiv \theta_s + \arg(\det \mathcal{Y}_u) + \arg(\det \mathcal{Y}_d)$$

$$= \theta_s - \arg(\det V_{CKM}) = \theta_s, \qquad (25)$$

where we have used $det(V_{CKM}) = 1$.

On the other hand, the physical vacuum angle in the SU(N') sector depends on the mass splitting mechanism for Q', u', and d'. If the mass splitting mechanism does not introduce new CP phases, which can be naturally realized with, e.g., a warped extra dimension compactified on an orbifold (see Appendix A), then again we have $\bar{\theta}' = \theta'_s$. Thus, we conclude that

$$\bar{\theta} = \bar{\theta}',$$
 (26)

in the unified model, assuming that the SU(3 + N')-preserving CP violation is sufficiently sequestered from the symmetry breaking. Once this relation holds at M_U , it is

 $^{^3}$ This assumes that there is an anomaly-free UV completion, where the local SU(3 + N') gauge anomalies cancel. This requires extra UV states which can be decoupled at M_U without affecting our arguments.

not spoiled at low energies since the physical theta terms are invariant under renormalization group flow.

Finally, we consider the effects of higher-dimensional CP-odd operators including the SU(3 + N')-breaking field Σ , which are expected to be induced at the Planck scale M_P (e.g., by virtual black holes). Among them, the following dimension-five operator gives the dominant effect,

$$\frac{c}{M_P} \text{Tr}(\Sigma F_{\mu\nu} \tilde{F}^{\mu\nu}), \tag{27}$$

where $F_{\mu\nu} \equiv F_{\mu\nu}^A T^A$ with T^A the generators of SU(3 + N') and c a dimensionless constant. This operator reduces to a theta term after Σ gets a VEV [see Eq. (7)], and thus could spoil the relation $\theta_s = \theta_s'$. This, however, causes no problem if $|c\langle \Sigma \rangle| < 10^{-10} M_P \simeq 2 \times 10^8$ GeV. This can be naturally realized for N' = 4, as can be seen in Table I. For N' = 3, the above limit gives $|c| \lesssim 10^{-2}$. Thus, we see that the theta relation in Eq. (9) can be well maintained in the IR, so that the axion can cancel both theta terms.

D. Gravitational violations of PQ symmetry

An immediate consequence of an electroweak scale axion is that gravitational violations of the PQ global symmetry become naturally suppressed [22]. Below the Planck scale, the effective PQ-violating terms are described by the Planck-scale-suppressed higher-dimensional operators,⁴

$$\mathcal{L} = \frac{\kappa}{M_P^{2m+n-4}} |\Phi|^{2m} \Phi^n + \text{H.c.},$$
 (28)

where κ is a dimensionless constant and m, n are integers satisfying $n \ge 1$ and $2m + n \ge 5$. Such an operator induces an effective θ angle [12–14],

$$\theta_{\text{eff}} \sim |\kappa| \left(\frac{f_a}{m_a}\right)^2 \left(\frac{f_a}{\sqrt{2}M_P}\right)^{2m+n-4},$$
 (29)

where we have omitted an $\mathcal{O}(1)$ factor for brevity. In particular, dimension-five operators (2m + n = 5) generate an effective θ angle of

$$\theta_{\rm eff} \sim 10^{-12} \times |\kappa| \cdot \left(\frac{f_a}{10 \text{ TeV}}\right)^3 \left(\frac{750 \text{ GeV}}{m_a}\right)^2.$$
 (30)

This value is sufficiently suppressed for the electroweak scale axion that it does not spoil the axion mechanism. This contrasts with the usual invisible axion models where since $f_a \gtrsim 10^9$ GeV, gravitational PQ-symmetry-violating terms to very high order ($n \gtrsim 10$) must be suppressed [12–14].

However, in the presence of extra Higgs fields which develop large VEVs, such as the SU(3+N') breaking Higgs field Σ , there could be other PQ-violating operators like $|\Sigma|^{2m}\Phi^n/M_P^{2m+n-4}$, which may spoil the PQ mechanism. We thus assume that such operators are sufficiently suppressed. Note however, that the SU(3+N') gauge group can be broken without the Σ field if we consider unification with an extra dimension compactified on an orbifold. In this case, the above problem can be avoided.

III. PHENOMENOLOGICAL CONSEQUENCES

A. The electroweak axion

Intriguingly, in our model, the value of the axion mass can be in the several hundred GeV range for a confinement scale, $\Lambda' \sim \text{TeV}$ and a PQ breaking scale, $f_a \sim 10 \text{ TeV}$. This axion is therefore quite "visible" and can be searched for in collider experiments. As shown in Table I, such a confinement scale is obtained with $N'=3,4,\ldots$ For concreteness, we choose N'=3 and assume that the QCD color group is embedded into SU(6) in what follows. Including the electroweak sector, the complete gauge group is $\text{SU}(6) \times \text{SU}(2)_L \times \text{U}(1)_Y$.

We consider a set of vectorlike quarks, Ψ transforming in the $\mathbf{6} \oplus \bar{\mathbf{6}}$ of SU(6). They are supposed to be singlets under the SU(2)_L gauge interaction. After SU(6) is broken to SU(3)_c × SU(3') we obtain a pair of QCD Dirac fermions, ψ transforming as $(\mathbf{3},\mathbf{1})_Y \oplus (\bar{\mathbf{3}},\mathbf{1})_{-Y}$, and a pair, ψ' transforming as $(\mathbf{1},\mathbf{3})_{Y'} \oplus (\mathbf{1},\bar{\mathbf{3}})_{-Y'}$ of the hidden color group, where Y and Y' are the Standard Model hypercharges. When integrated out, these fermions generate the effective axion couplings to gluons and photons:

$$\mathcal{L}_{a} = n_{F}^{\prime} \frac{\alpha_{s}}{8\pi} \frac{a}{f_{a}} G_{\mu\nu}^{a} \tilde{G}^{a\mu\nu} + 6n_{F}^{\prime} (Y^{2} + Y^{\prime 2}) \frac{\alpha_{Y}}{8\pi} \frac{a}{f_{a}} B_{\mu\nu} \tilde{B}^{\mu\nu},$$
(31)

where $\alpha_s \equiv g_s^2/(4\pi)$, $\alpha_Y \equiv g_Y^2/(4\pi)$ with g_Y the coupling constant of the U(1)_Y gauge interaction, and $B_{\mu\nu}$ the hypercharge field strength tensor. Note that we have moved to the basis where the gauge fields are canonically normalized. Only ψ contributes to the first term on the right-hand side of (31), while both ψ and ψ' generate the second term. We also note that in the electroweak symmetry-breaking basis,

⁴Here we assume that the PQ symmetry is broken only through higher-dimensional operators, though renormalizable operators can also be present if, for instance, wormhole effects are sizable [13].

⁵Note that even though Y = Y' when the U(1) subgroup of SU(6) is broken in the way described after (7), we allow the more general possibility that $Y \neq Y'$ which can occur when a linear combination of the U(1) subgroup of SU(6) and an additional U(1) is broken to give the usual U(1)_Y hypercharge below the unification scale. For example, this occurs when the scalar of the three-index antisymmetric tensor is charged under the additional U(1).

$$\alpha_Y B_{\mu\nu} \tilde{B}^{\mu\nu} = \alpha_{\rm EM} [F_{\mu\nu} \tilde{F}^{\mu\nu} - 2 \tan \theta_W F_{\mu\nu} \tilde{Z}^{\mu\nu} + \tan^2 \theta_W Z_{\mu\nu} \tilde{Z}^{\mu\nu}], \tag{32}$$

where $\alpha_{\rm EM}$ denotes the fine-structure constant, θ_W is the weak-mixing angle, and $F_{\mu\nu}, Z_{\mu\nu}$ are the field strength tensors for the photon and Z-boson, respectively.

An electroweak scale axion, a is produced at the LHC via the gluon fusion process. The production cross section is given by

$$\sigma(pp \to a) = \frac{k_g}{m_a s} C_{gg} \Gamma(a \to gg), \tag{33}$$

where m_a is the axion mass, \sqrt{s} is the center-of-mass energy of the pp collision, and C_{gg} is the gluon luminosity factor defined by

$$C_{gg} = \frac{\pi^2}{8} \int dx_1 dx_2 \delta(x_1 x_2 - m_a^2/s) g(x_1) g(x_2), \quad (34)$$

with g(x) the gluon parton distribution function (PDF). The so-called k factor, k_g is a multiplicative factor that parametrizes higher-order QCD corrections. The partial decay width of the axion into a pair of gluons, $\Gamma(a \to gg)$ is given by

$$\Gamma(a \to gg) = \frac{\alpha_s^2}{32\pi^3} \frac{n_F'^2 m_a^3}{f_a^2}.$$
 (35)

Notice that $\Gamma(a \to gg)$, and thus the production cross section is inversely proportional to the square of f_a/n_F' .

Once produced, the axion decays into gg, $\gamma\gamma$, $Z\gamma$, or ZZ. The partial decay widths of $\gamma\gamma$, $Z\gamma$, and ZZ are

$$\Gamma(a \to \gamma \gamma) = \frac{9\alpha_{\rm EM}^2}{64\pi^3} (Y^2 + Y'^2)^2 \frac{n_F'^2 m_a^3}{f_a^2}, \qquad (36)$$

$$\Gamma(a \to Z\gamma) \simeq 2 \tan^2 \theta_W \Gamma(a \to \gamma\gamma),$$
 (37)

$$\Gamma(a \to ZZ) \simeq \tan^4 \theta_W \Gamma(a \to \gamma \gamma),$$
 (38)

respectively. Note that these decay widths are related to each other via $\tan \theta_W \simeq 0.55$. In particular, the ZZ decay mode is significantly suppressed by a factor of $\tan^4 \theta_W$ compared to the diphoton decay channel.

In our minimal model, we have assumed that the electroweak axion has no coupling to W bosons. However W-boson couplings can be generated by introducing vectorlike fermions charged under $SU(2)_L$. Furthermore, since the Standard Model quarks and leptons are not charged under the PQ symmetry, as in the original KSVZ model, there are no tree-level axion couplings to Standard Model fermions. These couplings are instead

induced at higher-loop level compared with the photon and gluon couplings, and thus negligible in the present analysis.

Besides the axion, the model also predicts colored vectorlike fermions at a mass scale $\sim hf_a$, where h is a Yukawa coupling. Depending on the value of h, these fermions may be near the TeV scale. Furthermore, the radial scalar mode, ρ will obtain a mass of order $\sqrt{\lambda_\Phi}f_a$, where λ_Φ is the quartic coupling of the complex scalar, Φ potential. Thus our model has quite minimal predictions, which can be probed at run II of the LHC.

B. The 750 GeV diphoton resonance

Recently, the ATLAS [15,17] and CMS [16,18] collaborations announced an excess of events around 750 GeV in the diphoton resonance searches at the 13 TeV LHC run. These excesses can be explained if the production cross section of the 750 GeV resonance times its decay branching fraction to diphotons is 5–10 fb. After the announcement, many possible explanations have been proposed [31–50].

Obviously the electroweak axion in our model can be a candidate for the 750 GeV resonance. Identifying the visible axion with the 750 GeV resonance requires that

$$m_a \sim \frac{(\Lambda')^2}{f_a} \sim 750 \text{ GeV},$$
 (39)

or equivalently

$$\Lambda' \sim \left(\frac{f_a}{1 \text{ TeV}}\right)^{\frac{1}{2}} \times 870 \text{ GeV}.$$
 (40)

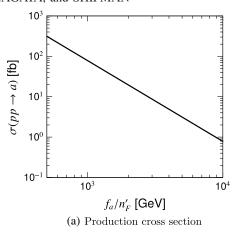
The 750 GeV axion is produced at the LHC via the gluon fusion process. The production cross section can be calculated using (33) where the numerical value of C_{gg} is evaluated using the MSTW2008NLO PDF data set [58] in Ref. [39] as $C_{gg} \simeq 2137$ (174) for $\sqrt{s} = 13$ TeV (8 TeV), and the k factor is taken to be $k_q \simeq 2$ [59].

In Fig. 2(a), we show the axion production cross section as a function of f_a/n_F' assuming $m_a=750$ GeV. Given that the observed diphoton rate implies a signal cross section of 5–10 fb, we see that the 750 GeV axion can explain the diphoton excess if $f_a/n_F' \sim 1$ TeV and the branching fraction of the axion into diphotons is sizable.

In Fig. 2(b), we show the axion branching ratios as functions of $Y^2 + Y'^2$ where black, red, blue, and green lines (from top to bottom) represent the branching fractions into dijet (a pair of gluons), diphoton, $Z\gamma$, and ZZ channels, respectively. From this figure, we find that a sizable rate

⁶For other models which consider the interplay between the 750 GeV resonance and a solution to the strong *CP* problem (or axion), see Refs. [27,38,41,51–57].

^{&#}x27;Photo-production is negligible unless the hypercharges Y and Y' are very large.



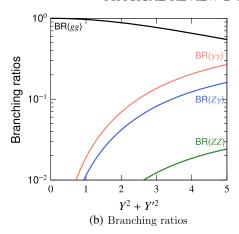


FIG. 2. The production cross section and branching ratios of the axion a, assuming $m_a = 750$ GeV. (a) Production cross section $\sigma(pp \to a)$ as a function of f_a/n_F' . (b) Branching ratios of a as functions of $Y^2 + Y'^2$.

into diphotons can be easily realized in our model; for instance, Y = Y' = 1 gives $BR(a \to \gamma \gamma) \simeq 7\%$. Note, however that if hypercharges Y and Y' are very large (or have (unusual) irrational values), stable charged particles (such as the lightest baryon composed of three ψ' s) may appear, which are cosmologically problematic. These charged particles can decay into Standard Model particles via interactions described by effective higher-dimensional operators. If Y and Y' are very large, such operators containing Standard Model fields must have correspondingly large dimensions since the hypercharges of the Standard Model particles are <1. Therefore, in order for the charged particles to have a sufficiently short lifetime, there must be a new scale below the unification scale, M_{II} at which these operators can be generated. Instead, the fact that $Y, Y' \sim 1$ gives rise to a sizable diphoton branching ratio suggests that there exists a simple UV model with operators generated at or above the UV scale which does not have charged stable particles and can explain the 750 GeV diphoton events.

For example, consider a set of vectorlike quarks $\psi_u^{(\prime)}$ and $\psi_d^{(\prime)}$ which have hypercharges $Y^{(\prime)}=\frac{2}{3}$ and $-\frac{1}{3}$, respectively. If $\psi_u^{(\prime)}$ is heavier than $\psi_d^{(\prime)}$, the lightest baryon is composed of one $\psi_u^{(\prime)}$ and two $\psi_d^{(\prime)}$ s, which is electrically neutral and thus can be a dark matter candidate, assuming it is stable. The heavier charged baryon, which is composed of two $\psi_u^{(\prime)}$ s and one $\psi_d^{(\prime)}$, can decay if we introduce, for instance, a charged scalar ϕ^+ with a PQ charge +1. This charged scalar can have a Yukawa coupling $\bar{\psi}_{uR}^{(\prime)}\psi_{dL}^{(\prime)}\phi^+$ as well as a coupling to the Standard Model sector via a dimension-five operator like $\phi^+\Phi^*\bar{u}_Rd_R$, which can be induced at M_U via a trilinear coupling $\phi^+\phi^-\Phi^*$ and a Yukawa coupling $\phi^+\bar{u}_Rd_R$ where ϕ^\pm are charged scalars with zero PQ charge and mass of $\mathcal{O}(M_U)$. The introduction of these fields and interactions does not spoil the relation $\bar{\theta}=\bar{\theta}'$ as they do

not induce mass terms for fermions.⁸ An alternative possibility is to embed our model into an $SU(2)_R$ gauge theory above M_U by putting $\psi_u^{(\prime)}$ and $\psi_d^{(\prime)}$ into a fundamental representation of $SU(2)_R$ with the Standard Model fields also embedded into $SU(2)_R$ representations in the usual manner. In this case, $\psi_u^{(\prime)}$ can decay into $\psi_d^{(\prime)}$ plus the Standard Model particles via the exchange of a $SU(2)_R$ gauge boson with an $\mathcal{O}(M_U)$ mass. Thus, we see that there are various possible ways to incorporate dark matter in a UV completion.

Next we evaluate the cross sections of the diphoton resonance events predicted in this model. We plot them as functions of f_a/n_F' in Fig. 3. This figure shows that the diphoton excess can be explained if $f_a/n_F' \sim 1$ TeV and the hypercharges are $\mathcal{O}(1)$. For example, when Y = Y' = 1, the best-fit cross section is achieved with $f_a/n_F' =$ 1–1.5 TeV. This corresponds to a total width Γ_{tot} = 3–6 MeV and predicts the $Z\gamma$ cross section $\simeq 1.5$ –3 fb and the dijet cross section \approx 32–65 fb. Notice that $n_F' \gg 1$ is possible as discussed in Sec. II. Thus, f_a can be as large as 10 TeV if one introduces a sufficient number of extra vectorlike fermions. This means that vectorlike quarks and the radial scalar mode ρ will have masses in the multi-TeV range depending on the values of the Yukawa coupling h and quartic coupling λ_{Φ} , respectively. If the vectorlike quarks are heavier than the CP-even scalar ρ , then it can only decay to axion pairs, otherwise the ρ will decay into (possibly) long-lived vectorlike quarks as well. If a glueball made of the SU(N') gluons has a mass smaller than half the ρ mass, then ρ can also decay into a pair of hidden glueballs at the one-loop level.

⁸Note that the unprimed fields will form visible baryons as well as R-hadron-like states with Standard Model quarks. These heavy bound states (≳TeV) can be made to decay promptly, and could eventually be detected at a future collider.

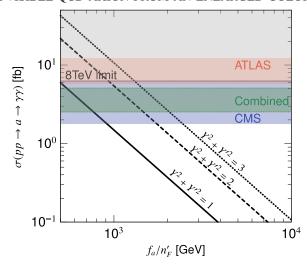


FIG. 3. Cross sections of the diphoton resonance events as functions of f_a/n_F' , where the black solid, dashed, and dotted lines show the cases of $Y^2 + Y'^2 = 1$, 2, and 3, respectively. The red (blue) shaded area reproduces the number of events observed by the ATLAS [15] (CMS [16]) collaboration. The gray-shaded region is disfavored by the 8 TeV results [60,61]. The green shaded region corresponds to the best fit cross section obtained in Ref. [62].

Visible vectorlike quarks can also be directly produced via strong interactions, and thus can be a good target at the LHC. They are observed as long-lived heavy hadrons, which may have an exotic electromagnetic charge depending on their hypercharge. Hidden vectorlike quarks are, on the other hand, produced only through the $U(1)_Y$ gauge interaction, and thus their production cross sections are rather small. Nevertheless, they may be probed at run II of the LHC since they yield quite distinct signatures. As soon as hidden vectorlike quarks are pair-produced, they annihilate promptly, and can be observed as dilepton, dijet, and diphoton resonances. They can also annihilate into hidden glueballs leading to a similar phenomenology as that considered in Ref. [71].

IV. CONCLUSION

In this paper, we have generalized the existing axion solution to allow for the possibility of a much heavier, visible axion. This is done by enlarging the QCD color group, $SU(3)_c$ to be SU(3+N') which is then broken to $SU(3)_c \times SU(N')$ at a UV scale, generating equal theta terms for the two gauge groups. Moreover due to the

unified structure, the CP-violating contributions from complex mass matrices are identical in the two sectors. This requires that the SU(3+N')-preserving CP violation is sufficiently sequestered from symmetry-breaking effects and no new phases are introduced when the unified partners of the Standard Model quarks are decoupled. In addition to the Standard Model quarks, there are extra vectorlike quarks charged under a global PQ symmetry. After the PQ symmetry is spontaneously broken at a scale f_a , the extra vectorlike quarks can be integrated out, generating a dimension-five axion coupling to gluons and, possibly photons. The unified origin of the theta and Yukawa terms then guarantees that after nonperturbative effects generate an axion potential, the two theta parameters can both be cancelled by a single axion.

Since the quark matter content is different between the two sectors, the SU(N') group can confine at a scale, Λ' much larger than in QCD. This then gives the dominant contribution to the axion mass, thereby untying the usual dependence between the axion mass m_a and the QCD confinement scale Λ_c . This gives rise to a model more flexible than the KSVZ invisible axion with regards to accommodating experimental data. For example, if $\Lambda' \sim$ TeV and the PQ breaking scale $f_a \sim 10$ TeV, then the axion obtains an electroweak scale mass. Thus, our model describes a "visible" axion which can be (or perhaps, already was) detected in experiments.

Although it is true that the construction we develop is more complicated and less elegant than the classical invisible axion it may open a window into a new corner of "beyond the Standard Model" physics. First of all, an electroweak axion is theoretically aesthetic because it helps to suppress gravitational violations of the global PQ symmetry. Secondly, it changes the pattern of expectation established from cosmology and astrophysics, completely opening up the axion "window." Finally, it is irresistible not to identify our visible axion as a candidate for the explanation of the 750 GeV diphoton peak at the LHC, assuming it survives with more experimental data. In the minimal model, it predicts decays to dijets, $Z\gamma$ and ZZ, as well as new states such as vectorlike quarks and a CP-even scalar mode with masses in the multi-TeV scale. Otherwise, if the signal disappears, the electroweak axion can still be searched for in future experiments together with the vectorlike quarks and the PQ scalar mode, in order to establish whether or not Nature prefers this more unified solution of the strong *CP* problem.

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⁹Phenomenological aspects of such particles were first discussed by Okun in Refs. [63,64], where these particles were dubbed "theta-leptons." More recently their collider phenomenology was discussed in Ref. [65], where they are referred to as "quirks" (see Refs. [66,67] for earlier work). Quirks have also recently been discussed in connection with the 750 GeV anomaly; see, for instance, Refs. [68–70].

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APPENDIX A: A POSSIBLE UV DESCRIPTION

Our low-energy model crucially depends on not introducing CP phases when the unified partners of the Standard Model quarks are decoupled. A UV framework to address this issue is to consider a warped extra dimension compactified on a Z_2 orbifold, where the SU(3 + N')gauge fields as well as the Standard Model quarks and their partners propagate in a CP-preserving bulk (with the $SU(2)_I \times U(1)$ symmetry implicitly assumed). The UV brane (identified with a scale near the Planck scale) is also assumed to be SU(3+N') symmetric, but CP is not conserved. It provides the source of CP violation including terms like in (6) and (27), as well as in the Higgs Yukawa coupling (24) to Standard Model quarks and their partners. Furthermore, the PQ-charged vectorlike quarks Ψ and the PQ scalar field Φ , are confined to the UV brane with the SU(3 + N')-symmetric Yukawa coupling (13).

Boundary conditions are then chosen to break the bulk gauge symmetry to $SU(3) \times SU(N')$ on the IR brane (identified with the M_U scale), so that only the $SU(3) \times SU(N')$ gauge fields and the Standard Model quarks have massless zero modes. This is similar to orbifold grand-unified models where only the Standard Model gauge bosons and the electroweak Higgs fields have massless zero modes [74–78]. We further assume that the IR brane preserves the CP symmetry so that the quark partner fields are projected out without introducing extra CP phases. Thus, the SU(3+N')-symmetric CP violation on the UV brane is "shined" onto the CP-preserving $SU(3) \times SU(N')$ IR brane, realizing the condition (26) at the scale M_U .

The warped dimension also admits a dual four-dimensional interpretation via the AdS/CFT correspondence. The source of CP violation is confined to an elementary sector containing SU(3+N') gauge fields, vectorlike fermions Ψ and the PQ complex scalar field Φ . The SU(3+N') elementary gauge fields weakly gauge the SU(3+N') global symmetry of some (unknown) strong "technicolor" dynamics. The strong dynamics preserves CP (via possibly massless "techniquarks") and spontaneously breaks the global symmetry to $SU(3) \times SU(N')$. The corresponding gauge fields remain massless and the Standard Model quark partners obtain a mass of order the confinement scale of the strong dynamics. The source of CP violation is again SU(3+N') symmetric, realizing the initial conditions at M_U for our visible axion model.

1. A field theory example of decoupling quarks

The orbifold decoupling of the partner quarks can be mimicked with the ordinary Higgs mechanism in field theory. We use the two-component notation in what follows. Suppose that at M_U the gauge group becomes $\mathrm{SU}(3+N')\times\mathrm{SU}(N')$ (besides $\mathrm{SU}(2)_L\times\mathrm{U}(1)$), where Q_{Li} and Q'_{Li} , u_{Ri} and u'_{Ri} , d_{Ri} and d'_{Ri} are embedded into fundamental representations of $\mathrm{SU}(3+N')$, Ψ_{Qi} , Ψ_{ui} , Ψ_{di} , respectively, with i the generation index. We also introduce anti-fundamental representations of $\mathrm{SU}(N')$, \bar{Q}'_{Li} , \bar{u}'_{Ri} , and \bar{d}'_{Ri} , and a Higgs field, Δ which transforms as antifundamental and fundamental representations under $\mathrm{SU}(3+N')$ and $\mathrm{SU}(N')$, respectively. Then, these fields have the following Yukawa terms 11 :

$$\mathcal{L}_{\text{Yukawa}} = \kappa_{Qij} (\bar{Q}'_{Li})_a \Delta^a{}_\alpha (\Psi_{Qj})^\alpha + \kappa_{uij} (\bar{u}'_{Ri})_a \Delta^a{}_\alpha (\Psi_{uj})^\alpha + \kappa_{dij} (\bar{d}'_{Ri})_a \Delta^a{}_\alpha (\Psi_{dj})^\alpha + \text{H.c.},$$
(A1)

where $\alpha = 1, ..., (3 + N')$ and a = 1, ..., N'. We first note that via field redefinitions of \bar{Q}'_{Li} , \bar{u}'_{Ri} , \bar{d}'_{Ri} , and Δ , it is only possible to make $\arg(\det \kappa_Q)$, $\arg(\det \kappa_u)$, and $\arg(\det \kappa_d)$ be zero, while the theta angle of SU(N') is in general nonzero. A zero SU(N') theta angle requires further UV assumptions (that mimic the CP invariance of the IR brane).

Next, working in this basis, we assume that the Higgs field, Δ develops the following VEV:

$$\langle \Delta^a{}_{\alpha} \rangle = V_{\Delta} \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & 0 & 1 & & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}, \quad (A2)$$

where V_{Δ} can always be taken to be real by using an SU(3+N') gauge transformation. In the dual CFT picture, this VEV corresponds to a condensate of "techniquarks," and since the strong "technicolor" dynamics preserves CP, no new phases are introduced. This VEV breaks the gauge group into $SU(3) \times SU(N')$. The upper three components of $\Psi_{Q,u,d}, Q_L, u_R$, and d_R , do not obtain a mass from the VEV, while the lower N' components, Q'_L, u'_R, d'_R , form vectorlike mass terms together with \bar{Q}'_L, \bar{u}'_R , and \bar{d}'_R , respectively. Since $\arg(\det \kappa_{Q,u,d}) = 0$, these mass terms do not contribute to the physical theta term. As a result, we can decouple the SU(3+N') partner fields of quarks without spoiling the relation $\bar{\theta}=\bar{\theta}'$.

¹⁰Note that on the IR brane the boundary gauge couplings can be different, but we assume that the bulk contribution dominates.

¹¹Note that we have omitted couplings of the barred fields with the Standard Model Higgs because these couplings are absent in the five-dimensional orbifold model.

APPENDIX B: DUAL INTERPRETATION

The PQ mechanism in four dimensions can also be understood in terms of the nondynamical Chern-Simons three-form in QCD and the screening of the corresponding background "electric" field. In this section, we reinterpret our model setup based on this dual description. However it is instructive to first consider a simpler two-dimensional model which has one U(1) gauge field. After that, it will become clear how $U(1)_{PQ}$ is broken, and the axion gets a mass, in the presence of two gauge fields. The generalization to the four-dimensional dual theory will then become apparent.

The standard Schwinger model [79] in two dimensions plus the axion, a has the Lagrangian

$$\mathcal{L} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{f^2}{2} (\partial_{\mu} a)(\partial^{\mu} a) + \frac{1}{2\pi} a \varepsilon^{\mu\nu} F_{\mu\nu}, \quad (B1)$$

where the θ term has been absorbed in the axion field and e is the U(1) coupling. A crucial point is that the gauge field has no physical propagating degree of freedom in two dimensions, and therefore there is only an instantaneous Coulomb interaction. The only physical degree of freedom is that described by a, which is massless at the Lagrangian level (due to the U(1)_{PQ} shift symmetry), but it obtains a mass quantum-mechanically. Simultaneously the Coulomb long-range potential (which grows linearly at large distances in two dimensions) gets screened.

First, note that one can always choose the gauge $A_1 \equiv 0$, and then the only remaining component of the gauge field is A_0 , which enters in the Lagrangian without a time derivative,

$$\mathcal{L}_{A_1=0} = \frac{1}{2e^2} (\partial_1 A_0)^2 + \frac{f^2}{2} (\partial_\mu a)(\partial^\mu a) - \frac{1}{\pi} a(\partial_1 A_0).$$
 (B2)

In this case, one can immediately eliminate A_0 through the classical equation of motion:

$$A_0 = \frac{e^2}{\pi} \partial_1^{-1} a, \tag{B3}$$

$$\mathcal{L}_{A_1=0} = \frac{f^2}{2} (\partial_{\mu} a)(\partial^{\mu} a) - \frac{e^2}{2\pi^2} a^2.$$
 (B4)

Hence, the axion mass becomes

$$m_a = \frac{e}{\pi f}. (B5)$$

The constraint (B3) can also be written as

$$\frac{1}{2} \left(\frac{1}{e} \partial_1 A_0 - \frac{e}{\pi} a \right)^2 \equiv 0. \tag{B6}$$

Note that A_0 is an auxiliary field and does not represent any physical degree of freedom in (B2), nor does it becomes a degree of freedom after elimination, as in (B3).

Next we consider adding a second gauge field, B_{μ} . The Lagrangian (B2) now becomes

$$\mathcal{L}_{A_1=0} = \frac{1}{2e^2} [(\partial_1 A_0)^2 + (\partial_1 B_0)^2] + \frac{f^2}{2} (\partial_\mu a) (\partial^\mu a) - \frac{1}{\pi} a [(\partial_1 A_0) + (\partial_1 B_0)]. \tag{B7}$$

The most crucial point is that the couplings of the both gauge fields A_{μ} and B_{μ} are the same. This is chosen to mimic the unified origin of the separate U(1) fields. The equations of motion for the auxiliary fields are now

$$A_0 = \frac{e^2}{\pi} \partial_1^{-1} a, \qquad B_0 = \frac{e^2}{\pi} \partial_1^{-1} a.$$
 (B8)

In fact, Eq. (B8) has an ambiguity which is sometimes formulated in terms of a constant electric field background in the vacuum. Such fields would require electric charges at the spatial boundary. If one has two distinct U(1) theories and assumes two distinct electric charges at the spatial infinities for two U(1)'s, then, effectively, this would correspond to different "primordial" θ 's in two U(1)'s. Then, of course, our axion will not be able to "screen" both. An analogous situation in four dimensions will be to have different θ 's in SU(3) and SU(N') if we ignore their unification. We cannot model a unifying non-Abelian group in the Schwinger two-dimensional model because, for non-Abelian groups, there is no θ in two dimensions. In this case, to model unification we can impose a Z_2 symmetry in the original Lagrangian. Then the boundary conditions at infinity should be Z_2 symmetric as well, implying that the electric background field in the bulk is one and the same for both U(1)'s.

Both auxiliary fields in Eq. (B8) are expressed in terms of one and the same physical field a, but there is no problem with this since A_{μ} and B_{μ} are auxiliary to begin with. Note that this is not the Higgs mechanism in which, if A_0 eats up a there is nothing left for B_0 to eat up.

Substituting Eq. (B8) in Eq. (B7), the axion mass-squared m_a^2 becomes twice as large, and Eq. (B6) is replaced by

$$\frac{1}{2} \left(\frac{1}{e} \partial_1 A_0 - \frac{e}{\pi} a \right)^2 \equiv 0,$$

$$\frac{1}{2} \left(\frac{1}{e} \partial_1 B_0 - \frac{e}{\pi} a \right)^2 \equiv 0.$$
(B9)

If we introduce probe electric charges, Q it is not difficult to see that both are screened at distances larger than $1/m_a$.

Finally, it is instructive to comment on the fourdimensional Yang-Mills theory and interpret the axion mechanism with an enlarged color group in the dual formulation introduced in Ref. [72]. We will focus on one aspect, namely, the integration constant ambiguities [73]. The essence of the effective low-energy dual formulation of Refs. [72,73] is as follows. One introduces a three-form gauge field,

$$C_{\alpha\beta\gamma} \propto \varepsilon_{\alpha\beta\gamma\mu} K^{\mu},$$
 (B10)

where K^{μ} is the conventional Chern–Simons current. Unlike the Schwinger model, the field $C_{\alpha\beta\gamma}$ is composite. However, in the effective low-energy description one can build the corresponding fully antisymmetric field tensor analogous to $F_{\mu\nu}$ in the Schwinger model, and, add its kinetic term. An analog of Eq. (B7) will take the form (symbolically)

$$\partial_{[\mu} C_{\alpha\beta\gamma]} \propto \varepsilon_{\alpha\beta\gamma\mu} a.$$
 (B11)

Using the gauge in which $C_{\alpha\beta\gamma}$ with the zero value of one of the subscripts vanishes, we obviously conclude that $C_{\alpha\beta\gamma}$ is nondynamical (much in the same way as A_0 in (B7)), and the solution of Eq. (B11) contains an integration constant. Note that nondynamical three-form C fields are sourced by domain walls. ¹² In Refs. [72,73], it is argued that, since at low energies we deal with two gauge groups, SU(3) and SU(N'), there are two independent integration constants. This is equivalent to having two distinct θ angles which would imply, in turn, that a single axion under consideration is unable to solve the CP problem.

To our mind the above argument does not take into account that both low-energy gauge groups are unified at high energies into an SU(3 + N') gauge group. This

provides us with a unified initial condition for the θ angle evolution. In the effective low-energy language of three-form fields, this would amount to equality of two integration constants. We do not know at the moment whether this equality is derivable in the effective description [72,73] $per\ se$.

The fact that the overall structure of the θ parameters (and the associated physical θ periodicity, related to the vacuum structure) depends on the topology in the space of fields at all energy-momentum scales, including arbitrarily high, was emphasized in [8,19]. In [19], it was explicitly noted that in the case of two group factors G_1 and G_2 (in our model, SU(3) and SU(N')) obtained from a unifying group G, i.e., $G_1 \times G_2 \subset G$ at a high scale, the number of independent θ angles is one rather than two because the G_1 and G_2 instantons can be deformed into one another by passing through configurations of arbitrarily large but finite action.

A very pedagogical example suggested in Ref. [19] is as follows. Consider the quantum-mechanical problem of a single particle on a circle S_1 assuming that the motion on the circle is free. The boundary conditions on the wave functions need not be periodic. They can be periodic up to a Bloch phase, provided that one and the same phase enters in the boundary conditions for all wave functions. This gives rise to the θ parameter.

Now, consider instead a particle on a sphere S_2 in a potential (defined on S_2) such that it has a deep and steep minimum along the sphere's equator. The depth of the trough can be arbitrarily large (but finite) so that one might naively say that the low-energy motion of the particle is equivalent to that on S_1 .

However, this would be the wrong answer, since no matter how high the barrier, the topology of the configurational space changes, and the Bloch boundary condition is impossible. Tails of the wave functions of the system "feel" that there is a continuous path from an effective S_1 to S_2 . The θ angle no longer exists. Therefore, considering only the low-energy limit tells us nothing about the disappearance of the Bloch boundary condition and the θ angle.

¹²Strictly speaking, in pure Yang-Mills there are no static domain walls since the vacuum is unique. However, if N is large, there are of order N quasivacua [80], which are split from the unique genuine vacuum by a small amount proportional to N^0 , while the vacuum energy density *per se* is proportional to N^2 (see Refs. [81,82] and references therein). The decay rate of the false vacua is exponentially suppressed.

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