Nucleon structure functions and longitudinal spin asymmetries in the chiral quark constituent model

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We have analyzed the phenomenological dependence of the spin independent $(F_1^{p,n} \text{ and } F_2^{p,n})$ and the spin dependent $(g_1^{p,n})$ structure functions of the nucleon on the Bjorken scaling variable x using the unpolarized distribution functions of the quarks q(x) and the polarized distribution functions of the quarks $\Delta q(x)$ respectively. The chiral constituent quark model, which is known to provide a satisfactory explanation of the proton spin crisis and related issues in the nonperturbative regime, has been used to compute explicitly the valence and sea quark flavor distribution functions of p and n. In light of the improved precision of the world data, the p and n longitudinal spin asymmetries $[A_1^p(x) \text{ and } A_1^n(x)]$ have been calculated. The implication of the presence of the sea quarks has been discussed for the ratio of polarized to unpolarized quark distribution functions for up and down quarks in the p and $n \frac{\Delta u^p(x)}{u^p(x)}, \frac{\Delta d^p(x)}{d^p(x)}, \frac{\Delta u^n(x)}{u^n(x)}, \text{ and } \frac{\Delta d^n(x)}{d^n(x)}$. The ratio of the n and p structure functions $R^{np}(x) = \frac{F_1^n(x)}{F_2^n(x)}$ has also been presented. The results have been compared with the recent available experimental observations. The results on the spin sum rule have also been included and compared with data and other recent approaches.

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I. INTRODUCTION

Several interesting studies have been carried out to understand the internal structure of the nucleon ever since the deep inelastic scattering (DIS) experiments revealed that the quarks are pointlike constituents [1]. These pointlike constituents were identified as the valence or constituent quarks with spin- $\frac{1}{2}$ in the naive quark model (NQM) [2-5]. Surprisingly, the measurements of polarized structure functions of proton in DIS experiments [6–9] showed that the total spin carried by the constituent quarks was very small (only about 30%) leading to the "proton spin crisis;" see Ref. [10] for a recent review. The polarized deep inelastic lepton-nucleon scattering is a useful probe of the spin structure of the nucleon and the measurements with proton, deuteron, and helium-3 targets have determined the unpolarized and polarized structure functions of the nucleon through the measurement of the longitudinal spin asymmetries with the target spin being parallel and antiparallel to the longitudinally polarized beam [7,11].

The data on the asymmetry of the nucleons as well as the ratio of neutron and proton unpolarized structure functions disagrees with the predictions of NQM. In addition to this, a major surprise has been revealed in the famous DIS experiments by the New Muon Collaboration (NMC) [12], Fermilab E866 [13], Drell-Yan cross section ratios of the NA51 experiments [14] and more recently by HERMES [15]. These experiments established the violation of Gottfried sum rule (GSR) $(\int_{0}^{1} [\bar{d}(x) - \bar{u}(x)] dx)$ [16]

confirming the sea quark asymmetry of the unpolarized quarks in the case of the nucleon. Recent measurements of the electromagnetic and weak form factors from the elastic scattering of electrons by SAMPLE at MIT-Bates [17], G0 at JLab [18], PVA4 at MAMI [19] and HAPPEX at JLab [20] have given clear signals for explicit contributions of nonvalence quarks in the spin structure of the nucleon. These results further confirm the nonperturbative origin of the sea quarks as the conventional perturbative production of the quark-antiquark pairs by gluons give nearly equal numbers of antiquarks.

Even though extensive studies have been carried out in the past 40 years, it is still a big challenge to perform the calculations from the first principles of quantum chromodynamics (QCD). Confinement has limited our knowledge on the composition of hadrons and internal structure continues to remain a major unresolved problem in high energy spin physics. In addition to this, to have a deeper understanding of the DIS results as well as the dynamics of the constituents of the nucleon, the "spin sum rule" [21] needs to be explained,

$$\frac{1}{2} = J_q + J_g = S_q + L_q + J_g, \tag{1}$$

where S_q is the spin polarization contribution of the quarks, L_q is the orbital angular momentum (OAM) of the quarks, J_g is the total angular momentum of the gluons. Recently, evidence for a nonzero contribution of gluon spin has been

found in the polarized proton-proton collisions [22]. Even though many experimental and theoretical efforts have been made to understand the contribution of the OAM part, a complete understanding does not seem to have been achieved so far.

Recently, the neutrino-induced DIS experiments [23] have emphasized that the sea quarks dominate for the values of Bjorken scaling variable x < 0.3 and precision data have been collected only in the low and moderate x regions due to experimental limitation. Further, the experiments CDHS [24], CCFR [25,26], CHARMII [27], NOMAD [28,29], NuTeV [30] and CHORUS [31] have pointed out the need for additional refined data renewing considerable interest in the nonvalence structure. In the absence of precise data above x > 0.4 which is a relatively clean region to test the valence structure of the nucleon, the parametrizations are quite unconstrained. The ongoing Drell-Yan experiment at Fermilab [32] and a proposed experiment at the J-PARC facility [33] are working towards extending the kinematic coverage.

Considerable progress in the past few years has been made to understand the origin of the sea quarks, however, there is no consensus regarding the various mechanisms which can contribute to it. The broader question of nonvalence quark contribution to the unpolarized distributions of sea quarks, sea quark asymmetry, structure function has been discussed in various models [34–45]. One of the most successful nonperturbative approaches is the chiral constituent quark model (χ CQM) [46,47]. The basic idea is based on the possibility that chiral symmetry breaking takes place at a distance scale much smaller than the confinement scale. The γ CQM uses the effective interaction Lagrangian approach of the strong interactions where the effective degrees of freedom are the valence quarks and the internal Goldstone bosons (GBs) which are coupled to the valence quarks [48–51]. The γ CQM successfully explains the spin structure of the nucleon [51], magnetic moments of octet and decuplet baryons [52], semileptonic weak decay parameters [53], magnetic moments of nucleon resonances and Λ resonances [54], quadrupole moment and charge radii of octet baryons [55], etc. On the other hand, the inclusion of Bjorken scaling variable x in the distribution functions has not yet been successfully derived from first principles. Instead, they are obtained by fitting parametrizations to data. Efforts have been made in developing a model with confining potential incorporating the x dependence in the valence quark distribution functions [3-5,42]. The x dependence in the quark distribution functions has also been derived in a physical model from simple assumptions [43,47]. In view of the above developments, it became desirable to extend the applicability of χ CQM by incorporating x dependence phenomenologically in the unpolarized and polarized quark distribution and nucleon structure functions whose knowledge would undoubtedly provide vital clues to the distribution of the valence and sea quarks in the kinematic range thus providing vital clues to the nonperturbative aspects of QCD.

The purpose of the present communication is to determine the unpolarized distribution functions of the quarks q and the polarized distribution functions of the quarks Δq using the chiral constituent quark model (χ CQM) which successfully accounts for the quantities affected by chiral symmetry breaking. The χ CQM allows us to understand the explicit contributions of the valence and the sea quarks. It would be significant to analyze the dependence of various quantities by phenomenologically incorporating the Bjorken scaling variable x since x < 0.3 is a relatively clean region to test the quark sea structure. In particular, we would like to understand in detail the spin independent structure functions $F_1^{p,n}(x)$ and $F_2^{p,n}(x)$, spin dependent structure functions $g_1^{p,n}(x)$. The p and n longitudinal spin asymmetries $A_1^p(x)$ and $A_1^n(x)$ come from the difference in cross sections in scattering of a polarized lepton from a polarized proton where the leptons are scattered with the same and unlike helicity as that of the proton. Further, it would be interesting to extend the calculations to compute the ratio of polarized to unpolarized quark distribution functions for up and down quarks in the p and $n \frac{\Delta u^p(x)}{u^p(x)}, \frac{\Delta d^p(x)}{d^p(x)}, \frac{\Delta u^n(x)}{u^n(x)}$, and $\frac{\Delta d^n(x)}{d^n(x)}$. The implications of the presence of the sea quarks can also be discussed for the ratio of the n and p spin independent structure functions $R^{np}(x) = \frac{F_2^n(x)}{F_2^p(x)}$. The role of valence and sea quarks and their orbital angular momentum can be discussed in the context of spin sum rule. The results can be compared with the recent available approaches and also provide important constraints on the future experiments to describe the role of nonvalence degrees of freedom.

II. UNPOLARIZED AND POLARIZED DISTRIBUTION FUNCTIONS OF QUARKS

The unpolarized distribution function of the quark (antiquark) $q_i(x)$ ($\bar{q}_i(x)$) is described as the probability of the *i*th quark (antiquark) carrying a fraction *x* of the nucleon's momentum. It can be calculated from the scalar matrix element of the nucleon using the operator $q\bar{q}$ measuring the sum of the quark and antiquark numbers as

$$\langle N|q\bar{q}|N\rangle,$$
 (2)

where $|N\rangle$ is the nucleon wave function. The operator $q\bar{q}$ is defined in terms of the number $n_{q(\bar{q})}$ of $q(\bar{q})$ quarks with electric charge $e_q(e_{\bar{q}})$. We have

$$q\bar{q} = \sum_{q=u,d,s} (n_q q + n_{\bar{q}}\bar{q}) = n_u u + n_{\bar{u}}\bar{u} + n_d d$$
$$+ n_{\bar{d}}\bar{d} + n_s s + n_{\bar{s}}\bar{s}.$$
(3)

The polarized distribution function of the *i*th quark $\Delta q_i(x)$ is defined as

NUCLEON STRUCTURE FUNCTIONS AND LONGITUDINAL ...

$$\Delta q_i(x) = q_i^{\uparrow}(x) - q_i^{\downarrow}(x), \qquad (4)$$

where $q_i^{\uparrow}(x)$ $(q_i^{\downarrow}(x))$ is the probability that the *i*th quark spin is aligned parallel or antiparallel to the nucleon spin. The polarized distribution function of the quarks can be calculated from the axial vector matrix element of the nucleon using the operator $q^{\uparrow}q^{\downarrow}$ measuring the sum of the quark with spin up and down as

$$\langle N|q^{\uparrow}q^{\downarrow}|N\rangle.$$
 (5)

Here $\mathcal{N} = q^{\uparrow}q^{\downarrow}$ is the number operator defined in terms of the number $n_{q^{\uparrow}(q^{\downarrow})}$ of $q^{\uparrow}(q^{\downarrow})$ quarks. We have

$$\begin{split} q^{\uparrow}q^{\downarrow} &= \sum_{q=u,d,s} (n_{q^{\uparrow}}q^{\uparrow} + n_{q^{\downarrow}}q^{\downarrow}) \\ &= n_{u^{\uparrow}}u^{\uparrow} + n_{u^{\downarrow}}u^{\downarrow} + n_{d^{\uparrow}}d^{\uparrow} + n_{d^{\downarrow}}d^{\downarrow} + n_{s^{\uparrow}}s^{\uparrow} + n_{s^{\downarrow}}s^{\downarrow}, \end{split}$$
(6)

with the coefficients of the $q^{\uparrow\downarrow}$ giving the number of $q^{\uparrow\downarrow}$ quarks.

III. CHIRAL CONSTITUENT QUARK MODEL

The QCD Lagrangian describes the dynamics of light quarks (u, d, and s) and gluons as

$$\mathcal{L} = i\bar{\psi}_L \mathcal{D}\psi_L + i\bar{\psi}_R \mathcal{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M\psi_L - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a,$$
(7)

where D^{μ} is the gauge-covariant derivative, M is the quark mass matrix, ψ_L and ψ_R are the left- and right-handed quark fields respectively, and $G^a_{\mu\nu}$ is the gluonic gauge field strength tensor. The Lagrangian in Eq. (7) does not remain invariant under the chiral transformation $(\psi \rightarrow \gamma^5 \psi)$ as the mass terms change sign as $\psi_L \rightarrow -\psi_L$ and $\psi_R \rightarrow \psi_R$. The Lagrangian will have global chiral symmetry of the $SU(3)_{L+R} \times SU(3)_R$ group if the mass terms are neglected. Around the scale of 1 GeV the chiral symmetry is believed to be spontaneously broken to $SU(3)_{L+R}$. As a consequence, there exists a set of massless particles, referred to as the Goldstone bosons (GBs), which are identified with the observed (π , K, η mesons). Within the region of QCD confinement scale ($\Lambda_{\text{OCD}} \simeq 0.1-0.3 \text{ GeV}$) and the chiral symmetry breaking scale $\Lambda_{\gamma SB}$, the constituent quarks, the octet of GBs (π , K, η mesons), and the *weakly* interacting gluons are the appropriate degrees of freedom.

The effective interaction Lagrangian between GBs and quarks in the leading order can now be expressed as

$$\mathcal{L}_{\rm int} = -\frac{g_A}{f_{\pi}} \bar{\psi} \partial_{\mu} \Phi \gamma^{\mu} \gamma^5 \psi, \qquad (8)$$

PHYSICAL REVIEW D 93, 114030 (2016)

where the field Φ describes the dynamics of the octet of GBs. The QCD Lagrangian is also invariant under the axial U(1) symmetry, which would imply the existence of a ninth GB. This breaking symmetry picks the η' as the ninth GB. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can now be expressed as

$$\mathcal{L}_{\text{int}} = g_8 \bar{\psi} \Phi \psi + g_1 \bar{\psi} \frac{\eta'}{\sqrt{3}} \psi$$
$$= g_8 \bar{\psi} \left(\Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi$$
$$= g_8 \bar{\psi} (\Phi') \psi, \qquad (9)$$

where $\zeta = g_1/g_8$, g_1 (g_8) is the coupling constant for the singlet (octet) GB and I is the 3 × 3 identity matrix.

The basic idea in the χ CQM [46] is the fluctuation process where the GBs are emitted by a constituent quark. These GBs further split into a $q\bar{q}$ pair, for example,

$$q^{\uparrow(\downarrow)} \to \mathrm{GB}^0 + q'^{\downarrow(\uparrow)} \to (q\bar{q}')^0 + q'^{\downarrow(\uparrow)}, \qquad (10)$$

where $q\bar{q}' + q'$ constitute the sea quarks [48,49,51]. The GB field can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} \frac{\pi^{0}}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^{+} & \alpha K^{+} \\ \pi^{-} & -\frac{\pi^{0}}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^{o} \\ \alpha K^{-} & \alpha \bar{K}^{0} & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}.$$
(11)

The transition probability of chiral fluctuation $u(d) \rightarrow d(u) + \pi^{+(-)}$, given in terms of the coupling constant for the octet GBs $|g_8|^2$, is defined as *a* and is introduced by considering nondegenerate quark masses $M_s > M_{u,d}$. The probabilities of transitions of $u(d) \rightarrow s + K^{+(0)}$, $u(d, s) \rightarrow u(d, s) + \eta$, and $u(d, s) \rightarrow u(d, s) + \eta'$ are given as $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively [48,49]. The probability parameters $\alpha^2 a$ and $\beta^2 a$ are introduced by considering nondegenerate GB masses M_K , $M_\eta > M_\pi$ and the probability $\zeta^2 a$ is introduced by considering $M_{\eta'} > M_K, M_\eta$.

The sea quark flavor distribution functions can be calculated in χ CQM by substituting for every valence (constituent) quark

$$q \to P_q q + |\psi(q)|^2, \tag{12}$$

where the transition probability of no emission of GB P_q can be expressed in terms of the transition probability of the emission of a GB from any of the u, d, and s quarks as follows:

$$P_q = 1 - P_{[q,GB]},$$
 (13)

with

$$P_{[u,GB]} = P_{[d,GB]} = \frac{a}{6} (9 + 6\alpha^2 + \beta^2 + 2\zeta^2), \text{ and}$$
$$P_{[s,GB]} = \frac{a}{3} (6\alpha^2 + 2\beta^2 + \zeta^2). \tag{14}$$

The transition probability of the q quark $|\psi(q)|^2$ calculated from the Lagrangian can be expressed as

$$\begin{split} |\psi(u)|^2 &= \frac{a}{36} [(63 + 6\beta + 12\zeta + 4\beta\zeta + 36\alpha^2 + 7\beta^2 + 16\zeta^2)u \\ &+ (9 + 6\beta + 12\zeta + 4\beta\zeta + \beta^2 + 4\zeta^2)\bar{u} \\ &+ (45 - 6\beta - 12\zeta + 4\beta\zeta + \beta^2 + 4\zeta^2)(d + \bar{d}) \\ &+ 4(-2\beta\zeta + 9\alpha^2 + \beta^2 + \zeta^2)(s + \bar{s})], \end{split}$$
(15)

$$\begin{split} |\psi(d)|^2 &= \frac{a}{36} [(63 + 6\beta + 12\zeta + 4\beta\zeta + 36\alpha^2 + 7\beta^2 + 16\zeta^2)d \\ &+ (9 + 6\beta + 12\zeta + 4\beta\zeta + \beta^2 + 4\zeta^2)\bar{d} \\ &+ (45 - 6\beta - 12\zeta + 4\beta\zeta + \beta^2 + 4\zeta^2)(u + \bar{u}) \\ &+ 4(-2\beta\zeta + 9\alpha^2 + \beta^2 + \zeta^2)(s + \bar{s})], \end{split}$$
(16)

$$|\psi(s)|^{2} = \frac{a}{9} [(4\beta\zeta + 18\alpha^{2} + 10\beta^{2} + 4\zeta^{2})s + (4\beta\zeta + 4\beta^{2} + \zeta^{2}9)\bar{s} + (-2\beta\zeta + 9\alpha^{2} + \beta^{2} + \zeta^{2})(u + \bar{u} + d + \bar{d})].$$
(17)

The spin structure of the nucleon after the inclusion of sea quarks generated through chiral fluctuation can be calculated by substituting for each valence (constituent) quark

$$q^{\uparrow\downarrow} \to P_q q^{\uparrow\downarrow} + |\psi(q^{\uparrow\downarrow})|^2, \tag{18}$$

where $|\psi(q^{\uparrow\downarrow})|^2$ is the probability of transforming the $q^{\uparrow\downarrow}$ quark after one interaction expressed by the functions

$$\begin{split} |\psi(u^{\uparrow\downarrow})|^2 &= \frac{a}{6}(3+\beta^2+2\zeta^2)u^{\downarrow\uparrow}+ad^{\downarrow\uparrow}+a\alpha^2s^{\downarrow\uparrow},\\ |\psi(d^{\uparrow\downarrow})|^2 &= au^{\downarrow\uparrow}+\frac{a}{6}(3+\beta^2+2\zeta^2)d^{\downarrow\uparrow}+a\alpha^2s^{\downarrow\uparrow},\\ |\psi(s^{\uparrow\downarrow})|^2 &= a\alpha^2u^{\downarrow\uparrow}+a\alpha^2d^{\downarrow\uparrow}+\frac{a}{3}(2\beta^2+\zeta^2)s^{\downarrow\uparrow}. \end{split}$$
(19)

IV. SPIN INDEPENDENT AND SPIN DEPENDENT STRUCTURE FUNCTIONS OF THE NUCLEON

The nucleon structure is conventionally parametrized by the spin independent structure functions $F_1(x)$ and $F_2(x)$, and by the spin dependent structure functions $g_1(x)$ and

PHYSICAL REVIEW D 93, 114030 (2016)

 $g_2(x)$, where x is the Bjorken scaling variable. One useful probe of the nucleon spin structure is the longitudinal spin asymmetry $A_1(x)$. The scattering of a polarized lepton from a polarized proton can be used to measure the spin dependent structure function g_1 from the difference in cross sections for leptons with the same and unlike helicity as that of the proton. The longitudinal spin asymmetries can be defined as

$$A_1(x) = \frac{\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow}}{\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}} \simeq \frac{g_1(x)}{F_2(x)}.$$
 (20)

The spin independent structure functions of the nucleon can be further defined in terms of the unpolarized distribution functions of the quarks defined in Sec. II as follows:

$$F_1^N(x) = \frac{1}{2} \sum_{u,d,s} e_i^2(q_i(x) + \bar{q}_i(x)),$$

$$F_2^N(x) = 2x F_1^N(x).$$
(21)

In the χ CQM, the unpolarized distribution function of the quarks can be defined in terms of the constituent or valence as well as the sea quark distribution functions as

$$q^{N}(x) = q_{V}^{N}(x) + q_{S}^{N}(x),$$
 (22)

where q = u, d, s. Since the antiquark distribution functions come purely from the sea quarks therefore we can replace the sea quark distribution functions with the antiquark distribution functions as

$$q^{N}(x) = q_{V}^{N}(x) + \bar{q}^{N}(x).$$
 (23)

Here we have the valence quark distribution functions for p and n as

$$\int_{0}^{1} u_{\rm V}^{p}(x)dx = 2, \quad \int_{0}^{1} d_{\rm V}^{p}(x)dx = 1, \quad \int_{0}^{1} s_{\rm V}^{p}(x)dx = 0,$$
$$\int_{0}^{1} u_{\rm V}^{n}(x)dx = 1, \quad \int_{0}^{1} d_{\rm V}^{n}(x)dx = 2, \quad \int_{0}^{1} s_{\rm V}^{n}(x)dx = 0,$$
(24)

and the sea quark distribution functions for p and n as

$$\begin{split} \bar{u}^{p} &= \frac{a}{12} \left(21 + \beta^{2} + 4\zeta + 4\zeta^{2} + \beta(2 + 4\zeta) \right), \\ \bar{u}^{n} &= \frac{a}{12} \left(33 + \beta^{2} - 4\zeta + 4\zeta^{2} + \beta(-2 + 4\zeta) \right), \\ \bar{d}^{p} &= \frac{a}{12} \left(33 + \beta^{2} - 4\zeta + 4\zeta^{2} + \beta(-2 + 4\zeta) \right), \\ \bar{d}^{n} &= \frac{a}{12} \left(21 + \beta^{2} + 4\zeta + 4\zeta^{2} + \beta(2 + 4\zeta) \right), \\ \bar{s}^{p} &= 3a \left(\alpha^{2} + \frac{1}{9} (\beta - \zeta)^{2} \right), \\ \bar{s}^{n} &= 3a \left(\alpha^{2} + \frac{1}{9} (\beta - \zeta)^{2} \right). \end{split}$$
(25)

NUCLEON STRUCTURE FUNCTIONS AND LONGITUDINAL ...

There are no simple or straightforward rules which could allow incorporation of x dependence in the valence quarks and the sea quarks. For the case of an unpolarized valence quark distribution function, we have incorporated the xdependence phenomenologically [3,4,47] as follows:

$$u_{\rm V}^{p}(x) = 8(1-x)^{3}\cos^{2}\phi + 4(1-x)^{3}\sin^{2}\phi + 8\sqrt{2}x^{4}(1-x)^{3}\cos\phi\sin\phi, d_{\rm V}^{p}(x) = 4(1-x)^{3}\cos^{2}\phi + 2(1-x)^{3}\sin^{2}\phi - 8\sqrt{2}x^{4}(1-x)^{3}\cos\phi\sin\phi.$$
(26)

For the case of unpolarized sea quark distribution function, we have for proton

$$\begin{split} \bar{u}^{p}(x) &= \bar{u}^{p}(1-x)^{10}, \\ \bar{d}^{p}(x) &= \bar{d}^{p}(1-x)^{7}, \\ \bar{s}^{p}(x) &= \bar{s}^{p}(1-x)^{8}, \end{split}$$
(27)

and neutron

$$\begin{split} \bar{u}^n(x) &= \bar{u}^n (1-x)^7, \\ \bar{d}^n(x) &= \bar{d}^n (1-x)^{10}, \\ \bar{s}^n(x) &= \bar{s}^n (1-x)^8. \end{split}$$
(28)

Using the unpolarized quark distribution functions from Eqs. (12) and (23), the structure function F_2 for the *p* and *n* Eq. (21) can be expressed as

$$F_{2}^{p}(x) = \frac{4}{9}x(u_{V}^{p}(x) + 2\bar{u}^{p}(x)) + \frac{1}{9}x(d_{V}^{p}(x) + 2\bar{d}^{p}(x) + s_{V}^{p}(x) + 2\bar{s}^{p}(x)),$$

$$F_{2}^{n}(x) = \frac{4}{9}x(u_{V}^{n}(x) + 2\bar{u}^{n}(x)) + \frac{1}{9}x(d_{V}^{n}(x) + 2\bar{d}^{n}(x) + s_{V}^{n}(x) + 2\bar{s}^{n}(x)).$$
(29)

The spin dependent structure function of the nucleon can similarly be defined in terms of the polarized distribution function of the quarks [Eq. (4)] as

$$g_1^N(x) = \frac{1}{2} \sum_{u,d,s} e_i^2 \Delta q_i(x).$$
(30)

The polarized distribution function of the quarks can also be defined in terms of polarized valence and sea quark distribution functions as

$$\Delta q^N(x) = \Delta q^N_V(x) + \Delta q^N_S. \tag{31}$$

Here we have the polarized valence quark distribution functions for p and n as

$$\Delta u_{\rm V}^{p} = \frac{4}{3}, \qquad \Delta d_{\rm V}^{p} = -\frac{1}{3}, \qquad \Delta s_{\rm V}^{p} = 0,$$

$$\Delta u_{\rm V}^{n} = -\frac{1}{3}, \qquad \Delta d_{\rm V}^{n} = \frac{4}{3}, \qquad \Delta s_{\rm V}^{n} = 0, \qquad (32)$$

and the polarized sea quark distribution functions for p and n as

$$\Delta u_{\rm S}^{p} = -\frac{a}{3} \left(7 + 4\alpha^{2} + \frac{4}{3}\beta^{2} + \frac{8}{3}\zeta^{2} \right),$$

$$\Delta u_{\rm S}^{n} = -\frac{a}{3} \left(2 - \alpha^{2} - \frac{1}{3}\beta^{2} - \frac{2}{3}\zeta^{2} \right),$$

$$\Delta d_{\rm S}^{p} = -\frac{a}{3} \left(2 - \alpha^{2} - \frac{1}{3}\beta^{2} - \frac{2}{3}\zeta^{2} \right),$$

$$\Delta d_{\rm S}^{n} = -\frac{a}{3} \left(7 + 4\alpha^{2} + \frac{4}{3}\beta^{2} + \frac{8}{3}\zeta^{2} \right),$$

$$\Delta s_{\rm S}^{p} = -a\alpha^{2}, \qquad \Delta s_{\rm S}^{n} = -a\alpha^{2}.$$
(33)

Following Brodsky *et al.* [56], for the polarized valence quark distribution functions of p and n we have parametrized

$$\Delta u_V^p(x) = \Delta u_V^p (1-x)^3,$$

$$\Delta d_V^p(x) = \Delta d_V^p (1-x)^3,$$

$$\Delta s_V^p(x) = \Delta s_V^p (1-x)^3,$$
(34)

$$\begin{aligned} \Delta u_{\rm V}^n(x) &= \Delta u_{\rm V}^n (1-x)^3, \\ \Delta d_{\rm V}^n(x) &= \Delta d_{\rm V}^n (1-x)^3, \\ \Delta s_{\rm V}^n(x) &= \Delta s_{\rm V}^n (1-x)^3, \end{aligned}$$
(35)

and for the polarized sea quark distribution functions of p and n we have parametrized

$$\Delta u_{\rm S}^{p}(x) = \Delta u_{\rm S}^{p}(1-x)^{5},$$

$$\Delta d_{\rm S}^{p}(x) = \Delta d_{\rm S}^{p}(1-x)^{5},$$

$$\Delta s_{\rm S}^{p}(x) = \Delta s_{\rm S}^{p}(1-x)^{5},$$
(36)

$$\begin{aligned} \Delta u_{\rm S}^n(x) &= \Delta u_{\rm S}^n(1-x)^5, \\ \Delta d_{\rm S}^n(x) &= \Delta d_{\rm S}^n(1-x)^5, \\ \Delta s_{\rm S}^n(x) &= \Delta s_{\rm S}^n(1-x)^5. \end{aligned}$$
(37)

The structure function g_1 for p and n can respectively be calculated using the above equations and are expressed as

HARLEEN DAHIYA and MONIKA RANDHAWA

1

$$g_1^p(x) = \frac{4}{9}(\Delta u^p) + \frac{1}{9}(\Delta d^p + \Delta s^p),$$

$$g_1^n(x) = \frac{4}{9}(\Delta u^n) + \frac{1}{9}(\Delta d^n + \Delta s^n).$$
 (38)

After having formulated the x dependence in the valence and sea quark distribution functions, we now consider the quantities which are measured at different x and can expressed in terms of the above-mentioned quark distribution functions. The proton and neutron longitudinal spin asymmetries are given by

1

$$A_{1}^{p}(x) = \frac{4\Delta u^{p}(x) + \Delta d^{p}(x)}{4u^{p}(x) + d^{p}(x)}, \ A_{1}^{n}(x) = \frac{4\Delta u^{n}(x) + \Delta d^{n}(x)}{4u^{n}(x) + d^{n}(x)}.$$
(39)

These expressions can be rearranged to obtain the explicit ratio of polarized to unpolarized quark distribution functions for up and down quarks in the proton and neutron as

$$\begin{aligned} \frac{\Delta u^{p}(x)}{u^{p}(x)} &= \frac{4}{15} A_{1}^{p}(x) \left(4 + \frac{d^{p}(x)}{u^{p}(x)} \right) - \frac{1}{15} A_{1}^{n}(x) \left(1 + 4 \frac{d^{p}(x)}{u^{p}(x)} \right), \\ \frac{\Delta d^{p}(x)}{d^{p}(x)} &= \frac{4}{15} A_{1}^{n}(x) \left(4 + \frac{u^{p}(x)}{d^{p}(x)} \right) - \frac{1}{15} A_{1}^{p}(x) \left(1 + 4 \frac{u^{p}(x)}{d^{p}(x)} \right), \\ \frac{\Delta u^{n}(x)}{u^{n}(x)} &= \frac{4}{15} A_{1}^{p}(x) \left(1 + 4 \frac{d^{n}(x)}{u^{n}(x)} \right) - \frac{1}{15} A_{1}^{n}(x) \left(4 + \frac{d^{p}(x)}{u^{p}(x)} \right), \\ \frac{\Delta d^{n}(x)}{d^{n}(x)} &= \frac{4}{15} A_{1}^{n}(x) \left(1 + 4 \frac{u^{n}(x)}{d^{n}(x)} \right) - \frac{1}{15} A_{1}^{p}(x) \left(4 + \frac{d^{n}(x)}{u^{p}(x)} \right). \end{aligned}$$

$$(40)$$

Another important quantity where the NQM disagrees with the data significantly is the ratio of the neutron and proton structure functions,

$$R^{np}(x) = \frac{F_2^n(x)}{F_2^p(x)}.$$
(41)

V. SPIN SUM RULE

The various terms in the spin sum rule $(S_q + L_q + J_g)$ can be expressed in terms of the quantities and χ CQM parameters discussed above. The spin contribution of the quarks S_q to p and n can be further expressed as a sum of the valence and sea contributions as

$$S_q^{p,n} = S_{qV}^{p,n} + S_{qS}^{p,n}, (42)$$

where

$$S_{qV}^{p,n} = \frac{1}{2} (\Delta u_{V}^{p,n} + \Delta d_{V}^{p,n} + \Delta s_{V}^{p,n}),$$

$$S_{qS}^{p,n} = \frac{1}{2} (\Delta u_{S}^{p,n} + \Delta d_{S}^{p,n} + \Delta s_{S}^{p,n}).$$
(43)

 $S_{qV}^{p,n}$ and $S_{qS}^{p,n}$ for the case of *p* and *n* can be calculated using the polarized valence quark distribution functions and the polarized sea quark distribution functions from Eqs. (32) and (33).

The total OAM carried by the quarks in the nucleon is given in terms of the transition probability of the emission of a GB from any of the *u*, *d*, and *s* quark $P_{[q,GB]}$ [57]. We have for the case of *p* and *n*

$$L_q^p = \sum_{q=u,d,s} \Delta q_V^p P_{[q,GB]},$$

$$L_q^n = \sum_{q=u,d,s} \Delta q_V^n P_{[q,GB]}.$$
 (44)

In the present context, the total orbital angular momentum can be expressed in terms of the χ CQM parameters as

$$L_q^{p,n} = \frac{a}{6}(9 + 6\alpha^2 + \beta^2 + 2\zeta^2).$$
(45)

There is no direct way to calculate the contribution of gluons to the spin sum rule in the χ CQM and it is already clear that gluon spin is not large enough to explain the spin problem. Therefore, we have not discussed the gluon contribution in the present work.

VI. RESULTS AND DISCUSSION

In order to study the phenomenological quantities pertaining to the valence and sea quarks distribution functions and further compare the χ CQM results with other model calculations and the available experimental data, we can study the x dependence of the spin independent and spin dependent structure functions. To this end, we first fix the γ CQM parameters which provide the basis to understand the extent to which the sea quarks contribute to the structure of the nucleon. The probabilities of fluctuations to pions, K, η , η' coming in the sea quark distribution functions are represented by a, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ respectively and can be obtained by taking into account strong physical considerations and carrying out a fine grained analysis using the well-known experimentally measurable quantities pertaining to the spin and flavor distribution functions. The hierarchy for the probabilities, which scale as $\frac{1}{M_{\pi}^2}$, can be obtained as

$$a > a\alpha^2 \ge a\beta^2 > a\zeta^2. \tag{46}$$

The mixing angle ϕ is fixed from the consideration of neutron charge radius [2]. The input parameters and their values have been summarized in Table I.

TABLE I. Input parameters.

Input parameters	Value
a	0.114
$a\alpha^2$	0.023
$a\beta^2$	0.023
$a\zeta^2$	0.002
ϕ	18^{0}

After having incorporated x dependence in the valence and the sea quark distribution functions, we now discuss the variation of all the related phenomenological quantities in the range $0 \le x \le 1$. In Fig. 1, we have presented the spin independent quark distribution functions for the case p $(xu^{p}(x), xd^{p}(x) \text{ and } xs^{p}(x))$ and n $(xu^{n}(x), xd^{n}(x) \text{ and} xs^{n}(x))$. The valence quarks distribution functions of p and n vary as

$$u_{\mathrm{V}}^{p}(x) > d_{\mathrm{V}}^{p}(x) > s_{\mathrm{V}}^{p}(x),$$

$$d_{\mathrm{V}}^{n}(x) > u_{\mathrm{V}}^{n}(x) > s_{\mathrm{V}}^{n}(x).$$

On the other hand, the sea quark distribution functions vary as

$$d_{\rm S}^{p}(x) > u_{\rm S}^{p}(x) > s_{\rm S}^{p}(x),$$

$$u_{\rm S}^{n}(x) > d_{\rm S}^{n}(x) > s_{\rm S}^{n}(x).$$

It is evident from Fig. 1 that there is u quark dominance in the case of p and d quark dominance in the case of n. Since the total quark distribution functions are dominated by the valence quarks, the overall variation of the quark distribution functions. The variation of sea quark distribution functions of p and n have been plotted in Fig. 2. Even though the variation of sea quarks if different, for example, $\bar{d}^p(x)$ dominates in the case of p and $\bar{u}^n(x)$ dominates in the case of n, but since the probability for the fluctuation of valence quarks to sea quarks depends upon the χ CQM parameter a

and this probability of the occurrence of sea quarks cannot be more than 10%–15%, therefore, $u^p(x)$ dominates in the case of p and $d^n(x)$ dominates in the case of n. This observation can also be directly related to the measurement of the Gottfried integral for the case of the nucleon which has shown a clear violation of GSR from $\frac{1}{3}$. The quark sea asymmetry $\int_0^1 (\bar{d}(x) - \bar{u}(x)) dx$ has been measured in the NMC and E866 experiments [12,13]. The NMC has reported $I_G^{pn} = \frac{1}{3} + \frac{2}{3} [\bar{u}^p - \bar{d}^p] = 0.266 \pm 0.005$ [12] and the E866 has reported $I_G^{pn} = 0.254 \pm 0.005$ [13]. A flavor symmetric sea $(\bar{u}^N = \bar{d}^N)$ would lead to $I_G^{pn} = \frac{1}{3}$. The χ CQM result for the case of the nucleon ($I_G^{pn} = 0.254$) is in good agreement with the available experimental data of E866 [13]. We have plotted some of the well-known experimentally measurable quantities, for example, $\bar{d}^p(x)$ – $\bar{u}^p(x)$ and $\bar{d}^p(x)/\bar{u}^p(x)$ in Fig. 3 and compared them with data [13]. It is clear from the plots that when x is small $\bar{d}^p(x) - \bar{u}^p(x)$ asymmetry is large implying the dominance of sea quarks in the low x region. In fact, the sea quarks dominate only in the region where x is smaller than 0.3. At the values x > 0.3, $\bar{d} - \bar{u}$ tends to 0 implying that there are no sea quarks in this region. To test the validity of the model as well as for the sake of completeness, we can present the results of our calculations for $\bar{d}^p(x) - \bar{u}^p(x)$ and $\overline{d}^p(x)/\overline{u}^p(x)$ whose data is available over a range of x or at an average value of x. We find a good overall agreement with the data in these cases also. The data for $\bar{d}^p(x) - \bar{u}^p(x)$ is available for the ranges x = 0-1 and x = 0.05-0.35and is given as $\int_{0}^{1} (\bar{d}^{p}(x) - \bar{u}^{p}(x)) dx = 0.118 \pm 0.012$ and $\int_{0.05}^{0.35} (\bar{d}^p(x) - \bar{u}^p(x)) dx = 0.0803 \pm 0.011$. We find that, in our model, $\int_0^1 (\bar{d}^p(x) - \bar{u}^p(x)) dx = 0.117$ and $\int_{0.05}^{0.35} (\bar{d}^p(x) - \bar{u}^p(x)) dx = 0.08 \text{ in these given } x \text{ ranges.}$ The valence quark distribution however is spread over the entire x region. Our results agree with the results of similar studies [58].

In Fig. 4, the ratio of polarized to unpolarized quark distribution functions for up and down quarks in the p and



FIG. 1. The unpolarized quark distribution functions for $p: xu^p(x), xd^p(x), xs^p(x)$ and $n: xu^n(x), xd^n(x), xs^n(x)$ as a function of x.



FIG. 2. The sea quark distribution functions for $p: x\bar{u}^p(x), x\bar{d}^p(x), x\bar{s}^p(x)$ and $n: x\bar{u}^n(x), x\bar{d}^n(x), x\bar{s}^n(x)$ as a function of x.

 $n \frac{\Delta u^p(x)}{u^p(x)}, \frac{\Delta d^p(x)}{d^p(x)}$ and $\frac{\Delta u^n(x)}{u^n(x)}, \frac{\Delta d^n(x)}{d^n(x)}$ have been presented. It is clear from the figure that $\frac{\Delta d^p(x)}{d^p(x)}$ and $\frac{\Delta u^n(x)}{u^n(x)}$ show constant values at lower and higher x and then suddenly fall off as $x \to 1$. This is unlike $\frac{\Delta u^{p}(x)}{u^{p}(x)}$ and $\frac{\Delta u^{n}(x)}{u^{n}(x)}$. As discussed for Fig. 1, the behavior of the unpolarized distribution functions of u^p and d^p is similar. They first rise at lower x and then fall with $x \rightarrow 1$. However, the behavior of polarized distribution functions Δu^p and Δd^p is different. In this case, Δu^p falls with respect to x in the positive direction whereas Δd^p rises in the negative axis. In the $\frac{\Delta u^p(x)}{u^p(x)}$ graph, both quantities in the numerator as well as the denominator are positive and fall with x whereas in the $\frac{\Delta d^{p}(x)}{d^{p}(x)}$ graph the numerator is positive while the denominator is negative and rising. The results agree with the very recent analysis performed by the Jefferson Lab Angular Momentum (JAM) collaboration to produce a new parametrization [59] and the ratio $\frac{\Delta d^p(x)}{d^p(x)}$ was found to remain negative across all x. The NQM has the following predictions for the abovementioned quantities,

$$\frac{\Delta u^{p}(x)}{u^{p}(x)} = \frac{2}{3},$$

$$\frac{\Delta d^{p}(x)}{d^{p}(x)} = -\frac{1}{3},$$

$$\frac{\Delta u^{n}(x)}{u^{n}(x)} = -\frac{1}{3},$$

$$\frac{\Delta d^{n}(x)}{d^{n}(x)} = \frac{2}{3}.$$
(47)

Since Δu and Δd denote the difference between the quark distributions polarized parallel and antiparallel to the polarized nucleon, the distribution when $x \to 1$ predicts that the structure functions should be dominated by valence quarks polarized parallel to the spin of the nucleon for the case of $\frac{\Delta u(x)}{u(x)}$ and by valence quarks polarized antiparallel to the spin of the nucleon for the case of $\frac{\Delta d(x)}{d(x)}$. Further, dramatically different behaviors for the $\frac{\Delta d(x)}{d(x)}$ ratio in different approaches allowed for $x \gtrsim 0.3$ highlights the critical need for precise data sensitive to the *d* quark



FIG. 3. The sea quark flavor asymmetries for the case of $p: \bar{d}^p(x) - \bar{u}^p(x)$ and $\bar{u}^p(x)/\bar{d}^p(x)$ as a function of x compared with the experimental data [13].



FIG. 4. The ratio of polarized to unpolarized distribution functions for the *p* and *n*: $\frac{\Delta u^{p}(x)}{u^{p}(x)}, \frac{\Delta u^{n}(x)}{u^{n}(x)}, \frac{\Delta d^{p}(x)}{d^{p}(x)}, \frac{\Delta d^{n}(x)}{d^{n}(x)}$ as a function of *x*.

polarization at large x values. Inclusion of nonzero orbital angular momentum could play an important role numerically. Further progress on this problem is expected with new data expected from several experiments at the 12 GeV energy upgraded Jefferson Lab [60] which aim to measure polarization asymmetries of protons up to $x \sim 0.8$.

In Fig. 5, we have plotted the spin independent structure functions $F_2^p(x)$ and $F_2^n(x)$ for the case of p and n. The plots clearly project out the distribution of the valence and sea quarks. The function has its peak at around $x \approx 0.25$.

Since the contribution of sea quarks decreases beyond this x, the function drops down to zero as $x \to 1$. There is no mechanism in NQM which can explain the contribution of sea quarks and it has the following predictions for the spin independent structure functions $F_1^p(x)$ and $F_1^n(x)$ at $x \to 1$:

$$F_1^p(x) = \frac{1}{2},$$

$$F_1^n(x) = \frac{1}{6}.$$
(48)



FIG. 5. The spin independent structure functions for p and n: $F_2^p(x)$ and $F_2^n(x)$.



FIG. 6. The spin dependent structure functions for p and n: $g_1^p(x)$ and $g_1^n(x)$ compared with the experimental data [7,9,61].

These results may also be related to the Gottfried integral determined from $\frac{F_2^p(x)-F_2^n(x)}{2x}$. The small *x* part is suppressed relative to the NQM prediction. As $x \rightarrow 1$, the distribution is dominated by the valence quarks and sea quark asymmetry reduces to zero. This is a clean region to test the valence structure of the nucleon. Measurements of the spin independent structure function in the presently inaccessible low x region will provide crucial information on the low *x* behavior of $F_1^p(x)$ and $F_1^n(x)$ and also allow access to the nonvalence contribution in this region.

In Fig. 6, we have plotted the spin dependent structure functions $g_1^p(x)$ and $g_1^n(x)$ for the case of p and n. For $g_1^p(x)$, we find that it constantly drops down to zero as x increases beyond x > 0.5 whereas for $g_1^n(x)$, it increases from -0.07 to 0 and again at x > 0.5 it becomes zero. The NQM predicts

$$g_1^p(x) = \frac{5}{9},$$

 $g_1^n(x) = 0.$ (49)

It is interesting to note that nonzero values of $g_1^n(x)$ for x < 0.5 clearly imply the presence of sea quarks. Even though the valence quark distribution is spread over the entire *x* region and the sea quark distribution decreases with the increasing value of *x*, the valence and sea quarks are polarized in opposite directions and they mutually cancel the effect of each other at higher values of *x*. When compared with the data [9], we find that our results do not agree with the data at low values of *x* but as the value of *x* increases the results are more close.

The results with the data [61] for the spin dependent structure function $g_1^p(x)$ however agree to a very large extent both at lower and higher values of x. The structure function $g_1^p(x)$ is important also in the context of the measured first moment,

$$\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2) dx$$

= $\frac{C_s(Q^2)}{9} g_A^0 + \frac{C_{ns}(Q^2)}{12} g_A^3 + \frac{C_{ns}(Q^2)}{36} g_A^8.$ (50)

It is related to the combinations of the axial-vector coupling constants: g_A^0 corresponding to the flavor singlet component, g_A^3 and g_A^8 corresponding to the flavor nonsinglet components usually obtained from the neutron β -decay and the semileptonic weak decays of hyperons respectively. Here C_s and C_{ns} are the flavor singlet and nonsinglet Wilson coefficients calculable from perturbative QCD. Very recently, a fairly good description of the singlet (g_A^0) and nonsinglet $(g_A^3 \text{ and } g_A^8)$ axial-vector coupling constants has been discussed in the χ CQM [51].

In Fig. 7, the results for $A_1^p(x)$ and $A_1^n(x)$ have been presented. The NQM predictions for these quantities are

$$A_1^p(x) = \frac{5}{9},$$

$$A_1^n(x) = 0.$$
 (51)

These results do not agree at all with the experimental results which show that $A_1^p(x)$ increases from 0 at $x \to 0$ to 1 at $x \to 1$ [7,61]. However, the $A_1^p(x)$ in χ CQM shows a peak at $x \simeq 0.5$. This low value of $A_1^p(x)$ at lower and higher values of x can be explained on the basis of the sea quarks as in the very low x regime the sea quarks are not highly polarized and at large x there are very few sea quarks and structure is dominated by the valence quarks. For the case of $A_1^n(x)$, the data [62] is negative at low x and becomes positive at large x. In χ CQM, the results agree with the data at some values of x and negative values are obtained. However, at large x, $A_1^n(x)$ continues to remain negative and becomes 0 only at $x \rightarrow 1$. This is because the d quarks dominate in the valence structure of the n and since they are negatively polarized they keep the values of $A_1^n(x)$ negative. These results agree with the LSS (BBS)



FIG. 7. The longitudinal spin asymmetries for p and n: $A_1^p(x)$ and $A_1^n(x)$ vs x compared with the experimental data [7,61,62].

parametrization where the Fock states with nonzero quark OAM are included [63] where they predict a zero significantly higher x.

The NQM prediction for the ratio of the neutron and proton structure functions is

$$R^{np}(x) = \frac{2}{3}.$$
 (52)

The data [64] however shows that R^{np} drops from 1 at $x \to 0$ to $\frac{1}{2}$ at $x \to 1$. From Fig. 8, we find that the χ CQM fits the experimental data quite reasonably. The higher values of R^{np} in the low x region are because of the dominance of the sea quarks in this region. As the value of x increases the valence quarks start dominating leading to the decrease in R^{np} .

After having examined the implications of Bjorken scaling variable x for spin independent and spin dependent structure functions, one would like to study the role of various terms in understanding the spin sum rule of the nucleon within the χ CQM. The results have been presented



FIG. 8. The ratio of the *n* and *p* spin independent structure functions $R^{np}(x) = F_2^n(x)/F_2^p(x)$ compared with the experimental data [64].

in Table II. The various contributions to the spin sum rule reveal several interesting points. In case the sum rule is to be explained in terms of the spin polarization contribution of the quarks S_q , the orbital angular momentum of the quarks L_q and the total angular momentum of the gluons J_g , then these should add on to give the total spin of the nucleon. The total angular momentum of the gluons cannot be calculated directly in the present context. It is clear from the results that the valence quark spin and the OAM of the quarks contribute in the same direction to the total proton spin. The sea quark contribution is also significant but in the opposite direction. Since the *u* quarks dominate in the valence structure of the proton, the valence spin of the *u* quarks, quark sea spin of the *u* quarks and the OAM of the *u* quarks are higher in magnitude as compared to that of the

TABLE II. The contributions of various terms to the spin sum rule.

Parameter	Data	NQM	χQM
$\Delta u_{ m V}^p$		1.333	1.333
$\Delta d_{ m V}^p$		-0.333	-0.333
$\Delta s_{ m V}^p$		0	0
$\Delta u_{ m S}^p$		0	-0.309
$\Delta d_{ m S}^p$		0	-0.065
$\Delta s_{\rm S}^p$		0	-0.023
$\Delta u^p = \Delta u^p_{\rm V} + \Delta u^p_{\rm S}$	0.85 ± 0.05 [70]	1.333	1.024
$\Delta d^p = \Delta d^p_{ m V} + \Delta d^p_{ m S}$	-0.41 ± 0.05 [70]	-0.333	-0.398
$\Delta s^p = \Delta s^p_{ m V} + \Delta s^p_{ m S}$	-0.07 ± 0.05 [70]	0	-0.023
$S_{qV}^{p} = \frac{1}{2} \left(\Delta u_{V}^{p} + \Delta d_{V}^{p} + \Delta s_{V}^{p} \right)$		0	0.5
$S_{qS}^{p} = \frac{1}{2} (\Delta u_{S}^{p} + \Delta d_{S}^{p} + \Delta s_{S}^{p})$		0	-0.199
$S_q^p = S_{qV}^p + S_{qS}^p$	0.30 ± 0.05 [8]	0.5	0.301
L^p_u			0.265
L_d^p			-0.066
L_s^p			0
$L^p_q = L^p_u + L^p_d + L^p_s$		0	0.199
J_g^p		0	0
$S_q^p + L_q^p + J_g^p$	0.5	0.5	0.5

d quarks. Further, even though the u quarks carry a comparatively larger amount of OAM as compared to the d quarks (0.265 as compared to -0.066), the total OAM reduces to 0.199 because of the opposite signs of the u and d quark contributions. The total angular momentum of the *u* quarks coming from the spin and OAM $(S_u^p + L_u^p)$ is 0.777 whereas the total angular momentum of the dquarks $(S_d^p + L_d^p)$ is -0.265. The contribution of the s quarks to the total angular momentum comes only from the spin part and is -0.012. Therefore, the proton spin is dominated by the *u* quark contribution from the spin as well as OAM. Our results are consistent with the results of Song [57] where it is shown that the quark spin is small, polarization of sea quarks is nonzero and negative and the OAM of sea quarks is parallel to the proton spin. The only difference is that in our model the total angular momentum of the proton has 60% contribution from the spin of the quarks whereas the OAM contribute 40% in contrast to the results of Song et al. [57] where they have around 40% contribution from the spin of the quarks and 60% from the OAM. Our results agree with the model calculations including two-body axial exchange currents necessary to satisfy the partial conservation of axial current (PCAC) condition [65] as well as with the calculations using spin flavor symmetry based parametrization of QCD [66]. It has been shown that the missing spin should be accounted for by the orbital angular momentum of the quarks and antiquarks [67,68] and the exploration of the angular momentum carried by the guarks and antiguarks is a major aim of the scientific program associated with the 12 GeV Upgrade at Jefferson Lab [60]. Recently, in a very interesting work, a qualitative interpretation of the positive and large u quark and small d quark orbital angular momenta in the proton has been suggested in terms of a prolate quark distribution corresponding to a positive intrinsic quadrupole moment [69].

VII. SUMMARY AND CONCLUSIONS

To summarize, the unpolarized distribution functions of the quarks q and the polarized distribution functions of the quarks Δq have been determined phenomenologically in the chiral constituent quark model (χ CQM). The χ CQM helps in understanding the dynamics of the constituents of the nucleon in terms of the explicit contributions of the valence and the sea quarks specifically for the quantities affected by chiral symmetry breaking. These quantities

have important implications in the nonperturbative regime of QCD. In light of precision data available for the low and moderate x region, we have analyzed the dependence of various quantities on the Bjorken scaling variable x by incorporating it phenomenologically as the x < 0.3 is a relatively clean region to test the quark sea structure. In particular, we have computed the spin independent structure functions $F_1^{p,n}(x)$ and $F_2^{p,n}(x)$ as well as the spin dependent structure functions $g_1^{p,n}(x)$. The implications of the model have also been studied for the p and nlongitudinal spin asymmetries $A_1^p(x)$ and $A_1^n(x)$. These asymmetries come from the difference in cross sections in scattering of a polarized lepton from a polarized proton where the leptons are scattered with the same and unlike helicity as that of the proton and one measures the spin dependent structure function g_1 via the longitudinal spin asymmetry. Further, the calculations have been extended to compute the explicit ratio of the polarized to unpolarized quark distribution functions for up and down quarks in the p and $n \frac{\Delta u^p(x)}{u^p(x)}$, $\frac{\Delta d^p(x)}{d^p(x)}$, $\frac{\Delta u^n(x)}{u^n(x)}$, and $\frac{\Delta d^n(x)}{d^n(x)}$. The u and dquarks have different polarizations and show interesting behavior owing to the dominance of the valence and sea quarks in the different x regions. The qualitative and quantitative role of sea quarks can be further substantiated by discussing the ratio of the n and p spin independent structure functions $R^{np}(x) = \frac{F_2^n(x)}{F_2^p(x)}$. The results have been compared with the recent available experimental observations and the scarcity of precise data at higher x does not allow to favor one model over another. Therefore, new experiments with extended x range are needed for profound understanding of the nonperturbative properties of QCD. At present, we do not have any deep understanding of the contribution of orbital angular momentum of quarks and the gluon spin, however theoretical studies do indicate that these contributions may not be negligible even in a more rigorous model. These results will provide important constraints on the future experiments to describe the explicit role of valence and nonvalence degrees of freedom.

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