PHYSICAL REVIEW D **93,** 114019 (2016)

Testing QCD factorization with phase determinations in $B \to K\pi$, $K\rho$, and $K^*\pi$ decays

T. N. Pham*

Centre de Physique Théorique, CNRS, Université Paris-Saclay Ecole Polytechnique, 91128 Palaiseau, Cedex, France

(Received 6 April 2016; published 20 June 2016)

The success of QCD factorization (QCDF) in predicting branching ratios for charmless B decays to light pseudoscalar and vector mesons and the small CP asymmetries measured at *BABAR*, Belle, and LHCb show that the phase in these decays, as predicted by QCDF, are not large. For a precise test of QCDF, one needs to extract from the measured decay rates the phase of the decay amplitude which appears in the interference terms between the tree and penguin contribution. Since the tree amplitude is known at the leading order in $\Lambda_{\rm QCD}/m_b$ and is consistent with the measured tree-dominated decay rates, the QCDF value for the tree amplitude can be used with the measured decay rates to obtain the phases in $B \to K\pi$, $K\rho$, and $K^*\pi$ decay rates. This is similar to the extraction of the final-state interaction phases in the interference term between $p\bar{p} \to J/\Psi \to e^+e^-$ and $p\bar{p} \to e^+e^-$ and in $J/\Psi \to 0^-0^-$ done previously. In this paper, we present a determination of the phase between the $I = 3/2$ tree and $I = 1/2$ penguin amplitudes in $B \to K\pi$, $K\rho$, and $K^*\pi$ decays using the measured decay rates and the QCDF $I = 3/2$ tree amplitude obtained from the $I = 2$ $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ tree-dominated decays and compare the result with the phase given by QCDF. It is remarkable that the phase extracted from experiments differs only slightly from the QCDF values. This shows that there is no large final-state interaction strong phase in $B \to K\pi$, $K\rho$, and $K^*\pi$ decays.

DOI: 10.1103/PhysRevD.93.114019

I. INTRODUCTION

QCD factorization (QCDF)[1,2] seems to be rather successful in predicting branching ratios and CP asymmetries for charmless B decays into light pseudoscalar and vector mesons. The small CP asymmetries measured at *BABAR*, Belle and LHCb show that the final-state interaction phase in these decays, as predicted by QCDF, is not large. For penguin-dominated charmless B decays into two light pseudoscalar and vector mesons, the phase appearing in the decay amplitude is the relative phase between the isospin $I = 3/2$ tree and $I = 1/2$ penguin amplitude, as in the $B \to K\pi$, $K\rho$, and $K^*\pi$ decays. Since all four modes for $B \to K\pi$, $K\rho$, and $K^*\pi$, respectively, have similar branching ratios, the interference terms are quite small, making a determination of these phases more difficult than for the Cabibbo-favored decays $D \to \bar{K}\pi$, $\bar{K}\rho$, and $\bar{K}^*\pi$, for which a large $\delta^{K\pi}_{3/2} - \delta^{K\pi}_{1/2} = (86 \pm 8°)$ has been obtained [3]. Since the tree amplitude is known at the leading order in $\Lambda_{\rm QCD}/m_b$ [2] and is consistent with the measured tree-dominated decay rates, knowledge of the tree amplitude then allows a simple determination of the phase in the decay amplitude using the measured decay rates. This is similar to the extraction of he final-state interaction phases in the interference term between $p\bar{p} \to J/\Psi \to e^+e^-$ and $p\bar{p} \to e^+e^-$ [4] and in the process $J/\Psi \to 0^-0^-$ via three-gluon and one-photon exchange interference terms [5]. By expressing the $B \to PP, PV$ decay amplitudes in terms of the $I = 1/2$ and $I = 3/2$ isospin amplitudes [6,7], the relative phase of the two isospin amplitudes can be obtained from the magnitudes of the isospin amplitudes and the decay rates, as knowledge of the three sides of the triangle formed with the decay amplitude and the other two sides, the two isospin amplitudes, allows a determination of the three angles of the triangle and the corresponding relative phases of the amplitudes. This is possible for the penguin-dominated $\Delta S = 1$, $B \to PP, PV$ decays for which all the decay rates have been measured, and since QCDF predictions for the $I = 2$ $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ tree-dominated decays agree rather well with experiments as shown in the table below and in [8], the $I = 2$ amplitudes in these decays could be taken as the $I = 3/2$ tree amplitudes in penguin-dominated $B \to PP, PV$ decays with $SU(3)$ breaking effects in the $B \to K, K^*$ form factors and decay constants involving the K, K^* meson taken into account [9]. With the $I = 3/2$ tree amplitude known, the three sides of the triangle formed with the decay rate, the $I = 1/2$ and $I = 3/2$ isospin amplitude allows a determination of the three angles and the relative phase between the sides. In this paper, we will present a determination of the relative phase between the $I = 3/2$ and $I = 1/2$ amplitudes using the QCDF $I = 3/2$ amplitude and the measured decay rates. It is remarkable that the phase extracted from experiments differs only slightly from the QCDF values. This shows that final-state interaction phases are not large in charmless

*pham@cpht.polytechnique.fr

$\Delta S = 1$ $B \to PP, PV$ decays. In the following section, we give amplitudes and branching ratios for the $B \to K\pi$, $K\rho$, and $K^*\pi$ decays in the QCD factorization approach. The determination of the phases of the decay amplitudes obtained from the measured decay rates and from the QCDF amplitudes and decay rates are given in Sec. III.

II. $\Delta S = 1$ $B \to PP, PV$ DECAY IN QCD FACTORIZATION

The $B \to M_1 M_2$, decay amplitude in QCDF for $B = B^-, \bar{B}^0$ is given by [10,11]

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{ps}^* \left(-\sum_{i=1}^{10} a_i^p \langle M_1 M_2 | O_i | B \rangle_H + \sum_i^{10} f_B f_{M_1} f_{M_2} b_i \right), \quad (1)$$

where the QCD coefficients a_i^p contain the vertex corrections, penguin corrections, and hard spectator scattering contributions, the hadronic matrix elements $\langle M_1 M_2 | O_i | B \rangle_H$ of the tree and penguin operators O_i are given by the factorization model [9,12], and b_i are the annihilation terms. The values for a_i^p, $p = u, c$, computed from the expressions in [10,11] at the renormalization scale $\mu = m_b$, with $m_b = 4.2$ GeV, are

$$\begin{aligned} a_4^c &= -0.031 - 0.010i + 0.0009\rho_H \exp(i\phi_H), \\ a_4^u &= -0.027 - 0.017i + 0.0009\rho_H \exp(i\phi_H), \\ a_6^c &= -0.045 - 0.003i, \qquad a_6^u = -0.042 - 0.013i, \\ a_8^c &= -0.0004 - 0.0001i, \qquad a_8^u = 0.0004 - 0.0001i, \\ a_{10}^c &= -0.0011 - 0.0001i - 0.0006\rho_H \exp(i\phi_H), \\ a_{10}^u &= -0.0011 + 0.0006i - 0.0006\rho_H \exp(i\phi_H) \end{aligned} \quad (2)$$

for $i = 4, 6, 8, 10$. For other coefficients, $a_i^u = a_i^p = a_i$:

$$\begin{aligned} a_1 &= 1.02 + 0.015i - 0.012\rho_H \exp(i\phi_H), \\ a_2 &= 0.156 - 0.089i + 0.074\rho_H \exp(i\phi_H), \\ a_3 &= 0.0025 + 0.0030i - 0.0024\rho_H \exp(i\phi_H), \\ a_5 &= -0.0016 - 0.0034i + 0.0029\rho_H \exp(i\phi_H), \\ a_7 &= -0.00003 - 0.00004i - 0.00003\rho_H \exp(i\phi_H) \\ a_9 &= -0.009 - 0.0001i + 0.0001\rho_H \exp(i\phi_H), \end{aligned} \quad (3)$$

where the complex parameter $\rho_H \exp(i\phi_H)$ represents the end-point singularity term in the hard-scattering corrections $X_H = (1 + \rho_H \exp(i\phi_H)) \ln(\frac{m_B}{\Lambda_h})$ [10,11].

For the annihilation terms, for $B \to PP$ decays, we have

$$\begin{aligned} b_2 &= -0.0041 - 0.0071\rho_A \exp(i\phi_A) \\ &\quad - 0.0019(\rho_A \exp(i\phi_A))^2, \\ b_3 &= -0.0071 - 0.016\rho_A \exp(i\phi_A) \\ &\quad - 0.0093(\rho_A \exp(i\phi_A))^2, \\ b_3^{ew} &= -0.00012 - 0.00016\rho_A \exp(i\phi_A) \\ &\quad + 0.000003(\rho_A \exp(i\phi_A))^2, \end{aligned} \quad (4)$$

where b_i are evaluated with the factor $f_B f_{M_1} f_{M_2}$ included and normalized relative to the factor $f_K F_0^{B\pi}(m_B^2 - m_\pi^2)$ in the factorizable terms, and ρ_A, like ρ_H, appears in the divergent annihilation term $X_A = (1 + \rho_A \exp(i\phi_A)) \ln(\frac{m_B}{\Lambda_h})$.

The $B \to K\pi$ decay amplitude with the factorizable part [9] and the annihilation term [10,11,13] is

$$\begin{aligned} A(B^+ \to K^+\pi^0) = &-i\frac{G_F}{2} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)(V_{ub}V_{us}^* a_1 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)[a_4 + a_{10} + (a_6 + a_8)r_\chi]) \\ &- i\frac{G_F}{2} f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2)\left(V_{ub}V_{us}^* a_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \times \frac{3}{2}(a_9 - a_7)\right) - i\frac{G_F}{2} f_B f_K f_\pi \\ &\times [V_{ub}V_{us}^* b_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \times (b_3 + b_3^{ew})] \end{aligned} \quad (5)$$

$$\begin{aligned} A(B^+ \to K^0\pi^+) = &-i\frac{G_F}{\sqrt{2}} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left[a_4 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8\right)r_\chi\right] \\ &- i\frac{G_F}{\sqrt{2}} f_B f_K f_\pi [V_{ub}V_{us}^* b_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \times (b_3 + b_3^{ew})], \end{aligned} \quad (6)$$

and for B^0,

$$A(B^0 \to K^+\pi^-) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)(V_{ub}V_{us}^* a_1 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)[a_4 + a_{10} + (a_6 + a_8)r_\chi]) - i\frac{G_F}{\sqrt{2}} f_B f_K f_\pi \left[(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left(b_3 - \frac{b_3^{ew}}{2}\right)\right] \tag{7}$$

$$A(B^0 \to K^0\pi^0) = i\frac{G_F}{2} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left[a_4 - \frac{1}{2}a_{10} + \left(a_6 - \frac{1}{2}a_8\right)r_\chi\right] - i\frac{G_F}{2} f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2)\left(V_{ub}V_{us}^* a_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \times \frac{3}{2}(a_9 - a_7)\right) + i\frac{G_F}{2} f_B f_K f_\pi \left[(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left(b_3 - \frac{b_3^{ew}}{2}\right)\right], \tag{8}$$

where $r_\chi = \frac{2m_K^2}{(m_b - m_d)(m_d + m_s)}$ is the chirally enhanced term in the penguin O_6 matrix element. We also need the $B^+ \to \pi^+\pi^0$ amplitude:

$$A(B^+ \to \pi^+\pi^0) = -i\frac{G_F}{2} f_\pi F_0^{B\pi}(m_\pi^2)(m_B^2 - m_\pi^2)\left(V_{ub}V_{ud}^*(a_1 + a_2) + (V_{ub}V_{ud}^* + V_{cb}V_{cd}^*)\frac{3}{2}(a_9 - a_7 + a_{10} + a_8 r_\chi)\right). \tag{9}$$

We see that the $B \to K\pi$ decay amplitudes consist of a QCD penguin (P) $a_4 + a_6 r_\chi$, a color-allowed tree (T) a_1, a color-suppressed tree (C) a_2, a color-allowed electroweak penguin(EW) $a_9 - a_7$, a color-suppressed electroweak penguin (EWC) $a_{10} + a_8 r_\chi$ term.

Similar expressions for the QCD coefficients for $B \to PV$ decays with hard-scattering corrections and annihilation terms used in the calculations are not shown here but can be found in [10,11,13,14]. For the CKM matrix elements, since the inclusive and exclusive data on $|V_{ub}|$ differ by a large amount and the higher inclusive data exceeds the unitarity limit for $R_b = |V_{ud}V_{ub}^*|/|V_{cd}V_{cb}^*|$ with the current value $\sin(2\beta) = 0.682 \pm 0.019$ [15], we shall determine $|V_{ub}|$ from the more precise $|V_{cb}|$ data [16]. As mentioned in [17], we have

$$|V_{ub}| = \frac{|V_{cb}V_{cd}^*|}{|V_{ud}^*|} |\sin\beta| \sqrt{1 + \frac{\cos^2\alpha}{\sin^2\alpha}}. \tag{10}$$

With $\alpha = (93.7 \pm 10.6)°$ [18] and $|V_{cb}| = (41.78 \pm 0.30 \pm 0.08) \times 10^{-3}$ [19], we find, neglecting the errors,

$$|V_{ub}| = 3.56 \times 10^{-3}, \tag{11}$$

in good agreement with the exclusive data in the range $|V_{ub}| = 3.33$–3.51 [19]. A recent UT fit also gives $|V_{ub}| = (3.61 \pm 0.12) \times 10^{-3}$ and $|V_{cb}| = (41.53 \pm 0.30 \pm 0.66) \times 10^{-3}$ close to the above values [20]. The measurements of the $B_s - \bar{B}_s$ mixing also allow the extraction of $|V_{td}|$ from $B_d - \bar{B}_d$ mixing data. The current determination [21] gives $|V_{td}/V_{ts}| = (0.208^{+0.008}_{-0.006})$ which in turn can be used to determined the angle γ from the unitarity relation [22]:

$$|V_{td}| = \frac{|V_{cb}V_{cd}^*|}{|V_{tb}^*|} \sin\gamma \sqrt{1 + \frac{\cos^2\alpha}{\sin^2\alpha}}. \tag{12}$$

With $|V_{tb}| = 1$, we find $\gamma = 67.6°$ which implies an angle $\alpha = 90.7°$, in good agreement with the new Belle value $\alpha = (93.7 \pm 10.6)°$ [18] mentioned above. The value $\gamma = 67.6°$ is also consistent with the current UT fit value $\gamma = (70.3 \pm 3.7)°$ [20]. In the following calculations, we shall use the unitarity triangle values for $|V_{ub}|$ and γ. For other hadronic parameters, we use the values in Table 1 of [11] and take $m_s(2\ \mathrm{GeV}) = 80$ MeV. For the $B \to \pi$ and $B \to K$ transition form factor, we use the current light-cone sum rules central value [23]:

$$F_0^{B\pi}(0) = 0.258, \qquad F_0^{BK}(0) = 0.33. \tag{13}$$

The computed branching ratios with $\rho_A = 1$, $\rho_H = 1$, $\phi_H = 0$, and $\phi_A = -55°$ as in scenario S4 of [11] are shown in Table I. As can be seen, QCDF with power corrections from penguin annihilation as in S4 [11,24] could bring the branching ratios closer to experiments. With a different choice of the annihilation parameters, as given in [25], one could increase further the predicted

TABLE I. The measured and computed QCDF branching ratios shown with the QCDF amplitudes for $B \to PV$ decays.

Decay	$A \times 10^8$ GeV(QCDF)	BR $\times 10^6$(QCDF)	BR $\times 10^6$(exp) [15,27]
$B^+ \to \pi^+\pi^0$	$2.162 - 1.112i$	5.535	5.5 ± 0.4
$B^+ \to \rho^0\pi^+$	0.925–2.752i	7.732	8.3 ± 1.2
$B^+ \to \rho^+\pi^0$	1.863–3.055i	11.744	10.9 ± 1.4
$B^+ \to K^+\pi^0$	$0.725 + 3.244i$	10.266	$12.94^{+0.52}_{-0.51}$
$B^+ \to K^0\pi^+$	$0.162 + 4.399i$	18.002	23.79 ± 0.75
$B^0 \to K^+\pi^-$	$0.887 + 4.180i$	15.782	$19.57^{+0.53}_{-0.52}$
$B^0 \to K^0\pi^0$	−0.016–2.817i	6.863	9.9 ± 0.5
$B^+ \to K^+\rho^0$	$1.422 + 0.4.483i$	2.052	3.7 ± 0.5
$B^+ \to K^0\rho^+$	2.463–0.363i	5.637	8.0 ± 1.5
$B^0 \to K^+\rho^-$	$2.608 + 0.466i$	5.943	7.0 ± 0.9
$B^0 \to K^0\rho^0$	$-2.164 + 0.411i$	4.107	4.7 ± 0.6
$B^+ \to K^{*+}\pi^0$	$-1.495 + 0.786i$	2.589	8.2 ± 1.8
$B^+ \to K^{*0}\pi^+$	−1.876–0.022i	3.206	$10.1^{+0.8}_{-0.9}$
$B^0 \to K^{*+}\pi^-$	$-1.657 + 0.946i$	3.084	8.4 ± 0.8
$B^0 \to K^{*0}\pi^0$	$1.003 + 0.128i$	0.867	3.3 ± 0.6

decay rates to values consistent with experiments. For the CKM-allowed tree-dominated decays, as shown in Table I and in [8], the predicted $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ decay rates agree well with experiments. Therefore. we can use the QCDF tree amplitude for $\Delta S = 1$ $B \to PP, PV$ in the determination of the phases of the decay amplitudes. For this purpose, one needs to express the $\Delta S = 1$ $B \to PP, PV$ decay amplitudes in terms of isospin amplitudes. Following [6,7], we have, for $B \to K\pi$, in the notation of [7],

$$\begin{aligned} A_{K^+\pi^0} &= \frac{2}{3}B_3 + \sqrt{\frac{1}{3}}(A_1 + B_1), \\ A_{K^0\pi^+} &= \frac{-\sqrt{2}}{3}B_3 + \sqrt{\frac{2}{3}}(A_1 + B_1), \\ A_{K^+\pi^-} &= \frac{\sqrt{2}}{3}B_3 + \sqrt{\frac{2}{3}}(A_1 - B_1), \\ A_{K^0\pi^0} &= \frac{2}{3}B_3 - \sqrt{\frac{1}{3}}(A_1 - B_1), \end{aligned} \tag{14}$$

with B_1, B_3 the $I = 1/2$ and $I = 3/2$ isospin amplitudes in terms of the decay amplitudes

$$\begin{aligned} A_1 &= \frac{\sqrt{6}}{4}(A_{K^0\pi^+} + A_{K^+\pi^-}) \\ B_1 &= \frac{1}{\sqrt{3}}A_{K^+\pi^0} + \frac{\sqrt{6}}{12}A_{K^0\pi^+} - \frac{\sqrt{6}}{4}A_{K^+\pi^-} \\ B_3 &= A_{K^+\pi^0} - \frac{1}{\sqrt{2}}A_{K^0\pi^+} \end{aligned} \tag{15}$$

with the expressions in QCDF given by

$$\begin{aligned} A_1 = {} & -i\frac{G_F}{2}f_K F_0^{B\pi}(m_K^2)\frac{\sqrt{3}}{2}(m_B^2 - m_\pi^2)\Bigg(V_{ub}V_{us}^* a_1 \\ & + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\Bigg[2a_4 + \frac{1}{2}a_{10} \\ & + \left(2a_6 + \frac{a_8}{2}\right)r_\chi\Bigg]\Bigg) - i\frac{G_F}{2}f_B f_K f_\pi \\ & \times\Bigg(V_{ub}V_{us}^* b_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \\ & \times\left(b_3 + \frac{3}{2}b_3^{ew}\right)\Bigg). \end{aligned} \tag{16}$$

For B_1, we have

$$\begin{aligned} B_1 = {} & -i\frac{G_F}{2}f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)\frac{\sqrt{3}}{2} \\ & \times\Bigg(V_{ub}V_{us}^*\frac{1}{3}a_1 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \\ & \times\left[\frac{1}{2}(a_{10} + a_8 r_\chi)\right]\Bigg) - i\frac{G_F}{2}f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2) \\ & \times\left(V_{ub}V_{us}^*\frac{2}{3}a_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)2(a_9 - a_7)\right) \\ & - i\frac{G_F}{2}f_B f_K f_\pi - [V_{ub}V_{us}^* b_2 \\ & + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)b_3^{ew}] \end{aligned} \tag{17}$$

and for B_3,

$$B_3 = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2) \times \left(V_{ub}V_{us}^* a_1 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) - \frac{3}{2}(a_{10} + a_8 r_\chi) \right) - i\frac{G_F}{\sqrt{2}} f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2) \times \left(V_{ub}V_{us}^* a_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*) \times \frac{3}{2}(a_9 - a_7) \right). \tag{18}$$

We see that B_3 does not contain the strong penguin a_4 and a_6 terms. In the $SU(3)$ limit, apart from the small electroweak penguin terms, the main contribution to B_3 comes from the large color-favored $(a_1 + a_2)$ term, as in the $B^+ \to \pi^+\pi^0$ decay, for which QCDF without the strong penguin contributions, is quite reliable, as can be seen from the good agreement with experiments for $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ decays shown in Table I. The relation between the B_3 and the $B^+ \to \pi^+\pi^0$ decay amplitude can also be obtained in a general proof based on a model-independent approach to charmless $B \to PP$ decays, given recently in [26]. In terms of the $SU(3)/U(3)$ invariant amplitudes, one has, putting aside the CKM factor,

$$T^{B_u}_{\pi^-\pi^0} = \frac{8}{\sqrt{2}} C^T_{\overline{15}}$$
$$T^{B_u}_{\pi^0 K^-} = \frac{1}{\sqrt{2}}(C^T_{\bar{3}} - C^T_{\bar{6}} + 3A^T_{\overline{15}} + 7A^T_{\overline{15}})$$
$$T^{B_u}_{\pi^-\bar{K}^0} = (C^T_{\bar{3}} - C^T_{\bar{6}} + 3A^T_{\overline{15}} - C^T_{\overline{15}}). \tag{19}$$

From Eq. (19), we get

$$B_3 = \sqrt{2}T^{B_u}_{\pi^0 K^-} - T^{B_u}_{\pi^-\bar{K}^0} = \frac{8}{\sqrt{2}} C^T_{\overline{15}} = T^{B_u}_{\pi^-\pi^0} \tag{20}$$

in agreement with QCDF in the $SU(3)$ limit. This relation can also be derived in a simple manner by using the topological amplitudes. We have [8]

$$A_{K^+\pi^0} = -\frac{1}{\sqrt{2}}(p' + t' + c'), \qquad A_{K^0\pi^+} = p'$$
$$A_{\pi^+\pi^0} = -\frac{1}{\sqrt{2}}(t + c)$$
$$B_3 = -(t' + c'), \tag{21}$$

showing $B_3 = \sqrt{2}A_{\pi^+\pi^0}$ in the $SU(3)$ limit.

Given QCDF for the CKM-favored tree-dominated decay amplitudes, the $SU(3)$ breaking effects can be automatically taken into account in the QCDF expressions for penguin-dominated decays. The point we made in this paper is that QCDF works well for processes with a large color-favored tree contribution, but without the strong penguin terms. The agreement with experiments for $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ measured branching ratios and the rather well-known short-distance Wilson coefficients for the tree operators shows that the central values for the form factors and decay constants involved are consistent with experiments and can be used in QCDF calculations with penguin-dominated decays. Thus, the uncertainties for the QCDF branching ratios depend only on the accuracy of the measured $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ branching ratios, which are 10%, while the theoretical errors and uncertainties in the current QCDF calculations are quite large [8,10,11]. This shows the advantage of using the measured $B^+ \to \pi^+\pi^0, \rho^0\pi^+, \rho^+\pi^0$ branching ratios to obtain the correct form factor values for QCDF calculations of the $B \to K\pi$, $K\rho$, $K^*\pi$ decay rates and, in particular, for the $I = 3/2$ isospin amplitude B_3, though the $SU(3)$ relation between B_3 and the $B^+ \to \pi^+\pi^0$ amplitude in Eq. (20) or Eq. (21) is useful for a qualitative argument that B_3 is exactly the $B^+ \to \pi^+\pi^0$ $I = 2$ amplitude in the $SU(3)$ limit. For the penguin-dominated decays, we do not expect QCDF to produce a correct penguin amplitude in the $B \to K\pi, K\rho$, $K^*\pi$ decays which could have power correction terms like the penguin annihilation mentioned in the literature [11,24,25], especially for the predicted $K^*\pi$ branching ratios which are below the measured values by more that 30%.

III. DETERMINATION OF PHASES OF THE $\Delta S = 1$ $B \to PP,PV$ DECAY AMPLITUDES

With the $I = 3/2$ amplitude given by QCDF, we now proceed to the determination of the relative phase between the tree and penguin amplitudes.

As shown in [7], by taking the sum of the B^+ and B^0 absolute square of the amplitudes $|A|^2$ or the decay rates, from Eqs. (14), we have

$$|A_1 + B_1|^2 = |A_{K^+\pi^0}|^2 + |A_{K^0\pi^+}|^2 - \frac{2}{3}|B_3|^2 \tag{22}$$

$$|A_1 - B_1|^2 = |A_{K^+\pi^-}|^2 + |A_{K^0\pi^0}|^2 - \frac{2}{3}|B_3|^2. \tag{23}$$

With the lengths of the sides $A_1 + B_1$ and $A_1 - B_1$ given by the decay rates of the four $B \to K\pi$ decay modes in Eqs. (22)–(23), the angles of the triangle formed with the decay amplitude, B_3, and with $A_1 + B_1$ and $A_1 - B_1$, respectively. This gives us the relative phase between the $I = 3/2$ tree and the $I = 1/2$ penguin amplitudes for a precise test of the QCDF. Clearly, isospin amplitudes are needed to obtain the phases in the $B \to K\pi$, $K\rho$, and $K^*\pi$ decays which are in the interference term between B_3 and $A_1 + B_1$ and between B_3 and $A_1 - B_1$, and each length $|A_1 + B_1|$ and $|A_1 - B_1|$ depends on the branching ratios of two decay modes. Let $\delta_{1,2}$ be the relative phase between B_3 and $A_1 + B_1$, and between B_3 and $A_1 - B_1$, respectively, from Eqs. (14), and using Eqs. (22)–(23), we have

$$\cos(\delta_1) = \frac{\sqrt{3}(2|A_{K^+\pi^0}|^2 - |A_{K^0\pi^+}|^2 - |B_3|^2/3)}{4|B_3||A_1 + B_1|} \quad (24)$$

$$\cos(\delta_2) = \frac{\sqrt{3}(|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2 + |B_3|^2/3)}{4|B_3||A_1 - B_1|}. \quad (25)$$

Since all four penguin-dominated decay modes have similar decay rates, the differences $|A_{K^0\pi^+}|^2 - 2|A_{K^+\pi^0}|^2$ and $|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2$ become small, and errors and uncertainties in the measured decay rates would make it difficult to obtain a correct value for $\cos(\delta_1)$ and $\cos(\delta_2)$. Another problem which could affect the phase determination is the consistency of the four measured decay rates imposed by an isospin relation between the decay rates which is given as [7,28], with QCDF values for $|B_3|^2$ and $\mathrm{Re}(B_3^* B_1)$:

$$|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2 = -[|A_{K^0\pi^+}|^2 - 2||A_{K^+\pi^0}|^2] - \left[\frac{4}{3}|B_3|^2 + \frac{8}{\sqrt{3}}\mathrm{Re}(B_3^* B_1)\right]_{K\pi}. \quad (26)$$

This relation gives a branching ratio 8.98×10^{-6} for $B^0 \to K^0\pi^0$ to be compared with the measured value of $(9.93 \pm 0.49) \times 10^{-6}$ which produces a cancellation in the quantity $|A_{K^+\pi^-}|^2 - 2|A_{K^0\pi^0}|^2$ in Eq. (25) and a phase δ_2 near 90°, which deviates largely from the phase between B_3 and $A_1 + B_1$, in contradiction with the isospin analysis. Since $|B_1|$ is small compared with $|A_1 + B_1|$ and $|A_1 - B_1|$, the difference $\delta_2 - \delta_1$ should be small. Using the above estimated branching ratio for $B^0 \to K^0\pi^0$, one would obtain $\delta_2 = 75.199°$, close to the value 77.296° for δ_1, consistent with isospin analysis. Thus, a correct value for δ_2 consistent with δ_1 requires a lower value for the $B^0 \to K^0\pi^0$ branching ratio. This lower value for $B^0 \to K^0\pi^0$ could turn out to be the correct value since, over the years, the $B^0 \to K^0\pi^0$ branching ratio has decreased to the present value.

The phases for $B \to K\rho$ and $B \to K^*\pi$ decays can be obtained from the above expressions by making a straightforward substitution with the $K\rho$ and $K^*\pi$ decay rates. In Table II, we give the relative isospin phases $\delta_{1,2}$ for $B \to K\pi$, $K\rho$, and $K^*\pi$ obtained from QCDF and from the measured decay rates.

As with the $B \to K\pi$ decays, the determination of δ_1 in the $B \to K\rho$ decays is also subject to large uncertainties. With almost a cancellation in the difference $(|A_{K^0\rho^+}|^2 - 2|A_{K^+\rho^0}|^2)$, one would get a value $\delta_1 = 98.791°$, very different from the value 110.638° for δ_2. In fact, using the isospin relation for $K\rho$ given as

$$(|A_{K^0\rho^+}|^2 - 2|A_{K^+\rho^0}|^2) = -(|A_{K^+\rho^-}|^2 - 2||A_{K^0\rho^0}|^2) + \left[-\frac{4}{3}|B_3|^2 - \frac{8}{\sqrt{3}}\mathrm{Re}(B_3^* B_1)\right]_{K\rho}, \quad (27)$$

we would get a branching ratio $(9.15 \pm 1.2) \times 10^{-6}$ for $B^+ \to K^0\rho^+$ higher than the measured value of $(8.0^{+1.5}_{-1.4}) \times 10^{-6}$. This predicted branching ratio then gives $\delta_1 = 109.217°$ close to the value 110.638° for δ_2, consistent with the fact that, as in the $B \to K\pi$ decays, since $|B_1|$ is small compared with the penguin amplitude $|A_1|$, δ_1 and δ_2 should be close to each other, as seen from the QCDF values given in Table II.

A similar problem also appears in the $B \to K^*\pi$ decay, as the isospin relation similar to that for $B \to K\rho$ in Eq. (27) would give a branching ratio $(6.3 \pm 2.2) \times 10^{-6}$ for the $B \to K^*\pi$ decay, lower than the measured value of $(8.2 \pm 1.9) \times 10^{-6}$. For this reason, the phases $\delta_{1,2}$ for the $B \to K^*\pi$ decay are obtained using only the $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$ decay rates and the isospin relation, as shown in Table II. We note that for the $B \to K\pi$ decays, the errors on the phases $\delta_{1,2}$, are around 15°. This could be due to the large cancellation between the measured branching ratios which, however, have small errors, on the order of a few percent. For this reason, we will not give errors on the phases for the $K\rho$ and $K^*\pi$ decays for which the errors are more than 10%. We note also that the errors for $B \to K\pi$ shown in Table II are comparable to the errors found in the determination of the relative phase between the three-gluon and the one-photon annihilation amplitudes of the $\psi(2S)$ decays to pseudoscalar meson pairs, for which a relative phase of $(-82 \pm 29)°$ or $(+121 \pm 27)°$ is found in [29]. What is remarkable with the result we found is that all the phases for the $B \to K\pi$, $K\rho$, and $K^*\pi$ decays obtained with the central values for the measured branching ratios consistently show only small deviations from the QCDF values. The implication of this result is that one may need power correction

TABLE II. The relative isospin phases given by QCDF and obtained from the measured decay rates for $B \to K\pi$, $K\rho$ and $K^*\pi$ decays. The numbers marked as "estimated" are the phases obtained with isospin relation as explained in the text. Errors are estimated to be in the range ±(10–15)°.

Decay	δ_1(deg)(QCDF)	δ_1(deg)(exp)	δ_2(deg)(QCDF)	δ_2(deg)(exp)
$B \to K\pi$	71.891	77.296 ± 15	68.968	75.199 ± 15 (estimated)
$B \to K\rho$	113.701	109.217 (estimated)	110.925	110.638
$B \to K^*\pi$	67.838	73.351 (estimated)	58.194	68.078

terms, probably of perturbative QCD origin, to bring QCDF values close to the measured decay rates, without the need for a strong phase from long-distance rescattering effects.

IV. CONCLUSION

With the tree amplitude known from the QCDF tree-dominated $B \to PP, PV$ decays, we are able to determine the relative phases of the tree-penguin interference term in the $B \to K\pi$, $K\rho$, and $K^*\pi$ decays. We find that the phases in the tree-penguin interference terms differ slightly from the QCDF phases, in particular with an uncertainty $\pm 15°$, more or less, for $B \to K\pi$. For the $K\rho$ and $K^*\pi$ decays, this uncertainty could be reduced considerably with more precise data with LHCb and the upcoming super Belle, which would allow a precise test of QCDF.

[1] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[2] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
[3] A. N. Kamal and T. N. Pham, Phys. Rev. D **50**, 6849 (1994).
[4] R. Baldini, C. Bini, and E. Luppi, Phys. Lett. B **404**, 362 (1997).
[5] M. Suzuki, Phys. Rev. D **60**, 051501 (1999).
[6] N. G. Deshpande, X. G. He, W. S. Hou, and S. Pakvasa, Phys. Rev. Lett. **82**, 2240 (1999).
[7] C. Isola and T. N. Pham, Phys. Rev. D **62**, 094002 (2000).
[8] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, Phys. Rev. D **91**, 014011 (2015).
[9] A. Ali, G. Kramer, and C. Lü, Phys. Rev. D **58**, 094009 (1998).
[10] M. Beneke and M. Neubert, Nucl. Phys. **B651**, 225 (2003).
[11] M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).
[12] B. Dutta, C. S. Kim, S. Oh, and G. H. Zhu, Eur. Phys. J. C **37**, 273 (2004).
[13] D. S. Du, H. Gong, J. F. Sun, D. S. Yang, and G. H. Zhu, Phys. Rev. D **65**, 094025 (2002).
[14] R. Alexan, P.-F. Giraud, V. Morénas, O. Pène, and A. S. Safir, Phys. Rev. D **67**, 094019 (2003).
[15] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).
[16] For a recent proposal to measure $|V_{ub}|$ in B_{l4} decays, see X. W. Kang, B. Kubis, C.Hanhart, and Ulf-G. Meißner, Phys. Rev. D **89**, 053015 (2014).
[17] T. N. Pham, Phys. Rev. D **77**, 014024 (2008).
[18] P. Vanhoefer *et al.* Belle Collaboration, Phys. Rev. D **93**, 032010 (2016).
[19] E. Barberio, in *Proceedings of XXIII International Symposium on Lepton and Photon Interactions at High Energy, 2007, Daegu, Korea*, edited by D. Son, and S. K. O. Daegu, (Kyungpook National University, South Korea, 2009). p 585
[20] M. Bona *et al.* (UTfit Collaboration), Summary of results at http://www.utfit.org/.
[21] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **97**, 062003 (2006).
[22] T. N. Pham, in *Proceedings of 2nd Workshop on the CKM Unitarity Triangle, Durham, 2003*, eConf Durham, UK, C034052, (2003), WG415.
[23] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005).
[24] T. N. Pham and Guohuai Zhu, Phys. Rev. D **69**, 114016 (2004).
[25] H. Y. Cheng and C. K. Chua, Phys. Rev. D **80**, 114008 (2009).
[26] Y. K. Hsiao, C. F. Chang, and X. G. He, arXiv:1512.09223.
[27] E. Barberio *et al.* (Heavy Flavor Averaging Group Collaboration), arXiv:0808.1297.
[28] H. Lipkin, Phys. Lett. B **445**, 403 (1999); M. Gronau and J. L. Rosner, Phys. Rev. D **59**, 113002 (1999).
[29] J. S. Bai *et al.* BES Collaboration, Phys. Rev. Lett. **92**, 052001 (2004).