

## Roles of scalar mesons in charmless $\Lambda_b$ decays

Y. K. Hsiao,<sup>1,2,3</sup> Yu-Heng Lin,<sup>3</sup> Yao Yu,<sup>1</sup> and C. Q. Geng<sup>1,2,3</sup>

<sup>1</sup>Chongqing University of Posts and Telecommunications, Chongqing 400065, China

<sup>2</sup>Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan

<sup>3</sup>Department of Physics, National Tsing Hua University, Hsinchu 300, Taiwan

(Received 31 March 2016; published 8 June 2016)

We first study the charmless two-body  $\Lambda_b$  decays with scalar mesons as the final states and predict that  $\mathcal{B}(\Lambda_b \rightarrow \Lambda f_0(980, 1500)) = (2.9 \pm 0.7, 12.4 \pm 3.8) \times 10^{-6}$  and  $\mathcal{B}(\Lambda_b \rightarrow p K_0^{*-}(800, 1430)) = (1.9 \pm 0.5, 14.1 \pm 4.5) \times 10^{-6}$ . With the resonant  $f_0(980, 1500) \rightarrow (\pi^+\pi^-, K^+K^-)$  and  $K_0^{*-} \rightarrow \bar{K}^0\pi^-$  decays, we then obtain  $\mathcal{B}(\Lambda_b \rightarrow \Lambda(\pi^+\pi^-, K^+K^-)) = (4.2 \pm 1.0, 3.5 \pm 0.7) \times 10^{-6}$  and  $\mathcal{B}(\Lambda_b \rightarrow p\bar{K}^0\pi^-) = (10.4 \pm 2.9) \times 10^{-6}$ , in comparison with the data of  $(4.6 \pm 1.9, 15.9 \pm 2.6) \times 10^{-6}$  and  $(12.6 \pm 4.0) \times 10^{-6}$ , respectively, from LHCb. Our results for  $\Lambda_b \rightarrow \Lambda\pi^+\pi^-$  and  $\Lambda_b \rightarrow p\bar{K}^0\pi^-$  would be regarded as the first evidence of scalar meson production in the antitriplet  $b$  baryon decays. The smaller predicted value of  $\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+K^-)$  indicates the existence of other resonant contributions to the decay, such as  $\Lambda_b \rightarrow K^-(N^{*+} \rightarrow)\Lambda K^+$ .

DOI: 10.1103/PhysRevD.93.114008

### I. INTRODUCTION

The LHCb Collaboration has recently measured the three-body  $\Lambda_b$  decays [1,2], given by

$$\begin{aligned}\mathcal{B}(\Lambda_b \rightarrow \Lambda\pi^+\pi^-) &= (4.6 \pm 1.2 \pm 1.4 \pm 0.6) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow \Lambda K^+K^-) &= (15.9 \pm 1.2 \pm 1.2 \pm 2.0) \times 10^{-6}, \\ \mathcal{B}(\Lambda_b \rightarrow p\bar{K}^0\pi^-) &= (12.6 \pm 1.9 \pm 0.9 \pm 3.4 \pm 0.5) \times 10^{-6}.\end{aligned}\quad (1)$$

However, the present available calculations in the literature show that  $\mathcal{B}(\Lambda_b \rightarrow \Lambda\rho^0, \rho^0 \rightarrow \pi^+\pi^-)$  is merely in the range of  $10^{-9}$ – $10^{-7}$  [3–5], which is much smaller than that in Eq. (1). Similarly, according to the measured  $\mathcal{B}(\Lambda_b \rightarrow \Lambda\phi) = (5.18 \pm 1.04 \pm 0.35_{-0.62}^{+0.67}) \times 10^{-6}$  [6] and the predicted  $\mathcal{B}(\Lambda_b \rightarrow pK^{*-}) = (2.5 \pm 0.3 \pm 0.2 \pm 0.3) \times 10^{-6}$  [7], the resonant vector  $\phi \rightarrow K^+K^-$  and  $K^{*-} \rightarrow \bar{K}^0\pi^-$  contributions lead to  $\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+K^-, p\bar{K}^0\pi^-) = (2.5 \pm 0.6, 1.7 \pm 0.3) \times 10^{-6}$ , which are also unable to explain the data in Eq. (1). Clearly, there must be some undiscovered contributions to these three-body  $\Lambda_b$  decays.

In this study, we propose to use the resonant scalar mesons as the dominant productions to resolve the deficits for the three-body decays in Eq. (1). Explicitly, we consider the scalar meson decays of  $f_0(980, 1500) \rightarrow (\pi^+\pi^-, K^+K^-)$  and  $K_0^{*-}(1430) \rightarrow \bar{K}^0\pi^-$  through the charmless two-body processes of  $\Lambda_b \rightarrow \Lambda f_0(980, 1500)$  and  $\Lambda_b \rightarrow pK_0^{*-}(1430)$  to produce the three-body  $\Lambda_b$  decays in Eq. (1). We will demonstrate that  $\mathcal{B}(\Lambda_b \rightarrow \Lambda\pi^+\pi^-, \Lambda K^+K^-)$  and  $\mathcal{B}(\Lambda_b \rightarrow p\bar{K}^0\pi^-)$  can be taken as the first evidence for scalar meson production in charmless two-body  $\Lambda_b$  decays. Our present study will be useful to distinguish the resonant contributions of the two-quark, tetraquark, and glueball bound states, similarly to the

tetraquark and scalar meson searches in the charmful two-body cases [8,9].

Note that theoretical studies are still controversial concerning the underlying structures of the scalar mesons [10,11]. For example,  $f_0(980)$  is one of the scalar mesons lighter than 1 GeV to be identified as either the two-quark or four-quark (tetraquark) bound state [12,13], while  $f_0(1500)$  and  $K_0^{*-}(1430)$  heavier than 1 GeV belong to the conventional  $q\bar{q}$  nonet, but with  $f_0(1500)$  identified to primarily consist of either the glueball or the  $s\bar{s}$  bound states [14]. Nevertheless, the scalar quark currents in the decaying processes favor the formation of scalar mesons due to the quantum numbers of  $J^{PC} = 0^{++}$ , causing the enhanced branching ratios compared to the decays with the recoiled vector mesons of  $\rho^0$ ,  $\phi$ , and  $K^{*-}$ .

### II. FORMALISM

In accordance with the diagrams depicted in Fig. 1, the amplitudes of  $\Lambda_b \rightarrow \mathcal{B}_n S$  via the effective Hamiltonian can be decomposed as the matrix elements of the  $\Lambda_b \rightarrow \mathcal{B}_n$  baryon transitions as well as the vacuum-to-scalar-meson productions ( $0 \rightarrow S$ ), given by [15]

$$\begin{aligned}\mathcal{A}(\Lambda_b \rightarrow \Lambda f_0) &= \frac{G_F}{\sqrt{2}} \{ \alpha_3 \langle f_0 | \bar{s}\gamma_\mu s | 0 \rangle \langle \Lambda | \bar{s}\gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle \\ &\quad + \alpha_6^s \langle f_0 | \bar{s}s | 0 \rangle \langle \Lambda | \bar{s}(1 - \gamma_5) b | \Lambda_b \rangle \}, \\ \mathcal{A}(\Lambda_b \rightarrow p S_0^-) &= \frac{G_F}{\sqrt{2}} \{ \alpha_1^q \langle S_0^- | \bar{q}\gamma_\mu u | 0 \rangle \langle p | \bar{u}\gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle \\ &\quad + \alpha_6^q \langle S_0^- | \bar{q}u | 0 \rangle \langle p | \bar{u}(1 - \gamma_5) b | \Lambda_b \rangle \}, \\ \mathcal{A}(\Lambda_b \rightarrow \Lambda a_0^0) &= \frac{G_F}{\sqrt{2}} \alpha_2 \langle a_0^0 | \bar{q}\gamma_\mu q | 0 \rangle \langle \Lambda | \bar{s}\gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle,\end{aligned}\quad (2)$$

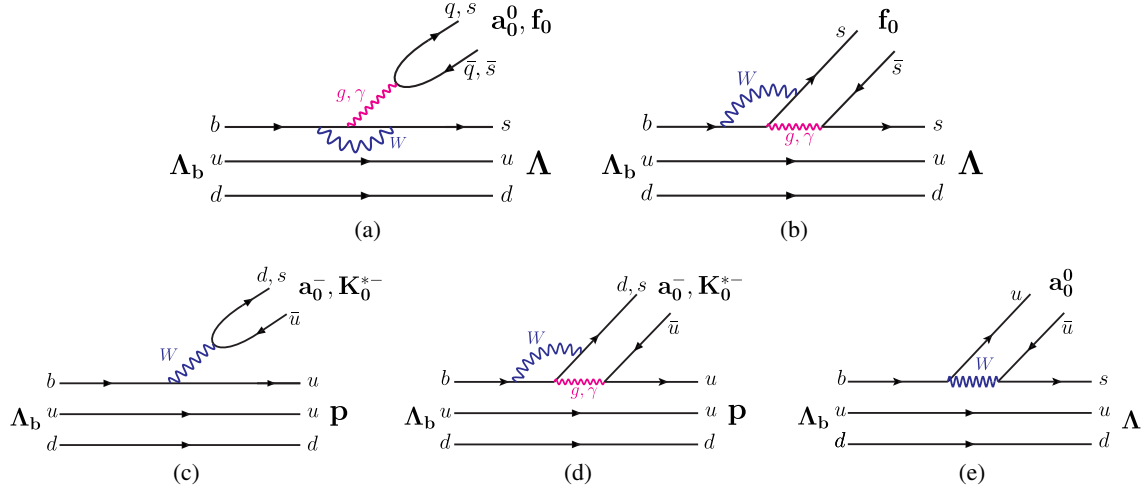


FIG. 1. Diagrams for the  $\Lambda_b^0 \rightarrow \mathcal{B}_n S$  decays, where  $S$  denotes the scalar mesons such as  $a_0^0$ ,  $f_0$ ,  $K_0^{*-}$ , and  $a_0^-$ .

where  $G_F$  is the Fermi constant,  $f_0 = f_0(980, 1500)$ ,  $S_0^- = (K_0^{*-}(800, 1430), a_0^-(980, 1450))$  for  $q = (s, d)$ , and  $a_0^0 = a_0^0(980, 1450)$  for  $q = u$  or  $d$ , while the parameters  $\alpha_i$  are given by

$$\begin{aligned} \alpha_1^q &= V_{ub}V_{uq}^*a_1 - V_{tb}V_{tq}^*a_4^q, \\ \alpha_2 &= V_{ub}V_{us}^*a_2 - V_{tb}V_{ts}^*3a_9/2, \\ \alpha_3 &= -V_{tb}V_{ts}^*(a_3 + a_4^s + a_5 - a_9/2), \\ \alpha_6^q &= V_{tb}V_{tq}^*2a_6^q, \end{aligned} \quad (3)$$

with  $V_{q_1 q_2}$  being the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, where  $a_i \equiv c_i^{\text{eff}} + c_{i\pm 1}^{\text{eff}}/N_c$  for  $i = \text{odd}$  (even) consists of the effective Wilson coefficients  $c_i^{\text{eff}}$  defined in Ref. [15] with the color number  $N_c$  fixed to be 3 in the naive factorization. Nonetheless, in the generalized factorization [15] used in this study, one is allowed to float  $N_c$  from 2 to  $\infty$  to estimate the nonfactorizable effects, which are taken as a part of the theoretical uncertainty. Note that although the use of the factorization method has been successful in various baryonic decays [16], the present case with scalar mesons such as  $f_0$  may contain some uncontrollable uncertainty due to the less-known meson structures.

The matrix elements of the  $0 \rightarrow S$  production are given by [17,18]

$$\langle S | \bar{q}_2 \gamma_\mu q_1 | 0 \rangle = f_S q_\mu, \quad \langle S | \bar{q}_2 q_1 | 0 \rangle = m_S \bar{f}_S, \quad (4)$$

with  $f_S$  and  $\bar{f}_S$  the decay constants, where  $q_\mu$  is the four-momentum vector. For the neutral scalar mesons, one has  $f_{f_0} = f_{a_0^0} = 0$  due to the charge conjugation invariance as well as the conservation of the vector current, such that the  $\alpha_{2,3}$  terms in Eq. (2) vanish, resulting in  $\mathcal{B}(\Lambda_b \rightarrow \Lambda a_0^0(980, 1450)) = 0$ . For the charged ones,  $f_S$

and  $\bar{f}_S$  are related as  $m_S f_S = (m_{q_2} - m_{q_1}) \bar{f}_S$  by using the equation of motion, such that  $f_{a_0^-} = \bar{f}_{a_0^-} (m_d - m_u) / m_{f_{a_0^-}}$  causes the suppressed  $\alpha_1^d$  term that contributes to  $\Lambda_b \rightarrow p a_0^-(980, 1450)$  in Eq. (2). The matrix elements of the  $\Lambda_b \rightarrow \mathcal{B}_n$  baryon transitions are parameterized as [7]

$$\begin{aligned} \langle \mathcal{B}_n | \bar{q} \gamma_\mu b | \Lambda_b \rangle &= \bar{u}_{\mathcal{B}_n} \left[ f_1 \gamma_\mu + \frac{f_2}{m_{\Lambda_b}} i \sigma_{\mu\nu} q^\nu + \frac{f_3}{m_{\Lambda_b}} q_\mu \right] u_{\Lambda_b}, \\ \langle \mathcal{B}_n | \bar{q} \gamma_\mu \gamma_5 b | \Lambda_b \rangle &= \bar{u}_{\mathcal{B}_n} \left[ g_1 \gamma_\mu + \frac{g_2}{m_{\Lambda_b}} i \sigma_{\mu\nu} q^\nu + \frac{g_3}{m_{\Lambda_b}} q_\mu \right] \gamma_5 u_{\Lambda_b}, \\ \langle \mathcal{B}_n | \bar{q} (1 - \gamma_5) b | \Lambda_b \rangle &= \bar{u}_{\mathcal{B}_n} (g_S \gamma_\mu - g_P \gamma_\mu \gamma_5) u_{\Lambda_b}, \end{aligned} \quad (5)$$

where  $f_i (g_i)$  ( $i = 1, 2, \text{ and } 3$ ) and  $g_{S(P)}$  are the form factors, where  $f_1 = g_1$  and  $f_{2,3} = g_{2,3} = 0$  are derived by the  $SU(3)$  flavor and  $SU(2)$  spin symmetries [19,20], in agreement with QCD models [21–23]. From the equation of motion, one obtains that  $g_S = a_S f_1$  and  $g_P = a_P g_1$  with  $a_{s,p} = (m_{\Lambda_b} \mp m_{\mathcal{B}_n}) / (m_b \mp m_q)$ .

### III. NUMERICAL RESULTS AND DISCUSSIONS

For our numerical analysis, the CKM matrix elements in the Wolfenstein parameterization are presented as

$$\begin{aligned} (V_{ub}, V_{tb}) &= (A\lambda^3(\rho - i\eta), 1), \\ (V_{ud}, V_{td}) &= (1 - \lambda^2/2, A\lambda^3), \\ (V_{us}, V_{ts}) &= (\lambda, -A\lambda^2), \end{aligned} \quad (6)$$

with  $(\lambda, A, \rho, \eta) = (0.225, 0.814, 0.120 \pm 0.022, 0.362 \pm 0.013)$  [10]. To estimate the nonfactorizable effects in the generalized factorization approach [15],  $a_i$  are taken as floating numbers for  $N_c$  from 2 to  $\infty$ . The specific values of  $a_i$  with  $N_c = 2, 3, \text{ and } \infty$  are given in Table I. With the double-pole momentum dependences, we have

TABLE I. The parameters  $a_i$  with  $N_c = 2, 3$ , and  $\infty$  to estimate the nonfactorizable effects in the generalized factorization.

$a_i$	$N_c = 2$	$N_c = 3$	$N_c = \infty$
$a_1$	0.98	1.05	1.17
$10^4 a_3$	$-13.1 - 15.6i$	72.4	$243.2 + 31.2i$
$10^4 a_4^d$	$-377.6 - 34.7i$	$-417.2 - 37.0i$	$-496.5 - 41.6i$
$10^4 a_4^s$	$-391.0 - 77.9i$	$-431.6 - 83.1i$	$-512.6 - 93.5i$
$10^4 a_5$	$-174.1 - 15.6i$	-65.8	$150.7 + 31.2i$
$10^4 a_6^d$	$-560.7 - 34.7i$	$-584.9 - 37.0i$	$-633.4 - 41.6i$
$10^4 a_6^s$	$-574.1 - 77.9i$	$-599.3 - 83.1i$	$-649.5 - 93.5i$
$10^4 a_9$	$-93.5 - 2.2i$	$-99.8 - 2.2i$	$-112.3 - 2.2i$

$f_1(g_1) = C_{B_s} / (1 - q^2/m_{\Lambda_b}^2)^2$  with  $C_p = \sqrt{3/2}C_\Lambda = 0.136 \pm 0.009$  [7,19]. The decay constants are scale ( $\mu$ )-dependent, adopted to be [18]

$$\begin{aligned} (\bar{f}_{f_0(980)}, \bar{f}_{f_0(1500)}) &= (460 \pm 25, 605 \pm 60) \text{ MeV}, \\ (\bar{f}_{K_0^*(800)}, \bar{f}_{K_0^*(1430)}) &= (420 \pm 25, 550 \pm 60) \text{ MeV}, \\ (\bar{f}_{a_0^-(980)}, \bar{f}_{a_0^-(1450)}) &= (450 \pm 25, 570 \pm 60) \text{ MeV}, \end{aligned} \quad (7)$$

with  $\mu = 2.1$  GeV, where the model with the  $q\bar{q}$  states for the scalar mesons has been assumed. The branching ratios for the scalar meson productions in the two-body  $\Lambda_b$  decays are shown in Table II.

In Table II, the three errors correspond to the uncertainties from the nonfactorizable effects in  $a_i$  with  $N_c = 2 - \infty$  illustrated in Table I, form factors, and decay constants, respectively. With the combined errors, we see that  $\mathcal{B}(\Lambda_b \rightarrow \Lambda f_0(980, 1500)) = (2.9 \pm 0.7, 12.4 \pm 3.8) \times 10^{-6}$  and  $\mathcal{B}(\Lambda_b \rightarrow p K_0^{*-}(800, 1430)) = (1.7 \pm 0.5, 14.1 \pm 4.5) \times 10^{-6}$ , which arise mainly from the  $\alpha_6^s$  terms due to the penguin contributions, where the  $\bar{s}u$  and  $\bar{s}s$  scalar currents favor the formations of  $f_0(980, 1500)$  and  $K_0^{*-}(800, 1430)$ , respectively. On the contrary, since the vector currents disfavor those of  $a_0^-(980, 1450)$ , which are in accordance

TABLE II. Our numerical results for the branching ratios of the two-body decays in units of  $10^{-6}$ , where the first, second, and third errors are from the nonfactorizable effects, form factors, and decay constants, respectively.

Decay modes	Our results
$\Lambda_b \rightarrow \Lambda f_0(980)$	$2.9_{-0.2}^{+0.5} \pm 0.4 \pm 0.3$
$\Lambda_b \rightarrow \Lambda f_0(1500)$	$12.4_{-1.0}^{+2.2} \pm 1.7 \pm 2.6$
$\Lambda_b \rightarrow p K_0^{*-}(800)$	$1.9_{-0.2}^{+0.4} \pm 0.3 \pm 0.2$
$\Lambda_b \rightarrow p K_0^{*-}(1430)$	$14.1_{-1.2}^{+2.5} \pm 1.9 \pm 3.2$
$\Lambda_b \rightarrow p a_0^-(980)$	$(7.8_{-0.6}^{+1.3} \pm 1.0 \pm 0.9) \times 10^{-2}$
$\Lambda_b \rightarrow p a_0^-(1450)$	$(6.3_{-0.5}^{+1.1} \pm 0.9 \pm 0.1) \times 10^{-1}$
$\Lambda_b \rightarrow \Lambda a_0^0(980, 1450)$	0

TABLE III. Our numerical results for the branching ratios of the three-body  $\Lambda_b$  decays in units of  $10^{-6}$ .

Decay modes	Our results	Data [1,2]
$\Lambda_b \rightarrow \Lambda \pi^+ \pi^-$	$4.2 \pm 1.0$	$4.6 \pm 1.9$
$\Lambda_b \rightarrow \Lambda K^+ K^-$	$3.5 \pm 0.7$	$15.9 \pm 2.6$
$\Lambda_b \rightarrow p \bar{K}^0 \pi^-$	$10.4 \pm 2.9$	$12.6 \pm 4.0$

with the decay constants of  $f_{a_0^-(980,1500)} \simeq (1.8, 1.1)$  MeV, we obtain  $\mathcal{B}(\Lambda_b \rightarrow p a_0^-(980, 1450))$  in the range of  $10^{-8} - 10^{-7}$ . Despite of the predictions of  $\mathcal{B}(\Lambda_b \rightarrow \Lambda a_0^0(980, 1450)) = 0$ , which are due to  $f_{a_0^0(980,1500)} = 0$ , the picture of the tetraquark state—that is,  $a_0^0(980) \equiv 1/\sqrt{2}(u\bar{u} - d\bar{d})s\bar{s}$ —may let the decays receive the  $s\bar{s}$  scalar current, such that whether or not the branching ratios of  $\Lambda_b \rightarrow \Lambda a_0^0(980, 1450)$  are equal to zero can be clear measurements to test the underlying structures of the scalar mesons.

With the two-body-decay branching ratios in Table II, connected to the resonant scalar meson data of  $\mathcal{B}(f_0(980) \rightarrow \pi^+ \pi^-, K^+ K^-) = (46 \pm 6, 16.1 \pm 7.2)\%$  [24],  $\mathcal{B}(f_0(1500) \rightarrow \pi^+ \pi^-, K^+ K^-) = (23.3 \pm 1.5, 4.3 \pm 0.5)\%$  [10], and  $\mathcal{B}(K_0^{*-}(1430) \rightarrow \bar{K}^0 \pi^-) = (62.0 \pm 6.6)\%$  [10], we show our results for the three-body  $\Lambda_b$  decays in Table III. It is interesting to see that our result for  $\Lambda_b \rightarrow \Lambda \pi^+ \pi^-$  fits the data very well, which would be regarded as the first evidence for scalar meson production in charmless two-body  $\Lambda_b$  decays. As a result, it is expected that we will find the resonant peaks for  $f_0(980)$  and  $f_0(1500)$  in the  $\pi\pi$  invariant mass spectrum. Like  $\Lambda_b \rightarrow \Lambda \pi^+ \pi^-$ , our result of  $\mathcal{B}(\Lambda_b \rightarrow p \bar{K}^0 \pi^-) = (10.4 \pm 2.9) \times 10^{-6}$  not only alleviates the theoretical deficit but also explains the data of  $(12.6 \pm 4.0) \times 10^{-6}$ . For  $\Lambda_b \rightarrow \Lambda K^+ K^-$ , our prediction due to the resonant scalar mesons seems much lower than that of the data. However, it still helps us to mimic the theoretical shortage, and leaves room for other contributions, such as the resonant  $\Lambda_b \rightarrow K^-(N^{*+} \rightarrow) \Lambda K^+$  decay. Since there exists the possible sign revealed in the  $m^2(\Lambda K^+)$  from the Dalitz plot [2], according to the study of the measured  $\mathcal{B}(\Lambda_b \rightarrow J/\psi(N^* \rightarrow) p K^+) \simeq (3.04 \pm 0.55) \times 10^{-4}$  [25,26], we estimate that  $\mathcal{B}(\Lambda_b \rightarrow K^-(N^{*+} \rightarrow) \Lambda K^+) \simeq 10 \times 10^{-6}$ , which can explain the deviation.

#### IV. CONCLUSIONS

In sum, we have studied the charmless two-body  $\Lambda_b$  decays with scalar mesons as final states. We predicted the first scalar meson productions in the  $\Lambda_b$  decays, such as  $\mathcal{B}(\Lambda_b \rightarrow \Lambda f_0(980, 1500)) = (2.9 \pm 0.7, 12.4 \pm 3.8) \times 10^{-6}$  and  $\mathcal{B}(\Lambda_b \rightarrow p K_0^{*-}(800, 1430)) = (1.9 \pm 0.5, 14.1 \pm 4.5) \times 10^{-6}$ . With the resonant  $f_0(980, 1500) \rightarrow \pi^+ \pi^-$ , we have obtained  $\mathcal{B}(\Lambda_b \rightarrow \Lambda \pi^+ \pi^-) = (4.2 \pm 1.0) \times 10^{-6}$ , which can explain the data of  $(4.6 \pm 1.9) \times 10^{-6}$  that was much

underestimated by the previous studies. Similarly, we have shown that the resonant scalar meson contributions from  $f_0(980, 1500)$  and  $K_0^*(1430)^-$  lead to  $\mathcal{B}(\Lambda_b \rightarrow \Lambda K^+ K^-) = (3.5 \pm 0.7) \times 10^{-6}$  and  $\mathcal{B}(\Lambda_b \rightarrow p \bar{K}^0 \pi^-) = (10.4 \pm 2.9) \times 10^{-6}$ , which alleviate the theoretical shortages compared to the current observations.

## ACKNOWLEDGMENTS

The authors would like to thank Dr. Eduardo Rodrigues for useful discussions. The work was supported in part by National Center for Theoretical Sciences, National Science Council (Grant No. NSC-101-2112-M-007-006-MY3) and MoST (Grant No. MoST-104-2112-M-007-003-MY3).

- 
- [1] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **04** (2014) 087.
- [2] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **05** (2016) 081.
- [3] X. H. Guo and A. W. Thomas, *Phys. Rev. D* **58**, 096013 (1998).
- [4] S. Arunagiri and C. Q. Geng, *Phys. Rev. D* **69**, 017901 (2004).
- [5] O. Leitner, Z. J. Ajaltouni, and E. Conte, [arXiv:hep-ph/0602043](https://arxiv.org/abs/hep-ph/0602043).
- [6] R. Aaij *et al.* (LHCb Collaboration), [arXiv:1603.02870](https://arxiv.org/abs/1603.02870).
- [7] Y. K. Hsiao and C. Q. Geng, *Phys. Rev. D* **91**, 116007 (2015); *Proc. Sci.*, FPCP2015 (2015) 073.
- [8] Y. K. Hsiao and C. Q. Geng, *Phys. Lett. B* **757**, 47 (2016).
- [9] A. Sharma, R. Dhir, and R. C. Verma, *Eur. Phys. J. C* **71**, 1 (2011).
- [10] K. A. Olive *et al.* (Particle Data Group), *Chin. Phys. C* **38**, 090001 (2014).
- [11] F. E. Close and N. A. Tornqvist, *J. Phys. G* **28**, R249 (2002).
- [12] S. Weinberg, *Phys. Rev. Lett.* **110**, 261601 (2013).
- [13] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **90**, 012003 (2014).
- [14] Y. K. Hsiao, C. C. Lih, and C. Q. Geng, *Phys. Rev. D* **89**, 077501 (2014).
- [15] A. Ali, G. Kramer, and C. D. Lu, *Phys. Rev. D* **58**, 094009 (1998).
- [16] Y. K. Hsiao and C. Q. Geng, *Phys. Rev. D* **93**, 034036 (2016).
- [17] W. Wang, Y. L. Shen, Y. Li, and C. D. Lu, *Phys. Rev. D* **74**, 114010 (2006).
- [18] H. Y. Cheng, C. K. Chua, and K. C. Yang, *Phys. Rev. D* **73**, 014017 (2006).
- [19] Y. K. Hsiao, P. Y. Lin, C. C. Lih, and C. Q. Geng, *Phys. Rev. D* **92**, 114013 (2015).
- [20] G. P. Lepage and S. J. Brodsky, *Phys. Rev. Lett.* **43**, 545 (1979); **43**, 1625(E) (1979).
- [21] A. Khodjamirian, Ch. Klein, Th. Mannel, and Y.-M. Wang, *J. High Energy Phys.* **09** (2011) 106; T. Mannel and Y. M. Wang, *J. High Energy Phys.* **12** (2011) 067.
- [22] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and P. Santorelli, *Phys. Rev. D* **88**, 114018 (2013).
- [23] Z. T. Wei, H. W. Ke, and X. Q. Li, *Phys. Rev. D* **80**, 094016 (2009).
- [24] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **89**, 092006 (2014).
- [25] Y. K. Hsiao and C. Q. Geng, *Phys. Lett. B* **751**, 572 (2015).
- [26] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 072001 (2015).