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White dwarfs in an ungravity-inspired model

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An ungravity-inspired model is employed to examine the astrophysical parameters of white dwarf stars (WDs) using polytropic and degenerate gas approaches. Based on the observed properties such as mass, radius, and luminosity of selected WDs, namely, Sirius B and ϵ Reticulum, bounds on the characteristic length and scaling dimension of the ungravity (UG) model are estimated. The UG effect on the Chandrasekhar limit for WDs is shown. The UG model is examined in the study of ultramassive WDs, e.g., EUVE J1746-706. The UG-inspired model implies that a new location for some WDs on the Hertzsprung-Russell diagram is found.

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I. INTRODUCTION

Gravity is a structural element of stellar dynamics. A change in the underlying gravity theory do have important implications for the astrophysical description (see, e.g., Refs. [1] and references therein). One interesting example is *ungravity* (UG) [2]. In its pristine form, UG arises from the assumption of coupling between spin-2 unparticles and the stress-energy tensor [3]. In this work, we shall consider the impact of an UG-inspired model on the astrophysics of white dwarfs (WDs). This work follows previous considerations on the effect of an UG-inspired model on the properties of the sun [4]. An UG-inspired model has also been recently considered to address the flyby anomaly [5].

As will be shown, an UG model allows for the prediction of ultramassive WDs (UWDs), i.e., WDs with masses above the Chandrasekhar limit ($M_{\rm Ch} \simeq 1.45 M_S$ with $M_S \simeq 2 \times 10^{33}$ g) such as WD $1143+321$ with $M = 1.52 M_S$ [6].[1] On the other hand, these astrophysical objects allow for setting bounds on the parameters of the UG model.

In this work, the stellar equilibrium equation for WDs is obtained by considering the polytropic and degenerate gas approaches. Bounds on the UG parameters for two typical WDs, namely, Sirius B (SIB) and ϵ Reticulum (or HD 27442 B, abbreviated here by HDB) are found. The effect of UG on the Chandrasekhar mass limit of WDs is examined and UWDs such as EUVE J1746-706 is considered. Our results generalize the study the of Ref. [10]. Furthermore, we show how UG affects the location of a few WDs in the Hertzsprung-Russell (H-R) diagram. This paper is organized as follows: in Sec. II, the UG model is concisely explained; in Sec. III, the equations of the polytropic and degenerate gas models are presented; in Sec. IV, the UG-modified equilibrium equations for WDs in the framework of both gas models are set up. Finally, our results are presented and discussed in Sec. V.

II. THE UG MODEL

The essential idea behind the UG model [2] is that a modification of the Newtonian gravitational potential is introduced through the coupling of spin-2 unparticles $O^U_{\mu\nu}$ [3] to the stress-energy tensor of Standard Model states, $T^{\mu\nu}$. The resulting stress-energy tensor has following form [2]:

$$\mathcal{T}^{\mu\nu} = T^{\mu\nu} + \left(\frac{\kappa_*}{\Lambda_U^{d_U-1}}\right) g^{\mu\nu} T^{\sigma\rho} O^U_{\sigma\rho}, \tag{1}$$

where d_U and Λ_U are the scaling dimension and the energy scale of O^U, respectively. In Eq. (1), $\kappa_* = \Lambda_U^{-1} (\frac{\Lambda_U}{M_U})^{d_{UV}}$ where M_U is the large mass scale and d_{UV} is the dimension of the hidden sector operators of the ultraviolet theory which posses an infrared fixed point [2]. In order to compute the effects of the unparticles to the lowest order correction to the Newtonian gravitational potential, the metric $g^{\mu\nu}$ is replaced by the Minkowski metric $\eta^{\mu\nu}$ in Eq. (1). The resulting Newtonian gravitational potential in the UG model framework then reads [2]

$$\phi_*(r) = -\frac{G_* M}{r}\left[1 + \left(\frac{R_*}{r}\right)^{\alpha-1}\right] \tag{2}$$

where G_* is the gravitational constant of UG, R_* is the length scale which characterizes the UG interactions, and α is associated with d_U through $\alpha = 2d_U - 1$. It is obvious,

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[1]Notice that UWDs in binary systems can have their masses above $M_{\rm Ch}$ by a small amount due to an accreted mass [7]. It has also been pointed out that highly magnetized WDs can have masses as large as $M = 2.58 M_S$, for extremely high magnetic fields $B_{\rm Max} \geq 10^{13}$ G [8], but these are much higher than the observed magnetic fields in WDs which are typically in the range between 10^3 G and 10^9 G [9].

from Eq. (2), that we can recover the ordinary Newtonian gravitational potential by choosing

$$G_* = \frac{G}{1 + (\frac{R_*}{R_0})^{\alpha-1}}, \tag{3}$$

where R_0 is the distance in which the UG potential, ϕ_*, matches the Newtonian one. As a good approximation, by considering the value of α near unity, we can write $G_* \simeq G/2$. Without loss of generality we set this approximation which allows for obtaining the bounds on the relevant parameters of the UG model as well as the effect of UG on the properties of WDs. Of course, the considered model is inspired on the original UG model, whose effects are expected to take place only at extremely short distances. Recent experiments set very stringent bounds on putative new interactions with ranges at submillimeter scale [11] (see also Ref. [12] for a comparison of the results of searches of new short range interactions and the bounds for ungravity arising from nucleosynthesis considerations); however, these do not conflict with our study as we will consider deviations of the Newtonian force at ranges in the interval $10^{-8}R_S \lesssim R_* \lesssim 10^2 R_S$, where R_S is the radius of the sun.

III. THE GAS MODELS

A WD is considered as a mixture gas of ions and electrons which deviates from an ideal gas. The equilibrium of this compact object is ensured by the pressure, P, of degenerate electrons rather than by high internal temperatures as in ordinary stars. The internal temperature of WDs is rather low by stellar standards (as high as 10^7 K) [13]. Their low luminosity (L), generally several orders of magnitude smaller than the one of the sun ($L_S = 3.846 \times 10^{33}$ erg/s), corresponds to typical surface temperatures of order of a few times 10^4 K [6]. Thus, WDs are too cold to ignite nuclear reactions. Their composition at birth is mostly ^{4}He, ^{12}C, and ^{16}O (with $\mu_e = \frac{A}{Z} = 2$). Some heavier elements can be produced during a pycnonuclear reaction process under which WDs evolve, on a very long timescale, through zero temperature nuclear reactions in which lattice vibrations yield a small, but finite probability of Coulomb barrier tunneling [6,14]. In this work, we assume that a WD is at zero-temperature and behaves as a neutral gas of noninteracting electrons and bounded nucleons in nuclei of which the composition parameter is $\mu_e = 2$. The electrons, whether they are relativistic or not, contribute virtually to the entire pressure of the WD, while the bounded nucleons contribute virtually to all the WD energy density, given by $\mu_e m_H c^2 n_e$, where m_H is the atomic mass of the hydrogen ion, and n_e as the density of electrons. We assume either the polytropic or the degenerate gas models to establish the Newtonian hydrostatic equilibrium (NHE) equation, for the WDs. In Sec. IV, the validity of the NHE equation for WDs will be investigated. For a static Newtonian star, the NHE equation is given by [13]:

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \tag{4}$$

where a further derivative with respect to r leads to the usual form of NHE equation:

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP(r)}{dr}\right) = -4\pi G\rho(r). \tag{5}$$

Next, we consider the two gas models.

A. Polytropic gas model

According to the polytropic gas model, the pressure depends on the density, ρ, as follows [13]:

$$P = K\rho^{(n+1)/n}, \tag{6}$$

where n is the polytropic index and K is a constant factor. With this equation of state (EOS), we can obtain the well-known form of the Lane-Emden (LE) equation. In order to do this, we introduce two dimensionless variables, θ, and, ξ, to express the density and radial distance with respect to the center of star values, respectively:

$$\rho = \rho_c\theta^n, \tag{7}$$

$$r = \beta_p\xi, \tag{8}$$

where ρ_c is the density at the center of a star and $\beta_p = [\frac{(n+1)K}{4\pi G}\rho_c^{(1-n)/n}]^{1/2}$. The pressure of a polytropic gas reads

$$P = P_c\theta^{n+1}, \tag{9}$$

where $P_c = K\rho_c^{(n+1)/n}$. Substituting Eqs. (7)–(9) into Eq. (5) yields the well-known LE equations:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n. \tag{10}$$

The above differential equation should be solved submitted to the following boundary conditions: $\theta(\xi = 0) = 1$ and $\theta'(\xi = 0) = 0$. The density and pressure of the star can be obtained through solution of the LE equation for each value of ξ. The first zero of the LE equation solutions (the value of $\theta(\xi) = 0$ for the first zero, indicated as ξ_{10}) allows for determining the relevant quantities of a star, such as its radius and mass. The radius of a star is obtained as

$$R = \beta_p\xi_{10}. \tag{11}$$

Using relation $dM(r) = 4\pi\rho(r)r^2dr$, together with Eqs. (7) and (8), as well as the LE equation, leads to

$$M(\xi_{10}) = 4\pi\rho_c\beta_p^3\left(-\xi^2\frac{d\theta}{d\xi}\right)\Big|_{\xi=\xi_{10}}. \quad (12)$$

Finally, eliminating ρ_c in Eq (12), we obtains a relation between the mass and the radius of the star [13]

$$4\pi M^{n-1}R^{3-n} = \left[\frac{(n+1)K}{G}\right]^n\left[\left(-\frac{d\theta_n}{d\xi}\right)_{\xi_{10}}\right]^{n-1}(\xi_{10})^{n+1}. \quad (13)$$

B. Degenerate gas model

We assume now that WDs are completely described as a electron-degenerate gas with densities in the range of 10^5–10^8 g/cm^3 [13]. On the other hand, WDs satisfy the degeneracy condition in which the temperature should be much smaller than the Fermi energy $E_F = \sqrt{p_F^2c^2 + E_{0e}^2}$ where p_F is the Fermi momentum and $E_{0e} \simeq 8\times10^{-6}$ erg ($T \simeq 6\times10^9$ K), the rest energy of electrons. With this assumption, the electron density distribution function can be given approximately by the Heaviside function and the ensued electron density as follows:

$$n_e = \frac{1}{\pi^2\hbar^3}\int_0^{p_F}p^2dp = \frac{E_{0e}^3}{3\pi^2(\hbar c)^3}x^3, \quad (14)$$

where $\hbar$ is the Planck constant and $x = p_F/m_ec$. The pressure of the electron gas is given by [13]

$$P = \frac{1}{3\pi^2\hbar^3}\int_0^{p_F}\frac{p^2}{\sqrt{m^2+\frac{p^2}{c^2}}}p^2dp = Af(x), \quad (15)$$

where $A = E_{0e}^4/24\pi^2(\hbar c)^3 \simeq 6.002\times10^{22}$ erg/cm^3 and

$$f(x) = x(2x^2-3)(x^2+1)^{1/2} + 3\sinh^{-1}(x). \quad (16)$$

As the gas is neutral, through Eq. (14) we can write the density of the WD in the degenerate gas model as [13]

$$\rho = Bx^3, \quad (17)$$

where $B = E_{0e}^3\mu_em_H/3\pi^2(\hbar c)^3 \simeq 9.74\times10^5\mu_e$ g/cm^3. This equation, together with Eq. (15) are known as the EOS of WDs in the framework of completely electron-degenerate gas model. In order to obtain the NHE equation in the degenerate gas model, we substitute Eqs. (15), (16), and (17) into Eq. (5) to obtain

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dX}{dr}\right) = -\frac{\pi GB^2}{2A}x^3 \quad (18)$$

where $X = \sqrt{x^2+1}$. By defining the new variable Φ as

$$X = X_c\Phi, \quad (19)$$

where X_c is the value of X at the center of star, and $\xi = r/\beta_d$ with $\beta_d = \sqrt{\frac{2A}{\pi GB^2X_c^2}}$. The NHE equation then reads

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right) = -(\Phi^2 - X_c^2)^{\frac{3}{2}}. \quad (20)$$

This LE equation for a degenerate gas, can be solved once ρ_c is known and boundary conditions specified: $\Phi(\xi=0) = 1$ and $\Phi'(\xi=0) = 0$. In contrast to Eq. (10), ξ_{10} is the first zero of Eq. (20) so that $X(\xi_{10}) = 1$. The radius of a WD is obtained as

$$R = \beta_d\xi_{10} = 7.77\times10^8\frac{1}{\mu_eX_c}\xi_{10}. \quad (21)$$

Similarly, the mass of WD can be obtained by

$$M(\xi_{10}) = 4\pi BX_c^3\beta_d^3\left(-\xi^2\frac{d\Phi}{d\xi}\right)\Big|_{\xi_{10}}. \quad (22)$$

IV. THE LE EQUATION FOR THE UG MODEL

In order to study the effect of UG on WDs, we must suitably adjust the LE equation. In this work, we use a method similar to the one of Refs. [1,4] to obtain the modified LE equation for both polytropic and degenerate gas models. We first argue that the NHE equation is a valid approximation of the most general Tolman-Oppenheimer-Volkoff (TOV) equation for a WD [15],

$$4\pi r^2dP(r) = -\frac{GM(r)dM(r)}{r^2}\left[1+\frac{P(r)}{\rho(r)c^2}\right]\left[1+\frac{4\pi r^3P(r)}{M(r)c^2}\right]\times\left[1-\frac{2GM(r)}{c^2r}\right]^{-1}, \quad (23)$$

where $dM(r) = 4\pi r^2\rho(r)dr$. For WDs, $P(r) \ll \rho(r)c^2$ in the nonrelativistic (low density) and the ultrarelativistic (high-density) limits. In order to show this, we focus on the EOS of WDs in the degenerate gas model (subsection B) at the two limits. In the nonrelativistic limit, $p_Fc \ll E_{0e}$ or equivalently $x \ll 1$; hence, $f(x) \sim \frac{8}{5}x^5$. Thus, from Eqs. (15) and (17), we can write

$$\frac{P}{\rho c^2} \sim \frac{8A}{5Bc^2}x^2 \simeq 5\times10^{-5}x^2 \ll 1. \quad (24)$$

Therefore, at the nonrelativistic regime or at the low-density regions ($x \ll 1$) we have $P \ll \rho c^2$. At the ultra-relativistic limit, $p_Fc \gg E_{0e}$ or equivalently $x \gg 1$, the expansion of $f(x)$ can be approximated by $\sim 2x^4$ and then

TABLE I. Relevant values for the selected WDs, i.e., SIB and HDB [16,17].

WD	$(M_0 \pm \Delta M_0)/M_S$	$(R_0 \pm \Delta R_0)/R_S$	$T_{\rm eff} \pm \Delta T_{\rm eff}(K)$	$(L_0 \pm \Delta L_0)/L_S$
SIB	1.02 ± 0.02	0.0081 ± 0.0002	25193 ± 37	0.0237 ± 0.0013
HDB	0.616 ± 0.022	0.0129 ± 0.0003	15310 ± 350	0.0082 ± 0.0011

$$\frac{P}{\rho c^2} \sim \frac{2A}{Bc^2} x \simeq 7 \times 10^{-6} \left(\frac{p_F c}{E_{0e}}\right) \ll 1. \tag{25}$$

Indeed, the density of WD is 10^9 g/cm^3, considering $\rho = \mu_e m_H n_e$, the Fermi momentum of electron, $k_{Fe} = (3\pi^2 n_e)^{1/3}$, is about 0.045 fm^{-1} and $p_F c/E_{0e} \sim 17$; hence, from Eq. (25), $P \ll \rho c^2$. Thus, we can neglect the second term in the first bracket in Eq. (23). Regarding the second bracket in Eq. (23), rearranging $M(r) \sim 4\pi r^3 \bar{\rho}/3$, where $\bar{\rho}$ is the average density up to radius r, then $4\pi r^3 P \sim 3(P/\bar{\rho}c^2)M(r)c^2 \ll M(r)c^2$, and hence we can ignore the second term of the second bracket of Eq. (23). Finally, for the third bracket, as no region of the star lies within its Schwarzschild radius, $\frac{2GM(r)}{rc^2} \ll 1$.

We now consider the UG hydrostatic equilibrium (UGHE) equation. We incorporate the UG-modified Newtonian gravitational potential, Eq. (2), in the NHE equation, Eq. (4), as follows:

$$\frac{dP(r)}{dr} = -\frac{G_* M(r)\rho(r)}{r^2}\left[1 + \left(\frac{R_*}{r}\right)^{\alpha-1}\right]. \tag{26}$$

By employing $dM(r) = 4\pi\rho(r) r^2 dr$, after a straightforward calculation, the UGHE equation becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP(r)}{dr}\right) = -4\pi G_* \rho(r)\left[1 + \alpha\left(\frac{R_*}{r}\right)^{\alpha-1}\right] + \frac{G_* M(r)}{R_*^3}\left[\alpha(\alpha-1)\left(\frac{R_*}{r}\right)^{\alpha+2}\right]. \tag{27}$$

It is clear that setting $\alpha = 1$ and $G_* = G/2$ in the UGHE equation leads to the NHE equation, Eq. (4).

In order to obtain LE-modified equation, we include the EOS of both gas models, i.e., Eqs. (7) and (8) for the polytropic gas model and Eqs. (15) and (17) for the degenerate gas model, in the UGHE equation. From Eq. (12) we obtain, after some manipulation, the modified LE equation for the polytropic gas model:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\theta}{d\xi}\right) = -\frac{G_*}{G}\left\{\left[1 + \alpha\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\right]\theta^n + \left[\alpha(\alpha-1)\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\left(\frac{1}{\xi}\frac{d\theta}{d\xi}\right)\right]\right\}. \tag{28}$$

From Eq. (22), we get, after some manipulation, the modified LE equation for the degenerate gas model:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d\Phi}{d\xi}\right) = -\frac{G_*}{G}\left\{\left[1 + \alpha\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\right](\Phi^2 - X_c^2)^{\frac{3}{2}} + \left[\alpha(\alpha-1)\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\left(\frac{1}{\xi}\frac{d\Phi}{d\xi}\right)\right]\right\}. \tag{29}$$

In Eqs. (28) and (29), $\xi_* = R_*/\beta_{p(d)}$ for the polytropic (degenerate) gas. Choosing $\alpha = 1$ and $G_* = G/2$, we recover the usual LE equations, Eqs. (10) and (20). The mass and radius of WDs are calculated by Eqs. (11) and (12) for the polytropic gas model or by Eqs. (21) and (22) for the degenerate gas model at ξ_{10}^*, the first zeros of the modified LE equations.

V. RESULTS AND DISCUSSION

We consider the UG model for two arbitrary WDs, i.e., SIB and HDB, in the framework of polytropic and degenerate gas models. Table I indicates the values of the mass (M_0), radius (R_0), and luminosity (L_0) in terms of the corresponding parameters of the sun (M_S, R_S, and L_S), along with data of their effective temperatures. The data of M_0, R_0, and $T_{\rm eff}$ arise from the gravitational redshift method, as quoted by Refs. [16,17]. The luminosity, L, is given by

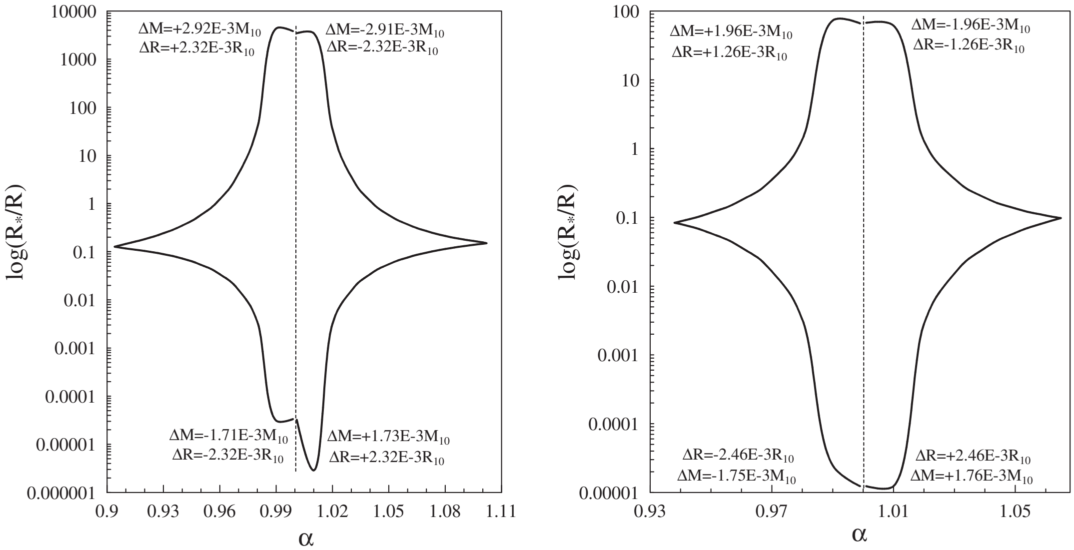
$$L = 4\pi R^2 \sigma T_{\rm eff}^4, \tag{30}$$

where σ is the Stefan-Boltzmann constant. Regarding the values of the effective temperature and radius of SIB and HDB, from the corresponding uncertainties, we obtain L_0 and ΔL_0.

Our method consists in using the uncertainties of the relevant quantities to obtain bounds on the characteristic length, R_*, and scaling dimension, α, of the UG-inspired model. In order to compute the astrophysical bounds on α and R_* and to get the new mass limit for WDs, we outline the adopted strategy. At first, we solve the LE equations, Eqs. (10) and (20), to obtain for the selected WDs mass, radius, and luminosity, denoted by M_{10}, R_{10}, and L_{10},

TABLE II. The computed values of the properties of the selected WDs (SIB and HDB).

Model	WD	M_{10}/M_S	R_{10}/R_S	L_{10}/L_S
Degenerate	SIB	1.0988	0.0080	0.0231
	HDB	0.6012	0.0127	0.0079
Polytropic	SIB	1.0201	0.0081	0.0237
	HDB	0.6162	0.0129	0.0082



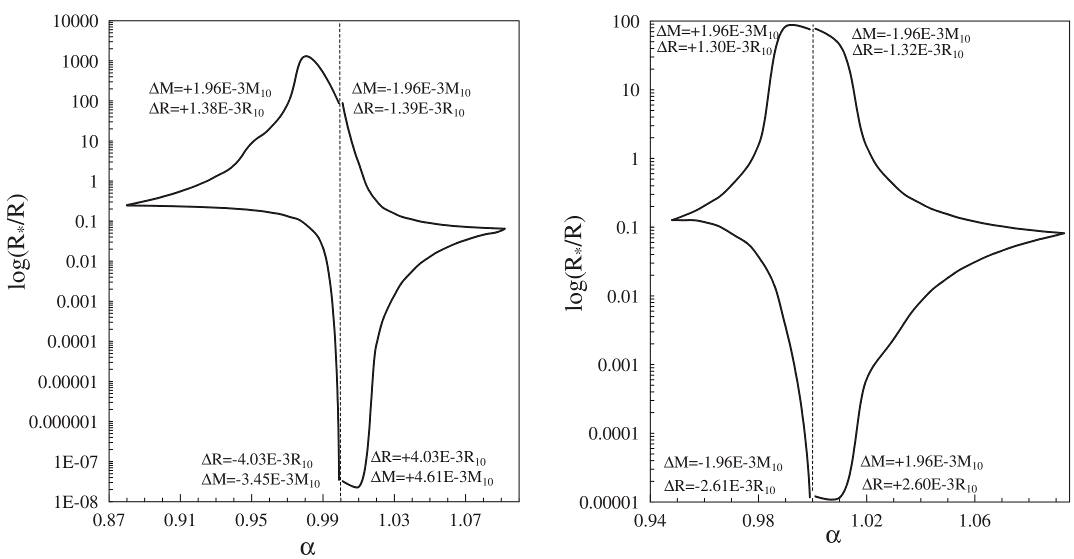
(a)HDB (b)SIB

FIG. 1. The allowed region for the UG parameters for (a) HDB and (b) SIB with the polytropic gas model. The characteristic length has been normalized by R, the radius of the relevant WD.

respectively. Then, by varying α and R_* within the LE-modified equations, Eqs. (28) and (29), we calculate the same observable parameters and accept those values that are compatible with the uncertainties (Table I). Next, the effect of UG on the Chandrasekhar limit mass is examined. Finally, we depict the effect of UG on the position of a few WDs in the H-R diagram.

We set the polytropic index of $n = 2.03(1.73)$ and the core density $\rho_c = 3.20 \times 10^7 (3.22 \times 10^6)$ g/cm^3 for SIB (HDB). Table II shows the calculated mass, radius, and luminosity.

For the same input parameters, that is, ρ_c and n, we solve Eqs. (28) and (29) for the different values of α and R_*. We select those solutions for which M, R, and L, calculated at ξ^*_{10}, remain within the observational range as illustrated by Table I, i.e., $[M_0 - \Delta M_0, M_0 + \Delta M_0]$, etc. In order to find the allowed region for R_* and α, we compute the upper and lower bounds on R_* denoted by R_*^+ and R_*^-, respectively. Figures 1 and 2 depict the allowed regions of R_* and α based on the degenerate and polytropic gas models, for HDB (SIB) [panels (a(b))]. In order to obtain $R_*^+(R_*^-)$, we use the upper (lower) values of M so that the values of R



(a)HDB (b)SIB

FIG. 2. The same as Fig. 1 for the degenerate gas model.

and L remain within the observational range (cf. Table I). It should be mentioned that, in each portion of the allowed regions, we set a fixed value for the uncertainty in M, R, and L. From Eqs. (11), (12), (21), (22), and (30), we obtain

$$\Delta R = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right) - 1\right] R_{10} \tag{31}$$

for the uncertainty in R,

$$\Delta L = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right)^2 - 1\right] L_{10} \tag{32}$$

for the uncertainty in L, and

$$\Delta M = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right)^2 \left(\frac{\eta'_{11}}{\eta'_{10}}\right) - 1\right] M_{10} \tag{33}$$

for the uncertainty in M. In Eq. (33), η' indicates $\theta'(\Phi')$, the derivative of the LE solution for the polytropic (degenerate) gas model. The ΔM and ΔR values shown in Figs. 1 and 2 are around $\alpha = 1$. Table III shows the astrophysical bounds on α and R_* with respect to data of SIB and HDB.

From values in Table III, we see that the allowed values of the UG parameters decrease when the central density, or equivalently the ratio M/R, increases. For example, in the framework of the polytropic model, when the core density increases an order of magnitude, α gets closer to unity by about 4%. A stronger behavior is found for R_*. For instance, based on the limit values of α for the polytropic model, R_* gets reduced by 60% with a tenfold increase in the density. As a result, by increasing the M/R, the allowed region for the UG parameters becomes smaller.

We now estimate the effect of UG on the Chandrasekhar mass limit, $M_{\rm Ch}$. At the ultrarelativistic limit, $x \gg 1$, from Eq. (16), $f(x) \sim 2x^4$. Hence, using Eqs. (15) and (17), we can write

$$P = \left(\frac{2A}{B^{4/3}}\right)\rho^{4/3}. \tag{34}$$

This EOS corresponds to a polytropic gas with $n = 3$. With the values of A, B, and μ_e, using Eq. (22), the Chandrasekhar mass limit reads

TABLE III. The astrophysical bounds on α and R_* with respect to the sample WD data.

Model	WD	α	$R_*(m)$	M/M_S	R/R_S	L/L_S
Degenerate	SIB	0.948	713.707	1.040	0.0079	0.0226
		1.093	460.951	1.000	0.0083	0.0248
	HDB	0.880	2261.582	0.638	0.0126	0.0078
		1.092	581.410	0.594	0.0132	0.0858
Polytropic	SIB	0.942	445.632	1.038	0.0079	0.0226
		1.065	550.077	1.000	0.0083	0.0248
	HDB	0.904	1141.932	0.638	0.0126	0.0078
		1.102	1345.948	0.594	0.0132	0.0858

$$M_{\rm Ch} = 0.721(-\xi^2\theta')|_{\xi_{10}} M_S, \tag{35}$$

where θ' indicates the derivative of the LE solution for the ultrarelativistic polytropic gas model. Hence, from the value of $\xi_{10} = 6.89679$ and $\theta'_{10} = -0.04243$, we obtain the well known result, $M_{\rm Ch} = 1.45 M_S$. When we switch on UG, the value of the first zero of the modified LE equation and of the corresponding derivative are changed and thus we can obtain new mass limits for WDs as a function of α and R_*. Figure 3 illustrates how the mass limit of WDs varies with R_* for different α's. As depicted in Fig. 3, it is possible to have WDs with masses greater than $M_{\rm Ch}$ for different values of α and R_*. As mentioned in Sec. I, the mass of WD 1143 + 321 is higher than $M_{\rm Ch}$ ($M = 1.52 M_S$ [6]). Thus, the existence of this WD can be accommodated within the UG model. As shown in Fig. 3, the curves get closer to ordinary gravity case when $\alpha \to 1^\pm$. Actually, it can be seen that the curves rotate clockwise (counter clockwise) around a point with $M = M_{\rm Ch}$ and $R_* \sim 10^{-5} R_S \simeq 7$ km for $\alpha \to 1^{+(-)}$ (but for $\alpha = 1.05$). It means for $\alpha \to 1^\pm$ we can recover the usual Chandrasekhar limit mass independently of the characteristic length of UG for $R_* \simeq 7$ km. This is achieved without any extra assumption beyond the choice $n = 3$, $\rho_c \simeq 10^{10}$ g/cm^3 and the ordinary boundary conditions to solve the LE equation, Eq. (28). Notice that Fig. 3 shows for $\alpha = 1$, that UG-inspired model also predicts that the mass limit for WDs is smaller than the usual value. Figure 3 and the corresponding data might be, thus, observationally useful.

Although UWDs ($M > 1.1 M_S$) are rather rare with respect to the ordinary WDs ($M \sim 0.6 M_S$), they can be observed through gravitational redshift measurements, radius estimates or surface gravity measurements [18],

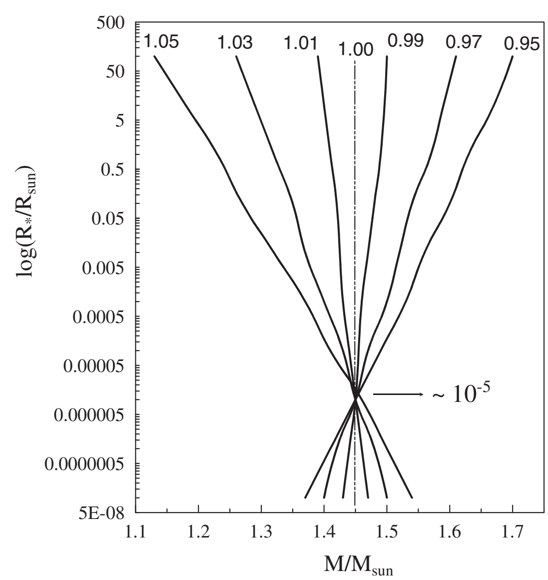


FIG. 3. The characteristic length of UG vs the mass limit of WDs for different R_* and α values.

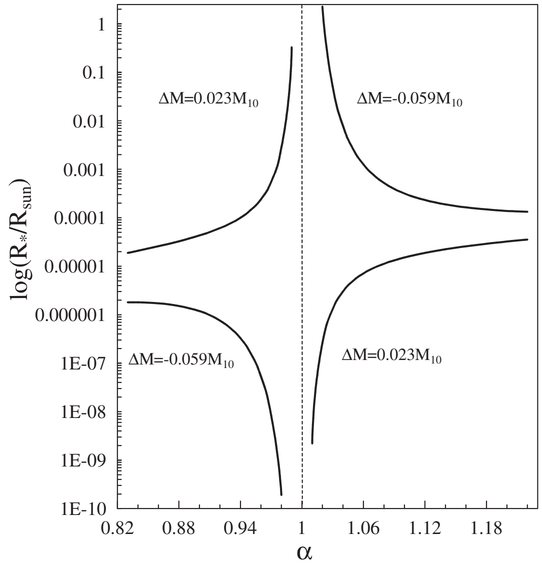


FIG. 4. The allowed region for UG for the EUVE J1746-706 WD, using the polytropic gas model.

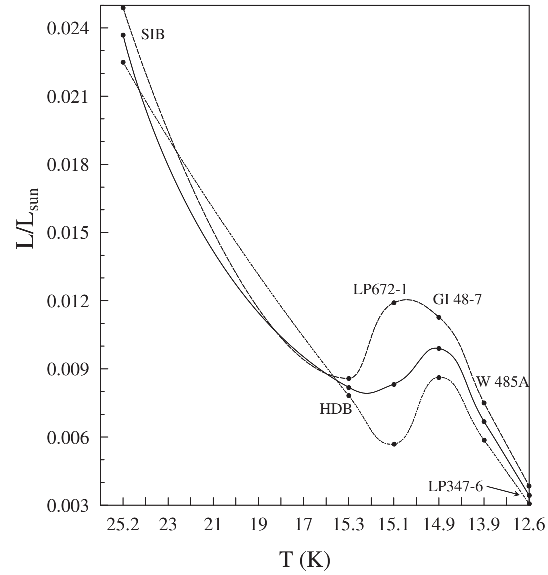


FIG. 5. The H-R diagram for a few WDs [16,17,20]. The solid, dashed, and dash-dotted curves corresponded to $\alpha = 1$, $\alpha > 1$, and $\alpha < 1$, respectively.

obtained, for instance, by surveys of the Extreme Ultraviolet Explorer (EUVE) [19]. We apply the UG model on an UWD, namely, EUVE J1746-706. According to the observational data, $M = 1.43M_S$, and $\Delta M = 0.06M_S$ [19], and for the UG polytropic gas model ($n = 3$), the $\alpha - R_*$ plot dependence is shown in Fig. 4. As depicted in Fig. 4, the tail of the curves is longer than the ordinary WDs. Although it seems that the curves in both region $\alpha < 1$ and $\alpha > 1$ do not meet each other unless for α very far from unity, we expect this behavior for curves of UWDs since we use NHE, Eq. (4), to get the modified LE equation, Eq. (28). We envisage that including general relativity corrections on UGHE might lead to a reliable bounds on the UG parameters for UWDs. It is worth mentioning that the obtained bounds for R_* and α are compatible with the ones obtained from the UG LE equation solutions applied for the sun using the 6% uncertainty on its core temperature [4].

At the final step, we show how the UG changes the location of WDs in the H-R diagram. In order to do this, we obtain the bound values of R_* and α for a few WDs with respect to their mass and radius and the corresponding uncertainties [16,17,20] and compute their luminosity. Table IV shows the bounds on R_* and α by considering the observational data. It should be pointed out that the calculations are performed in the framework of the polytropic model with $n = 2$. The luminosity of the selected WDs can be computed by knowing their radius and surface temperature. Figure 5 illustrates the position of WDs in the H-R diagram for $\alpha = 1$ (solid curve), $\alpha > 1$ (dashed curve), and $\alpha < 1$ (dash-dotted curve). It is clear that all curves for

TABLE IV. The astrophysical bounds on α and R_* with respect to the selected WD data [16,17,20].

WD	Alt ID	$M_0 \pm \Delta M$	$R_0 \pm \Delta R$	$T_{\rm eff} \pm \Delta T_{\rm eff}(K)$	α	$R_*(m)$
0642-166	Sirius B	1.02 ± 0.02	0.0081 ± 0.0002	25193 ± 37	0.975	278.5
					1.113	592
0416-594	ε Ret B	0.62 ± 0.022	0.0129 ± 0.0003	15310 ± 350	0.917	1178.8
					1.089	1366.8
1105-048	LP 672 − 1	0.45 ± 0.094	0.0133 ± 0.0026	15141 ± 88	0.530	548.7
					1.380	347.4
1143 + 321	G148-7	0.71 ± 0.072	0.0149 ± 0.0010	14938 ± 96	0.768	1124.5
					1.255	1845.2
1327-083	W485	0.53 ± 0.079	0.0141 ± 0.00085	13920 ± 167	0.846	17338
					1.305	2489.3
2341 + 322	LP 347 − 6	0.56 ± 0.022	0.0124 ± 0.0007	12300 ± 148	0.790	1039.6
					1.230	1573.6

different values of α and R_* are between the dashed and dash-dotted curves. Once again, we can see the role played by UG on the luminosity of the WDs.

In conclusion, we have considered the UG hydrostatic equilibrium equation in the framework of polytropic and degenerate gas models for selected WDs, from which we obtain bounds on the characteristic length, R_*, and scaling dimension, α, of the UG model. For ultramassive WDs, in order to get reliable bounds on the UG parameters, one may include general relativity corrections in the UG hydrostatic equilibrium equation. The effect of the UG shows that WDs heavier than the Chandrasekhar mass limit might exist. The location of WDs in the H-R diagram is also shown to be affected by UG.

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