Gravitational collapse of Hagedorn fluids

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We consider a toy model for the relativistic collapse of a homogeneous perfect fluid that takes into account an equation of state for high density matter, in the form of a Hagedorn phase, and semiclassical corrections in the strong field. We show that collapse reaches a critical minimum size and then bounces. We discuss the conditions needed for the collapse to halt and form a compact object. We argue that implications of models such as the one presented here are of great importance for astrophysics as they show that black holes may not be the only final outcome of collapse of very massive stars.

I. INTRODUCTION

Relativistic gravitational collapse is at the foundation of black hole physics. It is widely believed that the collapse of a sufficiently massive star will inevitably lead to the formation of a black hole. This idea is rooted in the simplest collapse model, developed by Oppenheimer and Snyder and independently by Datt (OSD) in 1939 [1]. The OSD model describes a spherical, nonrotating, homogeneous matter cloud (made of pressureless, "dust" particles) that collapses under its own weight. In the OSD model, after the boundary of the cloud passes the Schwarzschild radius, the black hole forms. All the matter eventually falls into the central singularity that remains hidden from far-away observers. Many analytical studies of relativistic spherical collapse have been developed, starting from the pioneering work of Oppenheimer and Snyder and Datt. These classical models are solutions of Einstein's equations for physical matter sources such as dust, perfect fluids, and fluids with only tangential pressures, with and without inhomogeneities (see, for example, Ref. [2]). In more recent times some attention has been devoted to the study of collapse models that take into account corrections to general relativity at high densities. These semiclassical models are solutions of Einstein's equations for effective matter sources that are composed of a physical part together with an unphysical part that describes modifications to general relativity in the strong field (see, for example, Ref. [3]).

Classical models, satisfying standard energy conditions, generically develop singularities [4]. However, it is usually believed that singularities should not form in the real Universe and that their appearance in solutions of Einstein's equations merely signals a breakdown of the theory in the strong field regime. Therefore, in order to avoid the formation of singularities, at some stage during collapse either the matter model must violate energy conditions or general relativity must not hold. Wheeler first recognized the importance of classical singularities as

possible windows in a regime where quantum gravity dominates. Singularities can be avoided in semiclassical models where repulsive forces, arising at high densities, balance the gravitational attraction, and bouncing scenarios are favored. For example, in the last few years there have been several studies of collapse in semiclassical models inspired by bounces in loop quantum cosmology (LQC) (see [5]). These bouncing models exhibit a minimum scale (related to the maximum allowed density, which is in turn related to the energy scale of quantum gravity) that suggests the possibility of the existence of small compact remnants. Exotic compact remnants as leftovers from gravitational collapse have been discussed for decades, and while their theoretical properties have been widely studied, their existence is still purely hypothetical. There exist many proposed objects, from gravastars [6] to quark stars [7] to boson stars [8] and Planck stars [9].

Here we present a simple toy model for collapse of a homogeneous perfect fluid that considers the equation of state for high density as well as semiclassical corrections that occur in the strong field. With respect to previously studied models the scenario presented here depends on two parameters, which are in turn related to two different energy scales. One is the characteristic scale of the Hagedorn phase, and the other is the energy scale of semiclassical corrections. Note that, depending on the chosen approach to modify general relativity, the latter may or may not be related to the Planck scale. In the present work we use a standard semiclassical approach that comes from loop quantum gravity (LQG). The model is particularly appealing because all physical quantities are well defined, physically meaningful and well behaved. The main result is that collapse reaches a critical size characterized by a maximum critical density and then bounces. Therefore, the collapsing phase that leads to the formation of a black hole is followed by an expanding phase that can be described as a white hole solution. The main difference with the standard OSD collapse model comes from the behavior of trapped surfaces. Trapped surfaces initially develop similarly to the classical case but "evaporate" before collapse reaches the

critical stage. One main consequence of the existence of a critical scale is that there is a minimum radius below which the horizon does not form at all. Such a radius is related to the critical density, and for quantum-gravitational effects, it is of the order of the Planck length. Therefore, this model suggests the possibility of the existence of exotic compact remnants as leftovers from collapse. The implications of such models for astrophysics are immediately clear: A sufficiently massive star that collapses under its own gravity may not necessarily end up in a black hole. In fact, we show here that its core may produce an extremely dense, not very massive, exotic compact object, while the outer layers and most of its mass are ejected in an explosion that is powered at the level of the quantum-gravity scale.

The paper is organized as follows: In Sec. II classical models for collapse of homogeneous perfect fluids are briefly reviewed. In Sec. III the Hagedorn equation of state (EOS) is introduced, and classical perfect fluid collapse with such EOS is investigated. In Sec. IV semiclassical collapse of the Hagedorn fluid is described. Finally, Sec. V is devoted to discussing possible astrophysical implications of the model. In the following, we use geometrical units for which $G = c = 1$.

II. PERFECT FLUID COLLAPSE

The OSD model can readily be extended to the case of collapse of a homogeneous perfect fluid sphere, with a linear equation of state relating the energy density $\rho(t)$ to the pressure $p(t)$, in the form $p = k\rho$. We use comoving coordinates $\{r, t\}$ that can be thought of as coordinates attached to the infalling particles of the cloud, for which the energy momentum tensor takes a diagonal form as $T^{\mu\nu} = \text{diag}\{\rho, p, p, p\}$. Standard energy conditions must be satisfied by energy density and pressure. Typically, we require the weak energy conditions (WEC), which can be written as $\rho > 0$ and $\rho + p \geq 0$ and imply $k \geq -1$. The speed of sound in the cloud c_s is defined as $c_s^2 = dp/d\rho = k$, so for c_s not to exceed the speed of light, we must require $k \leq 1$. Finally, in order to have positive pressures, one should impose $k > 0$. Note, however, that in certain cases negative values of k may be considered. For example, this is the case of the dark energy EOS which requires $k = -1$ and for which the density reduces to the cosmological constant. The amount of matter enclosed within the radius r at the time t is described via the Misner-Sharp mass of the system $F(r, t)$, which for a homogeneous perfect fluid, in order to satisfy regularity requirements at the center, can be written as $F(r, t) = r^3 M(t)$ [10]. In the case of dust ($p = 0$) we have $M(t) = M_0$, and the amount of matter within the comoving radius r remains unchanged during collapse. In the case of a perfect fluid we must set an initial condition for M as $M(0) = M_0$. Then, from the behavior of $M(t)$ we see that during collapse there can be an inflow or an outflow of matter across the shell r. Collapse is described by the

adimensional scale factor $a(t)$, which is related to the physical area radius R by $R(r, t) = ra(t)$. Therefore, once an initial scaling condition at $t = 0$ is chosen [in our case we set $a(0) = 1$] collapse proceeds as long as $\dot{a} < 0$. Then the metric is written as

$$ds^2 = -dt^2 + \frac{a^2}{1 - br^2} dr^2 + r^2 a^2 d\Omega^2, \qquad (1)$$

where $d\Omega^2$ is the line element on the unit two-sphere and b is an integration constant that can be thought of as a condition imposed on the initial velocity of the particles. For the sake of clarity, in the following we restrict our attention to the case of marginally bound collapse given by $b = 0$.

Homogeneous models can be thought of as representing the inner core of the collapsing object, with inhomogeneities becoming more important as one moves away from the center. Therefore, an increasing mass function M implies that the outer shells are falling onto the inner shells. Matching conditions should be imposed at the boundary of the cloud r_b, where the star's surface matches with a known exterior metric. In the following, we assume that the homogeneous approximation is valid only in the vicinity of the core and that radial inhomogeneities will change the density and pressure profiles at greater radii. Therefore, in the following analysis we will not concern ourselves with the matching conditions at the boundary of the star. Note, however, that matching across a (possibly varying) boundary surface $r_b(t)$ with a generalized Vaidya exterior is always possible (see [11]).

Also, in the present model we limit ourselves to isotropic pressures. The question of whether anisotropies become important towards the formation of the singularity was first addressed by Belinskii, Khalatnikov, and Lifshitz (BKL) [12]. During collapse, at least initially, spatial derivatives are less dominant with respect to time derivatives and anisotropies can be neglected. The BKL conjecture states that as one approaches the singularity, a regime is reached where anisotropies dominate. However, the BKL scenario is a conjecture related to classical singularities in GR, and it is not clear how it would translate in a quantum framework (see, for example, Ref. [13]). Diverging curvature is a fundamental part of the conjecture, and being close to the singularity is the key ingredient so that spatial gradients can dominate. Therefore, in bouncing scenarios, where the singularity is never reached, the role of anisotropies may become less important. In the following, we assume that as collapse proceeds we can always find a radius small enough so that anisotropies can be neglected.

The density and pressure of the cloud are given through Einstein's equations by

$$\rho = \frac{3M}{a^3}, \qquad p = -\frac{\dot{M}}{a^2 \dot{a}}, \qquad (2)$$

where dotted quantities denote derivatives with respect to t, and the remaining equations reduce the system to two differential equations for $M(t)$ and $a(t)$. Given the monotonic behavior of a, it is always possible to use a as a variable in place of t and invert the equations to obtain $M(a)$ and $t(a)$. The differential equation for M comes from the equation of state, by using Eq. (2), and takes the form

$$\frac{dM}{da} = -\frac{3kM}{a}. \tag{3}$$

Note that M decreasing in a corresponds to M increasing in t. Then when $k > 0$ the pressure is positive and diverges as collapse approaches the singularity. The differential equation for a, which is the true equation of motion for the system, comes from the Misner-Sharp mass equation that in the marginally bound case reduces to $M(t) = a(t)\dot{a}(t)^2$ and can be written as

$$\frac{dt}{da} = -\sqrt{\frac{a}{M}}, \tag{4}$$

with the minus sign chosen in order to describe collapse. The classical collapse scenario has several features that, from a physical point of view, are not desirable. Most notably, the scale factor $a = (1 - 3(k+1)\sqrt{M_0}t/2)^{2/3(k+1)}$ reaches zero size in a finite time $t_s = 2/3(k+1)\sqrt{M_0}$, thus indicating the occurrence of a space-time singularity where energy density and pressure diverge.

III. HAGEDORN PHASE

A first step to improve on the classical fluid model is to introduce an EOS that saturates the number of states as the density increases. In string theory and high energy physics the term Hagedorn temperature T_H is used to indicate the temperature corresponding to a stage where ordinary matter is forced to convert into quark matter. At this point, as energy is added to the system, collisions between hadrons no longer increase the temperature but they create more and more particles. Then new quark-antiquark pairs can be spontaneously generated from vacuum, thus providing arbitrary new degrees of freedom for the system. In a sense, one can think of a system reaching the Hagedorn phase as allowed to store any arbitrary amount of energy without further increasing its temperature. In this sense, the Hagedorn temperature associated with this state is the maximum temperature that can be reached, in principle, by matter (see, for example, Ref. [14]). Measurements of neutron star masses suggest that Hagedorn-type equations of state are ruled out for neutron stars, but they may still be valid for more dense exotic compact objects (see, for example, Ref. [15]). Also, the Hagedorn EOS has been investigated in cosmological models (see, for example, Ref. [16]) in connection with a cyclic universe and the possible existence of primordial black holes. Within

classical collapse models there exist some approaches to relativistic collapse of Hagedorn fluids in the Vaidya space-time (see, for example, Ref. [17]). However, classical collapse of a fluid sphere with an equation of state of this kind has not been investigated before. Close to the Hagedorn phase the pressure increases less and less, regardless of how much energy is added to the system. Then the fluid's heat capacity diverges at the temperature T_H, indicating that T_H is a limiting temperature and it can be reached by the system only by providing an infinite amount of energy.

Such a situation may be effectively represented classically by the choice of an equation of state of the form

$$p = \frac{k\rho}{1 + k\rho/p_0}. \tag{5}$$

The parameter p_0 describes the maximum pressure that can be achieved by the system and $p \to p_0$ as the density increases to infinity. Typically, for an astrophysical object one can think that the above equation of state will become important above nuclear densities, namely, for ρ greater than 10^{14} gr/cm^3. This equation of state has been used in [18] to describe a free bosonic string in Minkowski space in the context of quantum gravity, but it has not been used in collapse scenarios. Note that we recover the linear equation of state in the low density limit (if we take p_0 going to infinity). Also it is worth noting that the equation of state is "soft" as the speed of sound c_s within the cloud goes to zero as the density increases. This is in contrast with other theoretical approaches to high density matter, such as the Zeldovich EOS, for which the fluid becomes stiff with $p \to \rho$ and the speed of sound tends to the speed of light as ρ increases. Weak energy conditions are always satisfied if we consider k and p_0 positive. In the case where $p_0 < 0$ the EOS is still physically reasonable if k is negative. In this case WEC are also satisfied for $k \geq -1$. On the other hand, if k is negative (positive) and p_0 positive (negative) the EOS does not have a clear physical interpretation since p diverges as $\rho \to -p_0/k$. However, one may still be tempted to consider the EOS at large densities where p tends to p_0 from above (below) as the density goes to infinity. In this case the WEC are satisfied when $p_0 \geq -k\rho/(k+1)$ and $p_0 \geq -k\rho$ [respectively, when $p_0 \leq -k\rho/(k+1)$ and $p_0 \leq -k\rho$; see Fig. 1].

By solving Einstein's equations for a fluid with the above EOS, we obtain a set of two differential equations. The equation for the scale factor a is again given by Eq. (4), while the equation for the mass function M becomes

$$\frac{dM}{da} = -\frac{3kMa^2}{a^3 + 3kM/p_0}. \tag{6}$$

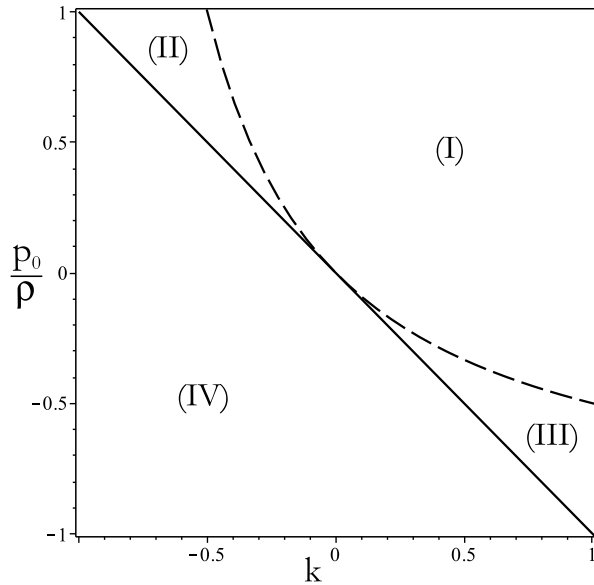The above equation can be integrated to obtain $M(a)$ implicitly via

FIG. 1. Plot of p_0/ρ as a function of $k \in [-1, 1]$. The weak energy condition $\rho + p \geq 0$ is satisfied in regions (I) and (IV) and violated in regions (II) and (III). The solid line is given by $-k$, while the dashed line is given by $-k/(k + 1)$ and for $\rho = \rho_0$ represents the region where collapse halts and $a_{\mathrm{cr}} = 0$.

$$\left(\frac{M_0}{M}\right)^{1/k} = \frac{3kM + p_0(k + 1)a^3}{3kM_0 + p_0(k + 1)}. \tag{7}$$

It is easy to see that the collapse scenario resulting from the choice of the equation of state (5) is very similar to the classical case with the linear equation of state. In the case where k and p are positive it forms a singularity that is covered by a horizon at all times. This is reasonable in light of the fact that no repulsive interactions are introduced as the density increases indefinitely. The main difference with the linear EOS model then is in the time of formation of the singularity that is now delayed (since pressures in this case are lower). As said, the EOS given in Eq. (5) is also suitable to describe negative pressures when $k < 0$. In this case it is worth asking if it is possible to construct a model that collapses to a finite compact remnant. The condition for the cloud to settle to an equilibrium configuration is given by $\dot{a} = \ddot{a} = 0$. In this case from

$$\ddot{a} = -\frac{a}{2}\left(\frac{\rho}{3} + p\right), \tag{8}$$

we see that, as ρ grows, p must tend to $-\rho/3$, which gives the equilibrium condition as $(3k + 1)p_0 + k\rho_0 = 0$. Then for $k \leq -1/3$ it is possible to construct models for which $\ddot{a}$ goes to zero. However, in order to also have $\dot{a} \to 0$ we must require $M \to 0$, and from Eq. (7) we see that this implies $a \to 0$. Therefore, no finite-size compact remnant can be constructed in the classical Hagedorn collapse model.

IV. SEMICLASSICAL EFFECTS

As it is well known, collapse does not halt in classical scenarios where energy conditions are always satisfied and the attractive nature of gravity leads inevitably to the formation of a space-time singularity. Nevertheless, one can expect that quantum corrections, which should appear in the strong field, will cause repulsive effects that may prevent the cloud from collapsing to a singularity. Several approaches in this direction have shown that taking these effects into account leads to a bouncing scenario where the cloud reaches a minimum size and then reexpands indefinitely [3]. The main idea is to treat the modifications to general relativity that must occur in the strong field limit as an effective matter source to be added to the energy-momentum tensor [19]. In this way one solves the usual Einstein's equations for an unphysical matter distribution (the effective energy-momentum tensor) that takes into account the modifications in the strong field. The effective density ρ_{eff} and effective pressure p_{eff} can thus violate energy conditions as they are not the physical density and pressure. One promising approach in this direction comes from loop quantum gravity and involves quadratic corrections to the energy density as ρ approaches a critical value ρ_0 [20]. In the semiclassical formalism this implies taking an effective density ρ_{eff} given by

$$\rho_{\mathrm{eff}} = \rho\left(1 - \frac{\rho}{\rho_0}\right), \tag{9}$$

where the parameter ρ_0 describes the maximum density that can be achieved by the system and signals the regime where quantum effects cannot be neglected. Typically, for a star one can think that quantum effects will become important around the Planck scale (namely, for $\rho > 10^{94}$ gr/cm^3). Therefore, by taking ρ_0 of the order of the Planck density we are constructing a semiclassical description of quantum-gravitational effects coming from a first-order approximation of collapse in LQG. Note that the energy scale of quantum corrections is several orders of magnitude higher than that of the Hagedorn phase. However, the constant ρ_0 is model dependent, as it comes from the specific approach chosen to deal with repulsive effects, and it need not necessarily be of the order of the Planck density. Then the above formalism may still be used with different values for the density parameter ρ_0 coming from different theoretical approaches. For example, in [21] it was suggested that the four-fermion interaction may halt collapse before the quantum-gravity regime and thus allow for the existence of compact objects. This approach has been used in [22] to describe semiclassical dust collapse. The choice of the effective density in turn leads to an effective pressure and an effective mass function as given by

$$M_{\text{eff}} = M\left(1 - \frac{\rho}{\rho_0}\right), \tag{10}$$

$$p_{\text{eff}} = p\left(1 - \frac{2\rho}{\rho_0}\right) - \frac{\rho^2}{\rho_0}. \tag{11}$$

The differential equation that must be satisfied by the mass function M is again given by Eq. (3), while the equation for the scale factor a now becomes $\dot{M}_{\text{eff}} = a\dot{a}^2$, which can be written as

$$\frac{dt}{da} = -\sqrt{\frac{a}{M}\frac{1}{\left(1 - \frac{3M}{\rho_0 a^3}\right)}}. \tag{12}$$

Note that as ρ_0 goes to infinity we recover the classical scenario. As mentioned, the final result is that the cloud reaches a minimum scale a_{cr} and then bounces back. At the time of the bounce the effective density and the effective mass function become zero, and thus quantum effects counterbalance the classical gravitational attraction, effectively "turning gravity off." This limit can be viewed as the semiclassical equivalent to approaching asymptotic freedom [23].

We now turn our attention to the semiclassical scenario for a fluid that has reached the Hagedorn phase. This means considering the mass function given by (6) and the scale factor from (12). The existence of a threshold near the Planck density for which modifications to the classical collapse scenario are necessary is not a new idea (see, for example, Ref. [24]). However, note that now there are two different scales at which corrections to the classical perfect fluid model become important, namely, the Hagedorn phase, determined by p_0, and the quantum-gravity phase, determined by ρ_0.

It is not difficult to see that when $k > 0$ and $p_0 > 0$ the semiclassical model results in a bouncing scenario. In fact, we see that as $a \to a_{\text{cr}}$ the collapsing velocity $\dot{a}$ goes to zero but the acceleration $\ddot{a}$ does not vanish, thus indicating the occurrence of the bounce (see Fig. 2). However, it should be noted that the possibility that the bounce leads to the formation of a baby universe may not be excluded, in principle. In this case the outer event horizon would remain unchanged, thus giving rise to a black hole for observers at infinity, while the expanding matter would be confined within the newly formed black hole. Such a scenario would require a phase transition for the collapsing matter to occur at the moment of the bounce. Mathematically this translates into the requirement that some suitable matching conditions be satisfied at the surface given by $t = t_B$. Such analysis is beyond the scope of this article, and it will be carried out elsewhere. On the other hand, in the bouncing scenario the black hole turns into a white hole after the bounce, and the expanding solution is simply described by the time reversal of the collapsing one. In this case it is

worth asking under which conditions a compact remnant may form. After a straightforward calculation we see that as a approaches the critical value, $\ddot{a}$ tends to the value

$$\ddot{a}_{\text{cr}} = \frac{a_{\text{cr}}}{2}(\rho + p). \tag{13}$$

Therefore, the fluid model must approach the behavior of a "dark energy" fluid with $p = -\rho$ for the repulsive effects to halt collapse. Note that the condition for equilibrium is more stringent in the semiclassical model with respect to the classical case. For the EOS considered here we get

$$\ddot{a} \to \frac{a_{\text{cr}}\rho_0}{2}\left(1 + \frac{kp_0}{p_0 + k\rho_0}\right), \tag{14}$$

from which we see that the condition $\ddot{a} = 0$ cannot be satisfied also in the case where $-1 < k < 0$ when $p_0 < 0$. We conclude that obtaining $\ddot{a} = 0$ is possible only by imposing $k < 0$ and $p_0 > 0$ or $k > 0$ and $p_0 < 0$. However, the behavior of the equation of state in these two cases is not physically very meaningful. Nevertheless, for the sake of argument, let us now focus for a moment on the case $k < 0$ with $p_0 > 0$. In this case the EOS (5) can be considered valid only at high densities, and p approaches the limiting value p_0 from above as ρ goes to infinity. If we want to satisfy the equilibrium condition we must choose the value of p_0 to be $p_0 = -k\rho_0/(1 + k)$, so that, as ρ grows approaching the critical value, the pressure balances the attraction giving $\ddot{a} = 0$ (see Fig. 1). Note that the above constraint, in order to have the value of p_0 several orders of magnitude lower than that of ρ_0, implies that k must be small. Conversely, assuming that we know the values of p_0 and ρ_0 we can evaluate the value of k for which collapse halts. With the above choice of p_0 we can achieve the equilibrium configuration, but from Eq. (7) we see that this implies $a_{\text{cr}} \to 0$. Therefore, we conclude that no compact remnant can be constructed in the semiclassical Hagedorn collapse model.

However, other repulsive effects, not due to gravity, may contribute to create an exotic object left over from collapse, much in the same way as neutron degeneracy pressure balances gravity in neutron stars. Then from the fact that the scale factor reaches the limiting value given by

$$a_{\text{cr}}^3 = \left(\frac{3M_0}{\rho_0}\right)^{\frac{1}{k+1}}\left(\frac{3kM_0 + (k+1)p_0}{k\rho_0 + (k+1)p_0}\right)^{\frac{k}{k+1}}, \tag{15}$$

we see that the critical density ρ_0 plays a crucial role in determining the size of the final remnant. The bigger the value of ρ_0, the smaller the critical scale of collapse.

The main consequence for astrophysical black holes can be inferred from the study of the behavior of the apparent horizon in the interior of the collapsing cloud. The condition for the formation of trapped surfaces in the classical scenario is given by
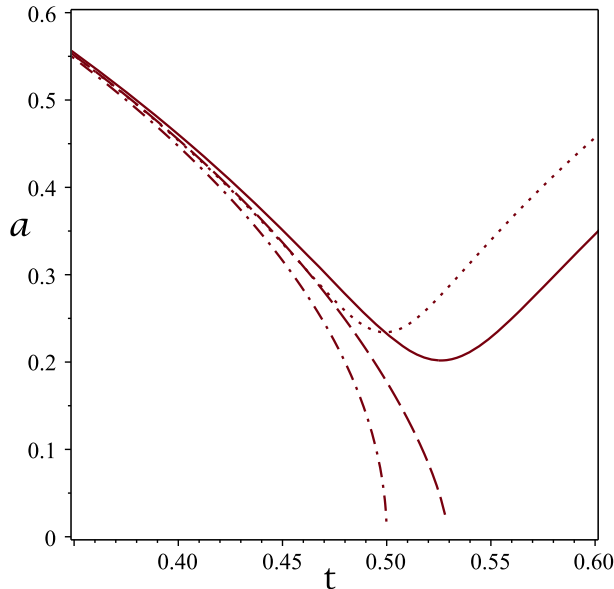
FIG. 2. Comparison of the scale factor in the four cases of classical or semiclassical collapse with linear EOS or Hagedorn EOS with the parameters chosen as $k = 1/3$, $p_0 = 50$ and $\rho_0 = 1000/3$ and the initial conditions taken as $a(0) = 1$, $M_0 = 1$. (i) The solid line is the semiclassical Hagedorn collapse, (ii) the dotted line is the semiclassical collapse with linear EOS, (iii) the dashed line is the classical Hagedorn collapse, and (iv) the dotted-dashed line is the classical collapse with linear EOS. Note that cases (i) and (ii) lead to a bounce when $a = a_{cr}$, while cases (iii) and (iv) lead to the formation of a singularity when $a = 0$.

$$1 - r^2 \frac{M}{a} = 0, \tag{16}$$

which gives the time at which the shell r becomes trapped. This can be expressed via the apparent horizon curve $r_{ah}(t) = \pm\sqrt{a(t)/M(t)}$. We see that, as a goes to zero, if M goes to a constant (or goes to zero slower than a), then r_{ah} goes to zero as well. The same can be seen from $M = a\dot{a}^2$ since we can write $r_{ah}(t) = 1/\dot{a}$, which goes to zero as $\dot{a}$ diverges. On the other hand, in the semiclassical models the condition for the formation of trapped surfaces is

$$1 - r^2 \frac{M_{eff}}{a} = 0, \tag{17}$$

where now a reaches a minimum value at which $\dot{a} = 0$ and $M_{eff} = 0$. Then it is easy to see that in this case the radius of the apparent horizon must go to infinity as the scale factor reaches the critical value a_{cr}. This means that r_{ah} will cross the boundary of the star at some point before the critical scale is reached, thus leaving the physics that occur for a close to a_{cr} not covered by any horizon (see Fig. 3). However, this feature may be due to the fact that we are neglecting inhomogeneities. In the more realistic case of inhomogeneous collapse the outer shells still gravitate as the central shell reaches the critical density. As a
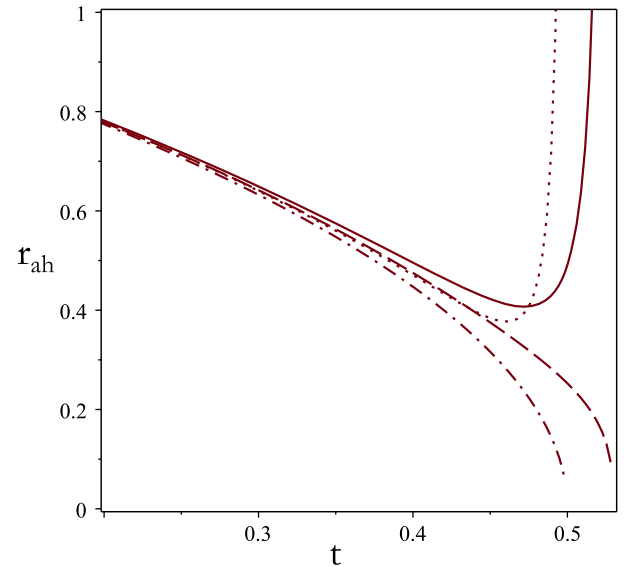

FIG. 3. Comparison of the apparent horizon in the four cases of classical or semiclassical collapse with linear EOS or Hagedorn EOS with the parameters chosen as $k = 1/3$, $p_0 = 50$ and $\rho_0 = 1000/3$ and the initial conditions taken as $a(0) = 1$, $M_0 = 1$. (i) The solid line is the semiclassical Hagedorn collapse, (ii) the dotted line is the semiclassical collapse with linear EOS, (iii) the dashed line is the classical Hagedorn collapse, and (iv) the dotted-dashed line is the classical collapse with linear EOS. Note that in cases (i) and (ii) the apparent horizon curve r_{ah} goes to infinity as $a \to a_{cr}$, while in cases (iii) and (iv) $r_{ah} \to 0$ as $a \to 0$.

consequence, the horizon does not disappear and the black hole turns into a white hole after the bounce (see, for example, Ref. [25]). In Fig. 4 we show the comparison between Penrose diagrams for collapse in these two cases. Note that if the boundary of the collapsing core is smaller than the minimum value of the apparent horizon curve, $a_{cr} < \min(r_{ah})$, then the collapsing object may not be covered by the horizon at any time (depending on what happens to trapped surfaces in the outer layers).

V. DISCUSSION

We have constructed a simple analytical toy model for relativistic collapse that includes a reasonable equation of state for high density matter as well as modifications to classical general relativity in the strong-field regime. In this model the pressure of the system has an upper limit related to the Hagedorn temperature, and the asymptotic safety regime is achieved at high densities. This implies that there are two scales as determined by the values of two parameters (namely, the maximum pressure and the maximum density). We have shown that under these circumstances no singularity is produced, and the collapse reaches a minimum size after which the cloud reexpands. Models such as the one presented here suggest that black holes, as defined mathematically in terms of singularity covered by an event horizon at all times, may not exist in nature. The
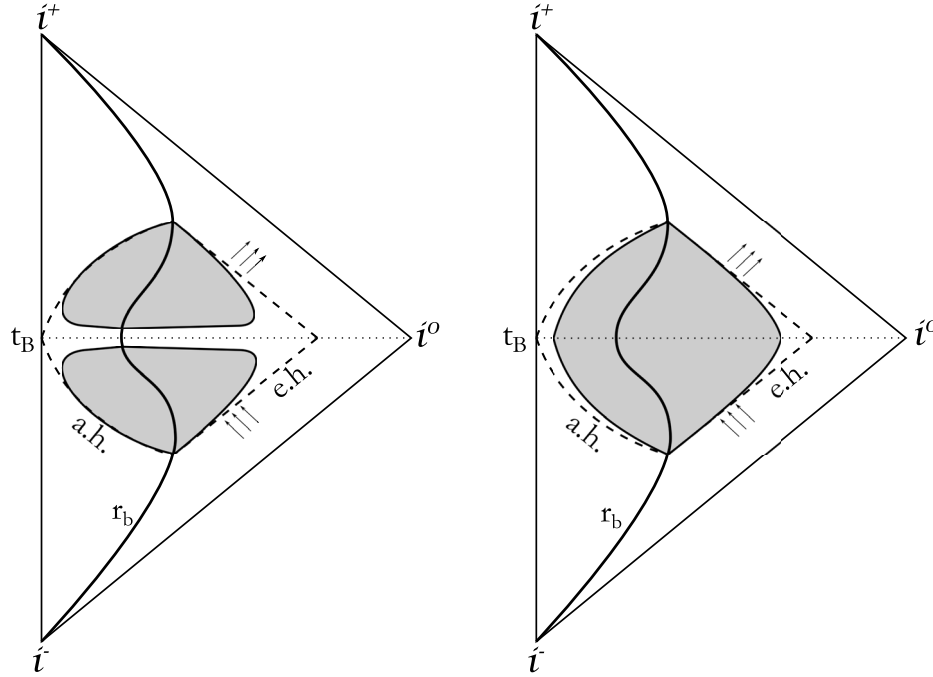
FIG. 4. The thick line represents the boundary of the cloud r_b; the grey area represents the trapped region. Dashed lines represent the classical apparent horizon (a.h.) and event horizon (e.h.), without semiclassical corrections. After the time of the bounce t_B the cloud expansion is described by the time reversal of the collapsing solution. Left panel: Penrose diagram of homogeneous collapse with semiclassical corrections. Every shell bounces at the same time t_B when the effective mass becomes zero. Right panel: Penrose diagram for the more realistic case of inhomogeneous collapse. As the central shell reaches the critical density, the outer shells still have nonzero effective mass. The total mass of the system decreases until the time of the bounce and then increases again. The outer horizon shrinks to a minimum radius due to the decrease of the effective mass. After the bounce the black hole turns into a white hole.

only horizons that may occur in realistic scenarios are apparent horizons, and these must be viewed as transient phenomena [26]. In the present model the black hole formation scenario turns into a white hole after the bounce. The most important consequence for astrophysics is that the physics of collapsing stars may be hiding quantum-gravitational effects at its core. And these effects may have detectable observational signatures. It has been suggested that such bouncing models may appear as powerful explosions in the Universe [27]. Therefore, if such a hypothesis were to be confirmed, these models would provide a cosmic laboratory to study quantum gravity, thus allowing us to probe energy scales that cannot be reached on Earth.

Furthermore, the existence of a minimum size at which collapse stops suggests the possibility of the existence of exotic compact remnants. Such objects would have to be smaller, denser and less massive than a neutron star, and they would be intrinsically quantum in nature. Note that the final size of such an object is related to the value of the maximum density parameter which may or may not be related to the Planck scale regime. At this point one is naturally led to wonder how such an object can be detected, if it exists, and what kind of observational features it would have. Studies of accretion disks around classical solutions with naked singularities indicate that the luminosity emitted by a disk around an exotic compact object may be higher than that emitted by the accretion disk around a black hole [28]. Nevertheless, this feature may be due to the presence of a singularity in the classical solution, while the true nature of the real objects may be entirely different. Detection of observable phenomena coming from exotic compact objects will be a challenge for future experiments, both in terms of the strength of the signals as well as the statistics of their occurrence. Nevertheless, future observations of strong gravity phenomena, via multi-messenger astronomy, will hopefully provide the much-needed experimental data to test general relativity in the strong field and to test the various hypotheses, regarding the final fate of collapse of very massive stars.

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