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Testing a two field inflation beyond the slow-roll approximation

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We consider a model of two field inflation, containing an ordinary scalar field and a Dirac-Born-Infeld (DBI) field. We work beyond the slow-roll approximation, but we assume a separable Hubble parameter. We then derive the form of potential in this framework and study the spectrum of the primordial perturbations in detail. We also study the amplitude of the non-Gaussianity of the primordial perturbations both in equilateral and orthogonal configurations in this setup. We test the model with recent observational data and find some constraints on the model parameters. Our study shows that for some ranges of the DBI parameter, the model is consistent with observations, and it is also possible to have large non-Gaussianity which would be observable by future improvements in experiments.

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I. INTRODUCTION

Inflationary cosmology has become a successful paradigm to understand the early stage of the Universe evolution, with its advantages of resolving the flatness, horizon and relics problems. Moreover, during inflation, the vacuum fluctuation of light scalar fields grow into super-Hubble density perturbations which are believed to be the origin of the structure formation in the Universe [1–8]. The paradigm of inflation is essentially related to a quasi-de Sitter universe, a homogeneous and isotropic universe that expands almost exponentially fast, with a nearly constant event horizon.

Recent observational data have detected a level of scale dependence in the primordial perturbations [9,10]. Although there is no direct signal for primordial non-Gaussianity in observation, the Planck team has obtained some tight limits on primordial non-Gaussianity [11]. Some inflationary models also predict a level of non-Gaussianity in the primordial perturbations mode [12–18]. In fact, the primordial non-Gaussianity carries a large amount of information on the cosmological dynamics deriving the initial inflationary expansion of the Universe. Thus, studying this feature of the perturbation modes is really an important and interesting issue, and any inflationary model which can show the non-Gaussianity and scale dependence of the primordial perturbation is in some sense more favorable on observational grounds.

It is well known that as a simplest realization inflation is derived by a single, slowly varying scalar field whose potential energy dominates the Universe expansion. However, a single field inflation model often suffers from fine tuning problems on the parameters of its potential, such as the mass and the coupling constant. It has been revealed that when a number of scalar fields are involved, they can relax many limits on the single scalar field inflation [19]. Although none of these fields can result inflation separately, they are able to work cooperatively to give an enough long inflationary stage [20–23]. Furthermore, there exists good reasons to believe that inflation might have been driven by more than one scalar field. First, there are many theories beyond the standard model of particle physics that involve multiple scalar fields, such as string theory, grand unified theories, supersymmetry, and supergravity [24–32]. Moreover, introducing one or more fields may provide attractive features. For example, hybrid inflation models [33], which involve two scalar fields, are able to result in sufficient inflationary expansion and match the observed power spectrum of density perturbations, while possessing more natural values for their coupling constants and happening at sub-Planckian field values [33–35]. Furthermore, the single field case is unusual in a sense since the evolving expectation value of the one field serves as a clock that determines when the inflation phase ends and the Universe returns to Friedmann-Robertson-Walker expansion [36]. However, with two or more fields, evolution of one field can be affected by the fluctuations in the other field(s), and so, the complex conditions under which inflation ends cannot be expressed in terms of one degree of freedom (for example some linear combination of the fields). For these reasons, the issue of multi-field inflation has became more important recently, and many authors have studied such models [37–43].

In this work, we consider an inflation model driven by two scalar fields, an ordinary scalar field with a canonical kinetic term and a Dirac-Born-Infeld (DBI) field with a noncanonical kinetic term. In fact, one of the fields describing the inflationary phase of the early Universe is expressed by the radial position of a D3-brane moving in a throat region of a warped compactified space. This proposal is based on the Dirac-Born-Infeld action [44,45] in which

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there is a speed limit upon the motion of the brane, affected by both its speed and the warp factor of the AdS_5 throat [46–48]. The effective action in a model with a DBI field contains a nonstandard kinetic term and also a function of the scalar field besides the potential that is related to the local geometry of the compact manifold traversed by the D3-brane [46]. Furthermore there is an interesting phenomenological feature in the DBI inflation such that it results non-Gaussian signatures in the cosmic microwave background [49,50]. In Ref. [51] the authors have studied a multi-field DBI inflation. They have shown that adiabatic and entropy modes in this setup propagate with the same effective sound speed and so get amplified at the sound horizon crossing. They have also found that for small sound speed, the amplitude of the entropy modes is much larger than the amplitude of the adiabatic modes. This feature can strongly affect both the observable curvature power spectrum and the amplitude of non-Gaussianities without changing the shape relative to the single field DBI case. The authors of Ref. [52] have studied a multi-field DBI inflation by considering some bulk fields present in generic flux compactification. They have investigated also the consequences of the bulk form fields on scalar cosmological perturbations, both at linear and nonlinear levels. As an important result, they have shown that the terms due to the fluctuations of the U(1) gauge field confined on the brane can be compensated exactly by the terms arising from the coupling between the bulk forms and the brane position scalar fields in the second and third order actions. Vector-type perturbations associated with the U(1) gauge field confined on the D3-brane are studied in this framework too, in order to see possible amplification of their quantum fluctuations. The gravitational wave constraints on DBI inflation have been studied in this setup when there is a transfer from entropy into adiabatic perturbations. As a result, an ultraviolet DBI multi-field scenario is compatible with data in contrast with the single field case which is in tension with the data. In Ref. [53] the leading order connected four-point function and the full quantum trispectrum of the primordial curvature perturbation are computed in multi-field DBI inflation models. They have shown that in the squeezed and counter-collinear limits the consistency relations hold as in single field models. They have shown also that adiabatic, mixed and purely entropic contributions have different momentum dependence in this setup. So the trispectrum has the potential to distinguish between the multi-field and single field DBI inflation models if the amount of the transfer from the entropy perturbations to the curvature perturbation is significantly large.

In Ref. [54] the authors have considered an inflationary model driven by an ordinary scalar field and a DBI field. They have studied the evolution of the nonadiabatic pressure perturbation during the inflation phase in this setup. Their analysis is based on the double quadratic potential [55] in the form $V(\phi,\chi) = \frac{1}{2}m_{\phi}^2(\phi^2 + \Gamma\chi^2)$. They have shown also that the evolution of the nonadiabatic pressure perturbation and also its final amplitude depend strongly on the kinetic terms. Here, we consider neither slow roll nor a separable potential; instead, to compare our inflationary model with observation, we suppose just a separable Hubble parameter. We study the spectrum of the primordial modes of perturbations in details. Non-Gaussian features of perturbations distribution parametrized by the quantity $f_{\rm NL}$ characterizing the bispectrum and generated by the evolution of scalar perturbations on super-Hubble scales are also treated carefully. We emphasize that our analysis is done beyond the slow-roll approximation, but we adopt a separable Hubble parameter. In other words, since the slow-roll condition can be temporarily violated during inflation (for example if fields start to decay during inflation as in staggered/cascade inflation [56–60], if a bump in the potential is encountered, and if it is necessarily violated at the end of inflation and during reheating), we go beyond this approximation, and our strategy is based on the first order Hamilton-Jacobi formalism developed by Salopek and Bond in Ref. [61], which allows us to express inflationary parameters in the model, without having to focus on a slow-roll regime (one can see [62] for application of this formalism to the single field case).

There are important parameters in an inflationary model such as the tensor-to-scalar ratio and the scalar spectral index which express the main properties of the cosmological perturbations. Therefore, confrontation of the inflation model with observation and constraining the model's parameters is an important task toward realization of more natural models. The constraints $r < 0.13$ and $n_s = 0.9636 \pm 0.0084$ are obtained from the combined WMAP9 + eCMB + BAO + H_0 data [63]. The conditions expressed by the joint Planck2013 + WMAP9 + BAO data are $r < 0.12$ and $n_s = 0.9643 \pm 0.0059$ [64]. Recently, the Planck collaboration released the constraints $r < 0.099$ and $n_s = 0.9652 \pm 0.0047$ from Planck TT, TE, EE + low P + WP data [9–11]. Thus, in order to compare our model with observational data, we study the behavior of the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck TT, TE, EE + low P data set and obtain some constraints on the parameters space of the model. Furthermore, we study numerically the non-Gaussianity feature of the model by studying the behavior of the orthogonal configuration versus the equilateral configuration in the background of the observational data. We show that for some ranges of the DBI parameter, our model is consistent with observation, and it is also possible to have large non-Gaussianity. We note that large non-Gaussianities would be observable by future improvements in experiments, and in this respect, this would be an important result in our study.

The paper is organized as follows: After introducing the setup in Sec. II, we investigate the linear perturbation of the model in Sec. III. By expanding the action up to the second order in perturbation, we obtain the two-point correlation functions which results in the amplitude of the scalar perturbation and its spectral index. Also, by studying the tensor part of the perturbed metric, we obtain the tensor perturbation and its spectral index as well. In order to investigate nonlinear perturbation in the model, in Sec. IV, the action is expanded up to the cubic order in perturbation. To study the non-Gaussian modes of the primordial perturbations, we consider the three-point correlation functions. In this section, the amplitude of the non-Gaussianity is obtained in the equilateral and orthogonal configurations and in $k_1 = k_2 = k_3$ limit. In other words, we focus on the possibility to obtain a large level of non-Gaussianity beyond the slow-roll inflation and derive some conditions to have large non-Guassianity. In Sec. V, we test our two field inflationary model in confrontation with the recently released observational data. Finally, we conclude in Sec. VI.

II. THE MODEL

We consider a model of inflation driven by two minimally coupled scalar fields, a scalar field with a canonical kinetic term and a DBI field, described by the action

$$S = \int \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\mathcal{R} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - f^{-1}(\chi)(1-\gamma^{-1}) - V(\phi,\chi)\right]d^4x, \tag{1}$$

where, $\mathcal{R}$ is the Ricci scalar and $M_{\rm pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass. Here, ϕ is the ordinary scalar field and χ is the DBI field, whereas V is the potential of the model which is a function of both fields. Also, $\gamma = \frac{1}{\sqrt{1-f(\chi)\partial_\alpha\chi\partial^\alpha\chi}}$ is the warp factor describing the shape of the extra dimensions, and $f^{-1}(\chi)$, which is the inverse brane tension, is related to the geometry of the throat in the original DBI framework [46,47].

We consider a spatially flat Friedmann-Robertson-Walker spacetime:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{2}$$

where $a(t)$ is the scale factor. The equations of motion for both fields are given by

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{3}$$

$$\ddot{\chi} + 3H\gamma^{-2}\dot{\chi} + \frac{1}{2}f_{,\chi}f^{-2}[1-3\gamma^{-2}+2\gamma^{-3}] + \gamma^{-3}V_{,\chi} = 0, \tag{4}$$

and Einstein's equations give

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{f(\chi)}(\gamma-1) + V\right), \tag{5}$$

$$-2\dot{H} = \dot{\phi}^2 + \gamma\dot{\chi}^2, \tag{6}$$

where a dot denotes the derivative with respect to the cosmic time t and a "," represents the derivative with respect to the scalar field. Here, $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

Following the notation of slow-roll parameters defined by $\epsilon \equiv -\frac{\dot{H}}{H^2}$ and $\eta \equiv -\frac{1}{H}\frac{\ddot{H}}{\dot{H}}$, we find these parameters in our setup as follows:

$$\epsilon = \frac{M_{\rm pl}^2(3\dot{\phi}^2 + 3\gamma\dot{\chi}^2)}{\dot{\phi}^2 + 2f^{-1}(\gamma-1) + 2V}, \tag{7}$$

$$\eta = \frac{-2(\dot{\phi}\ddot{\phi} + \gamma\dot{\chi}\ddot{\chi} + \frac{1}{2}\gamma^3(f\ddot{\chi} + \frac{f_{,\chi}}{2}\dot{\chi}^2)\dot{\chi}^3)}{H(\dot{\phi}^2 + \gamma\dot{\chi}^2)}. \tag{8}$$

It is important to note that we have not used the slow-roll conditions in the calculation of ϵ and η. Up to now, we obtained the main equations of this inflationary setup. In the next section, in order to test this inflationary model, we study the linear perturbation of the primordial fluctuations. To this end, we calculate the spectrum of perturbations produced due to quantum fluctuations of the fields about their homogeneous background values.

III. LINEAR PERTURBATIONS

Now, we study linear perturbations of the two field model introduced in the previous section. To this end, we expand the action up to the second order of fluctuations within the Arnowitt-Deser-Misner (ADM) formalism in which we can eliminate one extra degree of freedom of perturbations at the beginning of the calculation by choosing a suitable gauge.

The space-time metric in the ADM formalism is

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{9}$$

with N being the lapse function and N_i the shift vector. By expanding the lapse function N and the shift vector N_i, as $N = 1 + 2\Phi$ and $N^i = \delta^{ij}\partial_j B$, the general perturbed form of the metric will be obtained. There is no need to compute N or N_i up to the second order, since the second order perturbation is multiplied by a factor which is vanishing using the first order solution. We also note that the contribution of the third order term vanishes. This is because it is multiplied by a constraint equation at the zeroth order obeying the equations of motion. Here, h_{ij} is written as $h_{ij} = a^2[(1-2\Psi)\delta_{ij} + 2\mathcal{T}_{ij}]$, where Ψ is the spatial curvature perturbation and $\mathcal{T}_{ij}$ is a spatial shear three-tensor which is symmetric and also traceless. So, the above perturbed metric (9) becomes

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dtdx^i + a^2(t)[(1-2\Psi)\delta_{ij} + 2\mathcal{T}_{ij}]dx^i dx^j. \quad (10)$$

In what follows, to study the scalar perturbations, we choose the uniform field gauge, $\delta\phi = 0$ (which fixes the time component of a gauge transformation vector ξ^μ), and the gauge $\mathcal{T}_{ij} = 0$. Finally, by considering the scalar part of the perturbations at the linear level, the perturbed metric can be rewritten as

$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j. \quad (11)$$

By expanding the action (1) up to the second order in the perturbations, we obtain

$$S_2 = \int dtd^3xa^3\left[-3M_{\rm pl}^2\dot{\Psi}^2 + \frac{M_{\rm pl}^2}{a^2}(2\dot{\Psi} - 2H\Phi)\partial^2 B - 2\frac{M_{\rm pl}^2}{a^2}\Phi\partial^2\Psi + 6M_{\rm pl}^2 H\Phi\dot{\Psi} + \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\gamma\dot{\chi}^2 + \frac{1}{2}f\gamma^3\dot{\chi}^4 - 3M_{\rm pl}^2H^2\right)\Phi^2 + \frac{M_{\rm pl}^2}{a^2}(\partial\Psi)^2\right]. \quad (12)$$

Variation of the action with respect to N and N_i yields the following constraints:

$$\frac{1}{a^2}\partial^2 B = 3\dot{\Psi} - \frac{1}{a^2 H}\partial^2\Psi + \frac{1}{M_{\rm pl}^2 H}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\gamma\dot{\chi}^2 + \frac{1}{2}f\gamma^3\dot{\chi}^4 - 3M_{\rm pl}^2H^2\right)\Phi, \quad (13)$$

$$\Phi = \frac{1}{H}\dot{\Psi}. \quad (14)$$

Making use of the above results and doing some integrations by parts, the second order action takes the form

$$S_2 = \int dtd^3xa^3\mathcal{W}\left[\dot{\Psi} - \frac{c_s^2}{a^2}(\partial\Psi)^2\right], \quad (15)$$

where

$$\mathcal{W} = \frac{\dot{\phi} + \gamma\dot{\chi}^2 + f\gamma^3\dot{\chi}^4}{2H^2}, \quad (16)$$

and the sound speed, c_s^2, is defined as

$$c_s^2 = \frac{M_{\rm pl}^2(\dot{\phi}^2 + \gamma\dot{\chi}^2)}{\dot{\phi} + \gamma\dot{\chi}^2 + f\gamma^3\dot{\chi}^4}. \quad (17)$$

Now, in order to obtain the quantum perturbations of Ψ, one can vary the action (15) and find the equation of motion of the curvature perturbation, Ψ, as follows:

$$\ddot{\Psi} + \left(3H + \frac{\dot{\mathcal{W}}}{\mathcal{W}}\right)\dot{\Psi} + c_s^2\frac{k^2}{a^2}\Psi = 0. \quad (18)$$

The solution of this equation, up to the lowest order in the slow-roll variables gives

$$\Psi = \frac{iHe^{-ic_s^2k\tau}}{2(c_s k)^{3/2}\sqrt{\mathcal{W}}}(1 + ic_s k\tau). \quad (19)$$

By computing the two-point correlation function in our setup, we are able to study the power spectrum of the curvature perturbation. The two-point correlation function of curvature perturbations can be derived by obtaining the vacuum expectation value at the end of inflation

$$\langle 0|\Psi(0,\mathbf{k}_1)\Psi(0,\mathbf{k}_2)|0\rangle = \frac{2\pi^2}{k^3}\mathcal{A}_s(2\pi)^3\delta^3(\mathbf{k}_1 + \mathbf{k}_2), \quad (20)$$

where $\mathcal{A}_s = \frac{H^2}{8\pi^2\mathcal{W}c_s^3}$ is the power spectrum of the scalar perturbations and is evaluated at $c_s k = aH$ (k is the comoving wave number). Its spectral index can be derived as follows:

$$n_s - 1 = -2\epsilon - \frac{1}{H}\frac{d}{dt}\ln c_s - \frac{1}{H}\frac{d}{dt}\ln\epsilon. \quad (21)$$

Let us now proceed further to obtain power spectrum of the gravitational waves in this model. We study the tensor perturbations of the form

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}^{TT})dx^i dx^j, \quad (22)$$

where h_{ij}^{TT} is transverse and traceless. It is known that h_{ij}^{TT} can be written in terms of the two polarization modes, as $h_{ij}^{TT} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$. We choose the normalization for the two matrices such that, in Fourier space,

$$e_{ij}^{(+)}(k)e_{ij}^{(+)}(-k)^* = 2, \quad (23)$$

$$e_{ij}^{(\times)}(k)e_{ij}^{(\times)}(-k)^* = 2, \quad (24)$$

and

$$e_{ij}^{(+)}(k)e_{ij}^{(\times)}(-k)^* = 0. \quad (25)$$

In this case the second order action for the gravitational waves can be expressed as

$$S_T = \int dtd^3xa^3\mathcal{W}_T\left[\dot{h}^2_{(+)} - \frac{c_T^2}{a^2}(\partial h_{(+)}) + \dot{h}^2_{(\times)} - \frac{c_T^2}{a^2}(\partial h_{(\times)})\right], \tag{26}$$

where $\mathcal{W}_T = \frac{M_{\rm pl}^2}{4}$ and $c_T^2 = 1$.

The power spectrum of tensor perturbations, which is obtained by strategy as performed for the scalar perturbations, is as follows:

$$\mathcal{A}_T = \frac{H^2}{2\pi^2\mathcal{W}_T}, \tag{27}$$

which results the following spectral index of gravitational waves:

$$n_T = \frac{d\ln\mathcal{A}_T}{dN} = -2\epsilon. \tag{28}$$

Another important inflationary parameter is the tensor-to-scalar ratio, which in this model takes the following form

$$r = \frac{\mathcal{A}_T}{\mathcal{A}_s} = 16c_s\epsilon. \tag{29}$$

This is the consistency relation in this model. Up to this point, we have calculated the primordial fluctuations in linear order. In what follows, we explore the non-Gaussianity of the density perturbations by studying the nonlinear perturbations.

IV. NONLINEAR PERTURBATIONS AND NON-GAUSSIANITY

Now, we study the non-Gaussianity of the primordial density perturbation which is another important aspect of an inflationary model. It follows that to compute the amount of non-Gaussianity in specific inflation models we need to go beyond the linear order perturbation theory. Since the two-point correlation function of the scalar perturbations gives no information about the non-Gaussianity of perturbations distribution, one has to study higher order correlation functions. The most appropriate correlation function to study the non-Gaussian feature of the primordial perturbations is the three-point correlation function. In order to calculate the three-point correlation function, the action (1) should be expanded up to the cubic order in the small fluctuations around the homogeneous background solution. Note that cubic terms obtained in this manner result in a change both in the ground state of the quantum field and also nonlinearities in the evolution. After expanding the action (1) up to the third order in perturbation, the next step is to eliminate the perturbation parameter Φ in the expanded action. By introducing an auxiliary field $\mathcal{Q}$ satisfying the following relation

$$B = -\frac{1}{H}\Psi + \frac{a^2}{M_{\rm pl}^2}\mathcal{Q}, \tag{30}$$

and

$$\partial^2\mathcal{Q} = \mathcal{W}\dot{\Psi}, \tag{31}$$

the third order action up to the leading order can be written as

$$\begin{aligned} S_3 = \int dtd^3x\Bigg[&-\frac{3M_{\rm pl}^2a^3}{c_s^2}\epsilon\left(\frac{1}{c_s^2}-1\right)\Psi\dot{\Psi}^2 \\ &+ aM_{\rm pl}^2\epsilon\left(\frac{1}{c_s^2}-1\right)\Psi(\partial\Psi)^2 + \frac{a^3M_{\rm pl}\epsilon}{Hc_s^2}\left(\frac{1}{c_s^2}-1-2\frac{\lambda}{\Sigma}\right)\dot{\Psi}^3 \\ &-2\frac{a^3\epsilon}{c_s^2}\dot{\Psi}(\partial_i\Psi)(\partial_i\mathcal{Q})\Bigg], \end{aligned} \tag{32}$$

where the parameters λ and Σ are defined as follows

$$\lambda = \frac{f\dot{\chi}^4}{4(1-f\dot{\chi}^2)^{\frac{3}{2}}} + \frac{f\dot{\chi}^6}{3(1-f\dot{\chi}^2)^{\frac{5}{2}}}, \tag{33}$$

$$\Sigma = \frac{1}{2}(\dot{\phi}^2 + \gamma\dot{\chi}^2 + f\gamma^3\dot{\chi}^4). \tag{34}$$

Now, having obtained the third order action, we can proceed to study the non-Gaussianity of the primordial perturbations by evaluating the three-point correlation functions. In order to calculate the three-point correlation function, we use the interaction picture where $H_{\rm int}$, the interacting Hamiltonian, is equal to the Lagrangian of the cubic action. The vacuum expectation value of the curvature perturbation for the three-point operator in the conformal time interval between the beginning of the inflation, τ_i, and the end of the inflation, τ_f, is given by the following expression [65–67]:

$$\begin{aligned} &\langle\Psi(\mathbf{k}_1)\Psi(\mathbf{k}_2)\Psi(\mathbf{k}_3)\rangle \\ &\quad = -i\int_{\tau_i}^{\tau_f} d\tau a\langle 0|[\Psi(\mathbf{k}_1)\Psi(\mathbf{k}_2)\Psi(\mathbf{k}_3), H_{\rm int}]|0\rangle. \end{aligned} \tag{35}$$

Solving the integral in the above equation results the following three-point correlation function of the curvature perturbation in the Fourier space:

$$\begin{aligned} &\langle\Psi(\mathbf{k}_1)\Psi(\mathbf{k}_2)\Psi(\mathbf{k}_3)\rangle \\ &\quad = (2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{A}_s^2\mathcal{F}_\Psi(k_1,k_2,k_3), \end{aligned} \tag{36}$$

where $\mathcal{A}_s$ is the power spectrum of perturbation some time after the Hubble radius crossing, and

$$\mathcal{F}_\Psi(k_1,k_2,k_3) = \frac{(2\pi)^4}{\prod_{i=1}^3 k_i^3}\mathcal{G}_\Psi. \tag{37}$$

We note that, in solving the integral of Eq. (35), we have used the approximation that the coefficients in the brackets of the Lagrangian (32) to be constants, because these coefficients would vary slower than the scale factor. Furthermore, the parameter $\mathcal{G}_\Psi$ is defined as

$$\mathcal{G}_\Psi = \frac{3}{4}\left(1-\frac{1}{c_s^2}\right)\mathcal{S}_1 + \frac{1}{4}\left(1-\frac{1}{c_s^2}\right)\mathcal{S}_2 + \frac{3}{2M_{\rm pl}}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)\mathcal{S}_3, \tag{38}$$

in which we have the following relations for the shape functions $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$, respectively,

$$\mathcal{S}_1 = \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3, \tag{39}$$

$$\mathcal{S}_2 = \frac{1}{2}\sum_i k_i^3 + \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3, \tag{40}$$

$$\mathcal{S}_3 = \frac{(k_1k_2k_3)^2}{K^3}, \tag{41}$$

and also

$$K = \sum_i k_i. \tag{42}$$

It is obvious from Eq. (36) that the three-point correlator depends on the three momenta k_1, k_2 and k_3. There are several different shapes of non-Gaussianities depending on these wave numbers satisfying the condition $k_1 + k_2 + k_3 = 0$ [68–72]. The simplest one is the so-called local shape [73–76], which has a peak in the squeezed limit ($k_3 \to 0$ and $k_1 \simeq k_2$). The second shape corresponds to the equilateral configuration [77] with a signal at $k_1 = k_2 = k_3$. There is another shape whose scalar product with the equilateral template vanishes and is called the orthogonal configuration [78]. A linear combination of the equilateral and orthogonal templates gives a shape corresponding to folded triangle [79] with a maximal signal in $k_1 = 2k_2 = 2k_3$ limit. We also note that the orthogonal configuration has a signal with a positive peak at the equilateral configuration and a negative peak at the folded configuration. From the bispectrum $\mathcal{G}_\Psi$ of the three-point correlation function of curvature perturbations, the nonlinear parameter characterizing the amplitude of non-Gaussianities is expressed by

$$f_{\rm NL} = \frac{10}{3}\frac{\mathcal{G}_\Psi}{\sum_{i=1}^3 k_i^3}. \tag{43}$$

As has been stated previously, purely adiabatic Gaussian perturbations give $f_{\rm NL} = 0$; however, the presence of non-Gaussian perturbations results in deviation from $f_{\rm NL} = 0$. Here, we study the amplitude of non-Gaussianity in the equilateral and orthogonal configurations. To this end, we should find the bispectrum $\mathcal{G}_\Psi$ in these configurations. In this regard, we follow [80–82] and introduce a shape $\mathcal{S}_*^{\rm equil}$ as

$$\mathcal{S}_*^{\rm equil} = -\frac{12}{13}(3\mathcal{S}_1 - \mathcal{S}_2). \tag{44}$$

Moreover, we define another shape which is exactly orthogonal to $\mathcal{S}_*^{\rm equil}$, as follows:

$$\mathcal{S}_*^{\rm ortho} = \frac{12}{14-13\beta}[\beta(3\mathcal{S}_1-\mathcal{S}_2) + 3\mathcal{S}_1 - \mathcal{S}_2], \tag{45}$$

where $\beta \simeq 1.1967996$. Finally, making use of these relations, the leading order bispectrum (38) can be written in terms of the equilateral basis, $\mathcal{S}_*^{\rm equil}$, and the orthogonal basis, $\mathcal{S}_*^{\rm ortho}$, as

$$\mathcal{G}_\Psi = \mathcal{C}_1\mathcal{S}_*^{\rm equil} + \mathcal{C}_2\mathcal{S}_*^{\rm ortho}, \tag{46}$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are coefficients which determine the magnitudes of the three-point correlation function coming from equilateral and orthogonal contributions, respectively, and are defined as

$$\mathcal{C}_1 = \frac{13}{12}\left[\frac{1}{24}\left(1-\frac{1}{c_s^2}\right)(2+3\beta) + \frac{\lambda}{12\Sigma}(2-3\beta)\right], \tag{47}$$

and

$$\mathcal{C}_2 = \frac{14-13\beta}{12}\left[\frac{1}{8}\left(1-\frac{1}{c_s^2}\right) - \frac{\lambda}{4\Sigma}\right]. \tag{48}$$

Here, λ and Σ are defined by Eqs. (33) and (34), respectively. Making use of Eqs. (44)–(48), and also by definition of the nonlinearity parameter (43), one can obtain the following expressions for amplitude of the non-Gaussianity in the equilateral and orthogonal configurations, respectively:

$$f_{\rm NL}^{\rm equil} = \left(\frac{130}{36\sum_{i=1}^3 k_i^3}\right)\left[\frac{1}{24}\left(1-\frac{1}{c_s^2}\right)(2+3\beta) + \frac{\lambda}{12\Sigma}(2-3\beta)\right]\mathcal{S}_*^{\rm equil}, \tag{49}$$

and

$$f_{\rm NL}^{\rm ortho} = \left(\frac{140-130\beta}{36\sum_{i=1}^3 k_i^3}\right)\left[\frac{1}{8}\left(1-\frac{1}{c_s^2}\right) - \frac{\lambda}{4\Sigma}\right]\mathcal{S}_*^{\rm ortho}. \tag{50}$$

As has been mentioned previously, the shape function in the equilateral configuration has a peak in $k_1 = k_2 = k_3$ limit, and also, the orthogonal shape has a signal with a positive peak at the equilateral configuration. Thus, the nonlinearity parameter in both configurations can be rewritten as

$$f_{\rm NL}^{\rm equil} = \frac{325}{18}\left[\frac{1}{24}\left(\frac{1}{c_s^2}-1\right)(2+3\beta) + \frac{\lambda}{12\Sigma}(2-3\beta)\right], \tag{51}$$

and

$$f_{\rm NL}^{\rm ortho} = \frac{10}{9}\left(\frac{65}{4}\beta+\frac{7}{6}\right)\left[\frac{1}{8}\left(1-\frac{1}{c_s^2}\right)-\frac{\lambda}{4\Sigma}\right]. \tag{52}$$

Up to this point, we have obtained the main equations of the two field inflation model. In the following section, we examine the model in confrontation with Planck 2015 TT, TE, EE + low P and Planck2015 TTT, EEE, TTE and EET joint data set to see the consistency of this model. We also obtain some constraints on the model's parameters space in this treatment.

V. OBSERVATIONAL CONSTRAINTS

In previous sections we have calculated the primordial fluctuations in both linear and nonlinear orders. An inflationary model is successful and viable if its perturbation parameters are consistent with observational data. So, in what follows we find some observational constraints on the parameters space of the model in hand. To this end, we should firstly define the form of the DBI function $f(\chi)$. Usually, $f(\chi)$, which is related to the geometry of the throat, is given in terms of the warp factor of the Anti de Sitter (AdS)-like throat. In the pure AdS_5, this function takes a simple form as $f(\chi) = \frac{\lambda}{\chi^4}$ [47].

We also emphasize that our method is based on the first order Hamilton-Jacobi formalism. As we have mentioned, slow roll is not the only possibility for successfully implementing models of inflation, and solutions beyond the slow-roll approximation have been found in particular situations. In fact, inflation is defined to be a period of accelerated expansion, ($\frac{\ddot{a}}{a} > 0$), indicating an equation of state in which vacuum energy dominates over the kinetic energy of the field(s). In the slow-roll limit the expansion of the Universe is of the de Sitter form, with the scale factor increasing exponentially in time ($H \simeq$ const, and $a \propto e^{Ht}$). With constant Hubble distance and exponentially increasing scale factor, comoving length scales that are initially smaller than the horizon, get rapidly redshifted toward outside the horizon. In general, the Hubble parameter H is not exactly a constant, but it varies slowly as the field(s) evolve along the potential V. A convenient approach to the more general case is to express the Hubble parameter directly as a function of the field(s) instead of as a function of time [62,83–86]. Thus, we continue our analysis by reformulating the equations of motion as first order Hamilton-Jacobi equations, following [20]. We concentrate on solutions satisfying the following sum separable Hubble parameter:

$$H = H_0 + H_1\phi + H_2\chi. \tag{53}$$

As has been suggested by Kinney [62,87], as long as t is a single-valued function of fields separately, one can express the Hubble parameter directly as a function of the fields instead of as a function of time. In this case one can use the Hamilton-Jacobi formalism to describe dynamics of inflation. For this reason, we use the ansatz (53) in our setup (see [88,89] for other occasions of the sum separable Hubble parameter in the literature).

Focusing on a homogeneous universe, the equations of motion for both ordinary and DBI fields can be written as a first order Hamilton-Jacobi system as (see Appendix A)

$$\dot{\phi} = -2\frac{\partial H}{\partial \phi}, \tag{54}$$

$$\dot{\chi} = -2c_\chi\frac{\partial H}{\partial \chi}, \tag{55}$$

where c_χ is the individual sound speed of the DBI field and is given by

$$c_\chi = \frac{1}{\sqrt{1+4fH_\chi^2}}. \tag{56}$$

The above Hamilton-Jacobi equations of motion have the solutions as follows (see Appendix B):

$$\phi(t) = -2H_1 t + \phi_0, \tag{57}$$

$$\chi(t) \simeq \frac{\chi_0\sqrt{\lambda}}{\chi_0 t + \sqrt{\lambda}}, \tag{58}$$

leading to the scale factor as

$$a(t) = a_0 \exp((H_0 + H_1\phi_0)t - H_1^2 t^2 + H_2\sqrt{\lambda}\ln(\chi_0 t + \sqrt{\lambda})), \tag{59}$$

where ϕ_0 and χ_0 correspond to the value of each fields at t_* (the time at which observable scales exit the horizon). Furthermore, one can easily obtain the corresponding potential of the scalar fields by using the Friedmann equation (5):

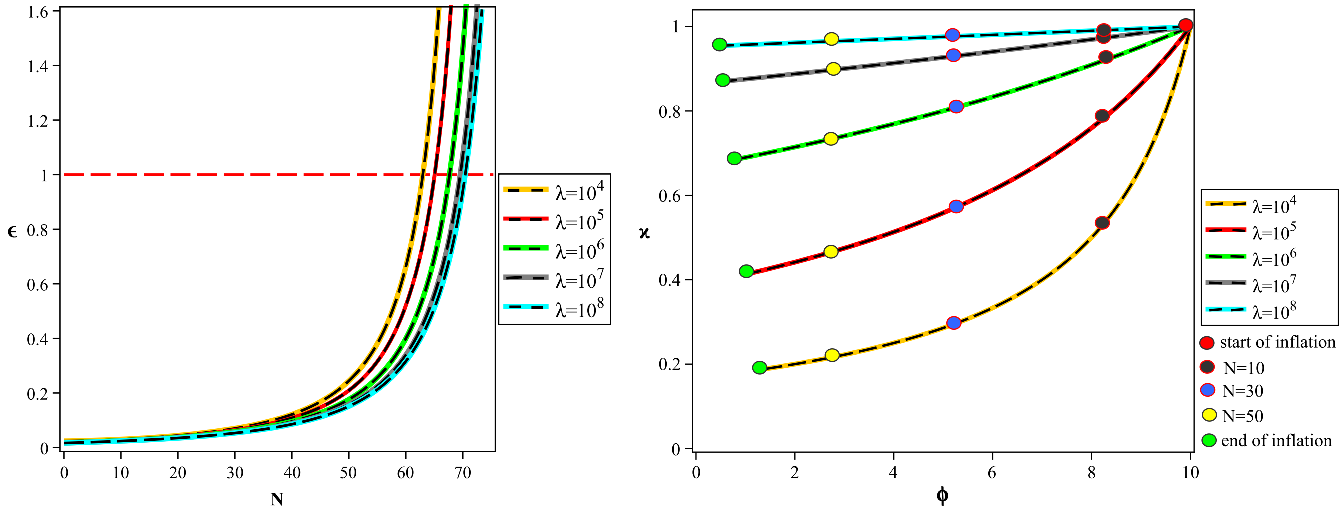

FIG. 1. Left Panel: Variations of ϵ versus N for different values of λ. Right Panel: Trajectories in the fields space, originating at $\phi_0 = 10$ and $\chi_0 = 1$ and ending at some values on N around $N \simeq 65$ e-folds of inflation when $\epsilon = 1$.

$$V(\phi,\chi) = 3(H_0 + H_1\phi + H_2\chi)^2 - 2H_1^2 - \frac{\chi^4}{\lambda}\left[\left(1 - \frac{4\lambda c_\chi^2 H_2^2}{\chi^4}\right)^{-1/2} - 1\right]. \quad (60)$$

After deriving the form of the potential, we explore the behavior of the tensor-to-scalar ratio versus the scalar spectral index and the orthogonal configuration versus the equilateral configuration in the background of the Planck2015 TTT, EEE, TTE and EET data in order to see the viability of this theoretical model in confrontation with the recent observations. We note that in our numerical study the values of H_0, H_1 and H_2 are chosen so that for a defined number of e-folds, the inflation phase terminates gracefully. For instance, we have set $H_0 = 10^{-6}$, $H_1 = 0.01$ and $H_2 = 0.01$. Note also that we considered the case where inflation is initially driven dominantly by the ordinary scalar field ϕ. The initial values for the two fields are $\phi_0 = 10$ and $\chi_0 = 1$.

Figure 1 in the left panel shows variations of the slow-roll parameter ϵ versus N for different values of λ. For instance, the condition $\epsilon = 1$ with $\lambda = 10^6$ is achieved for $N = 67$. In the right panel of this figure, the trajectories in the fields space are depicted. For relatively small values of λ, the field ϕ plays the central role. For large values of λ the DBI field takes more important role in the dynamics of inflation relative to its role in the case with smaller λ. The left panel of Fig. 2 shows the behavior of the tensor-to-scalar ratio

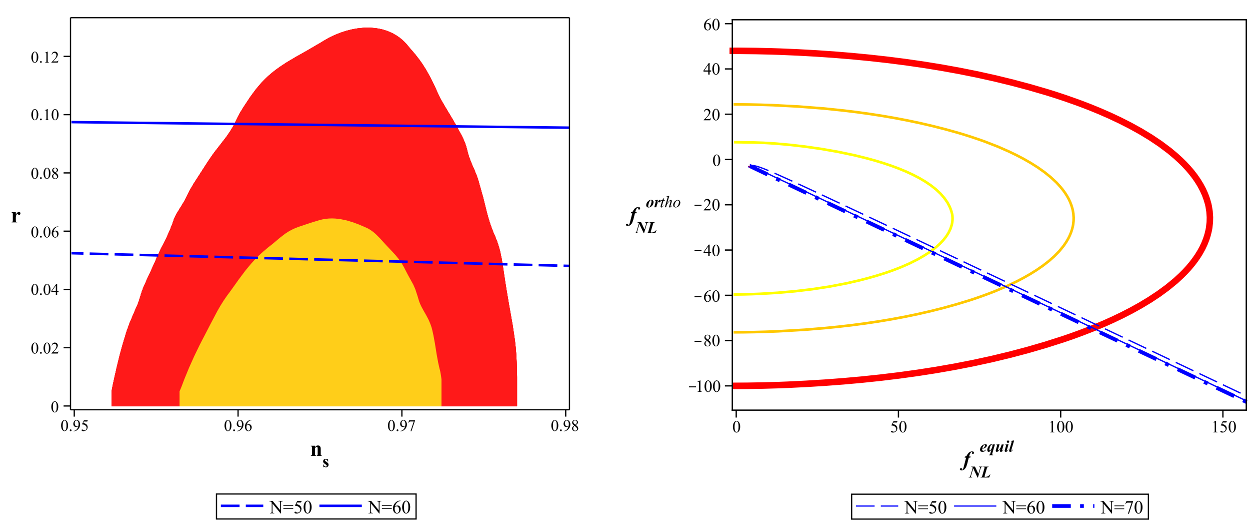

FIG. 2. Tensor-to-scalar ratio versus the scalar spectral index in the background of Planck2015 TT, TE, EE + low P data (left panel) and the amplitude of the orthogonal versus equilateral configuration of non-Gaussianity in the background of Planck2015 TTT, EEE, TTE and EET data (right panel). Note that these figures are plotted with $N = 50$, 60 and 70, for the geometric function of the DBI field as λ/χ^4.

TABLE I. The ranges of λ in which the values of the inflationary parameters r and n_s and also, $f_{\rm NL}^{\rm ortho}$ and $f_{\rm NL}^{\rm equi}$ are compatible with the 95% C.L. of the Planck2015 TT, TE, EE + low P and Planck2015 TTT, EEE, TTE and EET joint data set, respectively.

N	r versus n_s	$f_{\rm NL}^{\rm ortho}$ versus $f_{\rm NL}^{\rm equi}$
50	$20400 < \lambda < 47000$	$\lambda < 2102800$
60	$88200 < \lambda < 95000$	$\lambda < 911800$
70	*not consistent*	$\lambda < 357260$

versus the scalar spectral index for inflationary model with two scalar fields (an ordinary field and a DBI field) for $N = 50$, 60 and 70. Our numerical analysis shows that, although this model is not consistent with the Planck2015 data set for $N = 70$, it will be consistent with observations if $20400 < \lambda < 47000$ for $N = 50$ and $88200 < \lambda < 95000$ for $N = 60$. The behavior of the orthogonal configuration versus the equilateral non-Gaussianity of this model is shown in the right panel of Fig. 2. This figure confirms that in some ranges of the geometric parameter of the DBI field, that is, λ, it is possible to have large non-Gaussianity. For instance, for $N = 60$ and $\lambda < 911800$, large non-Gaussianity can be realized in this setup. The ranges of λ in which the values of the inflationary parameters r and n_s and also $f_{\rm NL}^{\rm ortho}$ and $f_{\rm NL}^{\rm equi}$ are compatible with the 95% confidence level of the Planck2015 TT, TE, EE + low P and Planck2015 TTT, EEE, TTE and EET joint data set, respectively, are shown in Table I.

Now we derive the form of the scale factor versus the involving fields in order to depict evolution in the fields space. Since $\frac{da}{a} = Hdt$, using the sum separable Hubble parameter (53), it can be written as

$$\frac{da}{a} = (H_0 + H_1\phi + H_2\chi)dt. \tag{61}$$

After integration we have

$$\ln\frac{a}{a_0} = \int H_0 dt + \int \frac{H_1\phi}{\dot{\phi}} d\phi + \int \frac{H_2\chi}{\dot{\chi}} d\chi, \tag{62}$$

which results in the following relation:

$$a(\phi,\chi) = a_0 \exp\left\{\frac{1}{4}(\phi_0^2 - \phi^2) - \frac{H_0}{2H_1}(\phi - \phi_0)\right\} \exp\left\{\frac{1}{2}\sqrt{\lambda H_2^2}\ln\left(\frac{4}{\chi^2}(2\lambda H_2^2 + \sqrt{\lambda H_2^2(\chi^4 + 4\lambda H_2^2)})\right)\right\}$$
$$\times \exp\left\{\frac{1}{4}(\sqrt{\chi_0^4 + 4\lambda H_2^2} - \sqrt{\chi^4 + 4\lambda H_2^2})\right\} \exp\left\{-\frac{1}{2}\sqrt{\lambda H_2^2}\ln\left(\frac{4}{\chi_0^2}(2\lambda H_2^2 + \sqrt{\lambda H_2^2(\chi_0^4 + 4\lambda H_2^2)})\right)\right\}. \tag{63}$$

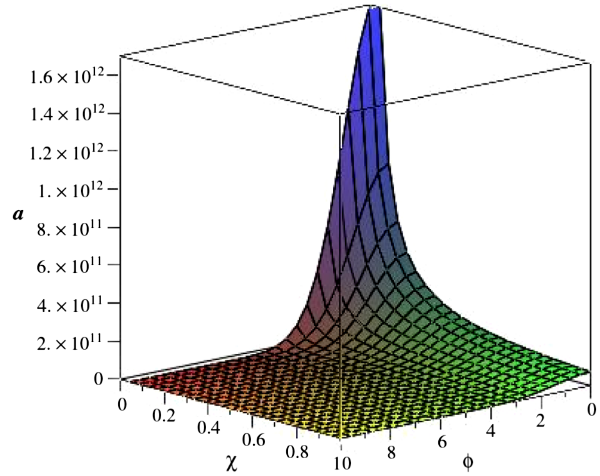

FIG. 3. Evolution in fields space for $\lambda = 10^5$.

Figure 3 shows the evolution in fields space for $\lambda = 10^5$.

VI. CONCLUSION

In this paper we have studied the dynamics of an inflationary model driven by two scalar fields, an ordinary scalar field with canonical kinetic term and a DBI field with noncanonical kinetic term. At first, we have obtained the main equations of the model. Then, we have studied the linear perturbations of this inflationary model using the ADM formalism. By expanding the action of the model up to the second order in perturbation, we have derived the two-point correlation functions which result in the amplitude of the scalar perturbation and its spectral index. Also, by studying the tensor part of the perturbed metric, we have obtained the tensor perturbation and its spectral index as well. The ratio between the amplitude of the tensor and scalar perturbations has been obtained in this setup. In order to study the non-Gaussian feature of the primordial perturbations in this setup, we have studied the nonlinear theory in details. To investigate nonlinear perturbation in the model, one has to expand the action up to the cubic order in perturbation and calculate the three-point correlation functions. Thus, by using the interacting picture we have computed the three-point correlation functions and the nonlinearity parameter in our setup. By introducing the shape functions as $\mathcal{S}_*^{\rm equil}$ and $\mathcal{S}_*^{\rm ortho}$, we have obtained the amplitude of the non-Gaussianity in the equilateral and orthogonal configurations. We have focused in the limit

$k_1 = k_2 = k_3$, in which both the equilateral and orthogonal configuration have peak.

After calculating the main perturbation parameters, we have tested our model with recent observational data. Note that we have worked beyond the slow-roll approximation, but we have assumed a separable Hubble parameter. In other words, since the slow-roll condition can be temporarily violated during inflation, we have gone beyond this approximation, and our method is based on the first order Hamilton-Jacobi formalism, which allows us to express inflationary parameters in the model, without having to focus on a slow-roll regime. We have also defined the form of the DBI function, $f(\chi)$ in terms of the warp factor of the AdS-like throat as $f(\chi) = \frac{\lambda}{\chi^4}$. Then, we have studied this inflationary model numerically and compared our model with the recently released observational data. To this end, we have studied the behavior of the tensor-to-scalar ratio versus the scalar spectral index in the background of the Planck2015 TT, TE, EE + low P data and obtained some constraints on model's parameters space. Furthermore, by studying the behavior of the orthogonal configuration versus the equilateral configuration in the equilateral limit and in the background of the Planck2015 TTT, EEE, TTE and EET data, the non-Gaussinaty feature of the primordial perturbations has been analyzed numerically. In this paper, we have shown that this inflationary model is observationally viable in some ranges of the DBI parameter. As an important result, we have shown that this model allows us to have large non-Gaussianity that would be observable by future improvements in experiments. On the other hand, the trajectories in the fields space have been depicted which indicate that for relatively small values of λ the field ϕ plays the central role in deriving inflation. However, for large values of λ, the DBI field takes a more important role in the dynamics of inflation relative to its role in the case with smaller λ.

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APPENDIX A: EQUATIONS OF MOTION IN HAMILTION-JACOBI FORMALISM

In this appendix we obtain the equations of motion (3) and (4) in Hamilton-Jacobi formalism. For an ordinary scalar field the Friedmann equation and the equation of motion are

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{A1}$$

and

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{A2}$$

respectively. Differentiating Eq. (A1) with respect to time results in

$$2HH_{,\phi}\dot{\phi} = \frac{1}{3}(\ddot{\phi} + V_{,\phi})\dot{\phi}. \tag{A3}$$

Using the equation of motion (A2) we can easily simplify the right-hand side of this relation as

$$2HH_{,\phi}\dot{\phi} = -H\dot{\phi}^2, \tag{A4}$$

and by substituting back into the definition of the Hubble parameter in the Friedmann equation, we find the following two first order equations which are entirely equivalent to the second order equation of motion (A2)

$$\dot{\phi} = -2H_{,\phi}, \tag{A5}$$

and

$$3H^2 = 2H_{,\phi}^2 + V, \tag{A6}$$

which are the Hamilton-Jacobi equations. In our case with an ordinary scalar field and a DBI field, by differentiating the Friedmann equation (5) with respect to time, we obtain

$$2H\dot{H} = \frac{1}{3}\Bigg((\ddot{\phi} + V_{,\phi})\dot{\phi} + \Bigg(\gamma^3\ddot{\chi} + \frac{1}{2}\gamma^3 f_{,\chi}f^{-2} + f_{,\chi}f^{-2} - \frac{3}{2}\gamma f_{,\chi}f^{-2} + V_{,\chi}\Bigg)\dot{\chi}\Bigg). \tag{A7}$$

We can also write

$$\dot{H} = H_{,\phi}\dot{\phi} + H_{,\chi}\dot{\chi}. \tag{A8}$$

Finally using equations of motion (3) and (4) one can easily find the following first order equations

$$\dot{\phi} = -2H_{,\phi}, \tag{A9}$$

$$\dot{\chi} = -\frac{2}{\gamma}H_{,\chi}, \tag{A10}$$

and also

$$3H^2 = 2H_{,\phi}^2 + f^{-1}\left(\frac{1}{c_\chi} - 1\right) + V. \tag{A11}$$

We note that according to definition of c_χ in Eq. (56) it is obvious that $c_\chi = \gamma^{-1}$, and we have therefore

$$\dot{\chi} = -2c_\chi H_{,\chi}. \tag{A12}$$

APPENDIX B: DERIVATION OF EQ. (58)

In order to derive Eq. (58), we note that since

$$\dot{\chi} = -2c_\chi \frac{\partial H}{\partial \chi}, \tag{B1}$$

one should solve the following integral to find $\chi(t)$

$$\int dt = \frac{-1}{2H_2} \int \sqrt{1 + \frac{4\lambda H_2^2}{\chi^4}} d\chi. \tag{B2}$$

The right-hand side integral cannot be solved analytically. To find an approximate analytical solution, we can neglect the unity in comparison with the second term in the square root (this is reasonable at least for sufficiently large λ):

$$\int dt \simeq \frac{-1}{2H_2} \int \sqrt{\frac{4\lambda H_2^2}{\chi^4}} d\chi, \tag{B3}$$

and it follows

$$\frac{1}{\chi(t)} - \frac{1}{\chi_0} \simeq \frac{t}{\sqrt{\lambda}}. \tag{B4}$$

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