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Soft Heisenberg hair on black holes in three dimensions

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Three-dimensional Einstein gravity with a negative cosmological constant admits stationary black holes that are not necessarily spherically symmetric. We propose boundary conditions for the near-horizon region of these black holes that lead to a surprisingly simple near-horizon symmetry algebra consisting of two affine $\hat{u}(1)$ current algebras. The symmetry algebra is essentially equivalent to the Heisenberg algebra. The associated charges give a specific example of "soft hair" on the horizon, as defined by Hawking, Perry and Strominger. We show that soft hair does not contribute to the Bekenstein-Hawking entropy of Bañados-Teitelboim-Zanelli black holes and "black flower" generalizations. From the near-horizon perspective the conformal generators at asymptotic infinity appear as composite operators, which we interpret in the spirit of black hole complementarity. Another remarkable feature of our boundary conditions is that they are singled out by requiring that the whole spectrum is compatible with regularity at the horizon, regardless of the value of the global charges like mass or angular momentum. Finally, we address black hole microstates and generalizations to cosmological horizons.

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I. INTRODUCTION

The Bekenstein-Hawking (BH) entropy formula for horizons of area A (G_N is Newton's constant),

$$S_{\mathrm{BH}} = \frac{A}{4G_N}, \tag{1}$$

has been a source of inspiration for approaches to quantum gravity and has led to derivations of the entropy (1) from microstate counting [1–5]. Many of these approaches exploit either the simplicity of (near-)extremal black holes or the power of conformal symmetries, or both. In generic situations, however, the horizon is nonextremal, not always due to a black hole, and the near-horizon symmetries are not necessarily conformal.

In particular, in flat space both the asymptotic symmetries [6–9] and [at least in three dimensions (3d)] the near-horizon symmetries [10,11] are related to the Bondi-van der Burg-Metzner-Sachs (BMS) algebra [6,7]. The importance of near-horizon BMS symmetries as a means to understand black holes was recently highlighted in [12]. For related works see [13–20].

In this article we explore the spacetime geometry around nonextremal horizons, which is universally approximated by the product of two-dimensional Rindler space [21] with a compact Euclidean manifold. For simplicity we work in 3d. In a corotating frame, the near-horizon metric in (ingoing) Eddington-Finkelstein coordinates is given by

$$\mathrm{d}s^2 = -2ardv^2 + 2dvdr + \gamma^2 d\varphi^2 + \cdots \tag{2}$$

where the constant a is the Rindler acceleration. The vanishing of the radial coordinate, $r = 0$, corresponds to the location of the horizon, v is the advanced time, and we assume periodicity of the angular coordinate $\varphi \sim \varphi + 2\pi$ so that the horizon is compact and has a total area given by $A = \oint d\varphi\gamma$. With no loss of generality we assume that a and γ are positive, and the ellipsis refers to higher order terms in the radial coordinate r or to rotation terms (we will be more explicit below).

One of our main goals is to explore the near-horizon behavior of the gravitational field. We propose a new set of boundary conditions consistent with (2), which leads to a very simple near-horizon symmetry algebra, the Heisenberg algebra. The associated charges provide a particular manifestation of "soft hair" in the sense of [12]. We show that the BH entropy is solely determined by the zero-mode charges and does not receive a contribution from the soft hair. We then establish how the near-horizon symmetries are linked to the ones at infinity [9,22] and interpret our results in the spirit of black hole

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complementarity [23–25]. We conclude with a discussion of black hole microstates and generalizations to cosmological horizons.

While some of the technical tools available to us are specific to 3d, we believe that the general lessons drawn from our derivations are dimension independent and thus shed new light on near-horizon symmetries, soft hair, microstate counting and black hole complementarity.

II. SOFT HAIRY BLACK HOLES

The behavior of the gravitational field in 3d general relativity with a negative cosmological constant $\Lambda = -\ell^{-2}$ around a nonextremal horizon can be described by a near-horizon metric in ingoing Eddington-Finkelstein coordinates [$\ell\rho = r$ and $f := 1 + \rho/(2a\ell)$],

$$\mathrm{d}s^2 = -2a\ell\rho f \mathrm{d}v^2 + 2\ell \mathrm{d}v\mathrm{d}\rho - 2\omega a^{-1}\mathrm{d}\varphi\mathrm{d}\rho + 4\omega\rho f \mathrm{d}v\mathrm{d}\varphi + \left[\gamma^2 + \frac{2\rho}{a\ell} f(\gamma^2 - \omega^2)\right]\mathrm{d}\varphi^2, \quad (3)$$

where ω and γ are arbitrary functions of φ. The metric deviates to leading order from (2) in the $g_{\rho\varphi}$ component, but this can always be gauged away. It turns out, however, to be convenient to keep it as it is.

The line element (3) is an exact solution of Einstein's equations in 3d since it has constant curvature. The geometry possesses an event horizon located at $\rho = 0$. Since it is not spherically symmetric, the solution generically describes a "black flower" [26]. In the case of constant ω and γ the solution (3) reduces to the Bañados-Teitelboim-Zanelli (BTZ) black hole [27,28]. The metric (3) does not obey Brown-Henneaux boundary conditions [22], which motivates us to propose boundary conditions that accommodate these solutions. This task becomes remarkably simple in the Chern-Simons formulation.

III. EINSTEIN GRAVITY AS A CHERN-SIMONS THEORY

While the metric formulation is closer to our physical and geometric intuition, the reformulation of 3d Einstein gravity as Chern-Simons theory is more powerful at a technical level, which is why we use it. The bulk action reads [29,30]

$$I_{\mathrm{CS}} = \frac{k}{4\pi}\int \left\langle \mathcal{A}\wedge d\mathcal{A} + \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\rangle. \quad (4)$$

The coupling constant is given by $k = \ell/(4G_N)$, and the connection $\mathcal{A}$ decomposes into two $sl(2,\mathbb{R})$ connections $A^\pm$ with generators $[L_n, L_m] = (n-m)L_{n+m}$ (n, $m = 0, \pm 1$) such that the bilinear form $\langle , \rangle$ is essentially the standard one for each $sl(2,\mathbb{R})$, $\langle L_1, L_{-1}\rangle = -1$, $\langle L_{\pm 1}, L_0\rangle = 0$, $\langle L_0, L_0\rangle = \frac{1}{2}$, with additional minus signs for A^-.

The metric is determined from the connections $A^\pm$ as

$$g_{\mu\nu} = \frac{\ell^2}{2}\langle (A^+_\mu - A^-_\mu)(A^+_\nu - A^-_\nu)\rangle. \quad (5)$$

First, we list the length dimensions that we are using. The quantities $v, \gamma, \omega, \ell, G_N$ have length dimension one, ρ, φ, k, $A^\pm$, L_n are dimensionless, and Rindler acceleration a has length dimension minus one.

IV. NEW BOUNDARY CONDITIONS

Based on the near-horizon behavior of the metric, one is naturally led to propose a new set of boundary conditions, which in terms of the gauge fields reads

$$A^\pm = b_\pm^{-1}(d + \mathfrak{a}^\pm)b_\pm \quad (6)$$

where $b_\pm = \exp(\pm\frac{1}{\ell\zeta^\pm}L_1)\cdot\exp(\pm\frac{\rho}{2}L_{-1})$. The auxiliary connection is given by

$$\mathfrak{a}^\pm = L_0(\pm\mathcal{J}^\pm d\varphi + \zeta^\pm dv) \quad (7)$$

with $\ell\mathcal{J}^\pm := \gamma \pm \omega$. The state-dependent functions $\mathcal{J}^\pm$ and the (arbitrary but fixed) chemical potentials $\zeta^\pm$ (see e.g. [31,32]) depend on φ and v in general.

The field equations $\mathcal{F} = d\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = 0$ hold exactly and yield $\partial_v\mathcal{J}^\pm = \pm\zeta^{\pm\prime}$, where the prime denotes differentiation with respect to φ.

For simplicity, we assume constant chemical potentials. Then the dynamical fields $\mathcal{J}^\pm$ become independent of the advanced time v, and in the particular case of $\zeta^\pm = -a$, from (5) one recovers the spacetime metric (3).

V. CANONICAL GENERATORS

Our next step is to determine the canonical generators $Q[\epsilon^+, \epsilon^-] = \mathcal{Q}^+[\epsilon^+] - \mathcal{Q}^-[\epsilon^-]$ for arbitrary transformations $\epsilon^\pm = \epsilon_i^\pm L_i$ that preserve the boundary conditions (6) and (7). In the Regge-Teitelboim approach [33,34] their variation is

$$\delta\mathcal{Q}^\pm[\epsilon^\pm] = \mp\frac{k}{4\pi}\oint d\varphi\eta^\pm\delta\mathcal{J}^\pm \quad (8)$$

with $\eta^\pm = \epsilon_0^\pm$. The most general transformations $\delta_{\epsilon^\pm}\mathfrak{a}^\pm = \mathrm{d}\epsilon^\pm + [\mathfrak{a}^\pm, \epsilon^\pm] = \mathcal{O}(\delta\mathfrak{a}^\pm)$ that preserve the boundary conditions (7) imply $\delta\mathcal{J}^\pm = \pm\eta^{\pm\prime}$, with $\partial_v\eta^\pm = 0$. The additional components $\epsilon_{\pm 1}^\pm$ generate trivial gauge transformations since they neither appear in the transformation laws of the dynamical fields nor in the variation of the global charges [35].

The global charges are obtained from functionally integrating (8), and they turn out to be finite, integrable and conserved in (advanced) time,

$$\mathcal{Q}^{\pm}[\eta^{\pm}] = \mp \frac{k}{4\pi} \oint \mathrm{d}\varphi \eta^{\pm}(\varphi) \mathcal{J}^{\pm}(\varphi). \tag{9}$$

We highlight that the surface integrals in (9) do not depend on the radial coordinate ρ, which implies that the boundary analysis actually holds for any fixed value $\rho = \rho_0$, regardless of whether ρ_0 is close to the horizon or infinity. As explained below, this is the key in order to establish the relationship between near-horizon symmetries and the ones at infinity.

VI. NEAR-HORIZON SYMMETRY ALGEBRA

The algebra of the global charges captures all boundary-condition-preserving transformations modulo trivial gauge transformations. It is determined by the relation $\delta_{\eta_2} Q[\eta_1] = \{Q[\eta_1], Q[\eta_2]\}$, where $\{,\}$ denotes Dirac brackets. Expanding in Fourier modes, $J_n^{\pm} = \frac{k}{4\pi} \oint d\varphi e^{in\varphi} \mathcal{J}^{\pm}(\varphi)$ leads to a remarkably simple symmetry algebra

$$[J_n^{\pm}, J_m^{\pm}] = \pm \frac{1}{2} kn \delta_{n+m,0}, [J_n^{+}, J_m^{-}] = 0, \tag{10}$$

where we made the usual replacement of Dirac brackets by commutators, $i\{,\} \to [,]$. The algebra (10) consists of two $\hat{u}(1)$ current algebras with levels $\pm k/2$.

Changing the basis according to $P_0 = J_0^{+} + J_0^{-}$, $P_n = \frac{i}{kn}(J_{-n}^{+} + J_{-n}^{-})$ if $n \neq 0$, $X_n = J_n^{+} - J_n^{-}$, it becomes apparent that the algebra (10) is equivalent to the canonical commutation relations for Casimir-Darboux coordinates (we set $\hbar = 1$)

$$[X_n, X_m] = [P_n, P_m] = [X_0, P_n] = [P_0, X_n] = 0, \tag{11}$$

$$[X_n, P_m] = i\delta_{n,m} \quad \text{if } n \neq 0, \tag{12}$$

where X_0 and P_0 are the two Casimirs and all other X_n, P_n form canonical pairs. Equation (12) is the Heisenberg algebra. Thus, we have obtained a surprisingly simple kinematical Hilbert space.

The near-horizon symmetry algebra (10) [or equivalently (11) and (12)] is a key result of our work.

VII. SOFT HAIR

The dynamics is governed by the Hamiltonian, whose corresponding surface integral is defined by $H \coloneqq Q[\epsilon^{\pm}|_{\partial_v}]$, with $\epsilon^{\pm}|_{\partial_v} = \mathfrak{a}_v^{\pm} = L_0 \zeta^{\pm}$ [30]. For the particular choice $\zeta^{\pm} = -a$ the Hamiltonian is given by $H = aP_0$, which commutes with all canonical coordinates X_n, P_n, so that we have trivial dynamics.

We now consider all vacuum descendants $|\psi(q)\rangle$ (labeled by a set q of arbitrary non-negative integer quantum numbers $N^{\pm}$, $n_i^{\pm}$ and $m_i^{\pm}$),

$$|\psi(q)\rangle = N(q) \prod_{i=1}^{N^+} (J_{-n_i^+}^{+})^{m_i^+} \prod_{i=1}^{N^-} (J_{-n_i^-}^{-})^{m_i^-} |0\rangle \tag{13}$$

with a normalization constant $N(q)$ such that $\langle\psi(q)|\psi(q)\rangle = 1$. Since H commutes with all generators $J_n^{\pm}$, we obtain the energy of the vacuum, $H|0\rangle = E_{\mathrm{vac}}|0\rangle$, for all descendants $|\psi(q)\rangle$.

$$E_{\psi} = \langle\psi(q)|H|\psi(q)\rangle = E_{\mathrm{vac}}\langle\psi(q)|\psi(q)\rangle = E_{\mathrm{vac}}. \tag{14}$$

This implies that all descendants of the vacuum have the same energy as the vacuum; i.e., they are "soft hair" in the precise sense of being zero-energy excitations [12]. In the derivation above we can replace the vacuum state $|0\rangle$ by any other state with the same conclusions.

VIII. SOFT HAIRY BLACK HOLE ENTROPY

Choosing constant chemical potentials $\zeta^{\pm}$, the general solution of the field equations with our boundary conditions (6) and (7) describes a stationary nonspherically symmetric black hole that carries all of the possible left and right $\hat{u}(1)$ charges. The simplest case is the spherically symmetric one that corresponds to the BTZ black hole, which only carries zero-mode charges, $J_0^{\pm} = \frac{1}{2\ell}(r_+ \pm r_-)$, where $r_{\pm}$ are the values of the surface radius at the outer or inner horizon [27,28]. Generic soft hairy black hole solutions can be obtained from the BTZ black hole by applying a generic "soft boost," i.e., acting on it with the full asymptotic symmetry group. Since soft boost generators commute with the Hamiltonian, they do not change the energy [36].

Soft hair charges do not contribute to the BH entropy, which can be readily computed from the Chern-Simons approach as in [37–40]. In fact, the entropy of a generic soft hairy black hole is found to be given by

$$S = 2\pi(J_0^{+} + J_0^{-}) = \frac{A}{4G_N}. \tag{15}$$

This result naturally motivates one to perform a microstate counting in the spirit of [1,3].

IX. LINKING NEAR-HORIZON AND ASYMPTOTIC SYMMETRIES

So far we have taken the perspective of a near-horizon observer. Here we translate our findings into the language of an asymptotic observer. Since the global charges (9) and their algebra (10) do not depend on the radial coordinate, the same structure arises at infinity. We clarify below how

our analysis manifests itself in terms of the standard variables in the asymptotic region.

Our near-horizon boundary conditions (6) and (7) are written such that the auxiliary connections $\mathfrak{a}^{\pm}$ are in diagonal gauge, while the standard asymptotic analysis uses the so-called highest weight gauge. Therefore, we transform the gauge fields in (6) and (7) to gauge fields $\hat{A}$ in the highest weight gauge [41].

For a generic choice of an unspecified chemical potential μ, the asymptotic form of the connection in the highest weight gauge is given by [32,40]

$$\hat{A} = \hat{b}^{-1}(d + \hat{\mathfrak{a}})\hat{b}, \qquad \hat{\mathfrak{a}}_{\varphi} = L_1 - \frac{1}{2}\mathcal{L}L_{-1},$$
$$\hat{b} = e^{\rho L_0}, \qquad \hat{\mathfrak{a}}_t = \mu L_1 - \mu' L_0 + \left(\frac{1}{2}\mu'' - \frac{1}{2}\mathcal{L}\mu\right)L_{-1}, \tag{16}$$

where $\mathcal{L}$ and μ are arbitrary functions of t, φ.

The problem reduces to finding a gauge transformation generated by a group element g, such that $\hat{\mathfrak{a}} = g^{-1}(d + \mathfrak{a})g$, followed by renaming the advanced time coordinate as $v = t$. We find $g = \exp(xL_1) \cdot \exp(-\frac{1}{2}\mathcal{J}L_{-1})$, where $x = x(v, \varphi)$ fulfills $\partial_v x - \zeta x = \mu$ and $x' - \mathcal{J}x = 1$, whose on-shell consistency implies

$$\mu' - \mathcal{J}\mu = -\zeta. \tag{17}$$

Therefore, the asymptotic chemical potential μ depends not only on the near-horizon chemical potential ζ but also on the near-horizon charge $\mathcal{J}$. The connections $\mathfrak{a}$ and $\hat{\mathfrak{a}}$ are mapped to each other provided that

$$\mathcal{L} = \frac{1}{2}\mathcal{J}^2 + \mathcal{J}'. \tag{18}$$

We now rephrase the gravity result (18) algebraically.

If η stands for the parameter generating an arbitrary $\hat{u}(1)$ transformation, $\delta\mathcal{J} = \eta'$, then Eq. (17) implies that the corresponding parameter ε in the highest weight gauge depends on the global charges and fulfills $\varepsilon' - \mathcal{J}\varepsilon = -\eta$. Hence, according to (18), the transformation law of $\mathcal{L}$ reads $\delta\mathcal{L} = 2\mathcal{L}\varepsilon' + \mathcal{L}'\varepsilon - \varepsilon'''$. Expanding in Fourier modes, Eq. (18) yields

$$kL_n = \sum_{p\in\mathbb{Z}} J_{n-p}J_p + iknJ_n. \tag{19}$$

This is a standard (twisted) Sugawara construction [42]. The generators L_n fulfill the Virasoro algebra with the Brown-Henneaux central extension [22]

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{1}{2}kn^3\delta_{n+m,0}. \tag{20}$$

What we have shown above is that the asymptotic symmetry algebra discovered in [22] is composite from the near-horizon perspective, which can be interpreted as an algebraic manifestation of black hole complementarity [23–25], in the sense that the same physics is most naturally described in very different terms for an asymptotic and a near-horizon observer.

Even though the spin-2 currents fulfill the Virasoro algebra (20), the corresponding global charges actually span the $\hat{u}(1)$ current algebra, which we now show explicitly. From the point of view of an observer at infinity, by virtue of Eqs. (17) and (18), the variation of the global charges reads

$$\delta\mathcal{Q} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\varepsilon\delta\mathcal{L} = -\frac{k}{4\pi}\oint \mathrm{d}\varphi\eta\delta\mathcal{J}. \tag{21}$$

The global charges satisfy the near-horizon symmetry algebra (10). Thus, in spite of the fact that the asymptotic conditions are written in the highest weight gauge, the global charges do not fulfill the Virasoro algebra with the Brown-Henneaux central extension because the chemical potential μ fulfills Eq. (17) for our boundary conditions, instead of being fixed at infinity without variation. In other words, in our case μ explicitly depends on the global charges, while what remains fixed at infinity is our chemical potential ζ.

One remarkable feature of our boundary conditions is that they are singled out by requiring that the whole spectrum is compatible with regularity of the fields, regardless of the value of the global charges. Indeed, if the chemical potential ζ is assumed to be constant, and the topology of the Euclidean manifold is that of a solid torus, where the contractible cycle corresponds to Euclidean time, regularity of the gauge field means that its holonomy around that cycle has to be trivial, implying

$$\mu\mu'' - \frac{1}{2}\mu'^2 - \mu^2\mathcal{L} = -2\pi^2/\beta^2, \tag{22}$$

where β is the length of the thermal cycle. The regularity condition (22) is solved automatically by virtue of the equations that define our boundary conditions, (17) and (18), provided that $\zeta^2 = 4\pi^2/\beta^2$. This last condition is easily obtained from solving the regularity condition directly in the diagonal gauge (7), and for $\zeta^2 = a^2$ it amounts to the Unruh temperature $T = 1/\beta = a/(2\pi)$ [43]. Remarkably, no global charges are involved in this relationship.

X. CONCLUDING REMARKS

One can use our near-horizon algebra (10) to provide a microstate counting of the entropy (15) [44]. Entropy can alternatively be calculated from composite algebras like the Virasoro algebra that we obtained in the last section. Indeed, in terms of the Virasoro zero modes $L_0^{\pm}$, using (19) entropy (15) can be written in Cardy form [45,46] as

$S = 2\pi\sqrt{kL_0^+} + 2\pi\sqrt{kL_0^-}$. Another way to perform a microstate counting through a composite algebra based on our near-horizon algebra (10) is to use the warped conformal algebra found in [10] [their Eq. (9)], which consists of a Virasoro and a $\hat{u}(1)$ current algebra. Introducing generators J_n and K_n as $J_n^{\pm} = \frac{1}{2}(J_n \pm K_n)$, the two algebras are related nonlinearly through $Y_n \sim \sum_{p\in\mathbb{Z}} J_{n-p}K_p$ and $T_n = J_n$. Using known results pertaining to two-dimensional field theories invariant under a single Virasoro and a $\hat{u}(1)$ current algebra, called warped conformal field theories [47], the cylinder partition function written as $Z(\beta,\theta) = \mathrm{Tr}e^{-\beta H + i\theta J}$, with $H = Q_{\partial_v}$ and $J = Q_{\partial_\varphi}$, enjoys the modular property $Z(\beta,\theta) = Z(2\pi\beta/\theta, -4\pi^2/\theta)$, which allows us to project the partition function on the ground state at small imaginary θ [47], yielding an entropy $S = 2\pi\beta H^{\mathrm{vac}}/\theta + i8\pi^2 J^{\mathrm{vac}}/\theta$. Assuming the vacuum state has no angular momentum, $J^{\mathrm{vac}} = 0$, and using $H = -\partial \ln Z/\partial\beta = 2\pi H^{\mathrm{vac}}/\theta$ establishes

$$S = \beta H = \frac{A}{4G_N} = S_{\mathrm{BH}}. \tag{23}$$

Interestingly, this result is independent of H^{vac}. With $\beta = 2\pi/a$ and $H = aP_0 = \frac{a}{8\pi G}\oint d\varphi\gamma = \frac{aA}{8\pi G}$, we recover the BH entropy law (1). This provides a microscopic explanation for the observation [10] that H is the product of black hole entropy and temperature.

Our results easily extend to the case of general relativity in 3d without a cosmological constant ($\ell \to \infty$) [44]. In particular, the flat limit of the metric (3) describes an interesting class of "soft hairy cosmological spacetimes" that contains the solutions discussed in [48]. Also in that case, we find that soft hair charges do not contribute to the entropy, which in turn agrees with the asymptotic state counting in [49,50]. The asymptotically flat structure can be recovered in the limit along the lines of [51–54]. We find the near-horizon symmetry algebra

$$[J_n, J_m] = [K_n, K_m] = 0, [J_n, K_m] = kn\delta_{n+m,0}. \tag{24}$$

The Hamiltonian again commutes with all the generators and therefore with all descendants of the vacuum. The centrally extended BMS currents [8,9] are recovered as composite operators constructed from (24).

It is clear that the specifics of our construction, including soft hair and black hole complementarity, certainly apply to different 3d gravity theories whose field equations are solved for constant curvature spacetimes, as is the case of the massive gravity theories discussed in [55–58], conformal gravity [52,59,60] and generalizations thereof, as well as for higher spin gravity in AdS [32,40,61–63] or in flat space [53,54,64,65]. It will be interesting to recover these physical features in four dimensions [6,7,10,12,66].

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