PHYSICAL REVIEW D **93,** 101501(R) (2016)

New results on charged compact boson stars

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(Received 26 February 2016; published 11 May 2016)

In this work we present some new results that we have obtained in a study of the phase diagram of charged compact boson stars in the theory involving massive complex scalar fields coupled to the U(1) gauge field and gravity in a conical potential in the presence of a cosmological constant Λ, which we treat as a free parameter taking positive and negative values and thereby allowing us to study the theory in de Sitter and anti de Sitter spaces, respectively. We obtain four bifurcation points (the possibility of more bifurcation points not being ruled out) in the de Sitter region. We present a detailed discussion of the various regions in our phase diagram with respect to four bifurcation points. Our theory is seen to have rich physics in a particular domain for positive values of Λ, which is consistent with the accelerated expansion of the Universe.

DOI: 10.1103/PhysRevD.93.101501

Introduced long ago [1–3], boson stars represent localized self-gravitating solutions, which are studied very widely in the literature [4–23]. Such theories are being considered in the presence of positive [14–16] as well as negative [17–20] values of the cosmological constant Λ. The theories with positive values of Λ [corresponding to de Sitter (dS) space] are relevant from an observational point of view, as they describe a more realistic description of the compact stars in the Universe since all the observations seem to indicate the existence of a positive cosmological constant. Such theories are also being used to model the dark energy of the Universe. However, the theories with negative values of Λ [corresponding to anti-de Sitter (AdS) space] are meaningful in the context of AdS/CFT correspondence [24–26].

In fact, the cosmological constant, the value of the energy density of the vacuum of space, is the simplest form of dark energy and it provides a good fit to many cosmological observations. A positive vacuum energy density resulting from a positive cosmological constant (implying a negative) pressure gives an accelerated expansion of the Universe consistent with the observations. Our theory is seen to have rich physics in a particular domain for positive values of Λ.

In a recent paper [15], we have studied the boson stars and boson shells in a theory of a complex scalar field coupled to a $U(1)$ gauge field A_μ and gravity in the presence of a fixed positive cosmological constant Λ (i.e., in de Sitter space). In the present work we study this theory of a complex scalar field coupled to a $U(1)$ gauge field A_μ and gravity in the presence of a potential: $V(|\Phi|) \coloneqq (m^2|\phi|^2 + \lambda|\phi|)$ (where m and λ are constant parameters) and a cosmological constant Λ, which we treat as a free parameter and which takes positive as well as negative values, thereby allowing us to study the theory in dS as well as in AdS space. We investigate the properties of the solutions of this theory and determine their domains of existence for some specific values of the parameters of the theory. Similar solutions have also been obtained by Kleihaus, Kunz, Laemmerzahl, and List, in a V-shaped scalar potential.

We construct the boson star solutions of this theory numerically, and we study their properties. We investigate in detail the phase diagram of the theory for the scalar and the vector fields. We obtain four bifurcation points (the possibility of more bifurcation points not being ruled out) in the dS region. We present a detailed discussion of the various regions in our phase diagram with respect to three bifurcation points.

We study the theory defined by the action

$$S = \int \left[\frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_M\right]\sqrt{-g}d^4x,$$

$$\mathcal{L}_M = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - (D_\mu\Phi)^*(D^\mu\Phi) - V(|\Phi|),$$

$$D_\mu\Phi = (\partial_\mu\Phi + ieA_\mu\Phi), \qquad F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu). \quad (1)$$

Here R is the Ricci curvature scalar, G is Newton's gravitational constant, and Λ is the cosmological constant. Also, $g = \det(g_{\mu\nu})$, where $g_{\mu\nu}$ is the metric tensor and the asterisk in the above equation denotes complex conjugation. Using the variational principle, equations of motion are obtained as

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$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu},$$
$$\partial_\mu(\sqrt{-g} F^{\mu\nu}) = -ie\sqrt{-g}[\Phi^*(D^\nu\Phi) - \Phi(D^\nu\Phi)^*],$$
$$D_\mu(\sqrt{-g} D^\mu \Phi) = 2m^2\sqrt{-g}\Phi + \frac{\lambda}{2}\sqrt{-g}\frac{\Phi}{|\Phi|},$$
$$[D_\mu(\sqrt{-g} D^\mu \Phi)]^* = 2m^2\sqrt{-g}\Phi^* + \frac{\lambda}{2}\sqrt{-g}\frac{\Phi^*}{|\Phi|}. \qquad (2)$$

The energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = \left[\left(F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) + (D_\mu\Phi)^*(D_\nu\Phi) + (D_\mu\Phi)(D_\nu\Phi)^* - g_{\mu\nu}((D_\alpha\Phi)^*(D_\beta\Phi))g^{\alpha\beta} - g_{\mu\nu}V(|\Phi|)\right]. \qquad (3)$$

To construct spherically symmetric solutions we adopt the static spherically symmetric metric with Schwarzschild-like coordinates:

$$ds^2 = [-A^2 N dt^2 + N^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \qquad (4)$$

This leads to the components of the Einstein tensor ($G_{\mu\nu}$),

$$G_t^t = \left[\frac{-[r(1-N)]'}{r^2}\right], \qquad G_r^r = \left[\frac{2rA'N - A[r(1-N)]'}{Ar^2}\right],$$
$$G_\theta^\theta = \left[\frac{2r[rA'N]' + [Ar^2N']'}{2Ar^2}\right] = G_\varphi^\varphi. \qquad (5)$$

Here the arguments of the functions $A(r)$ and $N(r)$ have been suppressed. For solutions with a vanishing magnetic field, the Ansätze for the matter fields have the form

$$\Phi(x^\mu) = \phi(r)e^{i\omega t}, \qquad A_\mu(x^\mu)dx^\mu = A_t(r)dt. \qquad (6)$$

We redefine $\phi(r)$ and $A_t(r)$ as

$$h(r) = (\sqrt{2}e\phi(r))/m, \qquad b(r) = (\omega + eA_t(r))/m. \qquad (7)$$

We introduce new dimensionless constant parameters:

$$\alpha = \frac{4\pi G m^2}{e^2}, \qquad \tilde{\lambda} = \frac{\lambda e}{\sqrt{2}m^3}, \qquad \tilde{\Lambda} = \frac{\Lambda}{m^2}. \qquad (8)$$

Introducing a dimensionless coordinate $\hat{r}$ defined by $\hat{r} = mr$ (implying $\frac{d}{dr} = m\frac{d}{d\hat{r}}$), Eq. (7) reads

$$h(\hat{r}) = (\sqrt{2}e\phi(\hat{r}))/m, \qquad b(\hat{r}) = (\omega + eA_t(\hat{r}))/m. \qquad (9)$$

Equations of motion in terms of $h(\hat{r})$ and $b(\hat{r})$ [where the primes denote differentiation with respect to $\hat{r}$ and sign(h) denotes the usual signature function] read

$$(AN\hat{r}^2h')' = \frac{\hat{r}^2}{AN}[A^2N(h + \tilde{\lambda}\text{sign}(h)) - b^2h], \qquad (10)$$

$$[(\hat{r}^2b')/A]' = [(\hat{r}^2h^2b)/(AN)]. \qquad (11)$$

We obtain

$$h'' = \left[\frac{\alpha\hat{r}h'}{A^2N}(A^2h^2 + 2A^2h\tilde{\lambda} + b'^2) - \frac{h'(1 + N - \tilde{\Lambda}\hat{r}^2)}{\hat{r}N} + \frac{A^2Nh + A^2N\tilde{\lambda}\text{sign}(h) - b^2h}{A^2N^2}\right], \qquad (12a)$$

$$b'' = \left[\frac{\alpha}{A^2N^2}\hat{r}b'(A^2N^2h'^2 + b^2h^2) - \frac{2b'}{\hat{r}} + \frac{bh^2}{N}\right],$$
$$A' = \left[\frac{\alpha\hat{r}}{AN^2}(A^2N^2h'^2 + b^2h^2)\right], \qquad (12b)$$

$$N' = \left[\frac{1 - N - \tilde{\Lambda}\hat{r}^2}{\hat{r}} - \frac{\alpha\hat{r}}{A^2N}(A^2N^2h'^2 + Nb'^2 + b^2h^2 + A^2Nh^2 + 2A^2Nh\tilde{\lambda})\right]. \qquad (12c)$$

For the metric function $A(\hat{r})$ we choose the boundary condition $A(\hat{r}_o) = 1$, where $\hat{r}_o$ is the outer radius of the star. For constructing globally regular ball-like boson star solutions, we choose

$$N(0) = 1, \qquad b'(0) = 0,$$
$$h'(0) = 0, \qquad h(\hat{r}_o) = 0, \qquad h'(\hat{r}_o) = 0. \qquad (13)$$

For the positive and negative $\tilde{\Lambda}$ we match, in the exterior region $\hat{r} > \hat{r}_o$, the Reissner-Nordström–de Sitter and Reissner-Nordström–anti-de Sitter solutions, respectively. The conserved Noether current is given by

$$j^\mu = -ie[\Phi(D^\mu\Phi)^* - \Phi^*(D^\mu\Phi)], \qquad D_\mu j^\mu = 0. \qquad (14)$$

The charge Q of the boson star is given by

$$Q = -\frac{1}{4\pi}\int_0^{\hat{r}_o} j^t\sqrt{-g}drd\theta d\phi, \qquad j^t = -\frac{h^2(\hat{r})b(\hat{r})}{A^2(\hat{r})N(\hat{r})}.$$

For all the gravitating solutions we obtain the mass parameter M (in the units employed):

$$M = \left(1 - N(\hat{r}_o) + \frac{\alpha Q^2}{\hat{r}_o^2} - \frac{\tilde{\Lambda}}{3}\hat{r}_o^2\right)\frac{\hat{r}_o}{2}. \qquad (15)$$

We now study the numerical solutions of Eqs. (12a)–(12c) with the boundary conditions defined by $A(\hat{r}_o) = 1$ and Eq. (13), and we determine their domain of existence for some specific values of the parameters of the theory. Our theory has three parameters: α, $\tilde{\lambda}$, and $\tilde{\Lambda}$. We study the theory for different values of $\tilde{\Lambda}$, giving it positive as well as negative values, keeping α and $\tilde{\lambda}$ fixed (namely, $\alpha = 0.2$ and $\tilde{\lambda} = 1.0$), and we discuss the corresponding physics as it is observed in our phase diagram.

We study the phase diagram of the theory involving the vector and scalar fields at the center of the boson star for different values of the cosmological constant $\tilde{\Lambda}$. We observe some interesting phenomena near some specific values of $\tilde{\Lambda}$ where the system is seen to have bifurcation points B_1, B_2, B_3, and B_4 which correspond to four different values of the cosmological constant $\tilde{\Lambda}$: $\tilde{\Lambda}_{c_1} \simeq 0.22521$, $\tilde{\Lambda}_{c_2} \simeq 0.52605$, $\tilde{\Lambda}_{c_3} \simeq 0.54076$, and $\tilde{\Lambda}_{c_4} \simeq 0.541250$, respectively (the possibility of more bifurcation points not being ruled out). The theory is seen to have rich physics in the domain $\tilde{\Lambda} = +0.500$ to $\tilde{\Lambda} \simeq +0.62$.

For a meaningful discussion, we divide our phase diagram into four regions denoted by IA, IB, IIA, and IIB in the vicinity of B_1 [as seen in Fig. 1(a)]. The asterisks seen in Fig. 1(a). coinciding with the axis $b(0)$ [i.e., corresponding to $h(0) = 0$], represent the transition points from the boson stars to boson shells.

Regions IA, IB, and IIA do not have any bifurcation points; however, region IIB is seen to contain rich physics evidenced by the occurrence of more bifurcation points in this region. For better detail, region IIB is also plotted in Fig. 1(b). Region IIB is further divided into the regions IIB1, IIB2, and IIB3 in the vicinity of B_2 as seen in Fig. 1(b).

Region IIB3 is seen to have a further bifurcation point B_3. In the vicinity of B_3 we further subdivide the phase diagram into the regions IIB3a, IIB3b, and IIB3c as seen in Fig. 1(b). Region IIB3b is seen to have closed loops, and the behavior of the phase diagram in this region is akin to that of region IIB2. Also, the insets in Figs. 1(b) and 1(c) represent part of the phase diagram with better precision.

Region IIB3c is again seen to have a further bifurcation point B_4, and in the vicinity of B_4, we again subdivide the phase diagram into the regions IIB3c1, IIB3c2, and IIB3c3 as seen in Fig. 1(c). Region IIB3c2 is again seen to have closed loops, and the behavior of the phase diagram in this region is akin to that of regions IIB2 and IIB3b. On the other hand, region IIB3c3 could, in principle, contain further bifurcation points, and the behavior of the phase diagram in this region is akin to that of regions IIB1, IIB3a, and IIB3c1.

Regions IA and IB could be divided into two subregions corresponding to positive and negative values of $\tilde{\Lambda}$, implying the dS and AdS regions corresponding to positive and negative values of $\tilde{\Lambda}$. In region IA, as we change the value of $\tilde{\Lambda}$ in the AdS region from $\tilde{\Lambda} = 0.000$ to

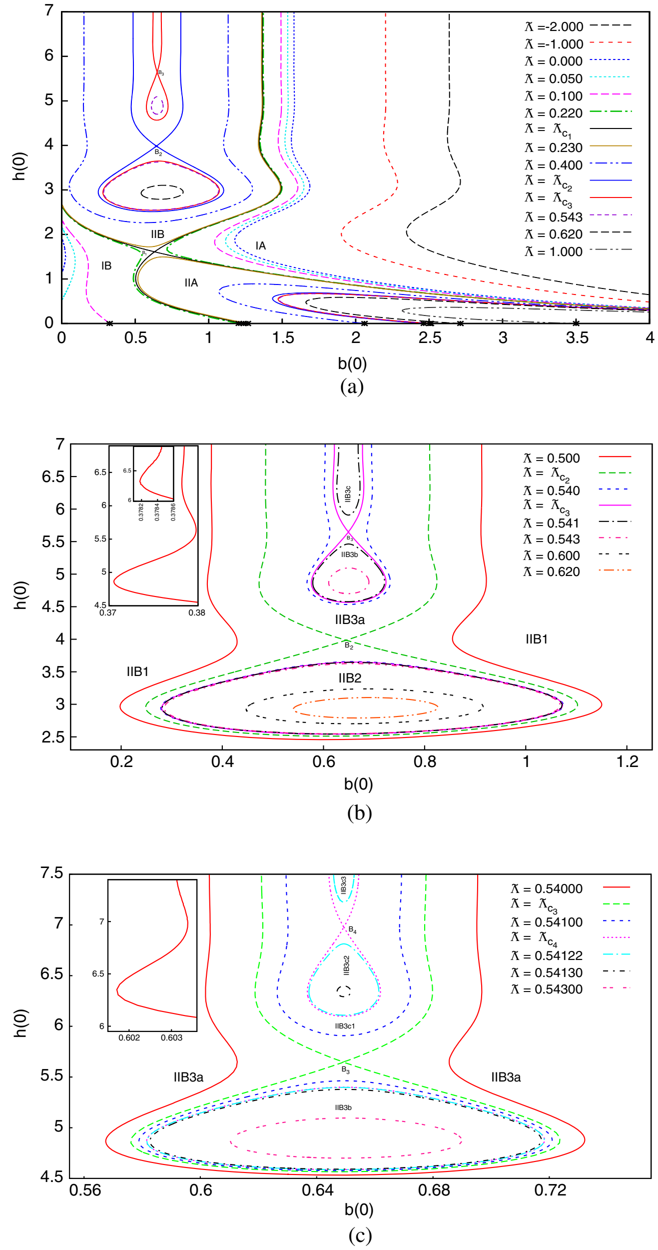

FIG. 1. (a) The phase diagram of the theory for the vector field at the center of the star $b(0)$ and the scalar field at the center of the star $h(0)$ for different values of the cosmological constant $\tilde{\Lambda}$ in the range $\tilde{\Lambda} = -2.000$ to $\tilde{\Lambda} = +1.000$. The points B_1, B_2, B_3, and B_4 represent the four bifurcation points. The entire region depicted in the phase diagram in panel (a) is divided into four regions: IA, IB and IIA, IIB in the vicinity of B_1. The region IIB of the phase diagram shown in (a) is separately depicted in detail in panels (b) and (c). The region IIB of the phase diagram is subdivided into three regions: IIB1, IIB2, and IIB3 in the vicinity of B_2. The region IIB3 is further subdivided into the regions IIB3a, IIB3b, and IIB3c in the vicinity of B_3. Similarly, the region IIB3c is subdivided into the regions IIB3c1, IIB3c2, and IIB3c3 in the vicinity of B_4 as depicted in panel (c). The asterisks shown in (a), corresponding to $h(0) = 0$, represent the transition points from the boson stars to the boson shells. The inset in panel (b) represents part of the phase diagram with better precision.

$\tilde{\Lambda} = -2.000$, we observe a continuous deformation of the curves in the phase diagram. In region IB, as we change the value of $\tilde{\Lambda}$ in the domain $\tilde{\Lambda} = 0.000$ to $\tilde{\Lambda} \simeq -0.02$, the theory is seen to have solutions for the boson stars only, without having transition points from boson stars to boson shells, and the curves corresponding to the solutions disappear in the phase diagram of the theory for the values $\tilde{\Lambda} \lesssim -0.02$.

As we change the value of $\tilde{\Lambda}$ in the dS region from $\tilde{\Lambda} = 0.000$ to $\tilde{\Lambda} = 1.000$, we observe a lot of new rich physics. While going from $\tilde{\Lambda} = 0.000$ to some critical value $\tilde{\Lambda} = \tilde{\Lambda}_{c_1}$, we observe that the solutions exist in two separate domains IA and IB [as seen in Fig. 1(a)]. However, as we increase $\tilde{\Lambda}$ beyond $\tilde{\Lambda} = \tilde{\Lambda}_{c_1}$, the solutions of the theory are seen to exist in regions IIA and IIB (instead of regions IA and IB).

As we increase the value of $\tilde{\Lambda}$ from one critical value $\tilde{\Lambda} = \tilde{\Lambda}_{c_1}$ to another critical value $\tilde{\Lambda} = \tilde{\Lambda}_{c_2}$, we notice that region IIA in the phase diagram shows a continuous deformation of the curves, and region IIB is seen to have its own rich physics as explained in the foregoing.

As we increase $\tilde{\Lambda}$ beyond $\tilde{\Lambda}_{c_2}$, we observe that in region IIA there is again a continuous deformation of the curves all the way up to $\tilde{\Lambda} = 1.000$. However, in the region IIB, we encounter another bifurcation point which divides region IIB into IIB1, IIB2, and IIB3. We observe that in region IIB1 there is a continuous deformation of the curves, and region IIB2 contains closed loops of the curves. Region IIB3 is subdivided into regions IIB3a, IIB3b, and IIB3c. Region IIB3a would have a continuous deformation of the curves, and region IIB3b is seen to contain closed loops. Region IIB3c (subdivided into regions IIB3c1, IIB3c2, and IIB3c3) has its own rich physics as depicted in Figs. 1(b) and 1(c) and as discussed in detail in the foregoing. Region IIB3c3 has its own rich physics in the sense that this region could, in principle, have further bifurcation points.

A plot of the vector field at the center of the star $b(0)$ versus the radius $\hat{r}_o$ of the boson star is depicted in Fig. 2(a). As before, the point B_1 corresponds to the bifurcation point, and the entire region depicted in Fig. 2(a) is divided into four regions, IA, IB and IIA, IIB, in the vicinity of the bifurcation point B_1. Region IIB, shown in Fig. 2(a), is separately depicted in detail in Fig. 2(b). The asterisks shown in Fig. 2(a) represent the transition points from the boson stars to the boson shells. The spiral behavior of the solutions is visible in regions IA and IIB. The inset in Fig. 2(b) represents part of region IIB with better precision.

In conclusion, we have studied a theory of a massive complex scalar field coupled to the U(1) gauge field and gravity with a conical potential in the presence of a cosmological constant Λ which takes positive as well as negative values. The theory is seen to have rich physics in

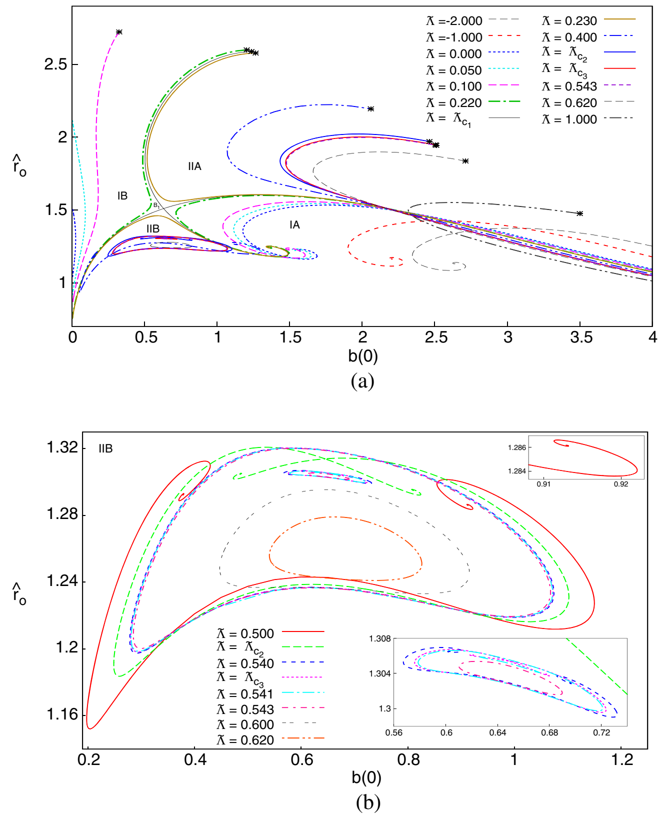

FIG. 2. (a) Plot of the vector field at the center of the star $b(0)$ versus the radius $\hat{r}_o$ of the boson star. The point B_1 corresponds to the bifurcation point, and the entire region depicted in panel (a) is divided into the four regions IA, IB and IIA, IIB in the vicinity of B_1. The region IIB shown in (a) is separately depicted in detail in panel (b). The asterisks shown in (a) represent the transition points from the boson stars to the boson shells. The spiral behavior of the solutions is visible in regions IA and IIB. The inset in panel (b) represents part of region IIB with better precision.

the domain $\tilde{\Lambda} = 0.5$ to $\tilde{\Lambda} \simeq 0.62$. Four bifurcation points, B_1, B_2, B_3, and B_4, have been obtained in the phase diagram, and the physical behavior of the phase diagram has been discussed in the various regions of the phase diagram. We have observed interesting physics near the four bifurcation points which correspond to the positive values of $\tilde{\Lambda}$.

Towards the end we make some interesting observations. Our theory has three free parameters. If we fix any two of them at some appropriate values and vary the third carefully, then we notice that the bifurcation phenomenon occurs, suggesting that the bifurcation phenomenon seems to be generic. In the present study we have fixed $\alpha = 0.2$ and $\tilde{\lambda} = 1.0$ and have studied the theory by varying the value of $\tilde{\Lambda}$ from -2.0 to 1.0 in the phase diagram.

We wish to emphasize that, in particular, if we fix $\tilde{\Lambda} = 0.541$, $\alpha = 0.2$, and $\tilde{\lambda} = 1.0$, for example, then we obtain closed loops in the phase diagram, and if we vary any one of the parameters, keeping the other two parameters fixed, then we obtain bifurcation points between these closed loops as seen in Fig. 1(b) for the variation of $\tilde{\Lambda}$.

The results of our preliminary investigations suggest that, in particular, if we fix $\tilde{\Lambda} = 0$ and $\alpha = 0.2$ and vary $\tilde{\lambda}$ carefully, then we notice the bifurcation phenomenon in a manner analogous to our present studies. Following the same logic, if we fix $\tilde{\Lambda}$ and $\tilde{\lambda}$ appropriately and vary α carefully, then we again expect to obtain a bifurcation phenomenon. These investigations are currently in progress, and the detailed results will be reported later in a separate communication. Nevertheless, we feel that the occurrence of the bifurcation phenomenon should be a generic feature of the theory.

ACKNOWLEDGMENTS

We thank Jutta Kunz and Burkhard Kleihaus for introducing us to this subject and for helpful educative discussions and encouragement. We also thank James Vary for very useful discussions. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-87ER40371 and by the U.S. National Science Foundation under Grant No. PHY-0904782.

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