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Chiral spin-3/2 particles in a medium

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We consider the propagation of a chiral spin-3/2 particle in a background medium using the thermal field theory (TFT) method, in analogy to the cases of a spin-1/2 fermion (e.g., a neutrino) and the photon. We present a systematic decomposition of the thermal self-energy, from which the dispersion relation of the modes that propagate in the medium are obtained. We find that there are several modes and in each case we obtain the equation for the dispersion relation as well as the corresponding spin-3/2 spinor. As an example of the general procedure and results, we consider a model in which the chiral spin-3/2 particle couples to a spin-1/2 fermion and a scalar particle, and propagates in a thermal background composed of such particles. The dispersion relations and corresponding spinors are detemined explicitly in this case from the 1-loop TFT expression for the self-energy. The results in this case share some resemblance and analogies with the photon and the chiral fermion cases but, as already noted, there are also differences. The present work provides the groundwork for considering problems related to the properties of chiral spin-3/2 particles in a medium, in analogy to the case of neutrinos for example, which can be relevant in physical contexts of current interest.

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I. INTRODUCTION

It is known that dispersion effects can have significant impact on the properties of elementary particles when they propagate through a background medium, such as the dispersion relation and the induced electromagnetic couplings of electrically neutral particles. The effects on photons and plasma physics have of course been known for a long time, and more recently it has been a crucial item in neutrino physics since the discovery of the Mikheyev-Smirnov-Wolfenstein effect [1]. From a modern point of view, the methods of thermal field theory (TFT) [2] have been useful for studying many problems associated with such effects in a variety of physical contexts. This view has been partially stimulated by the original work of Weldon [3,4] showing the convenience of the covariant TFT calculations in this kind of problems, in particular in systems involving chiral fermions at finite temperature.

In the case of neutrinos there is an extensive literature on the effects of the background medium on their properties and propagation. Apart from the dispersion relation [5,6], the medium also induces electromagnetic couplings [7] that can lead to effects in astrophysical and/or cosmological settings, as well as neutrino collective oscillations [8] that have been the subject and significant work in the context of instabilities in supernovas [9].

Here we consider the propagation of a chiral spin-3/2 particle λ_L^μ in a background medium, using the same TFT techniques. It has been shown recently [10] that the theory of a gauged massless chiral Rarita-Schwinger (RS) field [11] is consistent with physical principles (e.g., no superluminal propagation and others). Thus it seems useful to look in a general way at the case of a massless chiral spin-3/2 particle propagating in a thermal background, in analogy to the case of a spin-1/2 fermion (e.g., a neutrino) or the photon, as mentioned above. Our aim is to provide a useful starting point for considering similar problems in the spin-3/2 case. Because the chiral λ_L^μ field can be thought of as a combination of a spin-1/2 chiral field and massless spin-1 field, the problem shares some analogies with both of those cases, although the details are different.

Our main result is a systematic decomposition of the thermal self-energy, from which the dispersion relation of the modes that propagate in the medium are obtained. We assume that the interactions of the λ_L particle with the thermal background particles are such that the thermal self-energy is transverse to the momentum four-vector of the propagating mode. A key element of the procedure is a general expression for the self-energy in terms of a set of scalar functions, each one corresponding to an independent tensor, consistent with the transversality condition, constructed using the momentum vectors available in the

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system (the momentum four-vector k^μ of the particle and the background velocity four-vector u^μ), as well as the gamma matrices, the metric tensor and the Levi-Civita tensor. On the basis of such a decomposition we find that there is a transverse mode, in the sense that its spin-3/2 spinor is transverse to k^μ, and two other modes that involve the longitudinal polarization vector. In each case we obtain the equation for the dispersion relation in terms of the self-energy scalar functions, as well as the corresponding spin-3/2 spinor. We illustrate the application of the procedure and the results by considering a model in which the λ^μ_L particle couples to a spin-1/2 fermion and a scalar particle, and propagates in a thermal background composed of such particles. From the 1-loop TFT expression for the self-energy, we determine the scalar functions referred to above, and in turn from them the dispersion relations and corresponding spinors. The results in this case share some resemblance and analogies with the photon and the chiral fermion case but, as already noted, there are also differences. Thus, the present work provides the groundwork for considering problems related to the properties of chiral spin-3/2 particles in a medium, in analogy to the case of neutrinos for example. The results presented here can be useful in problems of current research interest that involve the thermal production of spin-3/2 particles in cosmological (e.g., gravitinos in the early Universe) or astrophysical contexts [12,13].

The rest of the paper is organized as follows. In Sec. II we introduce the self-energy function and the λ^μ_L effective equation of motion in a medium. In Sec. III we write down the decomposition of the thermal self-energy in terms of a set of independent structure tensors and the corresponding scalar functions, consistent with the transversality condition. The tensors are constructed from the momentum of the particle k^μ and the background velocity four-vector u^μ, as well as the gamma matrices, the metric tensor and the Levi-Civita tensor. In Sec. IV we set down the conventions that we use for the spin-3/2 basis spinors, in terms of the spin-1 polarization vectors and the spin-1/2 chiral spinors. In Sec. V the equations for the dispersion relations are obtained, and finally in Sec. VI the equations are solved explicitly in the example mentioned above.

II. EQUATION OF MOTION AND THE SELF-ENERGY IN THE MEDIUM

The free-field part of the chiral (massless) RS Lagrangian in coordinate space is given by

$$L^{(0)}_\lambda = -\overline{\lambda}^\mu_L\{i\partial\!\!\!/\lambda_{L\mu} - \gamma_\mu i\partial\cdot\lambda_L - i\partial_\mu\gamma\cdot\lambda_L + \gamma_\mu i\partial\!\!\!/\gamma\cdot\lambda_L\}, \tag{2.1}$$

and in momentum space it translates to

$$L^{(0)}_\lambda(k) = \overline{\lambda}^\mu_L(k)L_{\mu\nu}\lambda^\nu_L(k), \tag{2.2}$$

where

$$L_{\mu\nu} = -[g_{\mu\nu}k\!\!\!/ - k_\mu\gamma_\nu - k_\nu\gamma_\mu + \gamma_\mu k\!\!\!/\gamma_\nu]. \tag{2.3}$$

Equations (2.1) and (2.2) can be rewritten in the alternative forms

$$L^{(0)}_\lambda = -\epsilon_{\mu\nu\alpha\beta}\overline{\lambda}^\mu_L\gamma^\alpha\partial^\beta\lambda^\nu_L, \tag{2.4}$$

and

$$L^{(0)}_\lambda(k) = \overline{\lambda}^\mu_L(k)\ell_{\mu\nu\lambda}(k)\gamma^\lambda\lambda^\nu_L(k), \tag{2.5}$$

respectively, where we have defined

$$\ell_{\mu\nu\lambda}(k) = i\epsilon_{\mu\nu\lambda\rho}k^\rho. \tag{2.6}$$

This follows from the identity [14]

$$\gamma_\alpha\gamma_\beta\gamma_\gamma = (g_{\alpha\beta}\gamma_\gamma - g_{\alpha\gamma}\gamma_\beta + g_{\beta\gamma}\gamma_\alpha) + i\epsilon_{\alpha\beta\gamma\lambda}\gamma^\lambda\gamma_5, \tag{2.7}$$

which implies, for example, that

$$L_{\mu\nu}L = \ell_{\mu\nu\lambda}\gamma^\lambda L, \tag{2.8}$$

where we have defined as usual $L = \frac{1}{2}(1-\gamma_5)$.

Chirality implies that, in the presence of the medium, the effective Lagrangian has a similar gamma matrix structure. In particular it can be written in the form

$$L_{\rm eff}(k) = \overline{\lambda}^\mu_L(k)[\ell_{\mu\nu\lambda} - \pi_{\mu\nu\lambda}]\gamma^\lambda\lambda^\nu_L(k), \tag{2.9}$$

where $\pi_{\mu\nu\lambda}$ is a tensor that depends on k^μ and the velocity four-vector of the background medium u^μ, but does not contain any γ matrices. Therefore the thermal self-energy $\Sigma_{T\mu\nu}$ must have the form

$$\Sigma_{T\mu\nu} = \pi_{\mu\nu\lambda}\gamma^\lambda L, \tag{2.10}$$

which in turn implies the convenient formula

$$\pi_{\mu\nu\lambda} = \frac{1}{2}{\rm Tr}(\gamma_\lambda\Sigma_{T\mu\nu}). \tag{2.11}$$

The dispersion relations of the propagating modes then follow from solving the effective RS equation in the medium,

$$[\ell_{\mu\nu\lambda} - \pi_{\mu\nu\lambda}]\gamma^\lambda\psi^\nu_L(k) = 0, \tag{2.12}$$

or equivalently

$$-[g_{\mu\nu}k\!\!\!/ - k_\mu\gamma_\nu - k_\nu\gamma_\mu + \gamma_\mu k\!\!\!/\gamma_\nu]\psi^\nu_L(k) - \pi_{\mu\nu\lambda}\gamma^\lambda\psi^\nu_L(k) = 0. \tag{2.13}$$

Our purpose in what follows is to use this equation as the starting point to calculate the dispersion relation of the λ propagating modes in a background medium as previously described.

III. GENERAL FORM OF $\pi_{\mu\nu\alpha}$

The tensor $\pi_{\mu\nu\lambda}$ introduced in Eq. (2.9) depends on the vectors k^μ and u^μ, but does not contain any γ matrices, and since we are assuming that the interactions of the λ_L particle are such that the thermal self-energy is transverse, it satisfies the transversality condition,

$$k^\mu \pi_{\mu\nu\alpha} = k^\nu \pi_{\mu\nu\alpha} = 0. \tag{3.1}$$

In analogy with the decomposition of the photon self-energy in a medium, it is also convenient here to introduce a similar notation. Thus we define

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \tag{3.2}$$

the transverse vector

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu, \tag{3.3}$$

and the tensors

$$\begin{aligned} R_{\mu\nu} &= \tilde{g}_{\mu\nu} - Q_{\mu\nu}, \\ Q_{\mu\nu} &= \frac{\tilde{u}_\mu \tilde{u}_\nu}{\tilde{u}^2}, \\ P_{\mu\nu} &= \frac{i}{\kappa} \epsilon_{\mu\nu\alpha\beta} k^\alpha u^\beta, \end{aligned} \tag{3.4}$$

where

$$\kappa = \sqrt{\omega^2 - k^2}, \tag{3.5}$$

with

$$\omega = k \cdot u. \tag{3.6}$$

The variables ω and κ are simply the energy and magnitude of the three-dimensional momentum vector, respectively, in the frame in which the medium is at rest, i.e., the frame in which $u^\mu = (1, 0)$.

With this notation at hand, it is clear that, aside from a term $\sim \epsilon_{\mu\nu\alpha\beta} k^\beta$ of similar form to the vacuum kinetic energy term, $\pi_{\mu\nu\alpha}$ can be written as a combination of terms of the following form:

$$T_{\mu\nu} a_\alpha, \qquad T_{\mu\alpha} \tilde{u}_\nu, \qquad T_{\alpha\nu} \tilde{u}_\mu, \qquad \epsilon_{\mu\nu\alpha\beta} k^\beta, \tag{3.7}$$

where T can be either R, Q or P, defined in Eq. (3.4), and a_α can be either k_α or u_α. Therefore we write

$$\pi_{\mu\nu\alpha} = \pi_0 \ell_{\mu\nu\alpha} + \pi'_{\mu\nu\alpha}, \tag{3.8}$$

with

$$\begin{aligned} \pi'_{\mu\nu\alpha} = {} & \pi_{R1} R_{\mu\nu} k_\alpha + \pi_{R2} R_{\mu\nu} u_\alpha + \pi_{R3} R_{\mu\alpha} \tilde{u}_\nu + \pi_{R4} R_{\alpha\nu} \tilde{u}_\mu \\ & + \pi_{P1} P_{\mu\nu} k_\alpha + \pi_{P2} P_{\mu\nu} u_\alpha + \pi_{P3} P_{\mu\alpha} \tilde{u}_\nu + \pi_{P4} P_{\alpha\nu} \tilde{u}_\mu \\ & + \pi_{Q1} Q_{\mu\nu} k_\alpha + \pi_{Q2} Q_{\mu\nu} u_\alpha, \end{aligned} \tag{3.9}$$

where we have used the fact that the terms $Q_{\mu\alpha}\tilde{u}_\nu$ and $Q_{\alpha\nu}\tilde{u}_\mu$ are of the same form as those as the $\pi_{Q1,2}$ terms above, and $\ell_{\mu\nu\alpha}$ is defined in Eq. (2.6).

IV. BASIS SPINORS

A. Spin-1 polarization vectors

It is useful to introduce the following notation, borrowed from the analogous discussions in the case of the photon propagating in a medium. Adopting the rest frame of the medium,

$$u^\mu = (1, \vec{0}), \tag{4.1}$$

and writing the momentum vector in the form

$$k^\mu = (\omega, \vec{k}), \tag{4.2}$$

we introduce the spin-1 polarization vectors in the usual way,

$$\begin{aligned} \epsilon^\mu_{1,2} &= (0, \hat{e}_{1,2}), \\ \epsilon^\mu_\pm &= \frac{1}{\sqrt{2}} (\epsilon^\mu_1 \pm i \epsilon^\mu_2), \end{aligned} \tag{4.3}$$

where $\hat{e}_{1,2}$ are such that

$$\hat{e}_{1,2} \cdot \vec{k} = 0, \qquad \hat{e}_2 = \vec{k} \times \hat{e}_1. \tag{4.4}$$

In addition we define

$$\epsilon^\mu_\ell = \frac{1}{\sqrt{-\tilde{u}^2}} \tilde{u}^\mu, \tag{4.5}$$

where $\tilde{u}_\mu$ has been defined in Eq. (3.3), which in the rest frame of the medium is given by

$$\epsilon^\mu_\ell = -\frac{1}{\sqrt{k^2}} (\kappa, \omega \hat{k}), \tag{4.6}$$

with κ defined in Eq. (3.5). These vectors are mutually orthogonal and satisfy the following relations:

$$\epsilon_\ell \cdot k = \epsilon_\pm \cdot k = \epsilon_\pm \cdot u = 0, \tag{4.7}$$

which in turn imply

$$R_{\mu\nu}\epsilon_{\pm}^{\nu} = \epsilon_{\pm\mu},$$
$$Q_{\mu\nu}\epsilon_{\ell}^{\nu} = \epsilon_{\ell\mu},$$
$$P_{\mu\nu}\epsilon_{\pm}^{\nu} = \pm\epsilon_{\pm\mu},$$
$$R_{\mu\nu}\epsilon_{\ell}^{\nu} = Q_{\mu\nu}\epsilon_{\pm}^{\nu} = P_{\mu\nu}\epsilon_{\ell}^{\nu} = 0. \tag{4.8}$$

$R^{\mu\nu}$ and $P^{\mu\nu}$ have the representation

$$R^{\mu\nu} = -(\epsilon_{+}^{\mu}\epsilon_{-}^{\nu} + \epsilon_{-}^{\mu}\epsilon_{+}^{\nu}),$$
$$P^{\mu\nu} = -(\epsilon_{+}^{\mu}\epsilon_{-}^{\nu} - \epsilon_{-}^{\mu}\epsilon_{+}^{\nu}), \tag{4.9}$$

and also useful are the relations

$$\epsilon_{\mu\nu\alpha\beta}k^{\alpha}\epsilon_{\pm}^{\beta} = \frac{\mp i\kappa}{\sqrt{-\tilde{u}^2}}(\epsilon_{\pm\mu}\epsilon_{\ell\nu} - \epsilon_{\ell\mu}\epsilon_{\pm\nu}). \tag{4.10}$$

They can be verified as follows. Consider for example, $\epsilon_{\mu\nu\alpha\beta}k^{\alpha}\epsilon_{+}^{\beta}$. Since it is an antisymmetric tensor and is transverse to k^{μ} and ϵ_{+}^{μ}, it must be proportional to the term given in the right-hand side of Eq. (4.10). The proportionality factor can be verified by multiplying both sides with u^{ν} and reducing the resulting expressions using the multiplication rules of the polarization vectors.

It should be kept in mind that in Eq. (4.5), and wherever ϵ_{ℓ}^{μ} appears, the assumption is that we are considering situations in which k^{μ} is such that

$$k^2 \neq 0. \tag{4.11}$$

B. Dirac spinors

Regarding the Dirac spinors we adopt, once and for all, the Weyl representation of the gamma matrices,

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}. \tag{4.12}$$

We denote by u_{L-} the left-handed chiral spinor,

$$u_{L-} = \begin{pmatrix} 0 \\ \xi_{-} \end{pmatrix}, \tag{4.13}$$

where ξ_{-} is the Pauli spinor with negative helicity; i.e., it satisfies

$$\hat{k}\cdot\vec{\sigma}\xi_{-} = -\xi_{-}. \tag{4.14}$$

Using the same notation, it is useful to introduce also the spinor

$$u_{L+} = \begin{pmatrix} 0 \\ \xi_{+} \end{pmatrix}, \tag{4.15}$$

where,

$$\hat{k}\cdot\vec{\sigma}\xi_{+} = \xi_{+}, \tag{4.16}$$

and similarly,

$$u_{R\pm} = \begin{pmatrix} \xi_{\pm} \\ 0 \end{pmatrix}. \tag{4.17}$$

The spinors satisfy the following relations (the right-handed counterparts satisfy analogous relations, but we will not need them in what follows)

$$ku_{L\pm} = (\omega \pm \kappa)u_{R\pm},$$
$$\not{u}u_{L\pm} = u_{R\pm}, \tag{4.18}$$

and

$$(\epsilon_{-}\cdot\gamma)u_{L-} = (\epsilon_{+}\cdot\gamma)u_{L+} = 0$$
$$(\epsilon_{\pm}\cdot\gamma)u_{L\mp} = \sqrt{2}u_{R\pm}. \tag{4.19}$$

From Eqs. (4.9) and (4.19) we can obtain other useful formulas, for example

$$(R^{\mu}{}_{\alpha}\gamma^{\alpha})u_{L\pm} = -\sqrt{2}\epsilon_{\pm}^{\mu}u_{R}^{(\mp)},$$
$$(P^{\mu}{}_{\alpha}\gamma^{\alpha})u_{L\pm} = \mp\sqrt{2}\epsilon_{\pm}^{\mu}u_{R}^{(\mp)}. \tag{4.20}$$

V. DISPERSION RELATIONS

A. Transverse mode

Let us consider the spin-3/2 spinors

$$U_{L\pm}^{\mu} = \epsilon_{\pm}^{\mu}u_{L\pm}. \tag{5.1}$$

Equations (4.7) and (4.19) imply that they satisfy

$$k\cdot U_{L\pm} = u\cdot U_{L\pm} = \gamma\cdot U_{L\pm} = 0, \tag{5.2}$$

which in turn can be used together with Eq. (2.8) to obtain

$$\ell_{\mu\nu\alpha}\gamma^{\alpha}U_{L\pm}^{\nu} = -g_{\mu\nu}\not{k}U_{L\pm}^{\nu}. \tag{5.3}$$

We now consider $\pi'_{\mu\nu\alpha}$ given in Eq. (3.9). With the help of Eqs. (4.8) and (4.19) it follows simply

$$(\pi'_{\mu\nu\alpha}\gamma^{\alpha})U_{L\pm}^{\nu} = g_{\mu\nu}[(\pi_{R1}\pm\pi_{P1})\not{k} + (\pi_{R2}\pm\pi_{P2})\not{u}]U_{L\pm}^{\nu}. \tag{5.4}$$

Combining this with Eqs. (3.8) and (5.3) we obtain

$$(\pi_{\mu\nu\alpha}\gamma^{\alpha})U_{L\pm}^{\nu} = -g_{\mu\nu}(a_{\pm}\not{k} + b_{\pm}\not{u})U_{L\pm}^{\nu}, \tag{5.5}$$

where

$$a_\pm = \pi_0 \mp \pi_{P1} - \pi_{R1},$$
$$b_\pm = \mp\pi_{P2} - \pi_{R2}. \tag{5.6}$$

Thus, if we set $\psi^\mu_L = U^\mu_{L-}$ in the equation of motion Eq. (2.12), using the above relations the equation becomes

$$\not{V} U^\mu_{L-} = 0, \tag{5.7}$$

or equivalently

$$\not{V} u_{L-} = 0, \tag{5.8}$$

where

$$V_\alpha = (1 - a_-)k_\alpha - b_- u_\alpha. \tag{5.9}$$

Recalling Eq. (4.18), Eq. (5.8) requires that ω satisfies

$$[1 - a_-(\kappa,\omega)](\omega - \kappa) - b_-(\kappa,\omega) = 0, \tag{5.10}$$

or equivalently

$$\omega = \kappa + \frac{b_-(\kappa,\omega)}{1 - a_-(\kappa,\omega)}, \tag{5.11}$$

where we have explicitly indicated the arguments of a_- and b_- to emphasize that these are implicit equations for $\omega(\kappa)$.

Another solution is $\psi^\mu_L = U^\mu_{L+}$, provided that

$$\not{V}' u_{L+} = 0, \tag{5.12}$$

where

$$V'_\alpha = (1 - a_+)k_\alpha - b_+ u_\alpha. \tag{5.13}$$

Using Eq. (4.18) as before, this yields a dispersion relation $\omega(\kappa) = -\overline{\omega}(\kappa)$, with $\overline{\omega}(\kappa)$ satisfying

$$[1 - a_+(\kappa,-\overline{\omega})]\overline{\omega} + b_+(\kappa,-\overline{\omega}) = [1 - a_+(\kappa,-\overline{\omega})]\kappa \tag{5.14}$$

or equivalently

$$\overline{\omega} = \kappa - \frac{b_+(\kappa,-\overline{\omega})}{1 - a_+(\kappa,-\overline{\omega})}. \tag{5.15}$$

This solution corresponds to the antiparticle propagating in the medium with a dispersion relation $\overline{\omega}(\kappa)$.

These equations for the dispersion relations [e.g., Eqs. (5.8) and (5.10)] resemble the corresponding formulas obtained for the neutrino (or more generally a chiral spin-1/2 fermion) case [3,6]. As we will see below, the equations involving the longitudinal spinor are more complicated.

B. Longitudinal modes

We seek the longitudinal solution as a combination of the spinors

$$U^\mu_{L1} = \epsilon^\mu_\ell u_{L-},$$
$$U^\mu_{L2} = \epsilon^\mu_- u_{L+}. \tag{5.16}$$

We thus write the solution in the form

$$\psi^\mu_L = \sum_{a=1,2} \alpha_a U^\mu_{La}, \tag{5.17}$$

with coefficients $\alpha_{1,2}$ to be determined. The next step is to derive the formulas for $\ell_{\mu\nu\alpha}\gamma^\alpha U^\nu_{L1,2}$ and $\pi_{\mu\nu\alpha}\gamma^\alpha U^\nu_{L1,2}$, the details of which are given in the Appendix. The results given there in Eqs. (A2), (A4), (A6) and (A7) can be summarized in a compact form by introducing the right-handed spinors

$$U^\mu_{R1} = \epsilon^\mu_\ell u_{R-},$$
$$U^\mu_{R2} = \epsilon^\mu_- u_{R+}. \tag{5.18}$$

Thus,

$$(\ell_{\mu\nu\alpha}\gamma^\alpha)U^\nu_{La} = \sum_b L_{ba} U_{Rb\mu},$$
$$(\pi_{\mu\nu\alpha}\gamma^\alpha)U^\nu_{La} = \sum_b \Pi_{ba} U_{Rb\mu}, \tag{5.19}$$

where

$$L_{11} = 0,$$
$$L_{12} = L_{21} = \sqrt{2k^2},$$
$$L_{22} = \omega + \kappa, \tag{5.20}$$

and

$$\Pi_{11} = (\omega - \kappa)\pi_{Q1} + \pi_{Q2},$$
$$\Pi_{12} = \sqrt{-2\tilde{u}^2}(\pi_{R4} - \pi_{P4}) + \sqrt{2k^2}\pi_0,$$
$$\Pi_{21} = \sqrt{-2\tilde{u}^2}(\pi_{R3} - \pi_{P3}) + \sqrt{2k^2}\pi_0,$$
$$\Pi_{22} = (\omega + \kappa)(\pi_0 + \pi_{R1} - \pi_{P1}) + (\pi_{R2} - \pi_{P2}). \tag{5.21}$$

The equation for the coefficients $\alpha_{1,2}$ introduced in Eq. (5.17) is then obtained by substituting that expression in Eq. (2.12) and using Eq. (5.19), which yields

$$\sum_b (L_{ab} - \Pi_{ab})\alpha_b = 0. \tag{5.22}$$

Therefore, the dispersion relations are obtained by solving

$$\mathrm{Det}(L - \Pi) = 0. \tag{5.23}$$

This yields in principle two dispersion relations with the coefficients $\alpha_{1,2}$ obtained for each of them from Eq. (5.22) and the corresponding spinors given by Eq. (5.17).

We obtain the antiparticle modes in a similar fashion by seeking the solution in the form

$$\psi'^{\mu}_{L} = \sum_{a=1,2} \alpha'_a U'^{\mu}_{La}, \tag{5.24}$$

where

$$\begin{aligned} U'^{\mu}_{L1} &= \epsilon^{\mu}_{\ell} u_{L+}, \\ U'^{\mu}_{L2} &= \epsilon^{\mu}_{+} u_{L-}. \end{aligned} \tag{5.25}$$

As in the previous case, the next step is to obtain the formulas for $\ell_{\mu\nu\alpha}\gamma^{\alpha}$ and $\pi_{\mu\nu\alpha}\gamma^{\alpha}$ acting on the spinors $U'^{\mu}_{L1,2}$, the details of which are given in the Appendix. Introducing the right-handed spinors

$$\begin{aligned} U'^{\mu}_{R1} &= \epsilon^{\mu}_{\ell} u_{R+}, \\ U'^{\mu}_{R2} &= \epsilon^{\mu}_{+} u_{R-}, \end{aligned} \tag{5.26}$$

the formulas analogous to Eq. (5.19) in the present case are

$$\begin{aligned} (\ell_{\mu\nu\alpha}\gamma^{\alpha}) U'^{\nu}_{La} &= \sum_b L'_{ba} U'_{Rb\mu}, \\ (\pi_{\mu\nu\alpha}\gamma^{\alpha}) U'^{\nu}_{La} &= \sum_b \Pi'_{ba} U'_{Rb\mu}, \end{aligned} \tag{5.27}$$

where

$$\begin{aligned} L'_{11} &= 0, \\ L'_{12} = L_{21} &= -\sqrt{2k^2}, \\ L'_{22} &= (\omega - \kappa), \end{aligned} \tag{5.28}$$

and

$$\begin{aligned} \Pi'_{11} &= (\omega + \kappa)\pi_{Q1} + \pi_{Q2}, \\ \Pi'_{12} &= \sqrt{-2\tilde{u}^2}(\pi_{R4} + \pi_{P4}) - \sqrt{2k^2}\pi_0, \\ \Pi'_{21} &= \sqrt{-2\tilde{u}^2}(\pi_{R3} + \pi_{P3}) - \sqrt{2k^2}\pi_0, \\ \Pi'_{22} &= (\omega - \kappa)(\pi_0 + \pi_{R1} + \pi_{P1}) + (\pi_{R2} + \pi_{P2}). \end{aligned} \tag{5.29}$$

The equation for the coefficients α'_a is

$$\sum_b (L'_{ab} - \Pi'_{ab})\alpha'_b = 0, \tag{5.30}$$

and in particular the dispersion relations are obtained by solving

$$\mathrm{Det}(L' - \Pi') = 0. \tag{5.31}$$

C. Discussion

Besides the dispersion relation, a quantity that is physically relevant is the proper normalization factor of the spinor solutions. In principle such factors can be determined by mimicking the procedure followed in the spin-1/2 case [3,6]. For the transverse mode the analogy with the spin-1/2 case is close, but for the longitudinal mode the treatment must take into account the fact that it involves two spinor solutions. In this work we have not considered the calculation of such normalization factors.

VI. EXAMPLE

A. Model

As a specific application of the previously developed formalism, here we consider a background medium composed of a spin-1/2 particle f and a scalar particle A, that interact with the λ particle with the interaction Lagrangian

$$L_{\mathrm{int}} = hA\overline{f}_R \sigma^{\mu\nu}\partial_\mu \lambda_{L\nu} + \mathrm{H.c.} \tag{6.1}$$

where the coupling parameter h is inversely proportional to some mass scale. With the conventions specified in Fig. 1 this interaction gives a term $iV_\mu(k)L$ in the Feynman diagrams, where

$$V_\mu(k) = ih\sigma_{\mu\alpha}k^{\alpha}. \tag{6.2}$$

L_{int} is invariant under the transformation $\lambda_{L\mu} \to \lambda_{L\mu} + \partial_\mu \epsilon$, where ϵ is a Dirac field, which manifests in the fact that

$$k \cdot V(k) = 0. \tag{6.3}$$

As a consequence of this, the λ self-energy is transverse, as we will confirm explicitly in the calculation below.

Before entering the details of the calculation we mention the following. The model interaction given in Eq. (6.1) is not renormalizable. Our attitude here is the usual one, namely that such an interaction can arise as an effective interaction due to the exchange of heavier particles in a more fundamental theory in which the heavier fields are integrated out. This is analogous, for example, to the Fermi four-fermion interaction for neutrinos. In that case the resulting effective theory is presumed to be valid for

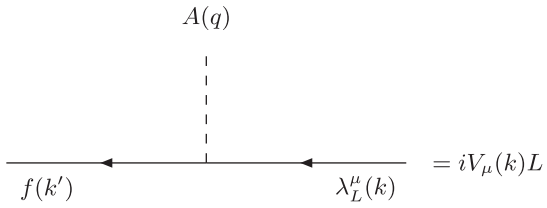

FIG. 1. Vertex diagram for the spin-3/2 particle $\lambda_{L\mu}$ with the spin-1/2 fermion f and the scalar particle A. The vertex function $V_\mu(k)$ is given in Eq. (6.2).

calculating tree-level amplitudes for external momenta much smaller than the heavy particle mass, including the lowest order thermal loops which are just tree-level amplitudes weighted by the appropriate thermal distributions. The results obtained this way are valid in environments in which the thermal distributions of the heavy particles (e.g., the W gauge bisons in the neutrino case) are negligible. In the case of neutrinos this approach leads to the Wolfenstein formula for the neutrino index of refraction. Our expectation is that similar considerations apply to the model interaction of Eq. (6.1) as well, and that the the lessons learned by considering this example will serve to guide the application to more general and/or fundamental interactions of the spin-3/2 particle.

B. One-loop thermal self-energy

Our problem at hand is to compute the thermal self-energy diagram depicted in Fig. 2 and then determine the dispersion relations for the $\lambda_{L\mu}$. In what follows we consider only the real part of the dispersion relation, for which we need to determine only the dispersive part of the thermal self-energy $\Sigma_{T\mu\nu}$. In the real-time formulation of TFT, which we will use, the f and A thermal propagators as well as the self-energy $\Sigma_{\mu\nu}$ in Fig. 2 are 2×2 matrices. The dispersive part of $\Sigma_{T\mu\nu}$ can be determined from the diagonal elements of the self-energy matrix, in particular from the 11 element,

$$-i\Sigma_{11\mu\nu} = \int \frac{d^4p}{(2\pi)^4} i\Delta_{F11}(p-k) iR\overline{V}_\mu(k) iS_{F11}(p) iV_\nu(k). \tag{6.4}$$

In Eq. (6.4) the vertex function V_μ has been defined in Eq. (6.2) and $\overline{V} = \gamma_0 V_\mu^\dagger \gamma_0$ while the propagators in Eq. (6.4) are given by

$$\begin{aligned} S_{F11}(p) &= S_{F0}(p) + S_T(p), \\ \Delta_{F11}(p) &= \Delta_{F0}(p) + \Delta_T(p), \end{aligned} \tag{6.5}$$

where S_{F0} and Δ_{F0} stand for the vacuum propagators

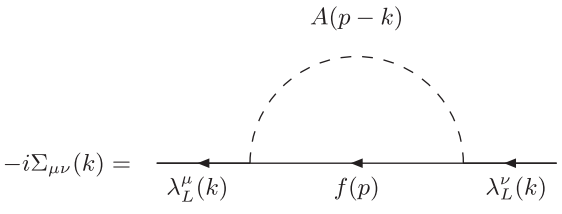

FIG. 2. Diagram for the self-energy of the spin-3/2 particle λ_L in the background of the spin-1/2 fermion f and the scalar particle A.

$$\begin{aligned} S_{F0}(p) &= \frac{1}{\not{p} - m_f + i\epsilon}, \\ \Delta_{F0}(p) &= \frac{1}{p^2 - m_A^2 + i\epsilon}, \end{aligned} \tag{6.6}$$

and the background-dependent parts are given by

$$\begin{aligned} S_T(p) &= 2\pi i\delta(p^2 - m_f^2)\eta_f(p\cdot u), \\ \Delta_T(p) &= -2\pi i\delta(p^2 - m_A^2)\eta_A(p\cdot u), \end{aligned} \tag{6.7}$$

with

$$\begin{aligned} \eta_f(x) &= \frac{\theta(x)}{e^{\beta x - \alpha_f} + 1} + \frac{\theta(-x)}{e^{-\beta x + \alpha_f} + 1}, \\ \eta_A(x) &= \frac{\theta(x)}{e^{\beta x - \alpha_A} - 1} + \frac{\theta(-x)}{e^{-\beta x + \alpha_A} - 1}. \end{aligned} \tag{6.8}$$

Here $1/\beta$ is the temperature and $\alpha_{f,A}$ are the chemical potentials of the fermion and scalar thermal background, respectively.

The substitution of Eq. (6.5) in Eq. (6.4) gives several contributions to the self-energy. The term that contains both S_T and Δ_T contributes only to the absorptive part of the self-energy, which we discard because we are considering only the real part of the dispersion relation as already mentioned. The term that contains neither S_T nor Δ_T corresponds to the pure vacuum contribution, which is not calculable in this model, but we neglect it assuming that it is unimportant relative to the background-dependent part. The remaining ones, that contain either S_T or Δ_T, are precisely the contributions to $\Sigma_{T\mu\nu}$ that we are after. In this way we then obtain

$$\Sigma_{T\mu\nu} = R(\Sigma_{\mu\nu}^{(f)} + \Sigma_{\mu\nu}^{(A)})L, \tag{6.9}$$

where

$$\begin{aligned} \Sigma_{\mu\nu}^{(f)} &= -\int \frac{d^4p}{(2\pi)^3} \frac{\delta(p^2 - m_f^2)\eta_f(p\cdot u)}{(p-k)^2 - m_A^2} \\ &\quad \times \overline{V}_\mu(k)(\not{p} + m_f)V_\nu(k), \\ \Sigma_{\mu\nu}^{(A)} &= \int \frac{d^4p}{(2\pi)^3} \frac{\delta(p^2 - m_A^2)\eta_A(p\cdot u)}{(p+k)^2 - m_f^2} \\ &\quad \times \overline{V}_\mu(k)(\not{p} + \not{k} + m_f)V_\nu(k). \end{aligned} \tag{6.10}$$

The fact that the vertex satisfies Eq. (6.3), in turn implies that

$$k^\nu \Sigma_{T\mu\nu} = 0 \tag{6.11}$$

as well, as we had already anticipated. In the following section we use the general decomposition of the self-energy

introduced in Sec. III and calculate the corresponding coefficients.

C. Calculation of the coefficients

In correspondence with Eq. (6.9) we write

$$\pi_{\mu\nu\alpha} = \pi^{(f)}_{\mu\nu\alpha} + \pi^{(A)}_{\mu\nu\alpha}, \tag{6.12}$$

where

$$\pi^{(f,A)}_{\mu\nu\alpha} = \frac{1}{2}\mathrm{Tr}(\gamma_\alpha \Sigma^{(f,A)}_{T\mu\nu}), \tag{6.13}$$

with $\Sigma^{(f,A)}_{T\mu\nu}$ given in Eq. (6.10). The resulting formulas $\pi^{(f,A)}_{\mu\nu\alpha}$ can be written in the form

$$\begin{aligned}\pi^{(f)}_{\mu\nu\alpha} &= -h^2 t_{\mu\nu\alpha\beta} I^\beta_f,\\ \pi^{(A)}_{\mu\nu\alpha} &= h^2 t_{\mu\nu\alpha\beta}[I^\beta_A + C_A k^\beta],\end{aligned} \tag{6.14}$$

where

$$t_{\mu\nu\alpha\beta} = \frac{1}{2}k^\lambda k^\rho \mathrm{Tr} L\gamma_\alpha\sigma_{\mu\lambda}\gamma_\beta\sigma_{\nu\rho}, \tag{6.15}$$

and

$$\begin{aligned}C_A &= \int \frac{d^3p}{(2\pi)^3 2E_A}\left\{\frac{f_A(p)}{(p+k)^2-m_f^2} + \frac{f_{\overline{A}}(p)}{(p-k)^2-m_f^2}\right\}\\ I^\mu_A &= \int \frac{d^3p}{(2\pi)^3 2E_A} p^\mu\{\frac{f_A(p)}{(p+k)^2-m_f^2} - \frac{f_{\overline{A}}(p)}{(p-k)^2-m_f^2}\}\\ I^\mu_f &= \int \frac{d^3p}{(2\pi)^3 2E_f} p^\mu\left\{\frac{f_f(p)}{(p-k)^2-m_A^2} - \frac{f_{\overline{f}}(p)}{(p+k)^2-m_A^2}\right\}.\end{aligned} \tag{6.16}$$

In these integrals $f_{A,f}(p)$ are the thermal distribution functions

$$\begin{aligned}f_A(p) &= \frac{1}{e^{\beta E_A - \alpha_A}-1},\\ f_f(p) &= \frac{1}{e^{\beta E_f - \alpha_f}+1},\end{aligned} \tag{6.17}$$

and the antiparticle counterparts $f_{\overline{A},\overline{f}}$ are obtained from them by making replacements $\alpha_{A,f} \to -\alpha_{A,f}$.

The integrals $I^\mu_{A,f}$ can be expressed in the form

$$I^\mu_X = A_X k^\mu + B_X u^\mu \qquad (X = f, A), \tag{6.18}$$

where the coefficients $A_{A,f}$, $B_{A,f}$ can be expressed in terms of scalar integrals by inverting the equations

$$\begin{aligned}k\cdot I_X &= k^2 A_X + \omega B_X,\\ u\cdot I_X &= \omega A_X + B_X,\end{aligned} \tag{6.19}$$

implied by Eq. (6.18). This procedure yields the formulas

$$\begin{aligned}A_X &= \frac{1}{\kappa}L^{(1)}_X,\\ B_X &= L^{(2)}_X - \frac{\omega}{\kappa}L^{(1)}_X,\end{aligned} \tag{6.20}$$

where

$$\begin{aligned}L^{(1)}_f &= \int \frac{d^3p}{(2\pi)^3 2E_f}\hat{k}\cdot\vec{p}\left\{\frac{f_f(p)}{(p-k)^2-m_A^2} - \frac{f_{\overline{f}}(p)}{(p+k)^2-m_A^2}\right\},\\ L^{(2)}_f &= \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left\{\frac{f_f(p)}{(p-k)^2-m_A^2} - \frac{f_{\overline{f}}(p)}{(p+k)^2-m_A^2}\right\},\end{aligned} \tag{6.21}$$

while the analogous formulas for $L^{(1,2)}_A$ are obtained from Eq. (6.21) by making the replacements $k \to -k$, $f_{f,\overline{f}} \to f_{A,\overline{A}}$ and $m_A \to m_f$ on the right-hand side.

Substituting Eq. (6.18) in Eq. (6.14), $\pi_{\mu\nu\alpha}$ is then given by

$$\pi_{\mu\nu\alpha} = h^2(C_A + A_A - A_f)t_{\mu\nu\alpha\beta}k^\beta + h^2(B_A - B_f)t_{\mu\nu\alpha\beta}u^\beta. \tag{6.22}$$

Straightforward evaluation of the trace in Eq. (6.15) yields

$$\begin{aligned}t_{\mu\nu\alpha\beta} = &-k^2\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta} - k^2\tilde{g}_{\mu\beta}\tilde{g}_{\alpha\nu} + k^2\tilde{g}_{\mu\nu}\tilde{g}_{\alpha\beta} - k_\alpha k_\beta \tilde{g}_{\mu\nu}\\ &- i[k_\alpha\epsilon_{\mu\nu\beta\rho}k^\rho + k_\beta\epsilon_{\mu\nu\alpha\rho}k^\rho].\end{aligned} \tag{6.23}$$

Rewriting this expression in terms of $R_{\mu\nu}$, $Q_{\mu\nu}$ and $\ell_{\mu\nu\alpha}$ by means of Eqs. (2.6) and (3.4), and then substituting the result in Eq. (6.22) we obtain the one-loop result for $\pi_{\mu\nu\alpha}$ in the form given in Eqs. (3.8) and (3.9), with the coefficients

$$\begin{aligned}\pi_0 &= -h^2k^2(C_A + A_A - A_f) - h^2\omega(B_A - B_f),\\ \pi_{R1} &= -h^2k^2(C_A + A_A - A_f) - 2h^2\omega(B_A - B_f),\\ \pi_{Q1} &= -h^2k^2(C_A + A_A - A_f),\\ \pi_{P1} &= h^2\kappa(B_A - B_f),\\ \pi_{R3} &= \pi_{R4} = \pi_{Q2} = -\pi_{R2} = -h^2k^2(B_A - B_f),\\ \pi_{P2} &= \pi_{P3} = \pi_{P4} = 0.\end{aligned} \tag{6.24}$$

D. Discussion

As a specific example let us consider the situation in which the mass of the scalar boson is much greater than the other relevant energy scales, i.e.,

$$T, \mu, \omega, \kappa, m_f \ll m_A, \tag{6.25}$$

so that, in particular, there are no A scalars in the background. Thus in this case,

$$C_A = L_f^{(1)} = L_A^{(1,2)} \simeq 0, \tag{6.26}$$

while

$$L_f^{(2)} = \frac{-1}{4m_A^2}(n_f - n_{\bar{f}}), \tag{6.27}$$

and therefore,

$$\begin{aligned} B_A &= A_f = A_A \simeq 0, \\ B_f &= \frac{-1}{4m_A^2}(n_f - n_{\bar{f}}). \end{aligned} \tag{6.28}$$

From Eq. (6.24),

$$\begin{aligned} \pi_0 &= \frac{1}{2}\pi_{R1} = h^2 B_f \omega, \\ \pi_{R3} &= \pi_{R4} = \pi_{Q2} = -\pi_{R2} = h^2 B_f k^2, \\ \pi_{P1} &= -h^2 B_f \kappa, \\ \pi_{Q1} &= \pi_{P2} = \pi_{P3} = \pi_{P4} = 0, \end{aligned} \tag{6.29}$$

from which we can now determine the dispersion relations.

1. Transverse mode

For the transverse modes, the parameters $a_\pm$, $b_\pm$ defined in Eq. (5.6) are then

$$\begin{aligned} a_\pm &= -h^2 B_f(\omega \mp \kappa), \\ b_\pm &= h^2 B_f k^2. \end{aligned} \tag{6.30}$$

Substituting Eq. (6.30) into the dispersion relation equation for the transverse modes, Eqs. (5.10) and (5.14), the B_f term cancels in both cases and the solutions are

$$\begin{aligned} \omega &= \kappa, \\ \overline{\omega} &= \kappa, \end{aligned} \tag{6.31}$$

for the particle and antiparticle, respectively. Therefore the dispersion relation for the transverse mode is not modified in the presence of the background. However, it should be remembered that this result holds under the conditions stated in Eq. (6.25), and for other conditions and/or regimes (e.g., ω, $\kappa \gg m_{A,f}$) the dispersion relation is in general modified. In the spirit of our presentation of this model being for illustrative purposes, here we do not pursue this further and turn instead to the longitudinal mode.

2. Longitudinal mode

For the longitudinal mode, using Eq. (5.29) the matrix Π defined in Eq. (5.21) is given by

$$\Pi = h^2 B_f(\omega + \kappa)\begin{pmatrix} \omega - \kappa & \sqrt{2k^2} \\ \sqrt{2k^2} & 2(\omega + \kappa) \end{pmatrix}. \tag{6.32}$$

Using Eqs. (5.20) and (6.32), the equation [Eq. (5.23)] for the longitudinal dispersion relation is

$$-k^2 x(1 - 2x) - 2k^2(1 - x)^2 = 0, \tag{6.33}$$

where we have defined

$$x = h^2 B_f(\omega + \kappa). \tag{6.34}$$

The solution with $k^2 \neq 0$ is $x = 2/3$, which gives the dispersion relation

$$\omega_\kappa = M - \kappa, \tag{6.35}$$

where

$$M \equiv \frac{2}{3h^2 B_f}. \tag{6.36}$$

For completeness we note that going back to Eqs. (5.17) and (5.22), the corresponding spinor is

$$\psi^\mu_{L\ell} = \alpha_1 \epsilon^\mu_\ell u_{L-} + \alpha_2 \epsilon^\mu_- u_{L+}, \tag{6.37}$$

where, up to a normalization factor,

$$\begin{aligned} \alpha_1 &= \frac{1}{\sqrt{2}}, \\ \alpha_2 &= \sqrt{1 - \frac{2\kappa}{M}}. \end{aligned} \tag{6.38}$$

In a similar fashion, from Eq. (5.29),

$$\Pi' = h^2 B_f(\omega - \kappa)\begin{pmatrix} \omega + \kappa & -\sqrt{2k^2} \\ -\sqrt{2k^2} & 2(\omega - \kappa) \end{pmatrix}. \tag{6.39}$$

Equations (5.30) and (5.31) have the solution

$$\omega'_\kappa = M + \kappa, \tag{6.40}$$

with the corresponding spinor

$$\psi'^\mu_{L\ell} = \alpha'_1 \epsilon^\mu_\ell u_{L+} + \alpha'_2 \epsilon^\mu_+ u_{L-}, \tag{6.41}$$

where again, up to a normalization factor

$$\alpha'_1 = \frac{1}{\sqrt{2}},$$
$$\alpha'_2 = -\sqrt{1+\frac{2\kappa}{M}}. \tag{6.42}$$

The proper interpretation of these branches in terms of particle-antiparticle or hole-antihole modes depends on the sign of M and the relative size of κ and M. For a specific example, suppose that the medium is such that $n_f > n_{\overline{f}}$. From Eqs. (6.28) and (6.36) it follows that $M < 0$. Then for $\kappa > |M|$ the dispersion relations

$$\omega'_\kappa = \kappa - |M|,$$
$$\omega_\kappa = -\kappa - |M| \equiv -\overline{\omega}_\kappa \tag{6.43}$$

resemble the dispersion relations for the neutrino (ω'_κ) and antineutrino ($\overline{\omega}_\kappa$) in an electron background [5,6].

Finally we mention the following. Consistency with the conditions stated in Eq. (6.25) imposes some requirements for this solution to be valid, but it is easy to see that they can be satisfied in this model example. For instance, remembering that h is inversely proportional to some mass scale, suppose that $h \sim 1/m_f$. The above result then gives $\omega_\kappa \sim O(m_A^2 m_f^2/n_f)$. Assuming that $T \gg m_f$, so that $n_f \sim T^3$, Eq. (6.25) is satisfied for

$$(m_A m_f^2)^{1/3} \ll T \ll m_A. \tag{6.44}$$

It has not been our intention in this section to study the model exhaustively. Rather, our purpose in going through these details has been to show that the solutions given above give results that are analogous to other well-studied systems and consistent with the assumptions that we have made in the model.

VII. CONCLUSIONS

In the present work we have used the methods of TFT to treat the propagation of a chiral spin-3/2 particle λ^μ_L in a background medium using the methods of TFT, in analogy with the familiar cases of a photon or a neutrino propagating in a matter background. The essential ingredient of the method is the general decomposition of the self-energy in terms of a set of scalar functions, each one corresponding to an independent tensor constructed using the momentum vectors available in the system (the momentum of the particle k^μ and the background velocity four-vector u^μ), as well as the gamma matrices, the metric tensor and the Levi-Civita tensor. Throughout this work we have assumed that the interactions of the λ_L particle with the thermal background particles are such that the thermal self-energy is transverse to k^μ. On the basis of such decomposition we showed that there is a transverse mode in which the spin-3/2 spinor is transverse to k, and two other modes that involve the longitudinal polarization vector. In each case we obtained the equation for the dispersion relation in terms of the self-energy scalar functions, as well as the corresponding spin-3/2 spinor. Finally, we illustrated the application of the formalism by computing the 1-loop TFT expression for the self-energy in a model in which the λ^μ_L propagates in a thermal background composed of spin-1/2 fermion and a scalar particle, and applying the general results to determine the dispersion relations and corresponding spinors. As we showed, the results so obtained share some resemblance and analogies with the photon and the chiral spin-1/2 fermion case, but there are some differences as well.

The present work provides the groundwork for considering problems involving a chiral spin-3/2 particle propagating in a medium, which can be relevant in cosmological or astrophysical contexts of current research interest. This work also opens the path to consider the effects of the medium on the electromagnetic properties of a massless chiral spin-3/2 particle, in analogy with the neutrino case [15]. Such induced electromagnetic couplings can have effects in the cosmological or astrophysical contexts in which the electromagnetic couplings are involved.

ACKNOWLEDGMENTS

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APPENDIX: FORMULAS FOR THE LONGITUDINAL SPINOR

1. Derivation of Eqs. (5.20) and (5.21)

From the equations satisfied by the spinors $U^\mu_{L1,2}$ defined in Eq. (5.16) we derive the formulas for $\mathscr{E}_{\mu\nu\alpha}\gamma^\alpha U^\nu_{L1,2}$ and $\pi_{\mu\nu\alpha}\gamma^\alpha U^\nu_{L1,2}$. We consider each of them one by one.

Consider $\epsilon^\mu_\ell u_{L-}$ first. From the definition of $\mathscr{E}_{\mu\nu\alpha}, P_{\mu\nu}$ and ϵ^μ_ℓ in Eqs. (2.6), (3.4) and (4.5),

$$\mathscr{E}_{\mu\nu\alpha}\epsilon^\nu_\ell = \sqrt{k^2}P_{\mu\alpha}, \tag{A1}$$

and using Eq. (4.20),

$$(\mathscr{E}_{\mu\nu\alpha}\gamma^\alpha)\epsilon^\nu_\ell u_{L-} = \sqrt{2k^2}g_{\mu\nu}\epsilon^\nu_- u_{R+}. \tag{A2}$$

Using the orthonormality relations of ϵ^μ_ℓ [e.g., Eq. (4.8)],

$$\pi'_{\mu\nu\alpha}\epsilon^\nu_\ell = \pi_{Q1}\epsilon_{\ell\mu}k_\alpha + \pi_{Q2}\epsilon_{\ell\mu}u_\alpha - \sqrt{-\tilde{u}^2}\pi_{R3}R_{\mu\alpha} - \sqrt{-\tilde{u}^2}\pi_{P3}P_{\mu\alpha}, \tag{A3}$$

and from Eqs. (4.18) and (4.20)

$$(\pi'_{\mu\nu\alpha}\gamma^\alpha)\epsilon^\nu_\ell u_{L-} = \{[(\omega-\kappa)\pi_{Q1} + \pi_{Q2}]\epsilon_{\ell\mu}u_{R-} + \sqrt{-2\tilde{u}^2}(\pi_{R3} - \pi_{P3})\epsilon_{-\mu}u_{R+}\}. \tag{A4}$$

For $\epsilon^\mu_- u_{L+}$, using Eq. (4.10),

$$\ell_{\mu\nu\alpha}\gamma^{\alpha}\epsilon_{-}^{\nu} = \frac{-\kappa}{\sqrt{-\tilde{u}^2}}(\epsilon_{-\mu}\epsilon_{\ell}\cdot\gamma - \epsilon_{\ell\mu}\epsilon_{-}\cdot\gamma), \quad \text{(A5)}$$

and from Eqs. (4.18) and (4.19)

$$(\ell_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{-}^{\nu}u_{L+} = \left\{(\omega+\kappa)\epsilon_{-\mu}u_{R+} + \sqrt{2k^2}\epsilon_{\ell\mu}u_{R-}\right\}. \quad \text{(A6)}$$

In a similar fashion, using Eqs. (4.7), (4.8), (4.18) and (4.19)

$$(\pi'_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{-}^{\nu}u_{L+} = \left\{[(\omega+\kappa)(\pi_{R1}-\pi_{P1})+(\pi_{R2}-\pi_{P2})]\epsilon_{-\mu}u_{R+} + \sqrt{-2\tilde{u}^2}(\pi_{R4}-\pi_{P4})\epsilon_{\ell\mu}u_{R-}\right\}. \quad \text{(A7)}$$

The results given in Eqs. (A2), (A4), (A6) and (A7) are summarized in Eq. (5.19).

2. Derivation of Eqs. (5.28) and (5.29)

In a similar fashion we derive the formulas for $\ell_{\mu\nu\alpha}\gamma^{\alpha}U'^{\nu}_{L1,2}$ and $\pi_{\mu\nu\alpha}\gamma^{\alpha}U'^{\nu}_{L1,2}$. Thus, using Eqs. (4.20) and (A1),

$$(\ell_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{\ell}^{\nu}u_{L+} = -\sqrt{2k^2}\epsilon_{+\mu}u_{R-}, \quad \text{(A8)}$$

and using Eqs. (4.10), (4.18) and (4.19)

$$(\ell_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{+}^{\nu}u_{L-} = (\omega-\kappa)\epsilon_{+\mu}u_{R-} - \sqrt{2k^2}\epsilon_{\ell\mu}u_{R+}. \quad \text{(A9)}$$

From Eq. (A3), and using Eqs. (4.18) and (4.20)

$$(\pi'_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{\ell}^{\nu}u_{L+} = \left\{[(\omega+\kappa)\pi_{Q1}+\pi_{Q2}]\epsilon_{\ell\mu}u_{R+} + \sqrt{-2\tilde{u}^2}(\pi_{R3}+\pi_{P3})\epsilon_{+\mu}u_{R-}\right\}, \quad \text{(A10)}$$

and finally from Eqs. (4.7), (4.8), (4.18) and (4.19),

$$(\pi'_{\mu\nu\alpha}\gamma^{\alpha})\epsilon_{+}^{\nu}u_{L-} = \left\{[(\omega-\kappa)(\pi_{R1}+\pi_{P1})+(\pi_{R2}+\pi_{P2})]\epsilon_{+\mu}u_{R-} + \sqrt{-2\tilde{u}^2}(\pi_{R4}+\pi_{P4})\epsilon_{\ell\mu}u_{R+}\right\}. \quad \text{(A11)}$$

The above results are summarized in Eq. (5.27).

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[14] We use the convention $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\epsilon^{0123} = +1$.

[15] For a review of the electromagnetic couplings of a neutrino in a background medium see for example Ref. [7].