

# Constraints on a $Z'$ boson within minimal flavor violation

C. S. Kim,<sup>\*</sup> Xing-Bo Yuan,<sup>†</sup> and Ya-Juan Zheng<sup>‡</sup>

*Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea*

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We explore a  $Z'$  boson coupled only with the Standard Model (SM) fermions ( $Z' f\bar{f}$ ) in the framework of minimal flavor violation. We study its effects on the processes with lepton flavor violation  $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$ ,  $\ell_j \rightarrow \ell_i \gamma$ ,  $\mu^- N \rightarrow e^- N$ , quark flavor-changing neutral currents  $b \rightarrow s \ell \bar{\ell}$ , neutral  $B$  and  $K$  meson mixing, and  $e^+ e^- \rightarrow f\bar{f}$  at the LEP experiment to constrain the parameter space of  $Z'$  mass and couplings. We find that among those relevant processes,  $\mu \rightarrow e \gamma$ ,  $\mu \rightarrow 3e$ ,  $\mu$ - $e$  conversion and  $e^+ e^- \rightarrow f\bar{f}$  can put more stringent bounds on  $Z' f\bar{f}$  couplings normalized by  $Z'$  mass. The implications for various processes are obtained, such as  $B$  and  $K$  mixing and  $B/K \rightarrow M \ell_1 \bar{\ell}_2$  decays. In addition, we also analyze  $Z'$  signatures at the LHC with  $\sqrt{s} = 8$  and 13 TeV.

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## I. INTRODUCTION

An additional spin-1 neutral gauge boson called  $Z'$  is known to appear in many scenarios beyond the Standard Model (BSM) [1], such as grand unified models [2], superstring-inspired models [3], models with extra dimensions [4], etc. Discovery of a  $Z'$  boson at present and future high-energy colliders could be one of the most illuminating signatures of BSM physics [5]. Experimental searches for a massive spin-1 resonance have been performed at the LEP [6], Tevatron [7], and the current LHC [8,9] experiments.

Interactions of  $Z'$  with the SM fermions depend on the parameters of specific models. An interesting case is that its interactions with the SM fermions are family nonuniversal [10]. A general theoretical framework for a family nonuniversal  $Z'$  boson has been investigated in Ref. [11], and the flavor-changing effects in such a scenario in both the quark [11,12] and lepton sectors [13–16] have been extensively explored. However, as is well known, the induced quark flavor-changing neutral currents (FCNC) mediated by the  $Z'$  boson are phenomenologically dangerous. Potentially large contributions to FCNC in the quark sector appear in many BSM scenarios, e.g., the two Higgs doublet model, technicolor, etc., which may result in severe phenomenological difficulties [17]. To avoid such large FCNC effects, solutions [18–20] such as the natural flavor conservation hypothesis [18] and the BGL model [19] have been invented. In this work, we want to investigate the FCNC effects within the hypothesis of minimal flavor violation (MFV) [21–23]. In the MFV hypothesis, it is assumed that all flavor-violating currents at low energy are controlled by the Yukawa couplings, so that all the FCNC interactions in quark sector are naturally suppressed by the Cabibbo-Kobayashi-Maskawa (CKM) [24] factors, as in the SM. It is also

possible to extend the MFV hypothesis to the leptonic sector [25–27]. However, the leptonic MFV is quite ambiguous because of the unknown mechanism responsible for the origin of neutrino mass. For a recent review on the MFV in both the quark and lepton sectors, we refer to Ref. [28]. The MFV hypothesis can be used in effective field theory to perform model-independent studies on possible BSM effects [23]. One can also implement MFV in a renormalizable model, which results in the SM-like flavor and  $CP$  violation at low energies [29].

In this paper, we investigate the possibility that a  $Z'$  boson couples to the quark and lepton with flavor-violating interactions within the most general MFV hypothesis. Instead of building a specific model, we adopt a bottom-up approach, where all the  $Z'$  interactions arise from some effective operators which involve both the SM fermions and a  $Z'$  boson, and at the same time satisfy the criterion of the MFV hypothesis. In quark sector, this scenario has been studied in the case of the SM  $Z$  boson with modified couplings to down-type quarks [30]. In our work, the MFV hypothesis is extended to both the quark and lepton sectors with the implementation of the  $Z'$  boson. Its effects on various processes such as lepton flavor-violating (LFV) transitions  $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$ ,  $\ell_j \rightarrow \ell_i \gamma$ , and  $\mu^- N \rightarrow e^- N$ , quark FCNC processes in neutral  $B$  and  $K$  meson mixing, and high-energy collisions  $e^+ e^- \rightarrow f\bar{f}$  at LEP are investigated in detail. Constraints on the  $Z'$  mass and its couplings to fermions are derived. Its implications to the LHC direct searches at  $\sqrt{s} = 8$  TeV (LHC run I) and 13 TeV (LHC run II) are also discussed.

It is also noted that current LHCb run I data present some deviations from the SM predictions [31]. The measured ratio  $R_K \equiv \mathcal{B}(B \rightarrow K \mu^+ \mu^-) / \mathcal{B}(B \rightarrow K e^+ e^-)$  shows 2.6 $\sigma$  departure from unity [32], which may indicate violation of lepton universality. In addition, some angular observables (mainly so-called  $P'_5$ ) in  $B \rightarrow K^* \mu^+ \mu^-$  decay differ from the SM predictions with a significance of about 3 $\sigma$  [33].

<sup>\*</sup> cskim@yonsei.ac.kr

<sup>†</sup> xbyuan@yonsei.ac.kr

<sup>‡</sup> yjzheng@yonsei.ac.kr

Many BSM scenarios are proposed to explain such anomalies [34–37], most of which contain a  $Z'$  boson. A general feature presented in these SM extensions is the  $Z'$  couplings with charged leptons and down-type quarks are typically family nonuniversal to explain the observed  $R_K$  and the anomalies in  $B \rightarrow K^* \mu^+ \mu^-$  decay, respectively. Similar features also present in the couplings within the MFV hypothesis. Therefore, the MFV  $Z'$  boson could be a candidate to explain the current LHCb anomalies, which will be investigated in detail here.

The paper is organized as follows: In Sec. II, we briefly describe the MFV hypothesis and introduce the theoretical framework for an MFV  $Z'$  boson. Some relevant processes in both the quark and lepton sectors are discussed in Sec. III. In Sec. IV, numerical analysis is performed. Then we give predictions on both low- and high-energy processes in Sec. V and conclude in Sec. VI.

## II. MINIMAL FLAVOR-VIOLATING $Z'$ BOSON

In the absence of Yukawa interactions, the SM quark sector exhibits a global flavor symmetry [21],

$$G_{\text{QF}} \equiv SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}, \quad (2.1)$$

plus three additional  $U(1)$  groups identified as baryon number, hypercharge, and the Peccei-Quinn symmetry [38]. The SM quark sector contains one  $SU(2)_L$  doublet  $Q_L$  and two  $SU(2)_L$  singlets  $U_R$  and  $D_R$ , all of which consist of three families. Under the flavor symmetry  $G_{\text{QF}}$ , they transform as

$$Q_L \sim (\mathbf{3}, \mathbf{1}, \mathbf{1}), \quad U_R \sim (\mathbf{1}, \mathbf{3}, \mathbf{1}), \quad D_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{3}). \quad (2.2)$$

The MFV hypothesis assumes that the dynamics of flavor and  $CP$  violation at low energy is determined by the structure of the Yukawa couplings [23]. Technically, the flavor symmetry group  $G_{\text{QF}}$ , which is explicitly broken by the Yukawa couplings  $Y_U$  and  $Y_D$  in the SM, can be formally recovered by promoting the Yukawa couplings to be spurion fields with the transformation property [23]

$$Y_U \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}), \quad \text{and} \quad Y_D \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}}). \quad (2.3)$$

Then it is possible to construct  $G_{\text{QF}}$  invariant effective operators from the SM fields and the spurions  $Y_U$  and  $Y_D$ , which could satisfy the criterion of MFV.

The MFV hypothesis can also be extended to the lepton sector. However, the mechanism responsible for neutrino masses is unknown at present. Thus, there is no unique way to introduce the MFV principle in the lepton sector. Various definitions of lepton MFV have been proposed in the literature [25–27], which depend on the specific BSM scenarios generating the sources of lepton flavor symmetry breaking, such as the seesaw mechanism [39–42]. Here we consider the realization of leptonic MFV within the

so-called minimal field content [25] with one left-handed lepton doublet  $L_L$  and one right-handed singlet  $e_R$ . The lepton flavor symmetry is [21]

$$G_{\text{LF}} \equiv SU(3)_L \otimes SU(3)_E, \quad (2.4)$$

plus two  $U(1)$  symmetries respecting lepton number  $U(1)_{\text{LN}}$  and the weak hypercharge. The Yukawa interaction, which generates lepton mass and breaks the lepton flavor symmetry, reads [25]

$$\begin{aligned} \Delta \mathcal{L} = & -\bar{e}_R \lambda_e H^\dagger L_L - \frac{1}{2\Lambda_{\text{LN}}} (\bar{L}_L^c \tau_2 H) g_\nu (H^T \tau_2 L_L) + \text{H.c.} \\ \xrightarrow{\text{sym.br.}} & -v \bar{e}_R \lambda_e e_L - \frac{v^2}{2\Lambda_{\text{LN}}} \bar{\nu}_L^c g_\nu \nu_L + \text{H.c.}, \end{aligned} \quad (2.5)$$

where  $\Lambda_{\text{LN}}$  denotes the scale of the lepton number symmetry  $U(1)_{\text{LN}}$  breaking, and the vacuum expectation value  $v = 174$  GeV. The charged lepton and neutrino Yukawa couplings  $\lambda_e$  and  $g_\nu$  are  $3 \times 3$  matrices in flavor space. In this case, the tiny neutrino masses are explained by the smallness of  $v/\Lambda_{\text{LN}}$ .

Considering effective couplings of a  $Z'$  boson to the SM fermions, the relevant effective operators satisfying the MFV hypothesis can be written as

$$\begin{aligned} \mathcal{O}_L^q &= (\bar{Q}_L \Delta_q \gamma^\mu Q_L) Z'_\mu \quad \text{and} \\ \mathcal{O}_L^e &= (\bar{L}_L \Delta_e \gamma^\mu L_L) Z'_\mu. \end{aligned} \quad (2.6)$$

In order to make them invariant under the quark and lepton flavor group  $G_{\text{QF}}$  and  $G_{\text{LF}}$ , the coupling matrices should have the form

$$\begin{aligned} \Delta_q &= \kappa_0 \mathbb{1} + \kappa_1 Y_U Y_U^\dagger + \cdots, \quad \text{and} \\ \Delta_e &= \lambda_0 \mathbb{1} + \lambda_1 g_\nu^\dagger g_\nu + \cdots, \end{aligned} \quad (2.7)$$

where  $\mathbb{1}$  denotes  $3 \times 3$  identity matrix in flavor space. In the series,  $\kappa_i$  and  $\lambda_i$  are unknown real coefficients, and the terms with higher orders of the spurions  $Y_U$ ,  $Y_D$ ,  $\lambda_e$ , and  $g_\nu$  are indicated by the ellipses. As in Ref. [43], the flavor-conserving term  $\mathbb{1}$  is also considered.

In the literature, there are some other treatments which can be used to realize the MFV hypothesis for the  $Z'$  couplings. In the expansion series Eq. (2.7), higher-order terms can be resummed by the Cayley-Hamilton identity and the series stop at the order  $\kappa_2 (Y_U Y_U^\dagger)^2$  and  $\lambda_2 (g_\nu^\dagger g_\nu)^2$  after neglecting the down-type fermion Yukawa couplings [44]. For the quark sector, it is also possible to use the approach of nonlinear parametrization to account for the higher-order contributions [45]. In addition, the operators with right-handed fields  $u_R$ ,  $d_R$ , and  $e_R$  can also be constructed to satisfy the MFV hypothesis. However, the corresponding flavor-violating couplings are suppressed by small down-type fermion Yukawa couplings such as  $\lambda_b$  and  $\lambda_\mu$ . In this

work, we concentrate on a  $Z'$  boson where the interactions with fermions satisfy the MFV hypothesis and originate from the effective operators  $\mathcal{O}_L^q$  and  $\mathcal{O}_L^\ell$  in Eq. (2.6). Our analysis can be straightforwardly extended to more general cases, such as including some of the above ingredients or considering lepton MFV with the seesaw mechanism.

In the following analysis, it is convenient to work in the Lagrangian,

$$\mathcal{L} = \Gamma_{\ell\ell'}^L (\bar{\ell}\gamma^\mu P_L \ell') Z'_\mu + \Gamma_{qq'}^L (\bar{q}\gamma^\mu P_L q') Z'_\mu + (L \rightarrow R), \quad (2.8)$$

where  $P_{L,R} = (1 \mp \gamma_5)/2$ ,  $q$  and  $q'$  ( $\ell$  and  $\ell'$ ) denote a pair of up- or down-type quarks (leptons). Then, the MFV operators of Eq. (2.6) result in the following couplings,

$$\begin{aligned} \Gamma_{\ell\ell'}^L &= \lambda_0 \delta_{\ell\ell'} + \lambda_1 \frac{\Lambda_{\text{LN}}^2}{v^4} \sum_{\nu_i} m_{\nu_i}^2 U_{\ell\nu_i} U_{\ell'\nu_i}^*, & \Gamma_{\ell\ell'}^R &= 0, \\ \Gamma_{qq'}^L &= \kappa_0 \delta_{qq'} + \kappa_1 \lambda_i^2 V_{iq}^* V_{iq'}, & \Gamma_{qq'}^R &= 0, \end{aligned} \quad (2.9)$$

where  $m_\nu$  denotes diagonal neutrino mass matrix, and  $U$  the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [46]. Numerically, the diagonal elements of the lepton coupling matrix  $\Gamma_{\ell\ell'}^L$  are almost universal. Therefore, we define a new coupling  $\bar{\lambda} = \Gamma_{ee}^L$  and take the approximation  $\Gamma_{\ell\ell}^L \approx \bar{\lambda}$  in the following discussion. In the case of normal hierarchy (NH) of neutrino mass spectrum, assuming  $m_1 = 0.2$  eV and  $\Lambda_{\text{LN}} = 10^{14}$  GeV, the coupling matrices read

$$\begin{aligned} |\Gamma_{\ell\ell'}^L| &= |\bar{\lambda}| \mathbb{1} + |\lambda_1| \begin{pmatrix} 0 & 0.23 & 0.31 \\ 0.26 & 0 & 1.28 \\ 0.31 & 1.28 & 0 \end{pmatrix} \times 10^{-2}, \\ |\Gamma_{qq'}^L| &= |\kappa_0| \mathbb{1} + |\kappa_1| \begin{pmatrix} 0.00007 & 0.00031 & 0.000760 \\ 0.00031 & 0.00144 & 0.003475 \\ 0.00760 & 0.03575 & 0.886868 \end{pmatrix}. \end{aligned} \quad (2.10)$$

The flavor-conserving couplings are almost universal in the lepton sector but rather hierarchical in the quark sector. For flavor-changing couplings, they are suppressed in both the quark and lepton sectors, which are the feature of MFV hypothesis.

As in many BSM scenarios, the mass of the  $Z'$  boson is a free parameter in our case. In this work, we focus on a TeV scale  $Z'$  boson, which may explain some current observed anomalies and could be detected at the LHC.

### III. PROCESSES TO CONSTRAIN THE PARAMETER SPACE OF $Z'$

Due to its family nonuniversal couplings, an MFV  $Z'$  boson may affect processes from low-energy flavor transitions all the way to high-energy collider processes. The most relevant processes of leptonic decays  $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$ ,

$\ell_j \rightarrow \ell_i \gamma$ ,  $\mu$ - $e$  conversion in the lepton sector, quark FCNC processes  $b \rightarrow s \ell^+ \ell^-$ , neutral  $B$  and  $K$  meson mixing  $B_s$ - $\bar{B}_s$ ,  $B_d$ - $\bar{B}_d$  and  $K^0$ - $\bar{K}^0$  processes in the quark sector, and  $e^+ e^- \rightarrow f \bar{f}$  at the LEP experiment are investigated in detail in this section.

#### A. Lepton flavor violation processes

##### 1. Leptonic decays $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$

Among LFV decays, the most important processes contain the decay of a charged lepton  $\ell_j$  into three charged leptons  $\ell_i$ ,  $\ell_k$  and  $\bar{\ell}_l$ , e.g.,  $\mu \rightarrow 3e$ . With the Lagrangian (2.8), the tree-level  $Z'$  exchange results in the following branching ratios [11,13,14],

$$\begin{aligned} \mathcal{B}(\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l) &= \frac{\tau_j m_j}{1536\pi^3} \left(\frac{m_j}{m_{Z'}}\right)^4 (|\Gamma_{ij}^L \Gamma_{kl}^L + \Gamma_{kj}^L \Gamma_{il}^L|^2 \\ &\quad + |\Gamma_{ij}^L \Gamma_{kl}^R|^2 + |\Gamma_{kj}^L \Gamma_{il}^R|^2 + (L \leftrightarrow R)), \\ \mathcal{B}(\ell_j \rightarrow \ell_i \ell_i \bar{\ell}_l) &= \frac{\tau_j m_j}{1536\pi^3} \left(\frac{m_j}{m_{Z'}}\right)^4 (2|\Gamma_{ij}^L \Gamma_{il}^L|^2 + |\Gamma_{ij}^L \Gamma_{il}^R|^2 \\ &\quad + (L \leftrightarrow R)), \end{aligned} \quad (3.1)$$

which are applied to the case of the two same-sign final leptons with same and different flavor, respectively.

##### 2. Leptonic decays $\ell_j \rightarrow \ell_i \gamma$

Another relevant decay is the loop-induced radiative decay  $\ell_j \rightarrow \ell_i \gamma$ , e.g.,  $\mu \rightarrow e \gamma$ . After neglecting the mass of the final leptons, the branching ratio reads [13,14]

$$\begin{aligned} \mathcal{B}(\ell_j \rightarrow \ell_i \gamma) &= \frac{\alpha_e \tau_j m_j}{9(4\pi)^4} \left(\frac{m_j}{m_{Z'}}\right)^4 \\ &\quad \times \left( \left| \sum_k \Gamma_{jk}^L \Gamma_{ki}^L - \frac{3m_k}{m_j} \Gamma_{kj}^L \Gamma_{ki}^R \right|^2 + (L \leftrightarrow R) \right), \end{aligned} \quad (3.2)$$

where the enhancement factor  $m_k/m_j$  is similar to its counterpart in quark sector  $b \rightarrow s \gamma$  decay.

##### 3. $\mu$ - $e$ conversion

For the  $\mu$ - $e$ - $Z'$  coupling, in particular, a strong bound comes from  $\mu$ - $e$  conversion in the nuclei. The experimental sensitivities are expected to improve by several orders of magnitude and reach about  $\mathcal{O}(10^{-17})$  in the near future [47]. The branching fraction for  $\mu$ - $e$  conversion in the atomic nuclei  $N$  reads [48]

$$\begin{aligned} \mathcal{B}(\mu^- N \rightarrow e^- N) &= \frac{\alpha_e^3 m_\mu^5}{(8\pi)^2 \Gamma_{\text{capt}} m_{Z'}^4} |F_p|^2 (|\Gamma_{e\mu}^L|^2 + |\Gamma_{e\mu}^R|^2) \\ &\quad \times |(2Z + N)(\Gamma_{uu}^L + \Gamma_{uu}^R) \\ &\quad + (Z + 2N)(\Gamma_{dd}^L + \Gamma_{dd}^R)|^2, \end{aligned} \quad (3.3)$$

where  $Z$  and  $N$  denote the atomic and neutron numbers, respectively.  $\Gamma_{\text{capt}}$  denotes the  $\mu$  capture rate,  $Z_{\text{eff}}$  the effective atomic number, and  $F_p$  the nuclear matrix element [49]. Unlike other LFV processes, the  $\mu$ - $e$  conversion in the nuclei involves interactions with light quarks, which could constrain the flavor-conserving  $u$ - $u$ - $Z'$ - and  $d$ - $d$ - $Z'$ -type couplings.

## B. Quark flavor-changing neutral current processes

### 1. $|\Delta F|=1$ transition: $b \rightarrow s\ell^+\ell^-$ processes

Generally, the effective Hamiltonian for  $b \rightarrow s\ell^+\ell^-$  transitions can be written as [50]

$$\mathcal{H}_{\text{eff}}^{\Delta F=1} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \mathcal{O}_i + \text{H.c.}, \quad (3.4)$$

plus small  $\mathcal{O}(V_{ub} V_{ts}^*)$  corrections, where explicit expressions of the four-quark operators  $\mathcal{O}_{1-6}$  can be found in Ref. [50]. In the SM, the electromagnetic dipole operator and semileptonic four-fermion operators play a leading role [51]:

$$\begin{aligned} \mathcal{O}_{7\gamma} &= \frac{e}{16\pi^2} m_b (\bar{s} \sigma_{\mu\nu} P_R b) F^{\mu\nu}, \\ \mathcal{O}_{9\ell} &= \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell), \\ \mathcal{O}_{10\ell} &= \frac{e^2}{16\pi^2} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell). \end{aligned}$$

In the Lagrangian Eq. (2.8), the Wilson coefficient  $C_7$  is affected at loop level while the semileptonic operators receive tree-level contributions from  $Z'$  exchange, which result in

$$\begin{pmatrix} C_{9\ell}^{\text{NP}} \\ C_{10\ell}^{\text{NP}} \end{pmatrix} = -\frac{\pi}{\sqrt{2}\alpha_e G_F V_{tb} V_{ts}^*} \frac{\Gamma_{sb}^L}{m_{Z'}^2} \begin{pmatrix} \Gamma_{\ell\ell}^R + \Gamma_{\ell\ell}^L \\ \Gamma_{\ell\ell}^R - \Gamma_{\ell\ell}^L \end{pmatrix}. \quad (3.5)$$

After neglecting the right-handed currents, there are only two model-independent parameters ( $C_{9e}^{\text{NP}} = -C_{10e}^{\text{NP}}$ ,  $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$ ) in  $b \rightarrow se^+e^-$  and  $b \rightarrow s\mu^+\mu^-$  transitions, which have been fit to current experimental data by several groups [52–54].

### 2. $|\Delta F|=2$ transition: Neutral $B$ and $K$ meson mixing

The FCNC processes  $B_s$ - $\bar{B}_s$ ,  $B_d$ - $\bar{B}_d$ , and  $K^0$ - $\bar{K}^0$  mixing play an important role in constraining possible BSM effects. In the SM,  $B_s$ - $\bar{B}_s$  mixing occurs via box diagrams by exchanging  $W^\pm$  boson. The mixing strength is described by the mass difference  $\Delta m_s = 2| \langle B_s | \mathcal{H}^{\Delta B=2} | \bar{B}_s \rangle |$  governed by the effective Hamiltonian [55]

$$\begin{aligned} \mathcal{H}_{\text{eff}}^{\Delta B=2} &= \frac{G_F^2}{16\pi^2} m_W^2 (V_{tb} V_{ts}^*)^2 C_1^{\text{VLL}} (\bar{s}^\alpha \gamma_\mu P_L b^\alpha) (\bar{s}^\beta \gamma^\mu P_L b^\beta) \\ &+ \text{H.c.} \end{aligned} \quad (3.6)$$

The Wilson coefficient  $C_1^{\text{VLL}}$  at matching scale  $\mu = \mu_W$  can be found in Ref. [56] and its QCD renormalization group evolution to  $B$  meson scale can be found in Refs. [55,57]. In our scenario, the left-handed current can modify the Wilson coefficient at high scale as [58]

$$C_{1,\text{NP}}^{\text{VLL}} = \frac{16\pi^2}{G_F} \frac{1}{m_W^2 (V_{tb} V_{ts}^*)^2} \frac{(\Gamma_{sb}^L)^2}{2m_{Z'}^2}. \quad (3.7)$$

Similar expressions hold for  $B_d$ - $\bar{B}_d$  and  $K^0$ - $\bar{K}^0$  mixing in both SM and BSM physics. In particular, the  $K^0$ - $\bar{K}^0$  mixing is more complicated. From the effective Hamiltonian, one can build two observables, mass difference  $\Delta m_K$  and  $CP$ -violating parameter  $\varepsilon_K$  [59]. However, compared to  $B$  meson mixing, these two observables suffer from large theoretical uncertainties, especially for  $\Delta m_K$  [59,60]. The uncertainties from short-distance and long-distance contributions to the mass difference have been discussed in Refs. [61–64] and Refs. [65–68], respectively. Recent lattice QCD calculations can be found in Refs. [69,70]. We refer to Ref. [71] for a recent review on  $B$  and  $K$  meson mixing.

It is also noted that the Wilson coefficients of Eq. (3.7) should run to the low scales  $\mu_K = 2$  GeV for  $K$  mixing and  $\mu_B$  for  $B$  mixing under QCD renormalization group evolution. The particular low-scale value should match the evaluation scale of the corresponding hadronic matrix element. The evolution from high scale to low scale should be done with the changing of the effective flavors  $n_f = 6 \rightarrow 4$  for  $K$  mixing and  $n_f = 6 \rightarrow 5$  for  $B$  mixing. All the relevant formulas can be found in Refs. [55,57,58,72].

## C. $e^+e^- \rightarrow f\bar{f}$ at the LEP

The LEP-II  $e^+e^- \rightarrow f\bar{f}$  data, where  $f$  denotes a quark or lepton flavor, were taken at the energies  $\sqrt{s}$  increasing from 130 to 209 GeV [6]. The cross sections and forward-backward asymmetries for various fermion pairs can be used to search for a TeV scale  $Z'$  boson. As a model-independent approach, the LEP Collaboration uses the following effective Lagrangian, i.e., contact interaction to constrain possible BSM effects [73],

$$\mathcal{L}_{\text{eff}} = \frac{4\pi}{(1 + \delta_f) \Lambda_{f,\pm}^2} \sum_{i,j=L,R} \eta_{ij} (\bar{e}_i \gamma_\mu e_i) (\bar{f}_j \gamma^\mu f_j), \quad (3.8)$$

where  $\delta_f = 1$  (0) for  $f = e$  ( $f \neq e$ ). The free parameters  $\Lambda_{f,\pm}$  encode possible BSM effects, which may constructively (+) or destructively (−) interfere with the SM contributions. Interactions with different chiralities and interferences correspond to the choices of  $\eta_{ij} = \pm 1$  or 0. With the notation in Eq. (2.8), the scale  $\Lambda_{f,\pm}$  and parameters  $\lambda_{ij}$  read



TABLE I. Input parameters used in the numerical analysis for the lepton sector, quark sector and  $B$  and  $K$  meson mixing. The mixing parameters in the lepton sector (values in brackets) correspond to NH [inverted hierarchy (IH)].

Lepton sector	$\sin^2 \theta_{12}$	$0.308_{-0.017}^{+0.017}$	[79]
	$\sin^2 \theta_{23}$	$0.437_{-0.023}^{+0.033}$ (0.455 $_{-0.031}^{+0.039}$ )	[79]
	$\sin^2 \theta_{13}$	$0.0234_{-0.0019}^{+0.0020}$ (0.0240 $_{-0.0022}^{+0.0019}$ )	[79]
	$\delta/\pi$	$1.39_{-0.27}^{+0.38}$ (1.31 $_{-0.33}^{+0.29}$ )	[79]
	$\Delta m_{21}^2$ [ $10^{-5}$ eV $^2$ ]	$7.54_{-0.22}^{+0.26}$	[79]
Quark sector	$ \Delta m^2 $ [ $10^{-3}$ eV $^2$ ]	$2.43_{-0.06}^{+0.06}$ (2.38 $_{-0.06}^{+0.06}$ )	[79]
	$ V_{us} f_+^{K \rightarrow \pi}(0)$	$0.21664 \pm 0.00048$	[80]
	$ V_{ub} $ (semi-leptonic)	$(3.70 \pm 0.12 \pm 0.26) \times 10^{-3}$	[80]
	$ V_{cb} $ (semi-leptonic)	$(41.0 \pm 0.33 \pm 0.74) \times 10^{-3}$	[80]
	$\gamma$ [ $^\circ$ ]	$73.2_{-7.0}^{+6.3}$	[80]
	$\bar{m}_c(\bar{m}_c)$	$(1.286 \pm 0.013 \pm 0.040)$ GeV	[80]
	$\bar{m}_b(\bar{m}_b)$	$(4.18 \pm 0.03)$ GeV	[81]
	$\bar{m}_t(\bar{m}_t)$	$(165.95 \pm 0.35 \pm 0.64)$ GeV	[80]
	$f_+^{K \rightarrow \pi}(0)$	$0.9641 \pm 0.0015 \pm 0.0045$	[80]
	$B$ and $K$ meson mixing	$f_K$	$(155.2 \pm 0.2 \pm 0.6)$ MeV
$f_{B_s}$		$(225.6 \pm 1.1 \pm 5.4)$ MeV	[80]
$f_{B_s}/f_{B_d}$		$1.205 \pm 0.004 \pm 0.007$	[80]
$\hat{B}_K$		$0.7615 \pm 0.0027 \pm 0.0137$	[80]
$\hat{B}_{B_s}$		$1.320 \pm 0.017 \pm 0.030$	[80]
$\hat{B}_{B_s}/\hat{B}_{B_d}$		$1.023 \pm 0.013 \pm 0.014$	[80]
$\eta_{cc}$		$1.87 \pm 0.76$	[63]
$\eta_{ct}$		$0.497 \pm 0.047$	[64]
$\eta_{tt}$		$0.5765 \pm 0.0065$	[62]
$\kappa_e$		$0.940 \pm 0.013 \pm 0.023$	[67]
$\varphi_e$		$(43.51 \pm 0.05)^\circ$	[67]

$$\Lambda_{f,\pm} = \left( \frac{4\pi m_{Z'}^2}{|\Gamma_{ee}^L \Gamma_{ff}^L|} \right)^{1/2} \quad \text{and}$$

$$\eta_{ij} = \begin{cases} -\text{sgn}(\Gamma_{ee}^L \Gamma_{ff}^L), & i = j = L \\ 0, & \text{others.} \end{cases} \quad (3.9)$$

In the case of hadron final states, since it is difficult to distinguish final jets originated from different flavors, the LEP Collaboration interprets the experimental data in several cases. We adopt the interpretation that possible new interactions only exist between electrons and a single up-type flavor. Since only  $u$  and  $c$  quarks can be produced at the LEP energies and the  $Z'$  couplings to them are almost universal, the LEP lower bound  $\Lambda_{uu,\pm}^{\text{LEP}}$  [6] can be converted to  $\Lambda_{u,\pm} > \Lambda_{uu,\pm}^{\text{LEP}}/\sqrt{2}$  without loss of generality.

In the case of  $f = \ell = e, \mu, \tau$ , additional  $u$ - and  $t$ -channel diagrams with LFV couplings also contribute to  $\Lambda_{\ell,\pm}$ . However, these LFV couplings are highly suppressed as shown in Eq. (2.10). Thus, Eq. (3.9) for  $\Lambda_{\ell,\pm}$  holds in a good approximation.

#### D. Other relevant processes

In this part, we discuss briefly about some other processes receiving contributions from the MFV  $Z'$  boson.

Due to its couplings to muons and neutrinos, the  $Z'$  boson can contribute to neutrino trident production  $\nu_\mu N \rightarrow \nu_\mu \mu^+ \mu^-$  [14,35,37]. Using combined measurements from CHARM-II [74], CCFR [75] and NuTeV [76], a bound on  $\bar{\lambda}$  and  $\lambda_1$  is derived, which turned out to be much weaker than the one from the LFV decays  $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$  and  $\ell_j \rightarrow \ell_i \gamma$ . Similarly, except for the  $\tau \rightarrow \mu \nu \bar{\nu}$  decay, the leptonic decays  $\ell_j \rightarrow \ell_i \nu \bar{\nu}$ <sup>1</sup> cannot put further constraints on the model parameters. For  $\tau \rightarrow \mu \nu \bar{\nu}$  in particular, its experimentally measured branching ratio is currently more than  $2\sigma$  above the SM prediction [35]. The allowed parameter space from this process is not compatible for the ones from other  $\ell_j \rightarrow \ell_i \nu \bar{\nu}$  modes. As described in Sec. II., the right-handed  $Z'$  couplings are not included in this scenario. Therefore, the MFV  $Z'$  considered in this paper cannot contribute to leptonic electric dipole moments [13]. Furthermore, the  $Z'$  effects on the anomalous magnetic moment  $a_\mu$  are always negative [14] and cannot relax the longstanding discrepancy between the SM and experiment [77]. At last, the bounds from conversion  $\mu^- e^+ \rightarrow \mu^+ e^-$  [78] are also very weak.

<sup>1</sup>For  $\mu \rightarrow e \nu \bar{\nu}$  decay, to avoid large corrections to the Fermi constant  $G_F$ , we demand  $|\mathcal{B}(\mu \rightarrow e \nu \bar{\nu})_{\text{exp}} - \mathcal{B}(\mu \rightarrow e \nu \bar{\nu})_{\text{SM}}| < 4 \times 10^{-5}$  as suggested in Ref. [14].

TABLE II. The SM predictions and experimental measurements for the observables used in the numerical analysis. The upper limits for LFV decays are values corresponding to 90% C.L.

Observable	SM	Experiment	Ref.
$\mathcal{B}(\mu \rightarrow e\gamma)$	...	$<5.7 \times 10^{-13}$	[81]
$\mathcal{B}(\mu\text{Ti} \rightarrow e\text{Ti})$	...	$<6.1 \times 10^{-13}$	[82]
$\mathcal{B}(\mu\text{Au} \rightarrow e\text{Au})$	...	$<7.0 \times 10^{-13}$	[81]
$\mathcal{B}(\mu^- \rightarrow e^- e^- e^+)$	...	$<1.0 \times 10^{-12}$	[81]
$\mathcal{B}(\tau \rightarrow \mu\gamma)$	...	$<4.4 \times 10^{-8}$	[81]
$\mathcal{B}(\tau^- \rightarrow \mu^- \mu^- \mu^+)$	...	$<2.1 \times 10^{-8}$	[81]
$\mathcal{B}(\tau^- \rightarrow \mu^- e^- e^+)$	...	$<1.8 \times 10^{-8}$	[81]
$\mathcal{B}(\tau \rightarrow e\gamma)$	...	$<3.3 \times 10^{-8}$	[81]
$\mathcal{B}(\tau^- \rightarrow e^- e^- e^+)$	...	$<2.7 \times 10^{-8}$	[81]
$\mathcal{B}(\tau^- \rightarrow e^- \mu^- \mu^+)$	...	$<2.7 \times 10^{-8}$	[81]
$\mathcal{B}(K_L \rightarrow e^\pm \mu^\mp)$	...	$<4.7 \times 10^{-12}$	[81]
$\Delta m_d$ [ps <sup>-1</sup> ]	$0.51 \pm 0.06$	$0.510 \pm 0.003$	[83]
$\Delta m_s$ [ps <sup>-1</sup> ]	$16.93 \pm 1.16$	$17.757 \pm 0.021$	[83]
$\Delta m_K$ [10 <sup>-3</sup> ps <sup>-1</sup> ]	$4.40 \pm 1.77$	$5.293 \pm 0.009$	[81]
$ \varepsilon_K $ [10 <sup>-3</sup> ]	$2.10 \pm 0.30$	$2.228 \pm 0.011$	[81]

#### IV. NUMERICAL ANALYSIS AND DISCUSSIONS

With the theoretical framework described in previous sections, we proceed to present our numerical analysis and discussions in this section. Table I shows the input parameters for various processes mentioned above. In Table II, we summarize the SM predictions and current experimental data for these processes. The theoretical uncertainties of the observables in  $B$  and  $K$  meson mixings are obtained, varying each input parameter within the  $1\sigma$  range and adding each individual uncertainty in quadrature.

As discussed in Sec. II, the relevant model parameters in our case contain the flavor-conserving (-changing) couplings  $\kappa_0$  ( $\kappa_1$ ) and  $\bar{\lambda}$  ( $\lambda_1$ ), which correspond to the quark and lepton sectors, respectively, and the  $Z'$  mass  $m_{Z'}$ . Since we concentrate on a TeV scale  $Z'$  boson, the mass effects are decoupled for all the processes except those at the LHC. Therefore, we choose the model parameter as

$$\left( \frac{\kappa_0}{m_{Z'}}, \frac{\kappa_1}{m_{Z'}}, \frac{\bar{\lambda}}{m_{Z'}}, \frac{\lambda_1}{m_{Z'}} \right). \quad (4.1)$$

The constraints on these parameters will be discussed in the following sections.

Compared to the lepton processes, the quark FCNC processes still suffer from large theoretical uncertainty due to hadronic inputs. In order to derive the allowed parameter space from these processes, we impose the experimental constraints in the same way as in Ref. [84]; i.e., for each point in the parameter space, a theoretical range is constructed from the prediction of the observable in that point together with the corresponding theoretical uncertainty. If this range overlaps with the  $2\sigma$  range of the experimental

measurement, then this point is regarded as allowed. To be conservative, the theoretical uncertainty is taken as twice the one listed in Table II. Since the main theoretical uncertainties arise from hadronic input parameters, which are common to both the SM and the MFV  $Z'$  boson, the relative theoretical uncertainty is assumed to be constant over the whole parameter space.

#### A. Bounds on $Z'$ couplings to leptons

The processes of lepton radiative decays  $\ell_j \rightarrow \ell_i \ell_k \bar{\ell}_l$  (e.g.,  $\mu \rightarrow 3e$ ) and  $\ell_j \rightarrow \ell_i \gamma$  (e.g.,  $\mu \rightarrow e\gamma$ ), collider processes  $e^+e^- \rightarrow \ell^+\ell^-$  at LEP, neutrino trident production  $\nu_\mu N \rightarrow \nu N \mu^+ \mu^-$ , and  $\mu^- e^+ \rightarrow \mu^+ e^-$  conversion involve only the  $Z'$  couplings to leptons  $\bar{\lambda}$  and  $\lambda_1$ , which controls the flavor-conserving and flavor-changing current, respectively. After considering the current experimental data of these processes, which are listed in Table II, it is found that the bounds on the  $Z'$  couplings are dominated by  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$  and the LEP processes  $e^+e^- \rightarrow \ell^+\ell^-$ . For NH and IH cases, Fig. 1 shows the allowed parameter space in the  $(\bar{\lambda}/m_{Z'}, \lambda_1/m_{Z'})$  plane. We can see that the upper bound on  $\bar{\lambda}$  and  $\lambda_1$  is provided by  $e^+e^- \rightarrow \ell^+\ell^-$  and a combination of  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$  decays, respectively. The latter process also puts a bound on the product of these two couplings, numerically as  $\bar{\lambda}\lambda_1/m_{Z'}^2 \lesssim 0.01 \text{ TeV}^{-2}$  for NH. Since the bounds from NH and IH are quite similar, we will only consider the NH case in the following analysis for simplicity.

#### B. Bounds on $Z'$ couplings to quarks

Constraints on the  $Z'$  couplings with quarks come from  $B_s-\bar{B}_s$ ,  $B_d-\bar{B}_d$ , and  $K^0-\bar{K}^0$  mixing. Due to relatively large theoretical uncertainties in the  $K^0-\bar{K}^0$  mixing discussed in Sec. III B 2, we adopt the conservative treatment in Ref. [60]; i.e., NP contributions to  $\Delta m_K$  are available within the 50% range of  $\Delta m_K^{\text{exp}}$ , and  $|\varepsilon_K|$  is allowed to vary within a 20% symmetric range. The experimental measurements on the observables in the  $B$  and  $K$  mixing are listed in Table II, which show good agreement with the SM predictions. Therefore, a stringent bound on the  $Z'$  parameter is found,

$$|\kappa_1/m_{Z'}| < 0.18 \text{ TeV}^{-1}, \quad (4.2)$$

which is dominated by  $\Delta m_s$  and slightly stronger than those from  $|\varepsilon_K|$  and  $\Delta m_d$ . Our numerics agree with the fit on the scale  $\Lambda$  of  $\Delta F = 2$  MFV effective operators [85].

#### C. Bounds on $Z'$ couplings with both lepton and quark

From Sec. III B 1,  $b \rightarrow s \ell^+ \ell^-$  processes involve both  $Z'$  couplings to quarks and leptons, which appear as  $\kappa_1 \bar{\lambda}$  in the amplitude. On the experimental side, recent measurements of these processes have shown deviations from the

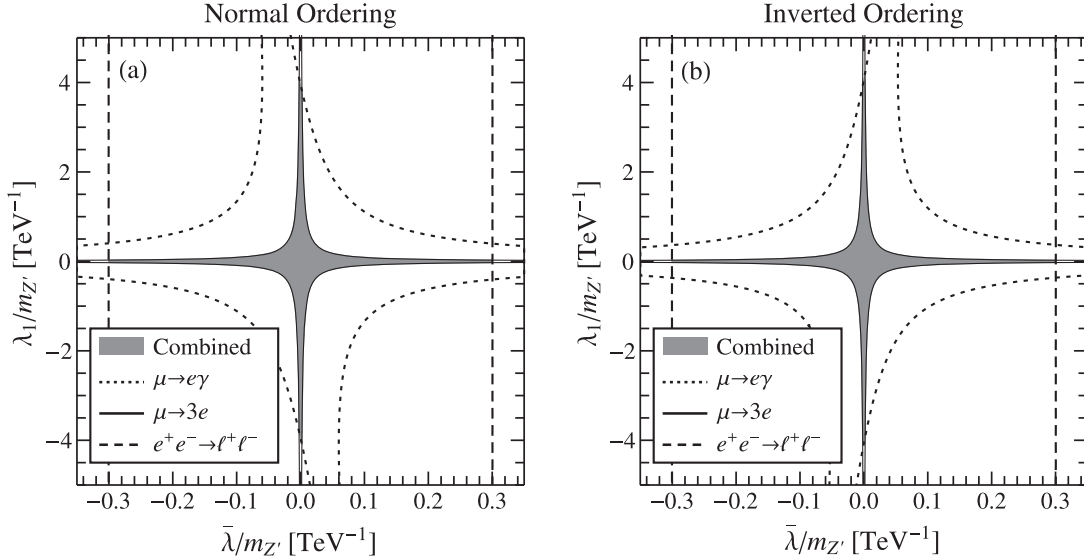


FIG. 1. Combined constraints on the  $Z'$  parameters, plotted in  $(\bar{\lambda}/m_{Z'}, \lambda_1/m_{Z'})$  plane. (a) and (b) denotes the results with NH and IH of neutrino masses, respectively. The gray region indicates the allowed parameter space after combining all the relevant processes, which are dominated by  $\mu \rightarrow e\gamma$  (dotted),  $\mu \rightarrow 3e$  (solid), and  $e^+e^- \rightarrow \ell^+\ell^-$  at LEP (dashed).

SM [31,86]. For example, the angular observable  $P'_5$  [87] in  $B \rightarrow K^*\mu^+\mu^-$  decay exhibits large deviations from the SM predictions in some bins [33]. The ratio  $R_K \equiv \mathcal{B}(B \rightarrow K\mu^+\mu^-)/\mathcal{B}(B \rightarrow Ke^+e^-)$  measured by LHCb shows  $2.6\sigma$  discrepancy from unity, which is predicted by the SM with very good accuracy and may give a hint of lepton flavor nonuniversality [32]. After model-independent global fits, BSM interpretations have been investigated by several groups [52–54]. In the following analysis, we adopt the recent results in Ref. [53], which include all available  $b \rightarrow s\mu^+\mu^-$  [33,86,88] and  $b \rightarrow se^+e^-$  [32,89,90] data. With the two model-independent Wilson coefficients ( $C_{9e}^{\text{NP}} = -C_{10e}^{\text{NP}}, C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$ ), the current anomalies can be explained with a nonvanishing contribution to the muon sector  $C_{9\mu}^{\text{NP}} \approx -1$  but nonsignificant NP contribution in the electron sector. In this case, the significance for deviation from lepton flavor universality is about  $1.2\sigma$ .

As discussed in Sec. III A, the collider processes  $e^+e^- \rightarrow q\bar{q}$  at LEP and  $\mu^-N \rightarrow e^-N$  conversion can constrain the production of  $Z'$  couplings  $\kappa_0\lambda_1$  and  $\kappa_0\bar{\lambda}$ , respectively. After considering the bounds obtained in the last two subsections, combined allowed regions as well as bounds from individual processes are shown in Fig. 2. From these plots, we observe that

- (i) As shown in  $(\kappa_1, \bar{\lambda})$  plane of Fig. 2(b), the current  $b \rightarrow s\ell^+\ell^-$  anomalies can be explained by an MFV  $Z'$  boson at  $2\sigma$  level after considering the constraints from  $B$  and  $K$  mixing as well as the combined constraints from all the lepton processes. In the solution, both the flavor-changing coupling  $\kappa_1$  and flavor-conserving coupling  $\bar{\lambda}$  have a lower bound, which would result in nonvanishing effects on the LFV decays as well as  $B$  and  $K$  mixing.

- (ii) In the  $(\kappa_0, \bar{\lambda})$  plane of Fig. 2(a), the combination of  $e^+e^- \rightarrow u\bar{u}$  and  $b \rightarrow s\ell^+\ell^-$  processes put upper bounds on  $\kappa_0$ , which is not constrained by other individual process. Since the allowed regions in this plane deviate from the  $\bar{\lambda} = 0$  axis, most parts of the parameter space may suggest signatures for  $pp \rightarrow Z' \rightarrow \ell^+\ell^-$  processes at LHC.
- (iii) In the  $(\kappa_1, \lambda_1)$  plane of Fig. 2(d), the combined constraints are much stronger than the one from  $K_L \rightarrow e^\pm\mu^\mp$  decay. Although large  $\lambda_1$  is still allowed,  $\kappa_1$  is stringently constrained by  $B$  and  $K$  mixing. Therefore, the resulting bound  $\kappa_1\lambda_1/m_{Z'}^2 < 0.7 \text{ TeV}^{-2}$  makes the branching ratios of relevant  $B$  and  $K$  LFV decays not very large.
- (iv) In the  $(\kappa_0, \lambda_1)$  plane of Fig. 2(c), the upper bounds on these two parameters are relatively loose. However, the production  $\kappa_0\lambda_1$  is strongly constrained as  $\kappa_0\lambda_1/m_{Z'}^2 < 0.001 \text{ TeV}^{-2}$  by  $\mu^- \text{Au} \rightarrow e^- \text{Au}$  due to its tiny experimentally allowed rates. Hence, an MFV  $Z'$  boson almost cannot produce LFV dilepton signatures at LHC.

## V. PREDICTIONS ON LOW- AND HIGH-ENERGY PROCESSES

Due to the current anomalies in  $b \rightarrow s\ell^+\ell^-$  transitions, some  $Z'$  couplings acquire nonvanishing values after the global fit. In this section, we discuss their impact on both low- and high-energy processes.

### A. Predictions on low-energy flavor processes

As shown in Fig. 2(b), the flavor-conserving coupling  $\bar{\lambda}$  should be nonzero after considering the  $b \rightarrow s\ell^+\ell^-$

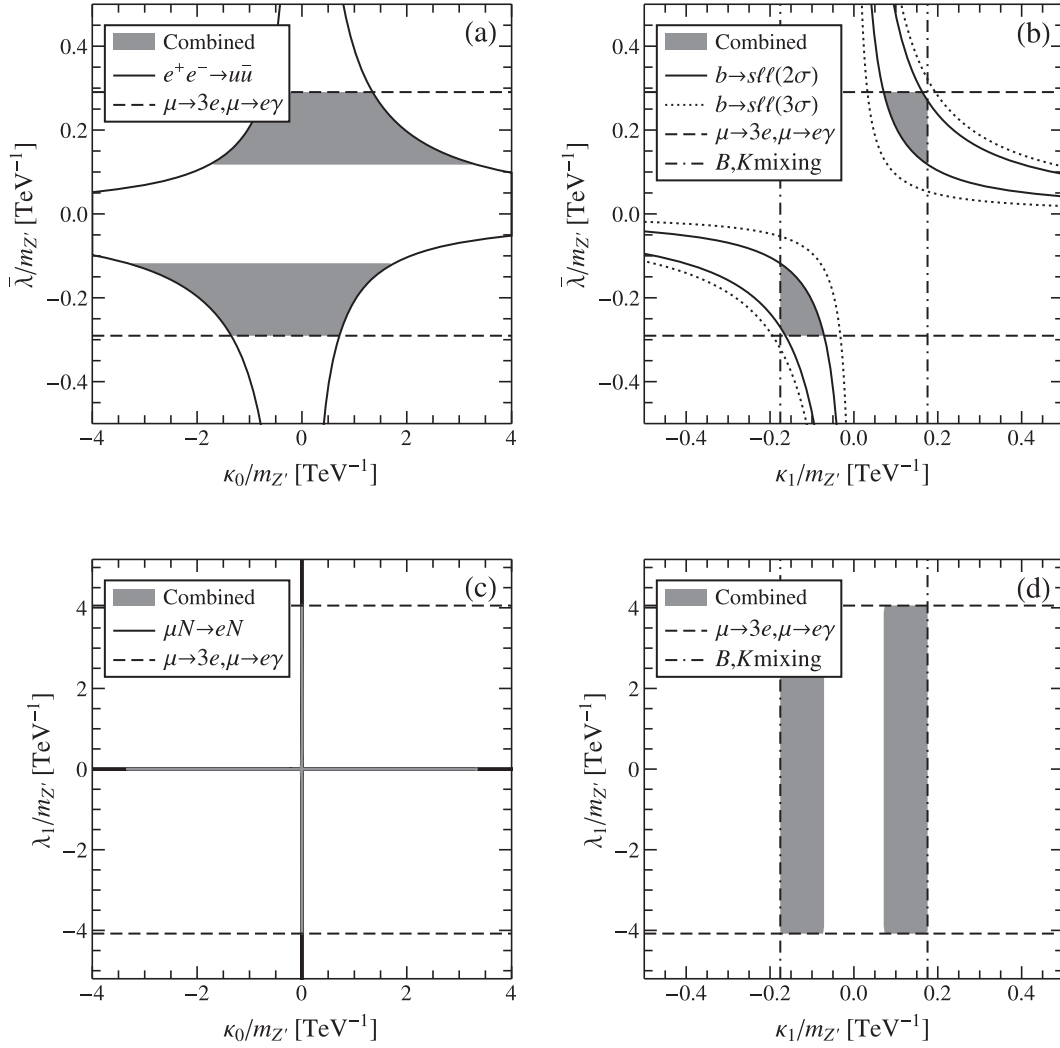


FIG. 2. Constraints for the  $Z'$  couplings in the planes of (a)  $(\kappa_0, \bar{\lambda})$ , (b)  $(\kappa_1, \bar{\lambda})$ , (c)  $(\kappa_0, \lambda_1)$ , and (d)  $(\kappa_1, \lambda_1)$ . Combined allowed regions are shown in gray. The various lines indicate individual bounds described in the legend.

processes. Therefore, there exists a lower bound on the branching ratios of  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$  for a given  $\lambda_1$ . In the combined allowed parameter space obtained in the previous section, the allowed range of  $\mathcal{B}(\mu \rightarrow 3e)$  and

$\mathcal{B}(\mu \rightarrow e\gamma)$  as a function of  $\lambda_1/m_{Z'}$  are shown in Fig. 3, where future experimental sensitivities on these two decays [91,92] are also presented. We note that for  $\lambda_1/m_{Z'} > 0.01 \text{ TeV}^{-1}$  the lower limit on the branching

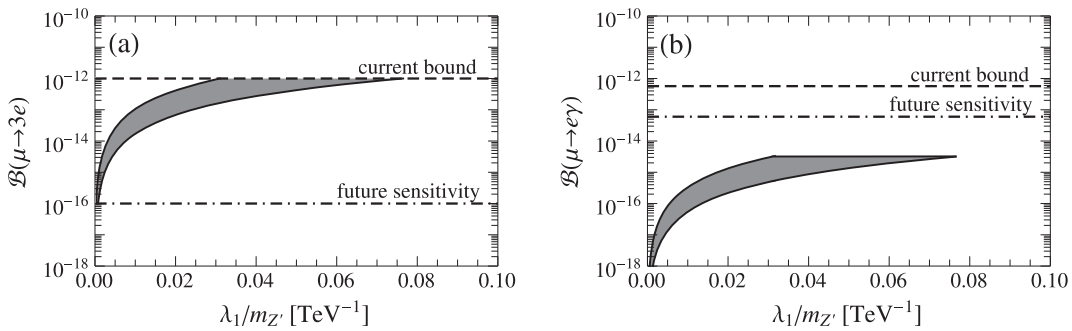


FIG. 3. MFV  $Z'$  predictions on  $\mathcal{B}(\mu \rightarrow 3e)$  and  $\mathcal{B}(\mu \rightarrow e\gamma)$  as a function of  $\lambda_1/m_{Z'}$ . The allowed range of the branching ratios are shown in gray, which are obtained from the combined allowed regions in Figs. 1 and 2. The dashed line denotes the current experimental upper bound, while the dot dashed line is for future sensitivity.



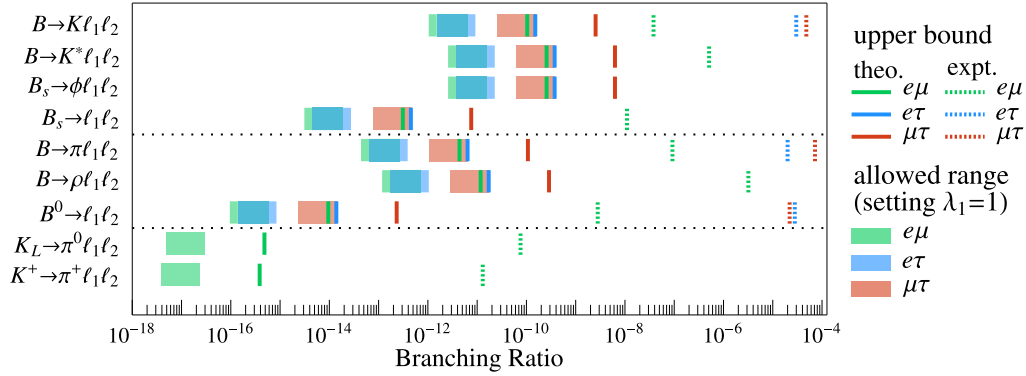


FIG. 4. Predictions on the branching ratios of various  $B$  and  $K$  meson LFV decays. The upper bounds from MFV  $Z'$  and current experimental limits are indicated by the solid and dashed line, respectively, for the final lepton pairs  $e\mu$  (green),  $e\tau$  (blue), and  $\mu\tau$  (red). In the case of  $\lambda_1 = 1$ , the allowed region for the branching ratios is shown in the rectangle regions.

ratio is  $\mathcal{O}(10^{-16})$  for  $\mu \rightarrow e\gamma$  decay, while  $\mathcal{O}(10^{-14})$  for  $\mu \rightarrow 3e$  decay, which is about two orders above the future experimental sensitivity. Therefore,  $\mu \rightarrow 3e$  decay can be very promising to probe the MFV  $Z'$  effects.

To explain the current  $b \rightarrow s\ell^+\ell^-$  anomalies, the FCNC coupling  $\kappa_1$  acquire a nonzero value. Since the mixing amplitudes are proportional to  $\kappa_1^2$ , the  $B$  and  $K$  mixing may be affected. In the allowed parameter space of Fig. 2(d), the predictions for various mixing observables are derived

$$\begin{aligned} 1.04 < \Delta m_s / \Delta m_s^{\text{SM}} < 1.22, \\ 1.05 < \Delta m_d / \Delta m_d^{\text{SM}} < 1.22, \\ 1.03 < |\epsilon_K| / |\epsilon_K^{\text{SM}}| < 1.17, \end{aligned} \quad (5.1)$$

where  $\Delta m_K$  is not presented due to minor  $Z'$  effects. All these observables are enhanced by the  $Z'$  effects with about 3% ~ 5%. As can be seen in Table II, the current uncertainties for these mixing observables are dominated by the theoretical calculation, which will be reduced from the current 10% to about 3% ~ 4% in the next few years [93]. Therefore, the  $B$  and  $K$  mixing could have a good opportunity to probe such an MFV  $Z'$  boson.

As shown in Fig. 2(d), large values of the LFV coupling  $\lambda_1$  is still allowed after the global fit. Together with the nonvanishing coupling  $\kappa_1$ , they could affect LFV decays of  $B$  and  $K$  mesons, such as  $B \rightarrow K^{(*)}e^+\tau^-$  and  $K_L \rightarrow \pi^0e^+\mu^-$  decays. The branching ratios of these processes are proportional to  $|\kappa_1\lambda_1/m_{Z'}^2|^2$ . Fig. 4 shows the predictions on these LFV decays from the allowed  $Z'$  parameter space in Fig. 2(d), as well as the current experimental upper limits [81,83]. Since the product  $\kappa_1\lambda_1$  is lower bounded for a nonvanishing value of  $\lambda_1$ , we also give the allowed region of the branching ratios in the case of  $\lambda_1/m_{Z'} = 1 \text{ TeV}^{-1}$ . We can see that the upper limits on the branching ratios are typically 4 ~ 5 orders lower than the current observed bounds.

In addition, our predictions are lower than the results from the effective theory analysis with MFV [94]. Within an MFV  $Z'$  boson, both  $\kappa_1$  and  $\lambda_1$  are bounded from the quark and lepton flavor-violating processes, which make the upper limit on the product  $\kappa_1\lambda_1$  stronger than that from  $K_L \rightarrow e^\pm\mu^\mp$  decay. In the effective field theory approach with MFV, bounds on the effective operators responsible to  $B$  and  $K$  meson LFV decays mainly arise from  $K_L \rightarrow e^\pm\mu^\mp$  decay [94]. Therefore, its predicted upper bounds on other  $B$  and  $K$  meson LFV decays are much higher than our results presented in Fig. 4.

## B. LHC signatures

In this part, we first introduce the current measurements and constraints on  $Z'$  boson from  $pp$  and  $p\bar{p}$  colliders. At the LHC, the CMS Collaboration analysed the dimuon (dielectron) mass spectra with the 8 TeV run I data corresponding to an integrated luminosity of 20.6 (19.7)  $\text{fb}^{-1}$ . A Sequential Standard Model (SSM)  $Z'_{\text{SSM}}$  resonance lighter than 2.90 TeV is excluded [9]. Lower limits on the energy scale parameter for the contact interaction  $\Lambda$  are found to be 12.0 (15.2) TeV for destructive (constructive) interference in the dimuon channel and 13.5 (18.3) TeV in the dielectron channel [9]. The ATLAS Collaboration searched for a high-mass resonance decaying into  $\tau^+\tau^-$  final state at  $\sqrt{s} = 8 \text{ TeV}$  with an integrated luminosity of 19.5–20.3  $\text{fb}^{-1}$ . Lower mass limit on the  $Z'_{\text{SSM}}$  boson is set to be 2.02 TeV at 95% C.L. [95]. At the Tevatron, both the D0 and CDF Collaborations searched for a heavy neutral gauge boson in the  $e^+e^-$  channel of the  $p\bar{p}$  collisions at  $\sqrt{s} = 1.96 \text{ TeV}$ . A lower mass limit of about 1 TeV for the SSM  $Z'$  boson is presented, respectively [96]. However, the above constraints on the  $Z'$  mass are model dependent and are typically sensitive to the free parameters such as its couplings to leptons, and therefore can still be loosened to some extent.

In Fig. 5, we show the cross section ratio of  $pp \rightarrow e^+e^-$  mediated by  $Z'$  and  $Z$ , respectively, with the center-of-mass

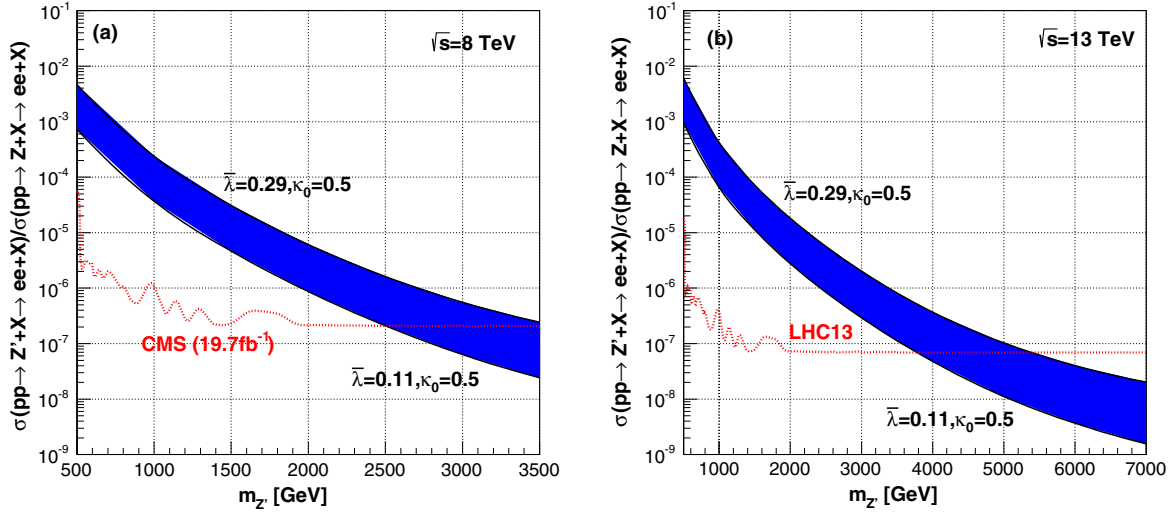


FIG. 5. The cross section ratios of  $pp \rightarrow e^+e^-$  through  $Z'$  and  $Z$  production for the two benchmark points of  $\bar{\lambda}$  and  $\kappa_0$ , (i)  $(\bar{\lambda}, \kappa_0) = (0.11, 0.5)$  and (ii)  $(\bar{\lambda}, \kappa_0) = (0.29, 0.5)$  at (a)  $\sqrt{s} = 8$  TeV and (b)  $\sqrt{s} = 13$  TeV. The experimental upper limits are shown by the red dotted lines, which are obtained from (a) the CMS dielectron channel with  $19.7 \text{ fb}^{-1}$  data at 8 TeV and (b) the projected LHC sensitivity at  $\sqrt{s} = 13$  TeV.

energy  $\sqrt{s} = 8$  and 13 TeV. The cross sections are computed by using `MadGraph5_aMC@NLO` [97] complemented with the Lagrangian Eq. (2.8) of the MFV  $Z'$  boson. The following kinematical cuts are imposed according to the CMS experiment [9],

$$\begin{aligned} p_T &> 10 \text{ GeV}, & |\eta| &< 2.5, \\ E_T &> 33 \text{ GeV}, & \Delta R &> 0.3, \end{aligned} \quad (5.2)$$

which correspond to the transverse momentum, the rapidity, the transverse energy and minimal separation of final state charged leptons. According to the allowed region of the  $Z'$  couplings to the lepton and quark shown in Fig. 2(a), we choose two benchmark points

$$\begin{aligned} \text{(i)} \quad \bar{\lambda} &= 0.11, & \kappa_0 &= 0.5, \\ \text{(ii)} \quad \bar{\lambda} &= 0.29, & \kappa_0 &= 0.5, \end{aligned} \quad (5.3)$$

where the values of  $\bar{\lambda}$  correspond to the upper and lower limits with  $\kappa_0 = 0.5$  obtained from a 1 TeV  $Z'$  boson. The blue-shaded region therefore satisfy the previous combined constraint. In the 8 TeV plot of Fig. 5(a), we also show the present CMS exclusion bound collected from the  $19.7 \text{ fb}^{-1}$  data in the dielectron channel. In addition, the expected LHC sensitivity at  $\sqrt{s} = 13$  TeV with integrated luminosity  $\mathcal{L} = 100 \text{ fb}^{-1}$  is shown in Fig. 5(b), which is estimated with the method in Ref. [98].

From Fig. 5, we can see that  $m_{Z'} < 2.5$  TeV is disfavored for point (i)  $(\bar{\lambda}, \kappa_0) = (0.11, 0.5)$  from the  $19.7 \text{ fb}^{-1}$  CMS data at LHC run I, while for point (ii)  $(\bar{\lambda}, \kappa_0) = (0.29, 0.5)$ ,  $Z'$  mass smaller than 3.5 TeV is excluded. At the 13 TeV LHC with  $100 \text{ fb}^{-1}$ , sensitivity to the cross

section ratio of  $pp \rightarrow e^+e^-$  mediated by the  $Z'$  and  $Z$  boson is expected to increase roughly by a factor of 2 compared to the current experimental bound at LHC run I. For the benchmark points (i) and (ii), the  $Z'$  mass below 3.8 and 5.4 TeV is, respectively, within the sensitivity reach of LHC run II.

## VI. CONCLUSIONS

In this work, the general family nonuniversal  $Z'$  model with a mass of the TeV scale has been investigated adopting the MFV hypothesis to avoid potentially large tree-level FCNCs in the quark sector. We also extend the general scenario to the lepton sector. Considering the MFV  $Z'$  couplings with fermions, their impact on various low- and high-energy processes has been studied in detail. It is found that lepton LFV decays  $\mu \rightarrow 3e$  and  $\mu \rightarrow e\gamma$ ,  $\mu$ - $e$  conversion in nuclei,  $b \rightarrow s\ell^+\ell^-$  transitions,  $B$  and  $K$  meson mixing, and the LEP processes  $e^+e^- \rightarrow f\bar{f}$  are more sensitive to such a  $Z'$  boson.

After a combined constraint from the current experimental data, the allowed parameter space is derived. We find that the MFV  $Z'$  boson can explain the current anomalies in  $b \rightarrow s\ell^+\ell^-$  transitions with a nonvanishing couplings  $\kappa_1$  and  $\bar{\lambda}$ , which controls the FCNC couplings in quark sector and flavor-conserving couplings in the lepton sector. The implications of these two nonzero couplings are investigated for various processes. In particular, the mass difference  $\Delta m_s$  in  $B_s - \bar{B}_s$  mixing and the parameter  $|\epsilon_K|$  in  $K^0 - \bar{K}^0$  mixing are enhanced by more than 3% ~ 5%, which are smaller than both experimental and theoretical uncertainties in the near future. At the same time, the various LFV  $B$  and  $K$  meson decays are less enhanced and

less promising in the near future. In addition, our predicted lower limit on the branching ratio of  $\mu \rightarrow 3e$  is  $\mathcal{O}(10^{-14})$  for  $\lambda_1/m_{Z'} > 0.01 \text{ TeV}^{-1}$ , which is much higher than the expected experimental sensitivity of near future.

At the LHC, the  $Z'$  boson can mediate a clear leptonic signal through the Drell-Yan channel  $pp \rightarrow Z' \rightarrow \ell^+\ell^-$ . After considering constraints from the LHC run I data, there are still allowed parameter space in our scenario. It is noted that, the lepton flavor-conserving coupling  $\bar{\lambda}$  acquired a nonzero value to explain the current  $b \rightarrow s\ell^+\ell^-$  anomaly. For a given quark flavor-conserving coupling  $\kappa_0$ , the cross section of  $pp \rightarrow Z' \rightarrow \ell^+\ell^-$  is lower bounded, which

make the collider signatures at LHC very predictive. In the near future, if the  $b \rightarrow s\ell^+\ell^-$  anomaly persists, direct searches at the LHC run II may have a good opportunity to distinguish various  $Z'$  scenarios, together with high-precision measurements at Belle II and LHCb.

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