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Impact of momentum space anisotropy on heavy quark dynamics in a QGP medium

Vinod Chandra[1,*] and Santosh K. Das[2,3,†]

[1] *Indian Institute of Technology Gandhinagar, VGEC Campus, Ahmedabad-382424, India*
[2] *Department of Physics and Astronomy, University of Catania, Via S. Sofia 64, 1-95125 Catania, Italy*
[3] *Laboratori Nazionali del Sud, INFN-LNS, Via S. Sofia 62, I-95123 Catania, Italy*
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Momentum space anisotropy present in the quark and gluon distribution functions in relativistic heavy ion collisions induces Chromo-Weibel instability in the hot QCD medium created therein. The impact of the Chromo-Weibel instability on the dynamics of a heavy quark (HQ) traversing in the QGP medium is investigated within the framework of kinetic theory by studying the momentum and temperature behavior of HQ drag and diffusion coefficients. The physics of anisotropy is captured in an effective Vlasov term in the transport equation. The effects of the instability are handled by making a relation with the phenomenologically known jet-quenching parameter in RHIC and LHC. Interestingly, the presence of instability significantly affects the temperature and momentum dependences of the HQ drag and diffusion coefficients. These results may have appreciable impact on the experimental observables such as the nuclear suppression factor, $R_{AA}(p_T)$, and the elliptic flow, $v_2(p_T)$, of heavy mesons in heavy ion collisions at RHIC and LHC energies which is a matter of future investigation.

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I. INTRODUCTION

The physics of the Chromo-Weibel instability [1] (non-Abelian analogue of Weibel instability [2]) during the hydrodynamic expansion of the QGP in heavy ion collisions may play a crucial role in understanding the space-time evolution and properties of a quark-gluon plasma medium. The momentum anisotropy present during the hydrodynamic expansion of the QGP may induce instabilities to the Yang-Mills field (Chromo field) equations. The Weibel type of instabilities can be seen in the expanding quark-gluon plasmas since the width of the momentum component in the direction of the expansion narrows by expansion, leading to an anisotropic momentum distribution. The instability in the rapidly expanding QGP in heavy ion collisions may lead to the plasma turbulence [3]. Recall that the plasma turbulence describes a random, nonthermal pattern of excitation of coherent color field modes in the QGP with a power spectrum similar to that of vortices in a turbulent fluid [3].

The prime goal here is to investigate the heavy quark dynamics in the presence of Chromo-Weibel instability. This could be done by first modeling the nonequilibrium momentum distribution functions that describe expanding anisotropic QGP followed by employing it to the kinetic theory description of heavy quark dynamics.

Hadrons containing HQs (c, $\bar{c}$, b, or $\bar{b}$) are of great interest in investigating the properties of the QGP since their physical properties get significantly modified while traveling through QGP. This fact has been reflected in the particle spectra at RHIC and the LHC energies. Further, HQ thermalization time is larger than gluons and light quarks, and they do not constitute the bulk of the QGP. Since their formation occurs in the early stages of the collisions, they can travel through the thermalized QGP medium and can retain the information about the interaction with them very efficiently. For instance, it is pertinent to ask whether a single $c\bar{c}$ can stay together long enough to form a bound state (say J/ψ) at the hadronization state. To address this, one is required to describe the dynamics of the HQs propagating through the QGP. Therefore, one can explore the physics of the HQ transport [4–22] in the QGP medium as follows. The nonequilibrated HQs can travel in the equilibrated QGP medium, and one has to deal with the problem within the framework of Langevin dynamics [23]. This is to say that the HQs perform random motion in the equilibrated QGP. Recall that the QGP goes through a hydrodynamic evolution before it reaches the hadronization and subsequently the hadrons freeze out.

The pertinent question to ask is whether HQs maintain equilibrium during this entire process of the space-time evolution or not. It has been observed [24] within the framework of Langevin dynamics and pQCD (perturbative QCD) that the HQs may not achieve the equilibrium in the RHIC and LHC energies.

The most important observable, which encodes the medium effects carried with them by the HQs while traveling in the QGP, is the nuclear modification factor, R_{AA}. It has been observed that their energy loss in the QGP due to gluon radiation is insufficient to describe the medium modification of the spectrum [25,26]. Therefore,

*vchandra@iitgn.ac.in
†dsantoshphy@gmail.com

one has to look at the collisions since they have different fluctuation spectrums than radiation and might contribute significantly as one thought of initially [27,28]. The collisional effects can be captured well in the HQ drag and diffusion coefficients which have been calculated within weak coupling QCD by several authors. The formalism and details are offered in Sec. II A.

The temperature, T, and chemical potential, μ_B, dependence of the drag and diffusion coefficient enter through the thermal distributions of light quarks and gluons. In the present calculation, we ignore the μ_B dependence in view of the fact that the QGP produced at RHIC and LHC energies at the midrapidity region has negligibly small net baryon density. But one has to implement the realistic QGP EOS in terms of the appropriate form of the thermal distribution functions. Lattice QCD EOS may be a good choice for the description of the equilibrated QGP. Additionally, it is important to address the role of the momentum anisotropies at RHIC and LHC in influencing the dynamics of the heavy quarks in the hot QCD medium. This is the main focus of the article.

The paper is organized as follows. Section II deals with the kinetic theory formulation of HQ dynamics in the background QGP medium in terms of drag and diffusion coefficients. Section III discusses the non(near)-equilibrium modeling of the degrees of freedom that describes the QGP medium in the presence of anisotropy. In Sec. IV, we present the results and related discussions. Finally, conclusions are presented in Sec. V.

II. HEAVY QUARK DRAG AND DIFFUSION IN THE HOT QCD MEDIUM

HQs play crucial roles in characterizing QGP as they are produced in the early stages of the heavy ion collisions and remain extant throughout the evolution and, hence, can capture the information of the entire evolution of the system. The dynamics of HQs while traveling in the QGP medium can be understood in terms of the drag and diffusion coefficients following Landau's prescription.

A. Heavy quark drag and diffusion

Let us consider the elastic interaction experienced by HQs while traversing into the hot QCD medium. Next, we consider the process $c(p) + l(q) \to c(p') + l(q')$ [l stands for gluon and light quarks and antiquarks; p denotes the momentum of an HQ and q of the bulk particles. Note that the sub(super)script, q, denotes the quarks and antiquarks].

B. HQ drag

The the drag coefficient, γ, can be calculated by using the following expression [29]:

$$\gamma = p_i A_i / p^2 \tag{1}$$

where A_i is given by

$$\begin{aligned} A_i = &\frac{1}{2E_p}\int \frac{d^3q}{(2\pi)^3 E_q}\int \frac{d^3p'}{(2\pi)^3 E'_p}\int \frac{d^3q'}{(2\pi)^3 E'_q} \\ &\times \frac{1}{g_Q}\sum \overline{|M|^2}(2\pi)^4\delta^4(p+q-p'-q') \\ &\times f(q)(1\pm f(q'))[(p-p')_i] \\ \equiv &\langle\langle (p-p')\rangle\rangle, \end{aligned} \tag{2}$$

with g_Q being the statistical degeneracy of the HQ propagating through QGP. The above expression indicates that the drag coefficient is the measure of the thermal average of the momentum transfer, $p - p'$, due to interaction of the heavy quarks with the bath particle weighted by the square of the invariant amplitude, $\overline{|M|^2}$. The factor $f(q)$ denotes the thermal distribution of the particles in the QGP. Here, $1 \pm f(p')$ is the momentum distribution with the Bose enhancement or Pauli-suppressed probability in the final state. Note that $f(q)$ will involve three types of thermal phase space distribution functions corresponding to the gluons (g), light quarks ($q \equiv$ up and down), strange quarks (s), and corresponding antiquarks. Hence, $f(q)$ jointly denotes this three phase space distribution as

$$f(q) \equiv \{f_g, f_q, f_s\}. \tag{3}$$

In the presence of initial momentum anisotropy, we need to model them appropriately by first setting up the transport equation and then solving it either analytically or numerically. In the present work, we consider the linearized transport equation and capture all the effects coming from the anisotropy as the first-order modification to the equilibrium distribution functions for quark-antiquark and gluons.

In view of the above, we consider the following decomposition for the $f(q)$ in three sector,

$$\begin{aligned} f_g &= f_0^g(q) + f_1^g(\vec{q},\vec{r}), \\ f_q &= f_0^q(q) + f_1^q(\vec{q},\vec{r}), \\ f_s &= f_0^s(q) + f_1^s(\vec{q},\vec{r}). \end{aligned} \tag{4}$$

Here, $q = |\vec{q}|$.

At this stage, we need the correct modeling of equilibrium (isotropic) distribution functions [(first term in the right-hand side of Eq. (4)] and the modifications induced by the anisotropy. This is presented in the next section.

C. HQ diffusion

Similar to the HQ diffusion coefficient, B_0 can be evaluated as

$$B_0 = \frac{1}{4}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(p.p')^2\rangle\rangle}{p^2}\right]. \quad (5)$$

With an appropriate choice of $\mathcal{F}(p')$, both the drag and diffusion coefficients can be evaluated from a single expression as follows:

$$\ll \mathcal{F}(p) \gg = \frac{1}{512\pi^4}\frac{1}{E_p}\int_0^\infty \frac{q^2 dq d(\cos\chi)}{E_q}\hat{f}(q) \times \frac{w^{1/2}(s, m_Q^2, m_p^2)}{\sqrt{s}}\int_1^{-1} d(\cos\theta_{\rm c.m.}) \times \frac{1}{g_Q}\sum \overline{|M|^2}\int_0^{2\pi} d\phi_{\rm c.m.}\mathcal{F}(p'), \quad (6)$$

where s is the Mandelstam variable and $w(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$ is the triangular function. In the next section, we present the modeling of nonequilibrium distribution functions for a rapidly expanding plasma in the presence of small momentum anisotropy. These distributions are employed to compute the HQ drag and diffusion coefficients.

III. MODELING MOMENTUM DISTRIBUTION FUNCTIONS FOR GLUONS AND QUARKS

A. The isotropic distributions

The equilibrium modeling of the momentum distribution functions employed here is based on the quasi-particle nature of the hot QCD medium (beyond T_c) [30]. The quasi-particle description employed here has been developed in the context of the recent (2 + 1)-flavor lattice QCD EOS [31] at physical quark masses. There are more recent lattice results with the improved actions and refined lattices [32], for which we need to re-look the model with a specific set of lattice data specifically to define the effective gluonic degrees of freedom. Therefore, we stick to the set of lattice data utilized in the model described in [30]. Here, the form of the equilibrium distribution functions, $f_{\rm eq} \equiv \{f_0^g, f_0^q, f_0^s\}$, (this notation is useful later while writing the transport equation in both the sector in compact notations) describing the strong interaction effects in terms of effective fugacities $z_{g,q}$ can be written as

$$f_0^{g/q} = \frac{z_{g/q}\exp[-\beta q]}{(1 \mp z_{g/q}\exp[-\beta q])},$$
$$f_0^s = \frac{z_q \exp[-\beta\sqrt{q^2 + m_s^2}]}{(1 + z_q\exp[-\beta\sqrt{q^2+m_s^2}])}, \quad (7)$$

where $q = |\vec{q}|$, m_s denotes the mass of the strange quark (which we choose to be 0.1 GeV), and $\beta = T^{-1}$ denotes the inverse of the temperature.

We use the notation $\nu_g = 2(N_c^2 - 1)$ for gluonic degrees of freedom, $\nu_q = 2 \times 2 \times N_c \times 2$ for light quarks, and $\nu_s = 2 \times 2 \times N_c \times 1$ for the strange quark for $SU(N_c)$. As we are working at zero baryon chemical potential, therefore quark and antiquark distribution functions are the same. Since the model is valid in the deconfined phase of QCD (beyond T_c), the mass of the light quarks can be neglected as compared to the temperature. As QCD is a $SU(3)$ gauge theory, $N_c = 3$ for our analysis.

Note that the effective fugacities ($z_{g/q}$) are not merely a temperature-dependent parameter which encodes the hot QCD medium effects. They lead to a nontrivial dispersion relation both in the gluonic and quark sectors as

$$\omega_g = q + T^2\partial_T \ln(z_g),$$
$$\omega_q = q + T^2\partial_T \ln(z_q),$$
$$\omega_s = \sqrt{q^2 + m^2} + T^2\partial_T \ln(z_q), \quad (8)$$

and this lead to the new energy dispersions for gluons (ω_g), light quark antiquarks (ω_q), and strange quark-antiquarks. For a detailed discussion of the interpretation and physical significance of z_g and z_q, we refer the reader to [30]. There are other quasi-particle descriptions in the literature, and those could be characterized as effective mass models [33,34], effective mass models with gluon condensate [35], and effective models with Polyakov loop [36]. Our model is fundamentally distinct from all these models. Another crucial point is regarding the definition of the energy momentum tensor, $T^{\mu\nu}$. As described in [37], in the presence of nontrivial temperature-dependent energy dispersion (as in all these quasi-particle models), we need to modify the definition of the $T^{\mu\nu}$ so that the trace anomaly effects in QCD can be accommodated in the definition. The modified $T^{\mu\nu}$ for the effective mass models is obtained in [37] and for the current model in [38]. To model the (non) near-equilibrium momentum distributions for the quarks and gluons, we exploit the role of momentum anisotropy present during the hydrodynamic expansion of the QGP in RHIC. The momentum anisotropy may induce Weibel-type instability in the hot QCD medium and may be responsible for anomalous transport processes in the QGP.

B. Chromo-Weibel instability and anomalous transport: Dupree-Vlasov equation

Recall that the momentum anisotropy present in quark and gluon momentum distribution functions induces instability in the Yang-Mills equations in a similar way as the Weibel instability in the case of electromagnetic (EM) plasmas. This instability, while coupled with the rapid expansion of the QGP, leads to anomalous transport and modulates the transport coefficients of the plasma substantially. This fact was realized by Dupree in the case of EM plasmas in 1954 [39] and was later generalized for the non-Abelian plasmas in [3,40]. In the context of QGP, the phenomenon of the anomalous transport is realized at the later stages of the collisions due to the hydrodynamic expansion of the QGP; one has appreciable momentum anisotropy present in the thermal distribution functions of quarks and gluons.

The first step towards estimating the near equilibrium momentum distributions of the quarks and gluons in rapidly expanding QGP with momentum anisotropy is to set up the Dupree-Vlasov equation (linearized version) and then solve it with the help of an ansatz to obtain the correction to the isotropic distribution functions. Next, we briefly outline the mathematical formalism in solving the transport equation.

C. Formalism

We start with the following ansatz for the nonequilibrium distribution function,

$$f(\vec{q},\vec{r}) = \frac{z_{g,q}\exp(-\beta u^\mu q_\mu)}{1 \pm z_{g,q}\exp(-\beta u^\mu q_\mu + f_1(\vec{q},\vec{r}))}, \tag{9}$$

where $z_{g,q}$ are the effective gluon, quark fugacities coming from the isotropic modeling of the QGP in terms of lattice QCD equation of state. The parameter β is the temperature inverse (in units of $K_B = 1$), and u^μ is the fluid 4-velocity considering the fluid picture of the QGP medium. Here, $f_1(\vec{q},\vec{r})$ denotes the effects from the anisotropy (momentum). To achieve the above-mentioned near equilibrium situation, f_1 must be a small perturbation. Under this condition, we obtain,

$$f(\vec{q},\vec{r}) = f_0(q) + f_0(1 \pm f_0(q))f_1(\vec{q},\vec{r}) + O(f_1(\vec{q},\vec{r})^2). \tag{10}$$

The *plus* sign is for gluons, and *minus* sign is for the quarks/antiquarks.

Next, the following form for the ansatz is considered for the linear order perturbation to the isotropic gluons and quarks distribution functions, respectively,

$$f_1(\vec{q},\vec{r}) \equiv f_1^{g,q} = -\frac{1}{\omega_{g,q}T^2} q_i q_j (\Delta_{g,q}(\vec{q})(\nabla u)_{ij}, \tag{11}$$

The quantities, $\Delta_{g,q}$, denotes the strength of the momentum anisotropy for the gluons and quarks, respectively. In the local rest frame of the fluid (LRF) $f_0 = f_{\rm eq} = (f_0^g, f_0^q)$, and considering longitudinal boost invariance, we obtain $\nabla\cdot\vec{u} = \frac{1}{\tau}$ and $\nabla u_{ij} = \frac{1}{3\tau}\mathrm{diag}(-1,-1,2)$, leading to

$$f_1^{g,q} = -\frac{\Delta_{g,q}(q)}{\omega_{g,q}T^2\tau}\left(q_z^2 - \frac{q^2}{3}\right). \tag{12}$$

We utilize the above anisotropic quasi-quark (antiquark) and quasi-gluon distribution functions to linearize the effective transport equation below.

1. Effective transport equation in turbulent chromo-fields

The evolution of the quasi-quark and quasi-gluon momentum distribution functions in the anisotropic QGP medium can be described by the Vlasov-Boltzmann equation [41]:

$$v^\mu \frac{\partial}{\partial x^\mu} f(\mathbf{r},\mathbf{q},t) + g\mathbf{F}^a\cdot\nabla_q f^a(\mathbf{r},\mathbf{q},t) = 0. \tag{13}$$

Here, $f(\mathbf{r},\mathbf{q},t)$ represents the parton distribution in phase space, which sums over all parton colors, $\mathbf{q}\equiv\vec{q}$ and $\mathbf{r}\equiv\vec{r}$. The quantity $f^a(\mathbf{r},\mathbf{q},t)$ denotes the color octet distribution function. Here, we have neglected the collision term. Both the distributions, f and f^a, are defined in the semiclassical formalism in [42] as the moments of the distribution function $\tilde{f}(\mathbf{r},\mathbf{q},Q,t)$ in an extended phase space that includes the color sector

$$f(\mathbf{r},\mathbf{q},t) = \int dQ\tilde{f}(\mathbf{r},\mathbf{q},Q,t), \tag{14}$$

$$f^a(\mathbf{r},\mathbf{q},t) = \int dQ Q^a\tilde{f}(\mathbf{r},\mathbf{q},Q,t). \tag{15}$$

Here, Q^a denotes the color charge, $v^\mu = \frac{q^\mu}{q^0}$, $q^\mu = (q^0 = E_q, \vec{q})$. Furthermore, the color Lorentz force is defined as

$$\mathbf{F}^a = E^a + \mathbf{v}\times B^a. \tag{16}$$

Note that the color octet distribution function, f^a, satisfies a transport equation which also involves coupling with the phase space distributions of higher color $SU(3)$ representations. In the present case, the near equilibrium consideration allows us to truncate this hierarchy by keeping only the lowest order term in the gradients for both f and f^a. In the case of equilibrium, the color octet distribution vanishes, $f^a \equiv 0$, implying that it is at least linear in perturbation. With all the above considerations, the transport equation for f^a reads [41,42],

$$v^\mu \frac{\partial f^a}{\partial x^\mu} + g f_{abc} A_\mu^b v^\mu f^c + \frac{gC_2}{N_c^2 - 1}\mathbf{F}^a\cdot\nabla_q f = 0, \tag{17}$$

where C_2 is quadratic Casimir invariant and f_{abc} denotes the structure constants of $SU(3)$. Here, A^μ represents the gauge field. Next, the prime goal is to solve Eq. (17) and obtain f^a in terms of f and finally solve Eq. (13), in the case of turbulent Chromo fields. At this juncture, we need to make additional assumptions about the field distribution in the Vlasov force term. In the presence of anisotropy (momentum), the color field is turbulent (random with a certain spatial and temporal correlation structure for fields at different space-time points). The octet distribution function in terms of the siglet one is obtained in [3]. We briefly outline the procedure here.

Next, Fourier transforming the f^a [the dependence on the space-time coordinate $x^\mu = (t, \mathbf{r})$], we obtain

$$f^a(\mathbf{q}, x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} f^a(\mathbf{q}, k). \tag{18}$$

Here, the singlet distribution, $f(\mathbf{q})$, is allowed to have arbitrary particle distribution in momentum space but neglects any space-time dependence.

Now, the solution of Eq. (17) is given by [43]

$$f^a(\mathbf{q},k) = -ig\frac{C_2}{N_c^2-1}(v\cdot k + i\epsilon)^{-1}\mathbf{F}^a(k)\cdot\nabla_q f(\mathbf{q}), \tag{19}$$

where $v\cdot k \equiv v^\mu k_\mu = k^0 - \mathbf{v}\cdot\mathbf{k}$.

Note that gauge connection, $U(x, x')$, is associated with the space-time derivative, which parallel transports the gauge field from one spatial point to another. In other words, $U(x, x')$ has the effect of introducing a path-ordered factor:

$$U_{ac}(x, x') = P\exp\left(-\int_{x'}^{x} f_{abc}A_\mu^b dx^\mu\right), \tag{20}$$

which parallel transports the gauge fields from x' to x (here, A_μ is the gauge field). With this consideration, returning back to coordinate space in Eq. (19), f^a reads

$$f^a(\mathbf{q}, x) = -ig\frac{C_2}{N_c^2-1}\int\frac{d^4k}{(2\pi)^4}\int d^4x' U_{ab}(x, x') \times \frac{e^{ik\cdot(x'-x)}}{v\cdot k + i\epsilon}\mathbf{F}^b(x')\cdot\nabla_q f(\mathbf{q}). \tag{21}$$

Substituting this solution for f^a into Eq. (13), the Vlasov force term takes the following form:

$$g\mathbf{F}^a(x)\cdot\nabla_q f^a(\mathbf{q}, x) = -\frac{ig^2C_2}{N_c^2-1}\mathbf{F}^a(x)\cdot\nabla_q\int\frac{d^4k}{(2\pi)^4} \times\int d^4x' U_{ab}(x, x')\frac{e^{ik\cdot(x'-x)}}{v\cdot k + i\epsilon} \times\mathbf{F}^b(x')\cdot\nabla_q f(\mathbf{q}). \tag{22}$$

The argument that has been invoked is that the soft chromo-fields are turbulent, and their action on the quasi-partons can be described by taking an ensemble average, which can be factorized in the form

$$\langle F_i^a(x)U_{ab}(x, x')F_j^b(x')f(\mathbf{q})\rangle = \langle F_i^a(x)U_{ab}(x, x')F_j^b(x')\rangle\bar{f}(\mathbf{q}). \tag{23}$$

Here, $F_i^{a,b}$ denotes the color electric and magnetic fields, and $U_{ab}(x, x')$ is the correction function/gauge connection. The further assumption that is invoked here is that the correlation functions of fields at different space-time points (x and x') depend only on $R \equiv |x - x'|$ and decay rapidly with correlation time τ_m and correlation length σ as

$$\langle E_i^a(x)U_{ab}(x,x')E_j^b(x')\rangle = \langle E_i^a E_j^a\rangle\Phi_\tau^{(\mathrm{el})}(|t-t'|)\tilde{\Phi}_\sigma^{(\mathrm{el})}(R),$$
$$\langle B_i^a(x)U_{ab}(x,x')B_j^b(x')\rangle = \langle B_i^a B_j^a\rangle\Phi_\tau^{(\mathrm{mag})}(|t-t'|)\tilde{\Phi}_\sigma^{(\mathrm{mag})}(R). \tag{24}$$

The Gaussian correlaters (given below) satisfy the above conditions,

$$\Phi_\tau^{(\mathrm{el/mag})}(|t-t'|) = \exp[(t-t')^2/2\tau^2_{\mathrm{el/mag}}], \tag{25}$$

$$\tilde{\Phi}_\sigma^{(\mathrm{el/mag})}(|\mathbf{x}-\mathbf{x}'|) = \exp[(\mathbf{x}-\mathbf{x}')^2/2\sigma^2_{\mathrm{el/mag}}]. \tag{26}$$

We also assume that the color electric and magnetic fields are uncorrelated. Next, performing first the integral over k^0, then over $\mathbf{k}$, finally, the integral over $\mathbf{x}'$ [3],

$$\langle g\mathbf{F}^a\cdot\nabla_p f^a\rangle = -\frac{g^2C_2}{N_c^2-1}\left[\tau_{\mathrm{m}}^{\mathrm{el}}\langle E_i^aE_j^a\rangle\frac{\partial^2}{\partial q_i\partial q_j} + \tau_{\mathrm{m}}^{\mathrm{mag}}\langle B_i^aB_j^a\rangle(\mathbf{v}\times\nabla_q)_i(\mathbf{v}\times\nabla_q)_j\right]\bar{f}(\mathbf{q}) \equiv -\nabla_q\cdot D(\mathbf{q})\cdot\nabla_q\bar{f}(\mathbf{q}). \tag{27}$$

Now, consider only the color magnetic field generated by plasma instability that points in the transverse direction,

$$\langle B_i^aB_j^a\rangle = \frac{1}{2}(\delta_{ij}-\delta_{iz}\delta_{jz}); \qquad \langle E_i^aE_j^a\rangle = 0. \tag{28}$$

Employing the notation $-i\mathbf{q}\times\nabla_q = \mathbf{L}^{(q)}$ for the generator of rotations in momentum space, we can now write the Vlasov term in the transverse color magnetic field as

$$\nabla_q\cdot D\cdot\nabla_q = -\frac{g^2C_2}{2(N_c^2-1)E_p^2}\langle B^2\rangle\tau_{\mathrm{m}}^{\mathrm{mag}}[(\mathbf{L}^{(q)})^2 - (L_z^{(q)})^2]. \tag{29}$$

Next, we consider the light-cone frame ($E = B$) and write $B^2 = (E^2 + B^2)/2$, and assuming same value for the relaxation times for E and B ($\tau_m^{\mathrm{mag}} \equiv \tau_m$), we obtain

$$\nabla_q \cdot D \cdot \nabla_q = -\frac{g^2 C_2}{4(N_c^2-1)E_p^2}\langle E^2+B^2\rangle\tau_{\rm m}[(\mathbf{L}^{(q)})^2-(L_z^{(q)})^2] \equiv \mathcal{F}_A. \qquad (30)$$

In our case, E_q is replaced by $\omega_{g,q}$ (energy dispersion for quasi-particle) as $\mathbf{v}=\mathbf{q}/\omega$, and for $\bar{f}(\vec{q})$, we employ $f(\vec{q})$ in Eq. (10).

Now, we can replace the Vlasov-Boltzmann equation in Eq. (13) by the ensemble averaged by the diffusive Vlasov-Boltzmann equation [3],

$$v^\mu \frac{\partial}{\partial x^\mu}\bar{f} - \mathcal{F}_A \bar{f} = 0. \qquad (31)$$

Here, $\bar{f}$ denotes the ensemble-averaged thermal distribution function of quasi-partons. In our case, $\bar{f} \equiv f(\vec{q},\vec{r})$ [given in Eq. (9)].

The force term ($\mathcal{F}_A$) in the case of chromo-electromagnetic plasma in the present case is

$$\mathcal{F}_A\bar{f}(q) \equiv \mathcal{F}_A f(\vec{q},\vec{r}) = \frac{g^2 C_2}{3(N_c^2-1)\omega_{g,q}^2}\langle E^2+B^2\rangle\tau_m \times \mathcal{L}^2 f_{\rm eq}(1\pm f_{\rm eq})q_i q_j(\nabla u)_{ij}, \qquad (32)$$

where C_2 is the Casimir invariants ($C_2 \equiv (N_c,(N_c^2-1)/2N_c)$ quadratic Casimirs of $SU(N_c)$). The quantities $\langle E^2\rangle$ and $\langle B^2\rangle$ are the color-averaged chromo-electric and chromo-magnetic fields (average over the ensemble of turbulent color fields [3]), and τ_m is the time scale (relaxation time) for the instability. Note that while obtaining the effective Vlasov-Dupree equation in Eq. (36), the operator $\mathcal{L}^2$ is defined as

$$\mathcal{L}^2 = [\vec{q}\times\partial_{\vec{q}}]^2 - [\vec{q}\times\partial_{\vec{q}}]_z^2. \qquad (33)$$

While obtaining the expression for the above force term, we first considered a purely chromo-magnetic plasma and then wrote the terms in light-cone frame [3,44].

Now, we start with the equilibrium distribution function (local) $f_{\rm eq} = 1/(z_{g,q}^{-1}\exp(\beta u.q)\mp 1)$, where $z_{g/q}$ is purely temperature dependent. The action of the drift operator on $f_{\rm eq}$ is given by

$$(v\cdot\partial)f_{\rm eq} = -f_{\rm eq}(1+f_{\rm eq})\{(q-\partial_\beta \ln(z_{g,q}))v\cdot\partial(\beta) + \beta(v\cdot\partial)(u\cdot q)\}, \qquad (34)$$

where we recognize that $q-\partial_\beta\ln(z_{g/q}) \equiv \omega_{g,q}$ is the modified dispersion relations.

After some straightforward computation [3,44], we obtain

$$(v\cdot\partial)f_{\rm eq}(q) = f_{\rm eq}(1\pm f_{\rm eq})\left[\frac{q_i q_j}{\omega_{g,q}T}(\nabla u)_{ij} - \frac{m_D^2\langle E^2\rangle\tau^{\rm el}\omega_{g,q}}{3T^2\partial\mathcal{E}/\partial T} + \left(\frac{q^2}{3\omega_{g,q}^2}-c_s^2\right)\frac{\omega_{g,q}}{T}(\nabla\cdot\vec{u})\right], \qquad (35)$$

where c_s^2 is the speed of sound, m_D^2 is the Debye mass, $\mathcal{E}$ is the energy density, and $\tau_{\rm el}$ is the time scale of the instability in chromo-electric fields. The physics of shear viscosity is mainly captured by the first term in the right-hand side of Eq. (35), and the other two terms lead to the thermal conductivity and bulk viscosities, respectively.

Finally, we obtain the effective transport equation as

$$\left\{\left(\frac{q^2}{3\omega_{g,q}}-c_s^2\right)\frac{\omega_{g,q}}{T}(\nabla\cdot\vec{u}) + \frac{q_i q_j(\nabla)_{ij}}{\omega_{g,q}T}\right\} f_0^{g,q}(1\pm f_0^{g,q}) = \frac{g^2C_2}{3(N_c^2-1)\omega_{g,q}^2}\langle E^2+B^2\rangle\tau_m\mathcal{L}^2 f_1^{g,q} f_0^{g,q}(1\pm f_0^{g,q}). \qquad (36)$$

The operator $\mathcal{L}^2$ is similar to the quadrapole operator, and the most peculiar thing about it is that it only acts nonvanishingly on the anisotropic piece of any function of momentum ($\vec{q}$). Note that the randomly distributed electric fields are well known to lead to an increase in the average energy of the plasma particles [45] and thus heating of the plasma. In contrast, the color magnetic fields only contribute to the isotropization of the momentum distribution and do not cause plasma heating [plasma heating effect is captured in the second term in the right-hand side of Eq. (35)]. Importantly, the first term in the left-hand side of Eq. (36) contributes to the physics of isotropic expansion (bulk viscosity effects), which is not taken into account in the present work.

Solving Eq. (36) for $\Delta_{g,q}$ analytically, we obtain the following expression [38,46]:

$$\Delta_{g,q} = 2(N_c^2-1)\frac{\omega_{g,q}T}{3C_{g,q}g^2\langle E^2+B^2\rangle_{g,q}\tau_m}. \qquad (37)$$

Next, we relate the unknown quantities in the denominator with the phenomenologically known parameter, the jet-quenching parameter, in both the gluonic and quark sector below.

2. *Relation to the jet-quenching parameter, $\hat{q}$*

The two most relevant transport coefficients related to anomalous transport due to the soft color fields are η and the jet-quenching parameter $\hat{q}$. Here, the strength of the anisotropy, $\Delta(\vec{q})$, is related to the physics of η. The $\hat{q}$ is proportional to the mean momentum square per unit length on the an energetic parton imparted by turbulent fields [47]. This fact has been employed to relate the two below.

In the QGP phase, $\hat{q}$ for both gluons ($\hat{q}_g$) and quarks ($\hat{q}_q$) has been estimated employing several different approaches [48]. The five distinct approaches mentioned in [48] are *viz.*, the GLV-CUJET Model [49], Higher Twist Berkeley Wuhan Model (HT-BW) [50], Higher-Twist-Majumder Model (HT-M) [51], MARTINI Model [52], and MCGILL AMY Model [53]. Combining all these models, one obtains the quark transport parameter $\hat{q}_q$ in the range,

$$\frac{\hat{q}_q}{T^3} = 4.6 \pm 1.2 \quad \text{at RHIC},$$
$$\frac{\hat{q}_q}{T^3} = 3.7 \pm 1.4 \quad \text{at LHC}. \tag{38}$$

The gluon-quenching parameter $\hat{q}_g$ is related to $\hat{q}_g$ by a factor of $\frac{9}{4}$ (in terms of Casimir invariants of the $SU(3)$ group),

$$\hat{q}_g = \frac{9}{4}\hat{q}_q. \tag{39}$$

A relevant point to be noted is that $\hat{q}$ for the QGP scales with T^3. If one considers the highest temperatures reached at central Au-Au at RHIC and Pb-Pb at LHC, $T = 370$ MeV and $T = 470$ MeV, respectively. The corresponding numbers for $\hat{q}_q$ for a 10 GeV quark jet are

$$\hat{q}_q = 1.3 \pm 0.3 \ \text{GeV}^2/\text{fm}; \qquad 1.9 \pm 0.7 \ \text{GeV}^2/\text{fm}, \tag{40}$$

for RHIC and LHC, respectively.

Let us now discuss the temperature variations at RHIC and LHC while obtaining $\hat{q}$ enlisted in Eq. (38). For Au-Au at 200 GeV/n, $T_0 = 346$–373 MeV and for Pb-Pb at 2.76 TeV/N, $T_0 = 447$–486 MeV with initial time $\tau_0 = 0.6$ fm/c for RHIC energy and $\tau_0 = 0.3$ fm/c for the LHC energy. In the present context, the unknown quantities $\langle E^2 + B^2 \rangle \tau_m$, which capture the physics of anisotropy and chromo-Weibel instability [1], can be written in terms of $\hat{q}$ both in gluonic and matter sectors as [54]

$$\hat{q} = \frac{2g^2 C_{g/f}}{2(N_c^2 - 1)} \langle E^2 + B^2 \rangle \tau_m, \tag{41}$$

where $C_g = N_c$, $C_f = \frac{(N_c^2-1)}{2N_c}$ for the gluons and quarks, respectively.

Invoking the definition of $\hat{q}$ from Eq. (41) in Eq. (37), we obtain the following expressions:

$$\Delta_{g,q} = \frac{4\omega_{g,q}^2 T}{9\hat{q}_{g,q}}. \tag{42}$$

Finally, we obtain the following near equilibrium distribution functions in terms of the jet-quenching parameter $\hat{q}$,

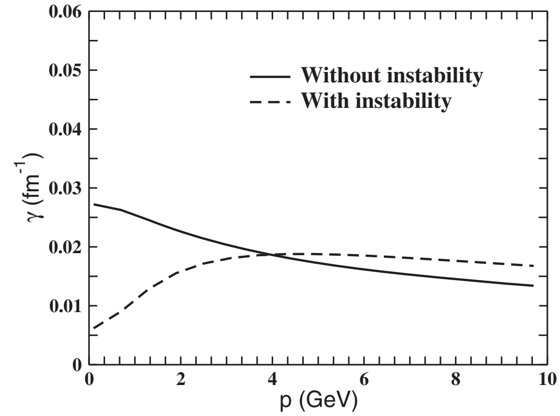

FIG. 1. Variation of the drag coefficient with momentum at RHIC energy.

$$f^{g,q}(\vec{q}) = f_0^{g,q} - f_0^{g,q}(1 \pm f_0^{g,q}) \frac{4\omega_g}{9\hat{q}_{g,q}(\tau T)} \left(q_z^2 - \frac{q^2}{3} \right). \tag{43}$$

IV. RESULTS AND DISCUSSIONS

The momentum variation of the drag and diffusion coefficients of the charm quark is depicted in Figs. 1 and 2 with and without instability at RHIC energy by invoking Eq. (43) in Eqs. (2) and (6), respectively.

The initial temperature (T_i) at RHIC energy is assumed to be equal to $T_i = 360$ MeV, and the $\hat{q}$ corresponding to the temperature 360 MeV is taken as 4.6. The initial thermalization time (τ_t) at RHIC energy is taken as 0.6 fm. The impact of instability is quite significant (mainly at low momentum range), which decreases the drag coefficient at low momentum, hence, allowing the heavy quarks to move freely. It is worth mentioning that the temperature dependence of the drag coefficient plays a significant role [19] to describe heavy quarks R_{AA} and v_2 simultaneously, which is currently a challenge to almost all the models of HQ

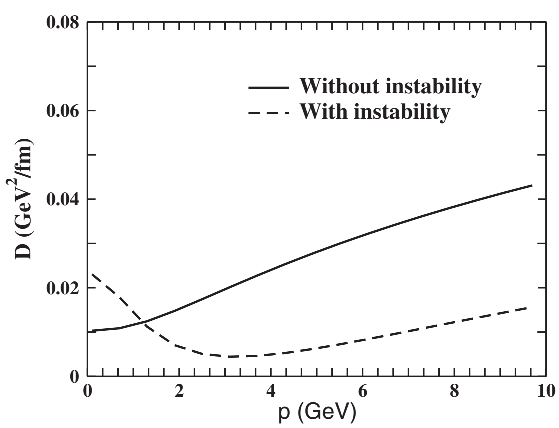

FIG. 2. Variation of the diffusion coefficient with momentum at RHIC energy.

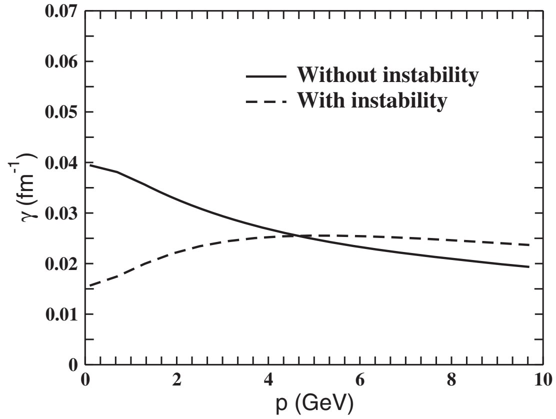

FIG. 3. Variation of the drag coefficient with momentum at LHC energy.

dynamics. A constant or weak temperature dependence of the drag coefficient is essential to reproduce the heavy quarks R_{AA} and v_2 simultaneously. In the presence of instability, the drag coefficient decreases at high temperature (at low momentum), and it does not affect the low temperature part of the drag coefficient. Hence, the presence of instability alters the temperature as well as the momentum dependence of the drag coefficient and may have a significance role on R_{AA} and v_2 relations. We address these aspects in future works. The variation of the corresponding diffusion coefficient with momentum is shown in Fig. 2 at the RHIC energy with and without instability. In the case of the diffusion coefficient, the impact of instability is noticeable throughout the momentum range considered in this work.

The momentum variation of drag and diffusion coefficients of charm quarks with and without instability at the LHC energy are displayed in Figs. 3 and 4, respectively, showing behavior qualitatively similar to that of the RHIC energy. In the case of LHC energy, we use $T = 480$ MeV and $\hat{q} = 3.7$. The initial thermalization time at LHC energy assumed to be $\tau_i = 0.3$ fm. At the qualitative front, HQ drag and diffusion coefficients both at RHIC and LHC show similar trends at lower as well as higher momentums. This may be due to that fact that the temperature dependence of $\hat{q}$ at RHIC and LHC is not very different.

It is worth mentioning that the allowed region of perturbative treatment in Eq. (10) depends on both the light quark/gluon momentum (q) and the value of $\hat{q}/T^3$. In terms of anisotropy ($\hat{q}/T^3$), we are already in the upper limit, just to highlight what could be the maximum effect. To be well within the allowed region of perturbative treatment and for a more realistic simulation, larger values of $\hat{q}/T^3$ is more appropriate. Keeping this view in mind, we choose $\hat{q}/T^3$ in the range of 10–20 for the analysis in the next section.

A. Impact of strength of the anisotropy

To explore the impact of the instability/anisotropy on the heavy quark dynamics, we vary the parameter $\hat{q}/T^3$ from 10–20 as shown in Fig. 5. As we increase the value of $\hat{q}/T^3$, conversely decreasing the strength of the anisotropy, the heavy quark drag coefficient, γ, at low p (less than 4 GeV) increases, in contrast with its behavior at high p (larger than 4 GeV). The impact is more pronounced at low momentum. The larger the strength of anisotropy is, the smaller the γ is, meaning that the anisotropy is creating relatively lesser hindrance for the HQs to travel in the QGP medium at low momentum, in contrast to the role played by the anisotropy at high p.

We vary the parameter $\hat{q}/T^3$ from 10–20 as shown in Fig. 6 for the diffusion coefficient. As we increase the value of $\hat{q}/T^3$, the heavy quark diffusion coefficient decreases (in low momentum) in contrast to the drag coefficient. The quantity Q in the figure legends (Figs. 5–8) is defined as $Q \equiv \hat{q}/T^3$.

Note that while deriving the anisotropic momentum distribution functions for the quasi-quarks and quasi-gluons, we ignored the collision term. This is based on

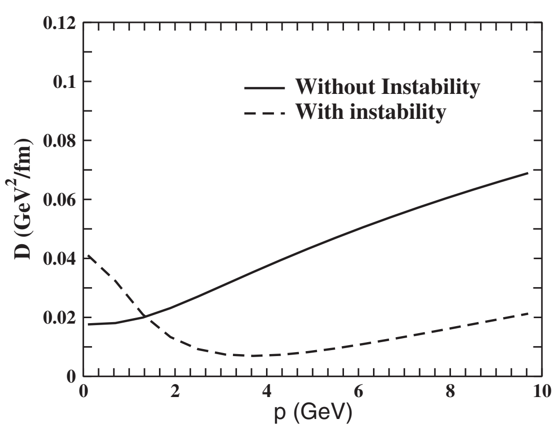

FIG. 4. Variation of the diffusion coefficient with momentum at LHC energy.

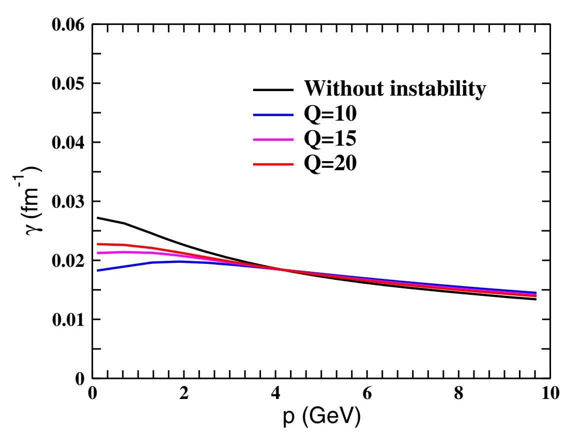

FIG. 5. Dependence on the strength of the anisotropy/instability of the drag coefficient at RHIC.

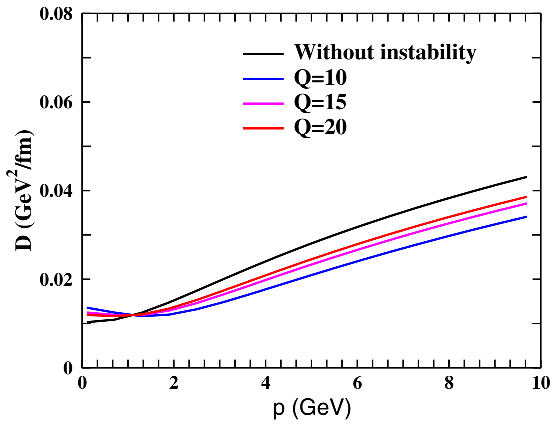

FIG. 6. Dependence on the strength of the anisotropy/instability of the diffusion coefficient at RHIC.

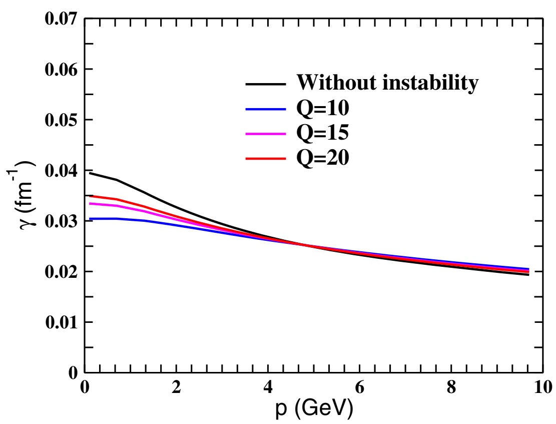

FIG. 7. Dependence on the strength of the anisotropy/instability of the drag coefficient at LHC.

the assumption that the anomalous process (the effect of turbulent chromo-fields) causes substantial suppression of the transport coefficients such as shear viscosity. The total viscosity is mainly decided by the anomalous process only

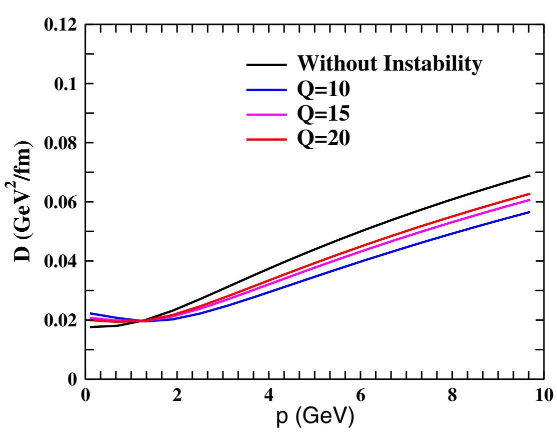

FIG. 8. Dependence on the strength of the anisotropy/instability of the diffusion coefficient at LHC.

as the transport rates are inverse additive not the viscosities. In contrast, for the HQs, the dynamics is primarily controlled by the collisional processes. It will be interesting to see the interplay of collisional and turbulent fields while exploring the dynamics of HQs along with contributions from the radiative processes in the near future.

V. CONCLUSIONS AND OUTLOOK

We have estimated the drag and diffusion coefficients of heavy quarks propagating through a QGP medium considering the role of momentum state anisotropy. The initial momentum anisotropy in the early stages coupled with the rapidly expanding QGP is modeled by setting up an effective transport equation, and its solution in near equilibrium approximation leads to the modeling of (non)near equilibrium distribution functions for quark-antiquarks and gluons. We have coupled these distribution functions to the kinetic theory description of heavy quark drag and diffusion coefficients and studied their temperature and momentum dependence.

We found that both at RHIC and LHC energies impact of the anisotropy on heavy quark transport is quite significant as compared to the case when HQs are moving in an isotropic QGP medium. The presence of anisotropy alters both the temperature as well as momentum dependences of the heavy quarks drag and diffusion coefficients. Moreover, the presence of anisotropy reduces the drag coefficient at the initial stage (at high temperature) of the QGP, whereas the impact of anisotropy will be very nominal at the later stages of the QGP (low temperature). Hence, the presence of anisotropy will make the temperature dependence of the drag coefficients smoother, which may help to develop larger v_2 than in the isotropic case for the same R_{AA} [19].

It is crucial to note that the nuclear modification factor, R_{AA}, is very sensitive to the early stages of the expansion (at high temperatures) where the energy density is the highest [55–58]. Therefore, collisions take place at a high rate in the early stages of the evolution. This translates into a strong initial suppression of R_{AA}, which subsequently gets saturated within 3–4 fm due to the radial flow that is able to compensate the energy loss. However, such a strong interaction in the early stages will not be accompanied by a buildup of v_2 because the bulk medium has not yet developed a sizable part of its elliptic flow. We intend to investigate this issue further in near future.

In the present case, we have only studied the impact of anisotropy on heavy quark collisional loss. Anisotropy may affect the heavy quark radiative loss. It will be interesting to study the impact of anisotropy on heavy quark radiation. The impact of these results on R_{AA} and v_2 will be a matter of future investigation. We also intend to explore the impact of bulk viscosity along the similar lines of the analysis.

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