Prompt charmonia production and polarization at LHC in the NRQCD with k_T-factorization. II. χ_c mesons

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(Received 13 March 2016; published 17 May 2016)

In the framework of the k_T-factorization approach, the production of prompt $\psi(2S)$ mesons in pp collisions at the LHC energies is studied. Our consideration is based on the off-shell amplitudes for hard partonic subprocesses $g^*g^* \to \chi_{cJ}$ and nonrelativistic QCD formalism for bound states. The transverse-momentum-dependent (unintegrated) gluon densities in a proton were derived from the Ciafaloni-Catani-Fiorani-Marchesini evolution equation or, alternatively, were chosen in accordance with the Kimber-Martin-Ryskin prescription. Taking into account both color-singlet and color-octet contributions, we deduce the corresponding nonperturbative long-distance matrix elements from the fits to the latest ATLAS data on χ_{c1} and χ_{c2} transverse-momentum distributions at $\sqrt{s} = 7$ TeV. We find that these distributions at small and moderate p_T are formed mainly by the color-singlet components. We successfully described the data on the relative production rates $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ presented by the ATLAS, CMS, and LHCb Collaborations. We find that the fit points to unequal wave functions of χ_{c1} and χ_{c2} states.

DOI: 10.1103/PhysRevD.93.094012

I. INTRODUCTION

Since it was first observed, charmonium production in hadronic collisions remains a subject of considerable theoretical and experimental interest. It provides a sensitive tool probing quantum chromodynamics (QCD) in both perturbative and nonperturbative regimes, as the production mechanism involves both short- and long-distance interactions. Two theoretical approaches for the nonperturbative part are known in the literature: the color-singlet (CS) model [1] and the color-octet (CO) model [2]. As we have explained in our previous paper [3], none of the existing theoretical approaches is able to describe all of the data in their integrity. Our present study is a continuation of the work [3], where the prompt $\psi(2S)$ production and polarization at the LHC has been considered. The motivation for the whole business has already been given there. Here we turn to the production of P-wave states. It is known that the feed-down contributions from χ_c and $\psi(2S)$ states due to their radiative decays $\chi_c \to J/\psi + \gamma$ and $\psi(2S) \to J/\psi + \gamma$ give a significant impact on the J/ψ polarization [4–8]. These mechanisms constitute about 30% of the visible J/ψ cross section at the LHC [6–8]. Therefore, a clear understanding of χ_c and $\psi(2S)$ production is a crucial component of any general description of J/ψ production. Another important issue concerns the relative production rate $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ at high transverse momenta. This ratio is sensitive to the CS and CO mechanisms and can provide information complementary to the study of the S-wave states [9,10].

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Below, we present a systematic analysis of the ATLAS [11], CMS [12], and LHCb [13] data collected at $\sqrt{s} = 7$ TeV for χ_{c1} and χ_{c2} the transverse-momentum distributions and for the ratio of the production rates $\sigma(\chi_{c2})/\sigma(\chi_{c1})$.

II. THEORETICAL FRAMEWORK

Our consideration is based on the off-shell gluon-gluon fusion subprocess that represents the true leading order in QCD:

$$g^*(k_1) + g^*(k_2) \to c\bar{c} \to \chi_{cJ}(p), \qquad (1)$$

where the four-momenta of all particles are indicated in parentheses. In general, the charmed quark pair is produced in a state ${}^{2S+1}L_J^{(a)}$ with spin S, orbital angular momentum L, total angular momentum J, and color a, which can be either identical to the final charmonium quantum numbers, as is accepted in the CS model, or different from those. In the latter case, the $c\bar{c}$ pair transforms into a physical charmonium state by means of soft (nonperturbative) gluon radiation, as is considered in the formalism of nonrelativistic QCD (NRQCD) [14,15]. The probability to form a given bound state is determined by the respective nonperturbative long-distance matrix elements (NMEs), which are assumed to be universal (process independent), not depending on the charmonium momentum and obeying a certain hierarchy in powers of the relative charmed quark velocity v.

The production of heavy $c\bar{c}$ pairs in a hard partonic subprocess is regarded as a purely perturbative stage and is considered in the framework of the k_T-factorization approach [16,17], where studying quarkonium production and polarization has a long history (see, for example, Refs. [18–28], and references therein). A detailed description and discussion of the different aspects of k_T factorization can be found in reviews [29]. Here we see certain advantages in the fact that, even with the leading-order (LO) matrix elements for a hard partonic subprocess, we can include a large piece of higher-order QCD corrections (all NLO + NNLO + … terms containing $\log 1/x$ enhancement) taking them into account in the form of transverse-momentum-dependent (TMD) gluon densities. The latter are obtained as numerical solutions to Balitsky-Fadin-Kuraev-Lipatov [30] or Catani-Ciafaloni-Fiorani-Marchesini (CCFM) [31] evolution equations and will be described below in more detail.

Summing over the initial gluon polarizations in (1) is done by using the spin density matrix $\overline{\epsilon^\mu \epsilon^\nu} = \mathbf{k}_T^\mu \mathbf{k}_T^\nu / |\mathbf{k}_T|^2$, where $\mathbf{k}_T$ is the component of the gluon momentum perpendicular to the proton beam direction [16,17]. In the collinear limit, when $|\mathbf{k}_T| \to 0$, this expression converges to the ordinary $\overline{\epsilon^\mu \epsilon^\nu} = -g^{\mu\nu}/2$, while, for nonzero $|\mathbf{k}_T|$, gluon polarization vectors acquire an admixture of longitudinal components. The evaluation of partonic amplitudes is straightforward and follows standard QCD Feynman rules in all other respects. Our results for perturbative production amplitudes squared and summed over polarization states agree with the ones in Ref. [32].

The formation of final state quarkonium from a $c\bar{c}$ pair (with any quantum numbers) is an essentially nonperturbative step. At the LO in the relative quark velocity v, the P-wave mesons χ_{cJ} with $J = 0$, 1, or 2 can be formed by a $c\bar{c}$ pair originally produced as color singlet ${}^3P_J^{(1)}$ or can evolve from an intermediate color-octet ${}^3S_1^{(8)}$ state. The corresponding amplitudes can be obtained from an unspecified $c\bar{c}$ case by applying the relevant projection operators [1]:

$$\Pi[{}^3S_1] = \hat{\epsilon}(S_z)(\hat{p}_c + m_c)/m^{1/2}, \tag{2}$$

$$\Pi[{}^3P_J] = (\hat{p}_{\bar{c}} - m_c)\hat{\epsilon}(S_z)(\hat{p}_c + m_c)/m^{3/2}, \tag{3}$$

where $m = 2m_c$ is the mass of the considered $c\bar{c}$ state, p_c and $p_{\bar{c}}$ are the four-momenta of the charmed quark and antiquark, respectively, $p_c = p/2 + q$, $p_{\bar{c}} = p/2 - q$, and q is the relative four-momentum of the quarks in the bound state. States with various projections of the spin momentum onto the z axis are represented by the polarization four-vector $\epsilon_\mu(S_z)$. The probability for the charmed quarks to form a meson depends on the real (for color singlets) or fictitious (for color octets) bound state wave functions $\Psi^{(a)}(q)$. The corresponding NMEs are related to the wave functions in the coordinate space $\mathcal{R}^{(a)}(x)$, which are the Fourier transforms of $\Psi^{(a)}(q)$, and their derivatives [2,14,15]:

$$\langle \mathcal{O}[{}^{2S+1}L_J^{(a)}] \rangle = 2N_c(2J+1)|\mathcal{R}^{(a)}(0)|^2/4\pi \tag{4}$$

for S waves and

$$\langle \mathcal{O}[{}^{2S+1}L_J^{(a)}] \rangle = 6N_c(2J+1)|\mathcal{R}'^{(a)}|^2/4\pi \tag{5}$$

for P waves. For more details, the reader can address the original papers [1,2,14,15] or our previous note [3]. The CS NMEs can be extracted from the measured $\chi_{c_2} \to \gamma\gamma$ decay width or obtained from the potential models [33–36]. In contrast with many other papers where the identity $\mathcal{R}'^{(a)}_{\chi_1}(0) = \mathcal{R}'^{(a)}_{\chi_2}(0)$ is assumed, we consider them as independent free parameters (as well as we do for the CO NMEs) and determine them from fits to the LHC data. Although the above identity directly follows from nonrelativistic potential models, we cannot exclude that it can be significantly modified by radiative corrections. We know, for example, that radiative corrections make as large as a factor of 2 effect on the wave function of the J/ψ meson, and we see no reason to take for granted that radiative corrections for χ_c mesons would make no difference between the states with different quantum numbers, $J = 1$, 2. This issue has been discussed in more detail in Ref. [27].

Finally, the χ_c meson production cross section is calculated as a convolution of the off-shell partonic cross sections and the TMD gluon densities in a proton:

$$\begin{aligned} \sigma(pp \to \chi_{cJ} + X) &= \int \frac{2\pi}{x_1 x_2 s F} f_g(x_1, \mathbf{k}_{1T}^2, \mu^2) f_g(x_2, \mathbf{k}_{2T}^2, \mu^2) \\ &\quad \times |\bar{\mathcal{A}}(g^* + g^* \to \chi_{cJ})|^2 d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 dy \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi}, \end{aligned} \tag{6}$$

where $f_g(x, \mathbf{k}_T^2, \mu^2)$ is the TMD gluon density, y the rapidity of the produced χ_c meson, and $\sqrt{s}$ the pp center-of-mass energy. The initial off-shell gluons carry longitudinal momentum fractions x_1 and x_2 (with respect to the parent protons) and nonzero transverse momenta $\mathbf{k}_{1T}$ and $\mathbf{k}_{2T}$ oriented at the azimuthal angles ϕ_1 and ϕ_2. The off-shell flux factor F is taken[1] in accordance with the general definition [37] as $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$, where $\hat{s} = (k_1 + k_2)^2$. This choice may seem unusual for the conventional parton model, where the initial gluons have no virtuality. In fact, the correct definition of the flux of virtual particles is questionable. Our choice is based on a comparison between an exact calculation of the QED process $e + e \to \chi_{cJ} + e + e$ (taken as a toy) and its equivalent photon approximation. The best agreement was observed with the photon flux definition $F = 2\lambda^{1/2}(\hat{s}, k_1^2, k_2^2)$ [24].

[1]The effect of the different forms of the flux factor on numerical predictions has been studied in Ref. [24].

In our numerical analysis, we tried several sets of TMD gluon densities. Two of them (A0 [38] and JH [39]) have been obtained from the CCFM equation where all input parameters have been fitted to the proton structure function $F_2(x, Q^2)$. Besides that, we used a parametrization obtained with the Kimber-Martin-Ryskin (KMR) prescription [40] which provides a method to construct TMD quark and gluon densities out of conventional (collinear) distributions. In that case, we used for the input the leading-order Martin-Stirling-Thorn-Watt set [41].

The renormalization and factorization scales μ_R and μ_F were set to $\mu_R^2 = m^2 + \mathbf{p}_T^2$ and $\mu_F^2 = \hat{s} + \mathbf{Q}_T^2$, where $\mathbf{Q}_T$ is the transverse momentum of the initial off-shell gluon pair. The choice of μ_R is rather standard for charmonium production, whereas the special choice of μ_F is connected with the CCFM evolution [38,39]. Following Ref. [42], we set the meson masses to $m_{\chi_{c1}} = 3.51$ GeV and $m_{\chi_{c2}} = 3.56$ GeV. We use the LO formula for the running coupling constant $\alpha_s(\mu_R^2)$ with $n_f = 4$ quark flavors and $\Lambda_{\mathrm{QCD}} = 200$ MeV, so that $\alpha_s(M_Z^2) = 0.1232$. The multidimensional integration has always been performed by means of the Monte Carlo technique using the routine VEGAS [43]. The full C++ code is available from the authors on request.

The production of χ_c mesons is followed by their radiative decays. Here we rely on the dominance of electric dipole transitions.[2] The hypothesis of $E1$ dominance is supported by the data taken by the E835 Collaboration at the Tevatron [44]. The corresponding decay amplitudes read [45]

$$\mathcal{A}(\chi_{c1} \to J/\psi + \gamma) \sim \epsilon^{\mu\nu\alpha\beta} k_\mu \epsilon_\nu^{(\chi_{c1})} \epsilon_\alpha^{(J/\psi)} \epsilon_\beta^{(\gamma)}, \tag{7}$$

$$\mathcal{A}(\chi_{c2} \to J/\psi + \gamma) \sim p^\mu \epsilon^{\alpha\beta}_{(\chi_{c2})} \epsilon_\alpha^{(J/\psi)} [k_\mu \epsilon_\beta^{(\gamma)} - k_\beta \epsilon_\mu^{(\gamma)}], \tag{8}$$

where $\epsilon^{\mu\nu\alpha\beta}$ is the fully antisymmetric Levi-Civita tensor, k is the final state photon four-momentum, $\epsilon_\mu^{(\chi_{c1})}$, $\epsilon_\mu^{(J/\psi)}$, and $\epsilon_\mu^{(\gamma)}$ are the polarization vectors of the respective spin-one particles, and $\epsilon_{\mu\nu}^{(\chi_{c2})}$ is the polarization tensor of the spin-two χ_{c2} meson. The absolute decay rates were normalized to the known branchings $B(\chi_{c1} \to J/\psi + \gamma) = 0.344$ and $B(\chi_{c2} \to J/\psi + \gamma) = 0.195$.

III. NUMERICAL RESULTS

The whole set of NMEs was determined from fitting the transverse-momentum distributions of χ_{c1} and χ_{c2} mesons measured by the ATLAS Collaboration at $\sqrt{s} = 7$ TeV [11]. The measurements were done at moderate and high transverse momenta $12 < p_T < 30$ GeV within the decay J/ψ rapidity region $|y^{J/\psi}| < 0.75$, where the NRQCD formalism is believed to be reliable. The combined fit of χ_{c1} and χ_{c2} data was performed under the requirement that all NMEs be strictly positive. Following the suggestion of Ref. [27], the χ_{c1} and χ_{c2} CS wave functions were treated as independent (not necessarily identical) parameters.[3] The results of our fit are displayed in Table I for three different gluon distributions together with two sets of NMEs taken from the literature. The calculated differential cross sections are presented in Figs. 1 and 2 as functions of the χ_{cJ} and J/ψ transverse momenta, respectively. The ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ is shown in Fig. 3 in comparison with the recent LHC data [11–13]. Using the fitted values of NMEs from Table I, we achieve a good simultaneous description of the measured χ_{c1} and χ_{c2} spectra and the ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ with each of the considered TMD gluon densities. We find that the χ_c production is dominated by the CS contributions, that agrees with earlier conclusions [20,21]. However, a small CO admixture improves agreement with the LHC data at high transverse momenta (see Fig. 1).

TABLE I. The NMEs for χ_c mesons and color-singlet wave functions $|\mathcal{R}'^{(1)}_{\chi_{c1}}(0)|^2$ and $|\mathcal{R}'^{(1)}_{\chi_{c2}}(0)|^2$ extracted from the fit of the ATLAS data [11]. The results obtained from the NLO NRQCD fits [9,10] are shown for comparison.

	$\lvert\mathcal{R}'^{(1)}_{\chi_{c1}}(0)\rvert^2/\mathrm{GeV}^5$	$\lvert\mathcal{R}'^{(1)}_{\chi_{c2}}(0)\rvert^2/\mathrm{GeV}^5$	$\langle\mathcal{O}^{\chi_{c0}}[^3S_1^{(8)}]\rangle/\mathrm{GeV}^3$
A0	3.85×10^{-1}	6.18×10^{-2}	8.28×10^{-5}
JH	5.23×10^{-1}	9.05×10^{-2}	4.78×10^{-5}
KMR	3.07×10^{-1}	6.16×10^{-2}	1.40×10^{-4}
[9]	7.50×10^{-2}	7.50×10^{-2}	2.01×10^{-3}
[10]	3.50×10^{-1}	3.50×10^{-1}	4.40×10^{-4}

The value of the CS wave function determined previously [10] from a combined fit to the Tevatron and LHC data is $|\mathcal{R}'^{(1)}(0)|^2 = 3.5 \times 10^{-1}$ GeV^5. It differs significantly from $|\mathcal{R}'^{(1)}(0)|^2 = 7.5 \times 10^{-2}$ GeV^5 obtained from the potential models [33–36]. The latter value is similar to the one extracted from the $\chi_{c2} \to \gamma\gamma$ decay width [42]. We note that the authors of Refs. [9,10] assume equal values of the wave functions for χ_{c1} and χ_{c2} mesons. On the other hand, our fitting procedure leads to unequal values of the χ_{c1} and χ_{c2} CS wave functions. This qualitatively agrees with the suggestions [27] that the ratio of the wave functions has to be modified as $|\mathcal{R}'^{(1)}_{\chi_{c1}}(0)|^2/|\mathcal{R}'^{(1)}_{\chi_{c2}}(0)|^2 \sim 5:3$. However, we find that the LHC data tend to support an even larger ratio, namely, $|\mathcal{R}'^{(1)}_{\chi_{c1}}(0)|^2/|\mathcal{R}'^{(1)}_{\chi_{c2}}(0)|^2 \sim 5:1$. Our fitted value of $|\mathcal{R}'^{(1)}_{\chi_{c2}}(0)|^2$ [but not $|\mathcal{R}'^{(1)}_{\chi_{c1}}(0)|^2$] is close to the estimations based on the potential models [33–36] and two-photon decay width [42].

[2]The same hypothesis has been used to study the production and polarization of Υ mesons at the Tevatron [25].

[3]The reasoning refers to the facts that treating quarks as spinless particles in the potential models [33–36] might be an oversimplification and that radiative corrections may be large.

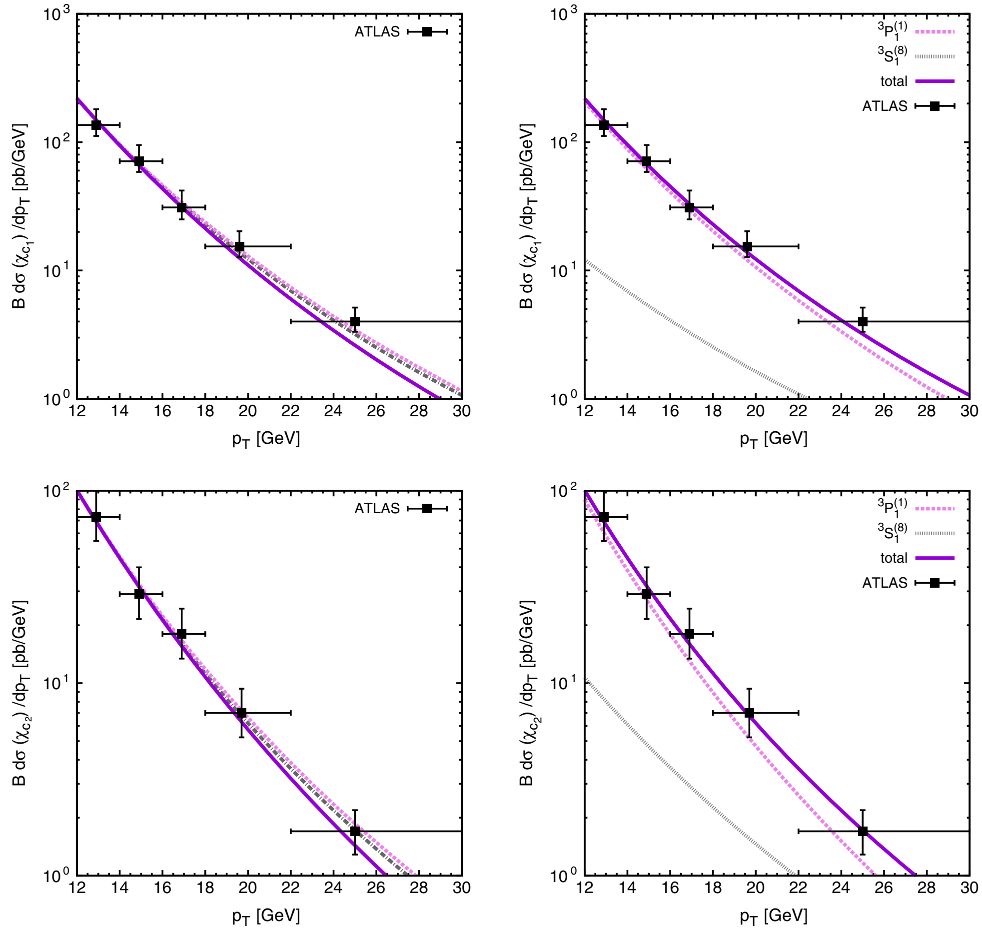

FIG. 1. The prompt χ_c production at the LHC calculated as a function of χ_c meson transverse momenta at $\sqrt{s} = 7$ TeV. Left panel: The dashed and dotted curves correspond to the color-singlet $^3P_J^{(1)}$ and color-octet $^3S_1^{(8)}$ contributions, respectively, calculated with the KMR gluon density. The solid curve represent the sum of CS and CO terms. Right panel: The solid, dashed, and dash-dotted curves correspond to the predictions obtained with the A0, JH, and KMR gluon densities, respectively. The experimental data are from ATLAS [39].

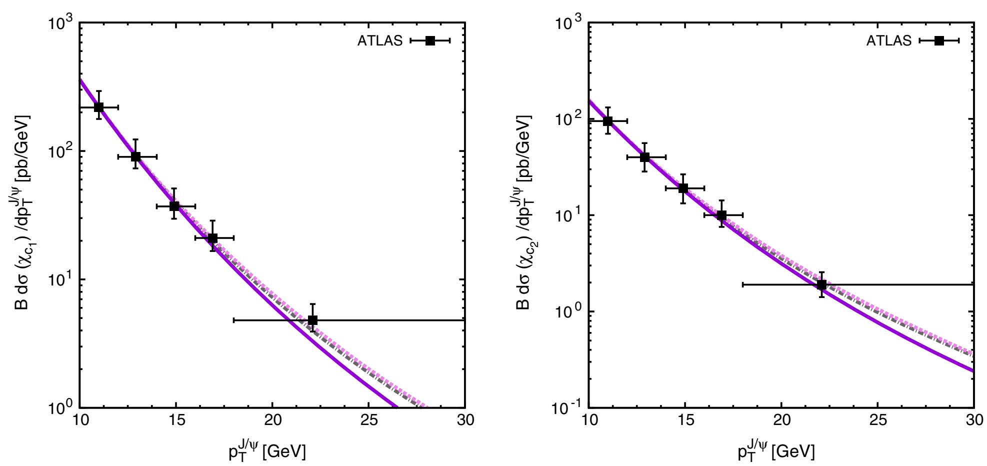

FIG. 2. The prompt χ_c production at the LHC calculated as a function of decay J/ψ transverse momenta at $\sqrt{s} = 7$ TeV. The solid, dashed, and dash-dotted curves correspond to the predictions obtained with the A0, JH, and KMR gluon densities, respectively. The experimental data are from ATLAS [39].

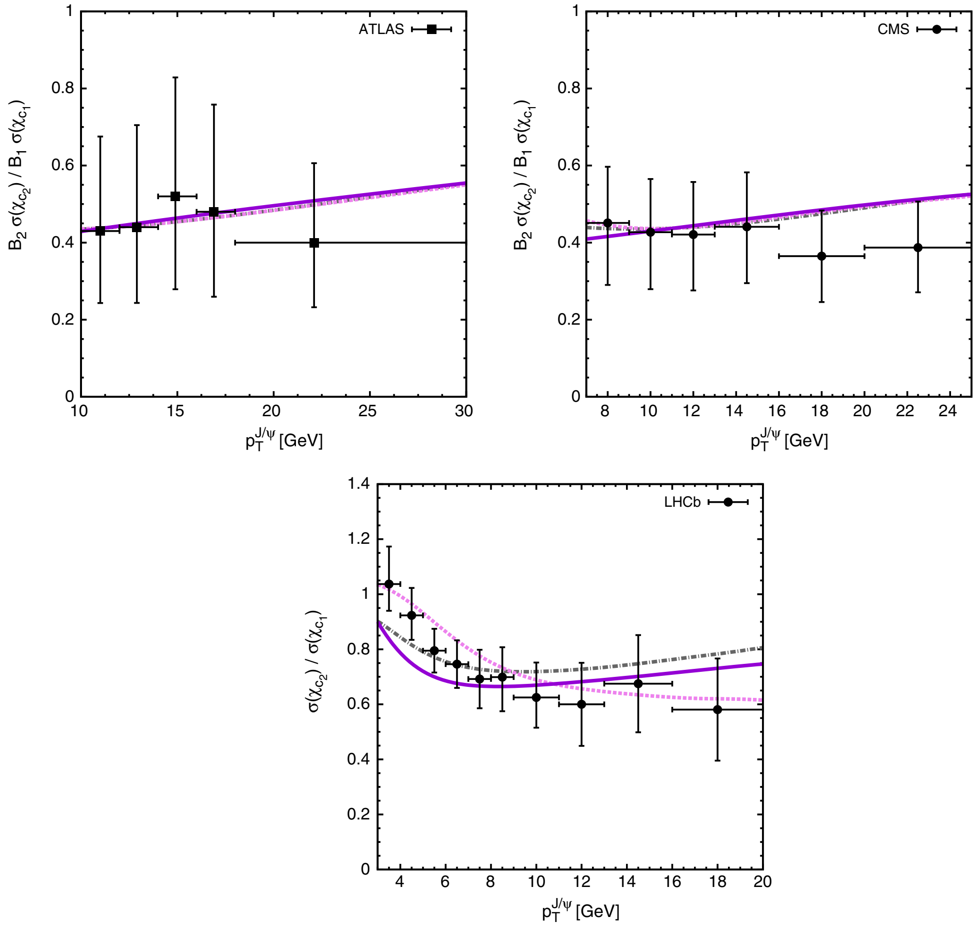

FIG. 3. The relative production rate $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ calculated as a function of J/ψ meson transverse momenta at $\sqrt{s} = 7$ TeV. The notation of all curves is the same as in Fig. 2. The experimental data are from ATLAS [39], CMS [40], and LHCb [41].

IV. CONCLUSIONS

We have considered prompt χ_c production in pp collisions at the energy $\sqrt{s} = 7$ TeV in the framework of the k_T-factorization approach incorporated with the nonrelativistic QCD formalism. Using the TMD gluon densities in a proton either derived from the CCFM equation or constructed with the Kimber-Martin-Ryskin method, we extracted the corresponding nonperturbative color-singlet and color-octet matrix elements from a combined fit to transverse-momentum distributions of χ_{c1} and χ_{c2} mesons provided by the latest ATLAS measurements. Using the fitted NMEs, we successfully described the data on the relative production rates $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ presented by the ATLAS, CMS, and LHCb Collaborations. We find that the χ_c production is dominated by the CS contributions. However, an admixture of CO contributions improves the description of the data at high transverse momenta. Our interpretation of the LHC data supports the idea of unequal values of the χ_{c1} and χ_{c2} CS wave functions.

ACKNOWLEDGMENTS

The authors are grateful to H. Jung for very useful discussions and important remarks. This research was supported by the FASI of Russian Federation (Grant No. NS-3042.2014.2). We are also grateful to DESY Directorate for the support in the framework of the Moscow-DESY project on Monte Carlo implementation for HERA-LHC.

[1] C.-H. Chang, Nucl. Phys. **B172**, 425 (1980); E. L. Berger and D. L. Jones, Phys. Rev. D **23**, 1521 (1981); R. Baier and R. Rückl, Phys. Lett. **102B**, 364 (1981); S. S. Gershtein, A. K. Likhoded, and S. R. Slabospitsky, Sov. J. Nucl. Phys. **34**, 128 (1981).
[2] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995).
[3] S. P. Baranov, A. V. Lipatov, and N. P. Zotov, Eur. Phys. J. C **75**, 455 (2015).
[4] CDF Collaboration, Phys. Rev. Lett. **99**, 132001 (2007).
[5] CDF Collaboration, Phys. Rev. Lett. **108**, 151802 (2012).
[6] CMS Collaboration, Phys. Lett. B **727**, 381 (2013).
[7] LHCb Collaboration, Eur. Phys. J. C **74**, 2872 (2014).
[8] CMS Collaboration, Phys. Rev. Lett. **110**, 081802 (2013).
[9] H.-F. Zhang, L. Yu, S.-X. Zhang, and L. Jia, Phys. Rev. D **93**, 054033 (2016).
[10] A. K. Likhoded, A. V. Luchinsky, and S. V. Poslavsky, Phys. Rev. D **90**, 074021 (2014).
[11] ATLAS Collaboration, J. High Energy Phys. 07 (2014) 154.
[12] CMS Collaboration, Eur. Phys. J. C **72**, 2251 (2012).
[13] LHCb Collaboration, J. High Energy Phys. 10 (2013) 115.
[14] G. Bodwin, E. Braaten, and G. Lepage, Phys. Rev. D **51**, 1125 (1995).
[15] P. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996); **53**, 6203 (1996).
[16] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983); E. M. Levin, M. G. Ryskin, Yu. M. Shabelsky, and A. G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991).
[17] S. Catani, M. Ciafaloni, and F. Hautmann, Nucl. Phys. **B366**, 135 (1991); J. C. Collins and R. K. Ellis, Nucl. Phys. **B360**, 3 (1991).
[18] S. P. Baranov, Phys. Lett. B **428**, 377 (1998).
[19] F. Yan and K. T. Chao, Phys. Rev. Lett. **87**, 022002 (2001).
[20] Ph. Hägler, R. Kirschner, A. Shäfer, L. Szymanowski, and O. V. Teryaev, Phys. Rev. D **63**, 077501 (2001).
[21] S. P. Baranov, Phys. Rev. D **66**, 114003 (2002).
[22] A. V. Lipatov and N. P. Zotov, Eur. Phys. J. C **27**, 87 (2003).
[23] S. P. Baranov and N. P. Zotov, J. Phys. G **29**, 1395 (2003).
[24] S. P. Baranov and A. Szczurek, Phys. Rev. D **77**, 054016 (2008).
[25] S. P. Baranov and N. P. Zotov, JETP Lett. **88**, 711 (2008).
[26] S. P. Baranov, A. V. Lipatov, and N. P. Zotov, Eur. Phys. J. C **71**, 1631 (2011).
[27] S. P. Baranov, Phys. Rev. D **83**, 034035 (2011).
[28] S. P. Baranov, A. V. Lipatov, and N. P. Zotov, Phys. Rev. D **85**, 014034 (2012).
[29] B. Andersson *et al.* (Small-x Collaboration), Eur. Phys. J. C **25**, 77 (2002); J. Andersen *et al.* (Small-x Collaboration), Eur. Phys. J. C **35**, 67 (2004); **48**, 53 (2006).
[30] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **44**, 443 (1976); **45**, 199 (1977); I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
[31] M. Ciafaloni, Nucl. Phys. **B296**, 49 (1988); S. Catani, F. Fiorani, and G. Marchesini, Phys. Lett. B **234**, 339 (1990); Nucl. Phys. **B336**, 18 (1990); **B445**, 49 (1995).
[32] B. A. Kniehl, D. V. Vasin, and V. A. Saleev, Phys. Rev. D **73**, 074022 (2006); V. A. Saleev, M. A. Nefedov, and A. V. Shipilova, Phys. Rev. D **85**, 074013 (2012).
[33] C. R. Munz, Nucl. Phys. **A609**, 364 (1996).
[34] D. Ebert, R. N. Faustov, and V. O. Galkin, Mod. Phys. Lett. A **18**, 601 (2003).
[35] G.-L. Wang, Phys. Lett. B **674**, 172 (2009).
[36] B.-Q. Li and K.-T. Chao, Commun. Theor. Phys. **52**, 653 (2009).
[37] E. Bycling and K. Kajantie, *Particle Kinematics* (Wiley, New York, 1973).
[38] H. Jung, arXiv:hep-ph/0411287.
[39] F. Hautmann and H. Jung, Nucl. Phys. **B883**, 1 (2014).
[40] M. A. Kimber, A. D. Martin, and M. G. Ryskin, Phys. Rev. D **63**, 114027 (2001); G. Watt, A. D. Martin, and M. G. Ryskin, Eur. Phys. J. C **31**, 73 (2003).
[41] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).
[42] PDG Collaboration, Chin. Phys. C **38**, 090001 (2014).
[43] G. P. Lepage, J. Comput. Phys. **27**, 192 (1978).
[44] E835 Collaboration, Phys. Rev. D **65**, 052002 (2002).
[45] P. Cho, M. Wise, and S. Trivedi, Phys. Rev. D **51**, R2039 (1995).