

Charmed partner of the exotic $X(5568)$ state and its propertiesS. S. Agaev,^{1,2} K. Azizi,³ and H. Sundu¹¹*Department of Physics, Kocaeli University, 41380 Izmit, Turkey*²*Institute for Physical Problems, Baku State University, Az-1148 Baku, Azerbaijan*³*Department of Physics, Doğuş University, Acibadem-Kadiköy, 34722 Istanbul, Turkey*

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The mass, decay constant, and width of a hypothetical charmed partner X_c of the newly observed exotic $X_b(5568)$ state are calculated using the technique of the QCD sum rule method. The $X_c = [su][\bar{c}\bar{d}]$ state with $J^P = 0^+$ is described, employing two types of the diquark-antidiquark interpolating currents. The evaluation of the mass m_{X_c} and decay constant f_{X_c} is carried out utilizing the two-point sum rule method by including vacuum condensates up to eight dimensions. The widths of the decay channels $X_c \rightarrow D_s^- \pi^+$ and $X_c \rightarrow D^0 K^0$ are also found. To this end, the strong couplings $g_{X_c D_s \pi}$ and $g_{X_c D K}$ are computed by means of QCD sum rules on the light-cone and soft-meson approximation.

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I. INTRODUCTION

The D0 Collaboration recently reported the observation of a narrow structure $X_b(5568)$ in the decay process $X_b(5568) \rightarrow B_s^0 \pi^\pm$, $B_s^0 \rightarrow J/\psi \phi$, $J/\psi \rightarrow \mu^+ \mu^-$, $\phi \rightarrow K^+ K^-$ based on $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV collected at the Fermilab Tevatron collider [1]. The new state $X_b(5568)$ is considered to possess quantum numbers $J^{PC} = 0^{++}$. The D0 Collaboration provides the value $m_{X_b} = 5567.8 \pm 2.9(\text{stat})_{-1.9}^{+0.9}(\text{syst})$ MeV for its mass and estimates $\Gamma = 21.9 \pm 6.4(\text{stat})_{-2.5}^{+5.0}(\text{syst})$ MeV for its decay width. As emphasized in Ref. [1], this is the first observation of a hadronic state with four different quark flavors. Thus, the X_b state is composed of b , s , u , and d quarks.

Suggestions concerning the possible quark-antiquark organization of X_b were made already in Ref. [1]. Thus, within the diquark-antidiquark model, the X_b state with positive charge, i.e., the particle X_b^+ , may be described as $[bu][\bar{d}\bar{s}]$ or $[su][\bar{b}\bar{d}]$ bound states, whereas X_b^- may have the structures $[bd][\bar{s}\bar{u}]$ or $[sd][\bar{b}\bar{u}]$. Alternatively, X_b may be considered as a molecule composed of B and \bar{K} mesons.

This is a valuable discovery, because the charmonium-like resonances that populate the XYZ family of “traditional” exotic states contain a $c\bar{c}$ charm quark-antiquark pair and hence, the number of quark flavors in these particles does not exceed three. Properties of known exotic states extracted from experimental data and theoretical calculations can be found in, for instance, review papers [2–9] and references therein.

The newly observed state $X_b(5568)$ has immediately attracted the interest of physicists and stimulated theoretical studies of X_b in the context of different approaches [10–18]. Thus, in Refs. [10,11] we have calculated the mass, decay constant, and width of the $X_b(5568)$ state within the diquark-antidiquark picture $X_b = [su][\bar{b}\bar{d}]$ considering the exotic state with positive charge. Our

predictions for the mass m_{X_b} and for the width of its decay $\Gamma(X_b^+ \rightarrow B_s^0 \pi^+)$ are in agreement with the experimental data. It is worth noting that in the context of the diquark model some parameters of X_b were also analyzed in Refs. [12–15]. In these works the authors used various versions for the diquark-antidiquark-type interpolating currents with different Lorentz structures. It is remarkable that the obtained values for m_{X_b} are in agreement with each other and also consistent with experimental data of the D0 Collaboration. The molecule picture for X_b was realized in Ref. [16], where the $X_b(5568)$ state was taken as the $B\bar{K}$ bound state. The questions of quark-antiquark organization of this particle and its partners were addressed in Ref. [17].

In the present work we are going to continue our investigation of the new family of the four-quark exotic states by considering the hypothetical charmed partner of the $X_b(5568)$ state, which is composed of the c , s , u , and d quarks. We assume that this state bears the same quantum numbers as its counterpart, i.e., $J^{PC} = 0^{++}$. We also accept that it has the internal structure $X_c = [su][\bar{c}\bar{d}]$ in the diquark-antidiquark model. Thus, the partner state X_c is a neutral particle. Our aim is to determine the parameters of the state X_c , i.e., to find its mass, decay constant, and widths of the strong $X_c \rightarrow D_s^- \pi^+$ and $X_c \rightarrow D^0 K^0$ decays. For these purposes, we apply methods presented in a rather detailed form in Refs. [10,11,19].

This work is structured in the following way. In Sec. II we introduce the interpolating currents employed in QCD sum rule calculations. Here we find the mass and decay constant of X_c using the two-point QCD sum rule approach. The widths of the strong decays $X_c \rightarrow D_s^- \pi^+$ and $X_c \rightarrow D^0 K^0$ are the subject of Sec. III. The explicit expression for the spectral density required in computation of the mass and decay constant of the exotic X_c is moved to the Appendix.

II. THE MASS AND DECAY CONSTANT OF X_c

As noted above, we use the two-point QCD sum rule approach in order to compute the mass and decay constant of the X_c state. To this end, we consider the two-point correlation function given as

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ J_{1(2)}^{X_c}(x) J_{1(2)}^{X_c \dagger}(0) \} | 0 \rangle, \quad (1)$$

where $J_{1(2)}^{X_c}(x)$ are the interpolating currents with required quantum numbers. We consider the X_c state as a particle with the quantum numbers $J^{PC} = 0^{++}$. Then in the diquark-antidiquark model, the current $J_1^{X_c}(x)$ is given by the following expression:

$$J_1^{X_c}(x) = \varepsilon^{ijk} \varepsilon^{imn} [s^j(x) C \gamma_\mu u^k(x)] [\bar{c}^m(x) \gamma^\mu C \bar{d}^n(x)]. \quad (2)$$

Alternatively, one may introduce the interpolating current

$$J_2^{X_c}(x) = \varepsilon^{ijk} \varepsilon^{imn} [s^j(x) C \gamma_5 u^k(x)] [\bar{c}^m(x) \gamma_5 C \bar{d}^n(x)]. \quad (3)$$

In Eqs. (2) and (3), $i, j, k, m,$ and n are color indexes and C is the charge conjugation matrix.

Let us note that the current $J_1^{X_c}(x)$ has been employed throughout in Refs. [10,11] for exploration of the exotic $X_b(5568)$ state. The sum rules derived in these works, after trivial replacements of corresponding parameters, can easily be applied to analyze the X_c state. Therefore, in what follows we concentrate on the current $J_2^{X_c}(x)$ omitting, in what follows, the subscript in its definition.

The representation of the function $\Pi(p)$ in terms of the physical quantities does not depend on the form of the interpolating current and is the same for both $J_{1(2)}^{X_c}(x)$,

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0 | J^{X_c} | X_c(p) \rangle \langle X_c(p) | J^{X_c \dagger} | 0 \rangle}{m_{X_c}^2 - p^2} + \dots, \quad (4)$$

where m_{X_c} is the mass of the X_c state, and dots stand for contributions of the higher resonances and continuum states. We define the decay constant f_{X_c} through the matrix element

$$\langle 0 | J^{X_c} | X_c(p) \rangle = f_{X_c} m_{X_c}. \quad (5)$$

Then, for the correlation function we obtain

$$\Pi^{\text{Phys}}(p) = \frac{m_{X_c}^2 f_{X_c}^2}{m_{X_c}^2 - p^2} + \dots \quad (6)$$

The Borel transformation applied to Eq. (6) yields

$$\mathcal{B}_{p^2} \Pi^{\text{Phys}}(p) = m_{X_c}^2 f_{X_c}^2 e^{-m_{X_c}^2/M^2} + \dots \quad (7)$$

The theoretical expression for the same function, $\Pi^{\text{QCD}}(p)$, has to be determined employing the quark-gluon degrees of freedom. Contracting the quark fields we find for the correlation function $\Pi^{\text{QCD}}(p)$

$$\begin{aligned} \Pi^{\text{QCD}}(p) &= i \int d^4x e^{ipx} \varepsilon^{ijk} \varepsilon^{imn} \varepsilon^{i'j'k'} \varepsilon^{i'm'n'} \\ &\times \text{Tr}[\gamma_5 \tilde{S}_d^{n'}(-x) \gamma_5 S_c^{m'm}(-x)] \\ &\times \text{Tr}[\gamma_5 \tilde{S}_s^{j'}(x) \gamma_5 S_u^{kk'}(x)], \end{aligned} \quad (8)$$

where $S_q^{ij}(x)$ and $S_c^{ij}(x)$ are the light ($q \equiv u, d$ or s) and c -quark propagators, respectively. In Eq. (8) we introduce the notation

$$\tilde{S}_q^{ij}(x) = C S_q^{ijT}(x) C.$$

In the x -space, the light quark propagator $S_q^{ij}(x)$ has the form

$$\begin{aligned} S_q^{ij}(x) &= i \delta_{ij} \frac{\not{x}}{2\pi^2 x^4} - \delta_{ij} \frac{m_q}{4\pi^2 x^2} - \delta_{ij} \frac{\langle \bar{q}q \rangle}{12} \\ &+ i \delta_{ij} \frac{\not{x} m_q \langle \bar{q}q \rangle}{48} - \delta_{ij} \frac{x^2}{192} \langle \bar{q}g\sigma Gq \rangle \\ &+ i \delta_{ij} \frac{x^2 \not{x} m_q}{1152} \langle \bar{q}g\sigma Gq \rangle \\ &- i \frac{g G_{ij}^{\alpha\beta}}{32\pi^2 x^2} [\not{x} \sigma_{\alpha\beta} + \sigma_{\alpha\beta} \not{x}] - i \delta_{ij} \frac{x^2 \not{x} g^2 \langle \bar{q}q \rangle^2}{7776} \\ &- \delta_{ij} \frac{x^4 \langle \bar{q}q \rangle \langle g^2 GG \rangle}{27648} + \dots \end{aligned} \quad (9)$$

For the c -quark propagator $S_c^{ij}(x)$, we employ the expression from Ref. [20],

$$\begin{aligned} S_c^{ij}(x) &= i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left[\frac{\delta_{ij}(\not{k} + m_c)}{k^2 - m_c^2} \right. \\ &- \frac{g G_{ij}^{\alpha\beta} \sigma_{\alpha\beta}(\not{k} + m_c) + (\not{k} + m_c) \sigma_{\alpha\beta}}{4(k^2 - m_c^2)^2} \\ &\left. + \frac{g^2}{12} G_{\alpha\beta}^a G^{a\alpha\beta} \delta_{ij} m_c \frac{k^2 + m_c \not{k}}{(k^2 - m_c^2)^4} + \dots \right]. \end{aligned} \quad (10)$$

In Eqs. (9) and (10)

$$G_{ij}^{\alpha\beta} \equiv G_a^{\alpha\beta} t_{ij}^a, \quad a = 1, 2, \dots, 8,$$

where i and j are color indexes and $t^a = \lambda^a/2$, with λ^a being the standard Gell-Mann matrices. The first term in Eq. (10) is the perturbative propagator of a massive quark; the next two terms are nonperturbative gluon corrections.

In the nonperturbative terms the gluon field strength tensor $G_{\alpha\beta}^a \equiv G_{\alpha\beta}^a(0)$ is fixed at $x = 0$.

The correlation function $\Pi^{\text{QCD}}(p^2)$ is given by a simple dispersion integral

$$\Pi^{\text{QCD}}(p^2) = \int_{(m_c+m_s)^2}^{\infty} \frac{\rho^{\text{QCD}}(s)}{s-p^2} + \dots, \quad (11)$$

where $\rho^{\text{QCD}}(s)$ is the corresponding spectral density. It can be computed using mathematical methods described in Refs. [10,19]. Therefore, here we omit details of calculations and provide explicit expressions for both $\rho^{\text{QCD}}(s)$'s in the Appendix.

By applying the Borel transformation on the variable p^2 to the invariant amplitude $\Pi^{\text{QCD}}(p^2)$, equating the obtained expression $\mathcal{B}_{p^2}\Pi^{\text{Phys}}(p)$, and subtracting the continuum contribution, we finally obtain the required sum rule. Thus, the mass of the X_c state can be evaluated from the sum rule

$$m_{X_c}^2 = \frac{\int_{(m_c+m_s)^2}^{s_0} ds s \rho^{\text{QCD}}(s) e^{-s/M^2}}{\int_{(m_c+m_s)^2}^{s_0} ds \rho^{\text{QCD}}(s) e^{-s/M^2}}, \quad (12)$$

whereas for the decay constant f_{X_c} we employ the formula

$$f_{X_c}^2 m_{X_c}^2 e^{-m_{X_c}^2/M^2} = \int_{(m_c+m_s)^2}^{s_0} ds \rho^{\text{QCD}}(s) e^{-s/M^2}. \quad (13)$$

The last two expressions are the sum rules needed to evaluate the X_c state's mass and decay constant, respectively. For numerical computation we need values of the quark, gluon, and mixed condensates. Additionally, QCD sum rules contain c and s quark masses. The values of the parameters used are given in Table I.

Sum rule calculations imply fixing regions for the parameters s_0 and M^2 , where they can be varied. For s_0 we employ

$$7.56 \text{ GeV}^2 \leq s_0 \leq 8.12 \text{ GeV}^2. \quad (14)$$

We find the range $2 \text{ GeV}^2 < M^2 < 4 \text{ GeV}^2$ as a reliable region for varying the Borel parameter. Here the effects of the higher resonances and continuum states, and the

TABLE I. Input parameters used in calculations.

Parameters	Values
m_c	$(1.275 \pm 0.025) \text{ GeV}$
m_s	$(95 \pm 5) \text{ MeV}$
$\langle \bar{q}q \rangle$	$(-0.24 \pm 0.01)^3 \text{ GeV}^3$
$\langle \bar{s}s \rangle$	$0.8 \langle \bar{q}q \rangle$
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	$(0.012 \pm 0.004) \text{ GeV}^4$
m_0^2	$(0.8 \pm 0.1) \text{ GeV}^2$
$\langle \bar{q}gGq \rangle$	$m_0^2 \langle \bar{q}q \rangle$

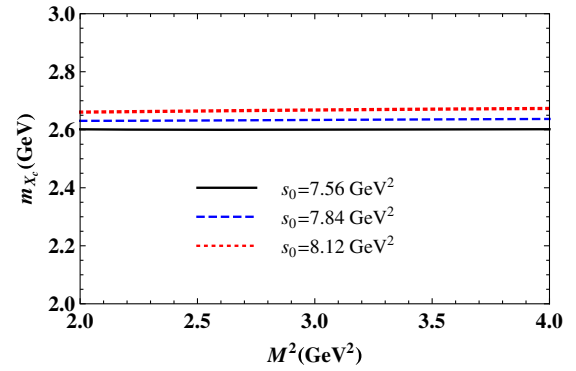


FIG. 1. The mass m_{X_c} as a function of the Borel parameter M^2 for different values of s_0 . In calculations the current $J_2^{X_c}$ is used.

contributions of the higher dimensional condensates, satisfy the well-known requirements of QCD sum rule calculations. It is not difficult to see that, in these intervals, the dependences of the mass and decay constant on M^2 and s_0 are very weak, and we expect that the sum rules give firm predictions (see Figs. 1 and 2). We estimate errors of the numerical computations by varying the parameters M^2 and s_0 within the accepted ranges, as well as taking into account uncertainties coming from other input parameters.

For the mass and decay constant of the X_c state we find

$$\begin{aligned} m_{X_c} &= (2590 \pm 60) \text{ MeV}, \\ f_{X_c} &= (0.20 \pm 0.03) \times 10^{-2} \text{ GeV}^4, \end{aligned} \quad (15)$$

when using the interpolating current $J_1^{X_c}$, and

$$\begin{aligned} m_{X_c} &= (2634 \pm 62) \text{ MeV}, \\ f_{X_c} &= (0.11 \pm 0.02) \times 10^{-2} \text{ GeV}^4 \end{aligned} \quad (16)$$

in the case of $J_2^{X_c}$. As is seen, for the mass of the X_c state the different interpolating currents lead to predictions that are very close to each other. The result for the mass of the X_c state obtained in Ref. [14],

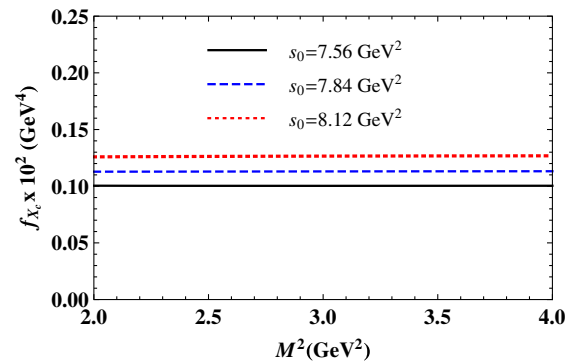


FIG. 2. The decay constant f_{X_c} vs Borel parameter M^2 for $J_2^{X_c}$. The values of the parameter s_0 are shown in the figure.

$$m_{X_c} = (2.55 \pm 0.09) \text{ GeV}, \quad (17)$$

is in agreement with our predictions, within errors.

III. THE STRONG DECAYS OF THE X_c STATE

Predictions for the mass of the X_c state obtained in the previous section allow us to continue our exploration by considering its possible decay channels and to calculate their decay widths. From the quark content and assigned quantum numbers, it is easy to conclude that the X_c state can decay into $D_s^-(s\bar{c}) + \pi^+(u\bar{d})$ or $D^0(u\bar{c}) + K^0(s\bar{d})$. In other words, $X_c \rightarrow D_s^- \pi^+$ and $X_c \rightarrow D^0 K^0$ transitions are kinematically allowed decay channels of the X_c state. Our aim in this section is to find widths of these decays. To this end, we calculate the strong couplings $g_{X_c D_s \pi}$ and $g_{X_c DK}$ using the QCD sum rule method on the light cone in conjunction with the soft-meson approximation [19].

We start our analysis from the decay $X_c \rightarrow D_s^- \pi^+$. In order to calculate the required strong coupling $g_{X_c D_s \pi}$ we consider the correlation function

$$\Pi(p, q) = i \int d^4 x e^{i p x} \langle \pi(q) | \mathcal{T} \{ J^{D_s}(x) J^{X_c \dagger}(0) \} | 0 \rangle. \quad (18)$$

Here the interpolating current $J^{X_c}(x)$ is given by Eq. (3), whereas for D_s^- we use

$$J^{D_s}(x) = \bar{c}^l(x) i \gamma_5 s^l(x). \quad (19)$$

It is not difficult to find $\Pi(p, q)$ in terms of the physical degrees of freedom:

$$\begin{aligned} \Pi^{\text{Phys}}(p, q) &= \frac{\langle 0 | J^{D_s} | D_s(p) \rangle \langle D_s(p) \pi(q) | X_c(p') \rangle}{p^2 - m_{D_s}^2} \\ &\times \frac{\langle X_c(p') | J^{X_c \dagger} | 0 \rangle}{p'^2 - m_{X_c}^2} + \dots, \end{aligned} \quad (20)$$

where by dots we denote contributions of the higher resonances and continuum states. Here p , q and $p' = p + q$, are the momenta of D_s , π , and X_c states, respectively. In order to finish computation of the correlation function we introduce the matrix elements

$$\begin{aligned} \langle 0 | J^{D_s} | D_s(p) \rangle &= \frac{f_{D_s} m_{D_s}^2}{m_c + m_s}, \\ \langle X_c(p') | J^{X_c \dagger} | 0 \rangle &= f_{X_c} m_{X_c}, \\ \langle D_s(p) \pi(q) | X_c(p') \rangle &= g_{X_c D_s \pi} p \cdot p', \end{aligned} \quad (21)$$

where f_{X_c} and m_{X_c} are the decay constant and mass of the X_c state, whereas f_{D_s} and m_{D_s} are the same parameters of the D_s meson.

We calculate $\Pi^{\text{Phys}}(p, q)$ in the soft-meson limit $q = 0$, and after some manipulations described in Refs. [11,19], for the Borel transformation of the correlation function find

$$\begin{aligned} \Pi^{\text{Phys}}(M^2) &= \frac{f_{D_s} f_{X_c} m_{X_c} m_{D_s}^2 g_{X_c D_s \pi}}{(m_c + m_s)} m^2 \\ &\times \frac{1}{M^2} e^{-m^2/M^2}, \end{aligned} \quad (22)$$

where $m^2 = (m_{X_c}^2 + m_{D_s}^2)/2$.

To proceed, we have to calculate $\Pi^{\text{QCD}}(p, q)$ in terms of the quark-gluon degrees of freedom and find the QCD side of the sum rule. Contractions of s - and c -quark fields in Eq. (18) yield

$$\begin{aligned} \Pi^{\text{QCD}}(p, q) &= \int d^4 x e^{i p x} \epsilon^{ijk} \epsilon^{imn} [\gamma_5 \tilde{S}_s^{lj}(x) \gamma_5 \\ &\times \tilde{S}_b^{ml}(-x) \gamma_5]_{\alpha\beta} \langle \pi(q) | \bar{u}_\alpha^k(0) d_\beta^n(0) | 0 \rangle, \end{aligned} \quad (23)$$

where α and β are the spinor indexes.

Skipping technical details, which can be found in Refs. [11,19], we provide a final expression for the spectral density, which is given as a sum of the perturbative and nonperturbative components,

$$\rho_{\text{coup}}^{\text{QCD}}(s) = \rho^{\text{pert}}(s) + \rho^{\text{n-pert}}(s), \quad (24)$$

where

$$\rho^{\text{pert}}(s) = \frac{f_\pi \mu_\pi}{16\pi^2 s} \sqrt{s(s - 4m_c^2)(s + 2m_c m_s - 2m_c^2)}, \quad (25)$$

and

$$\begin{aligned} \rho^{\text{n-pert}}(s) &= \frac{f_\pi \mu_\pi}{72} \{ 6 \langle \bar{s}s \rangle [-2m_c \delta(s - m_c^2) + sm_s \delta^{(1)}(s - m_c^2)] \\ &+ \langle \bar{s}g\sigma Gs \rangle [6(m_c - m_s) \delta^{(1)}(s - m_c^2) \\ &- 3s(m_c - 2m_s) \delta^{(2)}(s - m_c^2) - s^2 m_s \delta^{(3)}(s - m_c^2)] \}. \end{aligned} \quad (26)$$

In Eq. (26) $\delta^{(n)}(s - m_c^2) = (d/ds)^n \delta(s - m_c^2)$, which appears when extracting the imaginary part of the pole terms.

As is seen, in the soft limit the spectral density depends only on the parameters f_π and μ_π through the pion's local matrix element

$$\langle 0 | \bar{d}(0) i \gamma_5 u(0) | \pi(q) \rangle = f_\pi \mu_\pi, \quad (27)$$

where

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} = -\frac{2 \langle \bar{q}q \rangle}{f_\pi^2}. \quad (28)$$

Continuum subtraction performed in the standard way leads to the final sum rule for evaluating of the strong coupling:

$$g_{X_c D_s \pi} = \frac{(m_c + m_s)}{f_{D_s} f_{X_c} m_{X_c} m_{D_s}^2 m^2} \left(1 - M^2 \frac{d}{dM^2} \right) M^2 \times \int_{(m_c + m_s)^2}^{s_0} ds e^{(m^2 - s)/M^2} \rho^{\text{QCD}}(s). \quad (29)$$

The width of the decay $X_c \rightarrow D_s^- \pi^+$ can be found by applying the standard methods and is given in Ref. [11]:

$$\Gamma(X_c \rightarrow D_s^- \pi^+) = \frac{g_{X_c D_s \pi}^2 m_{D_s}^2}{24\pi} \lambda(m_{X_c}, m_{D_s}, m_\pi) \times \left[1 + \frac{\lambda^2(m_{X_c}, m_{D_s}, m_\pi)}{m_{D_s}^2} \right], \quad (30)$$

where

$$\lambda(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2 b^2 + a^2 c^2 + b^2 c^2)}}{2a}.$$

Equations (29) and (30) are final expressions that will be used for numerical analysis of the decay channel $X_c \rightarrow D_s^- \pi^+$.

The investigation of the transition $X_c \rightarrow D^0 K^0$ can be carried out in the same manner as for the decay $X_c \rightarrow D_s^- \pi^+$. One only needs to replace the parameters of the particles in accordance with the prescriptions $\pi \rightarrow K$ and $D_s \rightarrow D$. Nevertheless, below we write down some key expressions.

Thus, the analysis of the vertex $X_c DK$, which is necessary to derive the sum rule for the coupling $g_{X_c DK}$, is founded on the correlation function

$$\Pi_K(p, q) = i \int d^4x e^{ipx} \langle K(q) | \mathcal{T} \{ J^D(x) J^{X_c^\dagger}(0) \} | 0 \rangle, \quad (31)$$

where for the D^0 meson we employ the interpolating current

$$J^D(x) = \bar{c}^l(x) i \gamma_5 u^l(x). \quad (32)$$

In the soft-meson limit $q = 0$, the Borel transformation of the correlation function $\Pi_K^{\text{Phys}}(p, q)$ is given by

$$\Pi_K^{\text{Phys}}(M^2) = \frac{f_D f_{X_c} m_{X_c} m_D^2 g_{X_c DK}}{(m_c + m_u)} m^2 \times \frac{1}{M^2} e^{-m^2/M^2}. \quad (33)$$

In the formula above $m^2 = (m_{X_c}^2 + m_D^2)/2$, and m_D and f_D are the mass and decay constant of the D meson, respectively.

In terms of the quark-gluon degrees of freedom, the same function is determined by means of the formula

TABLE II. Input parameters used in the coupling calculations.

Parameters	Values
m_{D_s}	(1968.30 ± 0.10) MeV
f_{D_s}	(260.1 ± 10.8) MeV
m_D	(1864.84 ± 0.05) MeV
f_D	(218.9 ± 11.3) MeV
m_K	497.61 MeV
f_K	156 MeV
m_π	139.57 MeV
f_π	131 MeV

$$\Pi_K^{\text{QCD}}(p, q) = \int d^4x e^{ipx} \epsilon^{ijk} \epsilon^{imn} [\gamma_5 \tilde{S}_u^{lj}(x) \gamma_5 \times \tilde{S}_c^{ml}(-x) \gamma_5]_{\alpha\beta} \langle K(q) | \bar{s}_\alpha^k(0) d_\beta^n(0) | 0 \rangle. \quad (34)$$

Its imaginary part gives us the spectral density $\rho_{\text{coup}}^{\text{QCD}}(s)$, which now depends on the K meson local matrix element

$$\langle 0 | \bar{d}(0) i \gamma_5 s(0) | K(q) \rangle = \frac{f_K m_K^2}{m_s + m_d}, \quad (35)$$

with m_K and f_K being the mass and decay constant of the K meson. The remaining analysis is the same as for the $X_c \rightarrow D_s^- \pi^+$ decay: after evident changes in the relevant final expressions, they can be utilized for studying the $X_c \rightarrow DK$ transition as well.

The QCD sum rules derived above contain, as input parameters, the masses and decay constants of the D_s , D , π , and K mesons. They are collected in Table II. It is worth noting that for the decay constants f_D and f_{D_s} we use the lattice result from Ref. [21].

The results of the numerical calculations of the strong couplings and decay widths are shown in Table III. We find that the transition $X_c \rightarrow DK$ may be viewed as the dominant decay channel of the X_c state. The total width of this particle computed by taking into account the explored decay channels is equal to

$$\Gamma_{X_c}^1 \simeq (63.4 \pm 14.2) \text{ MeV}, \quad (36)$$

and

$$\Gamma_{X_c}^2 \simeq (53.7 \pm 11.6) \text{ MeV}, \quad (37)$$

TABLE III. The sum rule predictions for the strong couplings and corresponding decay widths.

	$J_1^{X_c}$	$J_2^{X_c}$
$g_{X_c D_s \pi}$	$(0.51 \pm 0.10) \text{ GeV}^{-1}$	$(0.51 \pm 0.11) \text{ GeV}^{-1}$
$\Gamma_{D_s \pi}$	$(8.0 \pm 2.0) \text{ MeV}$	$(8.2 \pm 2.1) \text{ MeV}$
$g_{X_c DK}$	$(1.57 \pm 0.34) \text{ GeV}^{-1}$	$(1.36 \pm 0.32) \text{ GeV}^{-1}$
Γ_{DK}	$(55.4 \pm 14.0) \text{ MeV}$	$(45.5 \pm 11.4) \text{ MeV}$

for the first and second interpolating currents, respectively. It is seen that results obtained for the total width of the X_c state using various interpolating currents, within errors, are compatible with each other; nevertheless the difference between the central values is sizable. The experimental exploration of the X_c state and its observation may extend our knowledge about the nature and internal structure of the new exotic states.

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APPENDIX: THE SPECTRAL DENSITY USED IN TWO-POINT SUM RULES

In this appendix we have collected the results of our calculations of the spectral density,

$$\rho^{\text{QCD}}(s) = \rho^{\text{pert}}(s) + \sum_{k=3}^{k=8} \rho_k(s), \quad (\text{A1})$$

used for evaluation of the X_c meson mass m_{X_c} and its decay constant f_{X_c} from the QCD sum rule. In Eq. (A1) by $\rho_k(s)$ we denote the nonperturbative contributions to $\rho^{\text{QCD}}(s)$. In calculations we have neglected the masses of the u and d quarks and taken into account terms $\sim m_s$. The explicit expressions for $\rho^{\text{pert}}(s)$ and $\rho_k(s)$ in the case of the current $J_2^{X_c}(x)$ are presented below as integrals over the Feynman parameter z . Note that in $\rho_8(s)$, we keep only the term containing the gluonic contribution.

$$\begin{aligned} \rho^{\text{pert}}(s) &= \frac{1}{6144\pi^6} \int_0^a \frac{dz z^4}{(z-1)^3} [m_c^2 + s(z-1)]^3 [m_c^2 + 3s(z-1)], \\ \rho_3(s) &= \frac{1}{64\pi^4} \int_0^a \frac{dz z^2}{(z-1)^2} [m_c^2 + s(z-1)] \{ \langle \bar{d}d \rangle m_c [m_c^2 + s(z-1)] + m_s (\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle) [m_c^2 + 2s(z-1)](z-1) \}, \\ \rho_4(s) &= \frac{1}{9216\pi^4} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \int_0^a \frac{dz z^2}{(z-1)^3} \{ 2m_c^4 [z(7z-15) + 9] + 3m_c^2 s(z-1) [z(13z-30) + 18] + 12s^2 (z-1)^3 (2z-3) \}, \\ \rho_5(s) &= \frac{m_0^2}{192\pi^4} \int_0^a \frac{dz z}{(1-z)} \{ 3m_c \langle \bar{d}d \rangle [m_c^2 + s(z-1)] + m_s (z-1) (\langle \bar{s}s \rangle - 3\langle \bar{u}u \rangle) [2m_c^2 + 3s(z-1)] \}, \\ \rho_6(s) &= \frac{g^2}{1296\pi^4} \int_0^a dz z (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2 + \langle \bar{s}s \rangle^2) [2m_c^2 + 3s(z-1)], \\ \rho_7(s) &= \frac{1}{576\pi^2} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle \int_0^a dz \{ 4m_c \langle \bar{d}d \rangle + m_s [\langle \bar{u}u \rangle (4z+2) - 3z \langle \bar{s}s \rangle] \}, \\ \rho_8(s) &= -\frac{11}{36864\pi^2} \left\langle \alpha_s \frac{G^2}{\pi} \right\rangle^2 \int_0^a dz z, \end{aligned} \quad (\text{A2})$$

where $a = (s - m_c^2)/s$.

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