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Classifying the bosonic quartic couplings

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The larger center-of-mass energy of the Large Hadron Collider Run 2 opens up the possibility of a more detailed study of the quartic vertices of the electroweak gauge bosons. Our goal in this work is to classify all operators possessing quartic interactions among the electroweak gauge bosons that do not exhibit triple gauge-boson vertices associated to them. We obtain all relevant operators in the nonlinear and linear realizations of the $SU(2)_L \otimes U(1)_Y$ gauge symmetry.

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I. INTRODUCTION

The recent discovery of a Higgs-like boson by the ATLAS and CMS collaborations [1] brings us a step closer to a full check of the standard model (SM). The SM has been subject to a large number of precision tests during the past decades [2] without any putative indication of deviations from its predictions for the particle couplings, which in the case of fermion-gauge interactions have been tested to close to per mil accuracy. Since the SM is a gauge theory based on the $SU(2)_L \otimes U(1)_Y$ group, it fixes completely the structure of the trilinear (TGC) and quartic (QGC) electroweak gauge-boson couplings. Therefore, it is important to establish whether these couplings indeed are in agreement with the SM predictions.

Precise knowledge of the gauge-boson self-interactions not only can serve to further establish the SM in the case of agreement with its predictions, but also any observed deviation can indicate the existence of new physics. For instance, new heavy bosons can generate a tree-level contribution to four gauge-boson couplings while its effect in the triple-gauge vertex would only appear at one loop [3] and consequently be suppressed with respect to the quartic one. Moreover, the comparison of deviations in TGC and QGC [4] can be used to determine whether the $SU(2)_L \otimes U(1)_L$ is linearly [5–13] or nonlinearly [14–16] realized in the low-energy effective theory of the electroweak breaking sector.

Presently, the trilinear gauge-boson couplings are known to agree with the SM within a few percent [17–19]. On the other hand, there are sparse direct data on anomalous QGC, and for a long time, the most stringent bound on QGC stemmed from their indirect effects to the Z physics via their one-loop contributions to the oblique corrections [20–23], a situation that is starting to change [24–27]. The LEP Collaborations directly probed $W^+W^-\gamma\gamma$ and $ZZ\gamma\gamma$ interactions in the reactions $e^+e^- \to W^+W^-\gamma$ [28] and $Z\gamma\gamma$ [29]. At the Tevatron, the D0 Collaboration studied the $W^+W^-\gamma\gamma$ vertex in diffractive events exhibiting dielectron and missing energy [30]. At the LHC, the ATLAS and CMS collaborations studied the production of $V\gamma\gamma$ with $V = Z$ or $W^\pm$ to constrain the $VV\gamma\gamma$ QGC [24]. Moreover, the ATLAS Collaboration analyzed the W^+W^- and $ZW^\pm$ pairs via vector-boson fusion to bound the QGC among four massive electroweak vector bosons [25] while CMS studied the $Z\gamma jj$, $W^\pm\gamma jj$ and $W^\pm W^\pm jj$ productions to probe QGC [26]. In addition to these inclusive processes, the CMS Collaboration also probed the $W^+W^-\gamma\gamma$ vertex through the exclusive $\gamma\gamma \to W^+W^-$ production [27].

The direct study of QGC requires either the production of three gauge bosons or the pair production of gauge bosons in vector-boson fusion [23]. Therefore, LHC Run 2 opens the possibility of testing systematically anomalous QGC due to the large center-of-mass energy. Here, we focus on genuine QGC, that is, QGC that do not have any TGC associated to them since the best bounds on the Wilson coefficients in the latter case are obtained from the direct study of TGC. In a scenario where the $SU(2)_L \otimes U(1)_Y$ is realized linearly, the lowest-order QGC are given by dimension-8 operators [21]. On the other hand, if the gauge symmetry is implemented nonlinearly, the lowest-order QGC appear at $\mathcal{O}(p^4)$ [4,16].

In the previous phenomenological [20–23,31–38] and experimental [24–30] analyses of QGC, just a partial list of effective operators has been considered. Our goal in this work is to classify all genuine QGC in the nonlinear and

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linear realizations of the gauge symmetry including up to two derivatives acting on the gauge-boson fields. This will facilitate the translation of bounds between different notations.

The outline of the paper is as follows. We start by listing the most general set of Lorentz structures which can be involved in quartic gauge-boson vertices containing up to two derivatives acting on the gauge fields in Sec. II. In Sec. III, we present the most general effective Lagrangian which generates QGC in scenarios in which the observed Higgs-like particle is indeed a fundamental state belonging to an $SU(2)_L$ doublet and for which the gauge symmetry is linearly realized. In these scenarios, QGC appear at dimension-8 independently on the number of derivatives, and we find a total of 10 independent operators and derive the relations between the coefficients of the generated Lorentz structures that this implies. Section IV contains the most general effective Lagrangian which gives rise to those QGC in scenarios with a dynamical light Higgs for which the electroweak symmetry realization is chiral. In this case, QGC with no derivatives and two derivatives appear at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$, respectively, and we find the same number of independent chiral operators as Lorentz structures. Finally, in Sec. V, we summarize our conclusions. The article is complemented with a set of Appendixes containing the most lengthy expressions as well as some technical details.

II. GENERAL LORENTZ STRUCTURES

Without loss of generality, initially we construct the possible Lorentz structures that are invariant under $U(1)_{\rm em}$ and which contain four vector bosons. We group the vertices in terms of the number of derivatives that they contain, and we restrict ourselves to vertices exhibiting zero or two derivatives or, equivalently, to operators with mass dimension up to 6.

A. Vertices with zero derivatives

Since the $U(1)_{\rm em}$ invariance requires that the photon field appears only as part of the electromagnetic field strength, the zero-derivative vertices do not contain photons; i.e., the QGC exhibit only massive electroweak gauge bosons. These structures are

$$\mathcal{Q}^{\partial=0}_{\rm WW,1} = W^{+\mu}W^-_\mu W^{+\nu}W^-_\nu, \quad \mathcal{Q}^{\partial=0}_{\rm WW,2} = W^{+\mu}W^{-\nu}W^+_\mu W^-_\nu,$$
$$\mathcal{Q}^{\partial=0}_{\rm WZ,1} = W^{+\mu}W^-_\mu Z^\nu Z_\nu, \quad \mathcal{Q}^{\partial=0}_{\rm WZ,2} = W^{+\mu}W^{-\nu}Z_\mu Z_\nu,$$
$$\mathcal{Q}^{\partial=0}_{\rm ZZ} = Z^\mu Z_\mu Z^\nu Z_\nu. \tag{1}$$

The first four structures in Eq. (1) modify the SM quartic couplings $W^+W^-W^+W^-$ and W^+W^-ZZ, while the last one leads to a QGC not present in the SM. The effective Lagrangian containing these five structures has the general form

$$\mathcal{L}^{\partial=0}_Q = \sum_{i=1}^{2} c_i^{0,WW}\mathcal{Q}^{\partial=0}_{WW,i} + \sum_{i=1}^{2} c_i^{0,WZ}\mathcal{Q}^{\partial=0}_{WZ,i} + c^{0,ZZ}\mathcal{Q}^{\partial=0}_{ZZ}. \tag{2}$$

B. Vertices with two derivatives

The quartic vertices containing two derivatives can be classified according to the number of photons in the vertex as $U(1)_{\rm em}$ requires that for each photon field at least one derivative must appear.

1. Vertices with two derivatives and two photons

The vertices with two derivatives and containing two photons are constructed using two photon field strengths plus two $W^\pm$ or Z fields. In this case, there are only four possible Lorentz structures:

$$\mathcal{Q}^{\partial=2}_{\gamma W,1} = F_{\mu\nu}F^{\mu\nu}W^{+\alpha}W^-_\alpha, \quad \mathcal{Q}^{\partial=2}_{\gamma W,2} = F_{\mu\nu}F^{\mu\alpha}W^{+\nu}W^-_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{\gamma Z,1} = F_{\mu\nu}F^{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{\gamma Z,2} = F_{\mu\nu}F^{\mu\alpha}Z^\nu Z_\alpha. \tag{3}$$

2. Vertices containing two derivatives and a single photon

In this group of QGC, one derivative appears in the photon field strength, while the other derivative can appear in the antisymmetric combination $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ with $V_\mu = Z_\mu$ or $W^\pm_\mu$ or in the symmetric combinations

$$X^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu + \partial_\nu W^\pm_\mu, \qquad Y_{\mu\nu} = \partial_\mu Z_\nu + \partial_\nu Z_\mu. \tag{4}$$

In this case, one can construct nine Lorentz structures corresponding to anomalous γZW^+W^- interactions,

$$\mathcal{Q}^{\partial=2}_{\gamma ZW,1} = F_{\mu\nu}Z^{\mu\nu}W^{+\alpha}W^-_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,2} = F_{\mu\nu}Z^{\mu\alpha}(W^{+\nu}W^-_\alpha + W^{-\nu}W^+_\alpha),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,3} = F^{\mu\nu}(W^+_{\mu\nu}W^-_\alpha Z^\alpha + W^-_{\mu\nu}W^+_\alpha Z^\alpha),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,4} = F^{\mu\nu}(W^+_{\mu\alpha}W^-_\nu Z^\alpha + W^-_{\mu\alpha}W^+_\nu Z^\alpha),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,5} = F^{\mu\nu}(W^+_{\mu\alpha}W^{-\alpha}Z_\nu + W^-_{\mu\alpha}W^{+\alpha}Z_\nu),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,6} = F_{\mu\nu}Y^{\mu\alpha}(W^{+\nu}W^-_\alpha + W^{-\nu}W^+_\alpha),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,7} = F_{\mu\nu}Z_\alpha(X^{+\mu\alpha}W^{-\nu} + X^{-\mu\alpha}W^{+\nu}),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,8} = F_{\mu\nu}Z^\nu(X^{+\mu\alpha}W^-_\alpha + X^{-\mu\alpha}W^+_\alpha),$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZW,9} = F_{\mu\nu}Z^\mu(X^{+\alpha}_\alpha W^{-\nu} + X^{-\alpha}_\alpha W^{+\nu}), \tag{5}$$

while there are three Lorentz structures associated to the coupling of a single photon to three Z's:

$$\mathcal{Q}^{\partial=2}_{\gamma ZZ,1} = F_{\mu\nu}Z^{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} = F_{\mu\nu}Z^{\mu\alpha}Z^\nu Z_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{\gamma ZZ,3} = F_{\mu\nu}Y^{\mu\alpha}Z^\nu Z_\alpha. \tag{6}$$

3. Vertices with two derivatives without photons

The W^+W^-ZZ quartic interactions exhibiting two derivatives are summarized by 29 distinct Lorentz structures,

$$\mathcal{Q}^{\partial=2}_{WZ,1} = W^{+\mu\nu}W^-_{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{WZ,2} = W^{+\mu\nu}W^-_{\mu\alpha}Z^\alpha Z_\nu, \quad \mathcal{Q}^{\partial=2}_{WZ,3} = W^{+\mu\nu}Z_{\mu\nu}Z^\alpha W^-_\alpha + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,4} = W^{+\mu\nu}Z_{\mu\alpha}Z_\nu W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,5} = W^{+\mu\nu}Z_{\mu\alpha}Z^\alpha W^-_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,6} = Z^{\mu\nu}Z_{\mu\nu}W^+_\alpha W^{-\alpha},$$
$$\mathcal{Q}^{\partial=2}_{WZ,7} = Z^{\mu\nu}Z_{\mu\alpha}W^+_\nu W^{-\alpha}, \quad \mathcal{Q}^{\partial=2}_{WZ,8} = X^{+\mu\nu}X^-_{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{WZ,9} = X^{+\mu\nu}X^-_{\mu\alpha}Z^\alpha Z_\nu, \quad \mathcal{Q}^{\partial=2}_{WZ,10} = X^{+\mu}_\mu X^{-\nu}_\nu Z^\alpha Z_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{WZ,11} = X^{+\mu}_\mu X^{-\nu\alpha}Z_\alpha Z_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,12} = X^{+\mu\nu}Y_{\mu\nu}Z^\alpha W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,13} = X^{+\mu\nu}Y_{\mu\alpha}Z^\alpha W^-_\nu + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,14} = X^{+\mu\nu}Y_{\mu\alpha}Z_\nu W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,15} = X^{+\mu}_\mu Y^\nu_\nu Z_\alpha W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,16} = X^{+\mu}_\mu Y^{\nu\alpha}Z_\nu W^-_\alpha + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,17} = X^+_{\mu\alpha}Y^\nu_\nu Z^\mu W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,18} = Y^{\mu\nu}Y_{\mu\nu}W^+_\alpha W^{-\alpha}, \quad \mathcal{Q}^{\partial=2}_{WZ,19} = Y^{\mu\nu}Y_{\mu\alpha}W^+_\nu W^{-\alpha},$$
$$\mathcal{Q}^{\partial=2}_{WZ,20} = Y^\mu_\mu Y^\nu_\nu W^+_\alpha W^{-\alpha}, \quad \mathcal{Q}^{\partial=2}_{WZ,21} = Y^\mu_\mu Y^{\nu\alpha}W^+_\nu W^-_\alpha, \quad \mathcal{Q}^{\partial=2}_{WZ,22} = W^{+\mu\nu}X^-_{\mu\alpha}Z^\alpha Z_\nu + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,23} = W^{+\mu\nu}Y_{\mu\alpha}Z_\nu W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,24} = W^{+\mu\nu}Y_{\mu\alpha}Z^\alpha W^-_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,25} = W^{+\nu\alpha}Y^\mu_\mu Z_\nu W^-_\alpha + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,26} = X^{+\mu\nu}Z_{\mu\alpha}Z_\nu W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,27} = X^{+\mu\nu}Z_{\mu\alpha}Z^\alpha W^-_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WZ,28} = X^{+\mu}_\mu Z_{\nu\alpha}Z^\nu W^{-\alpha} + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WZ,29} = Y^{\mu\nu}Z_{\mu\alpha}W^+_\nu W^{-\alpha} + \text{H.c.}, \tag{7}$$

where H.c. stands for the Hermitian conjugate. Correspondingly, there are 18 $W^+W^-W^+W^-$ effective vertices containing two derivatives that are given by

$$\mathcal{Q}^{\partial=2}_{WW,1} = W^{+\mu\nu}W^-_{\mu\nu}W^{+\alpha}W^-_\alpha, \quad \mathcal{Q}^{\partial=2}_{WW,2} = W^{+\mu\nu}W^-_{\mu\alpha}W^{+\alpha}W^-_\nu, \quad \mathcal{Q}^{\partial=2}_{WW,3} = W^{+\mu\nu}W^-_{\mu\alpha}W^+_\nu W^{-\alpha},$$
$$\mathcal{Q}^{\partial=2}_{WW,4} = W^{+\mu\nu}W^+_{\mu\nu}W^{-\alpha}W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,5} = W^{+\mu\nu}W^+_{\mu\alpha}W^{-\alpha}W^-_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,6} = X^{+\mu\nu}X^-_{\mu\nu}W^{+\alpha}W^-_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{WW,7} = X^{+\mu\nu}X^-_{\mu\alpha}W^{+\alpha}W^-_\nu, \quad \mathcal{Q}^{\partial=2}_{WW,8} = X^{+\mu\nu}X^-_{\mu\alpha}W^{-\alpha}W^+_\nu, \quad \mathcal{Q}^{\partial=2}_{WW,9} = X^{+\mu}_\mu X^{-\nu}_\nu W^{+\alpha}W^-_\alpha,$$
$$\mathcal{Q}^{\partial=2}_{WW,10} = X^{+\mu}_\mu X^{-\nu\alpha}W^+_\nu W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,11} = X^{+\mu\nu}X^+_{\mu\nu}W^{-\alpha}W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,12} = X^{+\mu\nu}X^+_{\mu\alpha}W^{-\alpha}W^-_\nu + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WW,13} = X^{+\mu}_\mu X^{+\nu}_\nu W^{-\alpha}W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,14} = X^{+\mu}_\mu X^{+\nu\alpha}W^-_\nu W^-_\alpha + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,15} = W^{+\mu\nu}X^-_{\mu\alpha}W^{+\alpha}W^-_\nu + \text{H.c.},$$
$$\mathcal{Q}^{\partial=2}_{WW,16} = W^{+\mu\nu}X^-_{\mu\alpha}W^{-\alpha}W^+_\nu + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,17} = X^{+\mu}_\mu W^-_{\nu\alpha}W^{+\nu}W^{-\alpha} + \text{H.c.}, \quad \mathcal{Q}^{\partial=2}_{WW,18} = X^{+\mu\nu}W^+_{\mu\alpha}W^{-\alpha}W^-_\nu + \text{H.c.}. \tag{8}$$

Finally, seven different Lorentz structures are required to describe $ZZZZ$ vertices with two derivatives: /new/c

$$\mathcal{Q}^{\partial=2}_{ZZ,1} = Z^{\mu\nu}Z_{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{ZZ,2}Z^{\mu\nu}Z_{\mu\alpha}Z^\alpha Z_\nu, \quad \mathcal{Q}^{\partial=2}_{ZZ,3} = Y^{\mu\nu}Y_{\mu\nu}Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{ZZ,4} = Y^{\mu\nu}Y_{\mu\alpha}Z^\alpha Z_\nu,$$
$$\mathcal{Q}^{\partial=2}_{ZZ,5} = Y^\mu_\mu Y^\nu_\nu Z^\alpha Z_\alpha, \quad \mathcal{Q}^{\partial=2}_{ZZ,6} = Y^\mu_\mu Y^{\nu\alpha}Z_\alpha Z_\nu, \quad \mathcal{Q}^{\partial=2}_{ZZ,7} = Z^{\mu\nu}Y_{\mu\alpha}Z^\alpha Z_\nu. \tag{9}$$

Altogether, any effective Lagrangian possessing two derivatives and any four gauge bosons can be written as a combination of the 70 Lorentz structures above as

$$\mathcal{L}^{\partial=2}_{Q} = \sum_{i=1}^{2} c_i^{2,\gamma W}\mathcal{Q}^{\partial=2}_{\gamma W,i} + \sum_{i=1}^{2} c_i^{2,\gamma Z}\mathcal{Q}^{\partial=2}_{\gamma Z,i} + \sum_{i=1}^{9} c_i^{2,\gamma ZW}\mathcal{Q}^{\partial=2}_{\gamma ZW,i} + \sum_{i=1}^{3} c_i^{2,\gamma ZZ}\mathcal{Q}^{\partial=2}_{\gamma ZZ,i}$$
$$+ \sum_{i=1}^{29} c_i^{2,WZ}\mathcal{Q}^{\partial=2}_{WZ,i} + \sum_{i=1}^{18} c_i^{2,WW}\mathcal{Q}^{\partial=2}_{WW,i} + \sum_{i=1}^{7} c_i^{2,ZZ}\mathcal{O}^{\partial=2}_{ZZ,i}. \tag{10}$$

It is interesting to notice that the quartic vertices in Eqs. (5), (6), (7), (8), and (9) containing $X^\pm_{\mu\nu}$ or $Y_{\mu\nu}$ have not been considered before in the literature.

In summary, we have found 5 Lorentz structures without derivatives and 70 with two derivatives. Next, we are going to build the lowest-dimension electroweak gauge invariant Lagrangian which can lead to genuine quartic gauge-boson vertices and map their coefficients to those of these Lorentz structures.

III. GENUINE QGC IN MODELS WITH AN ELEMENTARY HIGGS: THE LINEAR LAGRANGIAN

Assuming that the new state observed in 2012 is indeed the SM Higgs boson and belongs to a light electroweak scalar doublet, we can construct an effective theory where the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is linearly realized [5–13] that can be expressed as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{n=5}^{\infty} \frac{f_n}{\Lambda^{n-4}} \mathcal{O}_n, \tag{11}$$

where the dimension-n operators $\mathcal{O}_n$ involve SM fields with couplings f_n and where Λ is a characteristic scale.

The basic blocks for constructing the effective Lagrangian leading to genuine QGC with the gauge symmetry realized linearly and their transformations are

$$\Phi, \quad \text{that transforms as} \quad \Phi' = U\Phi \tag{12}$$

$$D_\mu\Phi, \quad \text{that transforms as} \quad D'_\mu\Phi' = UD_\mu\Phi \tag{13}$$

$$\hat{W}_{\mu\nu} \equiv \sum_j W^j_{\mu\nu}\frac{\sigma^j}{2}, \quad \text{that transforms as} \quad \hat{W}'_{\mu\nu} = U\hat{W}_{\mu\nu}U^\dagger \tag{14}$$

$$B_{\mu\nu}, \quad \text{that transforms as} \quad B'_{\mu\nu} = B_{\mu\nu}, \tag{15}$$

where Φ stands for the Higgs doublet, $W^i_{\mu\nu}$ is the $SU(2)_L$ field strength, and $B_{\mu\nu}$ is the $U(1)_Y$ one. Here, we denote an arbitrary gauge transformation by U. According to our conventions, the covariant derivative is given by $D_\mu\Phi = (\partial_\mu + igW^j_\mu\frac{\sigma^j}{2} + ig'B_\mu\frac{1}{2})\Phi$, and σ^j stand for the Pauli matrices.

Notice that the covariant derivative of Φ and also the field strength tensors contain terms with at least one weak gauge boson when we substitute Φ by its vacuum expectation value, v. Therefore, the lowest-dimension operators that lead to genuine quartic interactions are dimension 8.[1] They can be classified in three groups:

(i) terms that contain four covariant derivatives of the Higgs field;
(ii) terms exhibiting two Higgs covariant derivatives and two field strength tensors;
(iii) terms presenting four field strength tensors.

Here, we focus on operators containing up to two derivatives, and therefore, we will not analyze the last class; however, we present these operators in the Appendix B for the sake of completeness.

A. Operators containing only $D_\mu\Phi$

There are three independent operators belong to this class[2]:

$$\begin{aligned}
\mathcal{O}_{S,0} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\mu\Phi)^\dagger D^\nu\Phi] = \frac{g^4v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,2} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,2} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right],\\
\mathcal{O}_{S,1} &= [(D_\mu\Phi)^\dagger D^\mu\Phi] \times [(D_\nu\Phi)^\dagger D^\nu\Phi] = \frac{g^4v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,1} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,1} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right],\\
\mathcal{O}_{S,2} &= [(D_\mu\Phi)^\dagger D_\nu\Phi] \times [(D^\nu\Phi)^\dagger D^\mu\Phi] = \frac{g^4v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,1} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,2} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right].
\end{aligned} \tag{16}$$

The corresponding effective Lagrangian can be mapped to that in Eq. (2) with coefficients related as

$$\begin{aligned}
c_1^{0,WW} &= \frac{g^4v^4}{16}\left[\frac{f_{S,1}}{\Lambda^4} + \frac{f_{S,2}}{\Lambda^4}\right], \qquad c_2^{0,WW} = \frac{g^4v^4}{16}\frac{f_{S,0}}{\Lambda^4}, \qquad c_1^{0,WZ} = \frac{g^4v^4}{16c_W^2}\frac{f_{S,1}}{\Lambda^4},\\
c_2^{0,WZ} &= \frac{g^4v^4}{16c_w^2}\left[\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,2}}{\Lambda^4}\right], \qquad c^{0,ZZ} = \frac{1}{4c_w^4}(c_1^{0,WW} + c_2^{0,WW}),
\end{aligned} \tag{17}$$

where s_w (c_w) stand for the sine (cosine) of the weak mixing angle θ_w verifying $\tan\theta_w = g/g'$. Notice that the linear realization of the symmetry leads to correlations between the Wilson coefficients appearing in Eq. (2).

B. Operators containing $D_\mu\Phi$ and field strength

This class possesses seven operators[3]:

$$\begin{aligned}
&\mathcal{O}_{M,0} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \times [(D_\beta\Phi)^\dagger D^\beta\Phi], \qquad \mathcal{O}_{M,1} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\beta}] \times [(D_\beta\Phi)^\dagger D^\mu\Phi], \qquad \mathcal{O}_{M,2} = [B_{\mu\nu}B^{\mu\nu}] \times [(D_\beta\Phi)^\dagger D^\beta\Phi],\\
&\mathcal{O}_{M,3} = [B_{\mu\nu}B^{\nu\beta}] \times [(D_\beta\Phi)^\dagger D^\mu\Phi], \qquad \mathcal{O}_{M,4} = [(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\mu\Phi] \times B^{\beta\nu}, \qquad \mathcal{O}_{M,5} = [(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}D^\nu\Phi] \times B^{\beta\mu} + \text{H.c.},\\
&\mathcal{O}_{M,7} = [(D_\mu\Phi)^\dagger\hat{W}_{\beta\nu}\hat{W}^{\beta\mu}D^\nu\Phi].
\end{aligned} \tag{18}$$

[1]At dimension 6, quartic vertices are generated, but they are always accompanied by triple gauge-boson vertices with related coefficients.

[2]We are using the conventions of Ref. [21]; however, the operator $\mathcal{O}_{S,2}$ has been included.

These seven operators involve 23 of the 70 possible Lorentz structures as explicitly given in Appendix A. In particular, none of the structures with symmetric gauge-boson tensors is generated. The corresponding effective Lagrangian can be written in the form of Eq. (10) where the 23 coefficients of the Lorentz structures can be expressed in terms of 7 of them. For example, these can be chosen to be $c_{1,2}^{2,WW}$, $c_{2,3,6,7}^{2,WZ}$, and $c_2^{2,\gamma W}$ with

$$
\begin{aligned}
c_1^{2,WW} &= \frac{g^2v^2}{8}\frac{2f_{M,0}}{\Lambda^4}, \qquad c_2^{2,WW} = -\frac{g^2v^2}{8}\frac{f_{M,1}}{\Lambda^4}, \qquad c_2^{2,WZ} = \frac{g^2v^2}{8}\frac{1}{c_w^2}\frac{-f_{M,1}+\frac{1}{2}f_{M,7}}{\Lambda^4}, \qquad c_3^{2,WZ} = \frac{g^2v^2}{8}\frac{s_w}{c_w}\frac{f_{M,4}}{\Lambda^4},\\
c_6^{2,WZ} &= \frac{g^2v^2}{8}\left[c_w^2\frac{f_{M,0}}{\Lambda^4} + 2s_w^2\frac{f_{M,2}}{\Lambda^4} - s_wc_w\frac{f_{M,4}}{\Lambda^4}\right], \qquad c_7^{2,WZ} = \frac{g^2v^2}{8}\left[-c_w^2\frac{f_{M,1}}{\Lambda^4} - s_w^2\frac{f_{M,3}}{\Lambda^4} - 2s_wc_w\frac{f_{M,5}}{\Lambda^4} + \frac{1}{2}c_w^2\frac{f_{M,7}}{\Lambda^4}\right],\\
c_2^{2,\gamma W} &= \frac{g^2v^2}{8}\left[-s_w^2\frac{f_{M,1}}{\Lambda^4} - c_w^2\frac{f_{M,3}}{\Lambda^4} + 2s_wc_w\frac{f_{M,5}}{\Lambda^4} + \frac{1}{2}s_w^2\frac{f_{M,7}}{\Lambda^4}\right],
\end{aligned} \tag{19}
$$

and the remaining 16 coefficients satisfy the following relations:

$$
\begin{aligned}
c_1^{2,WZ} &= \frac{1}{2c_w^2}c_1^{2,WW}, \qquad c_1^{2,\gamma Z} = \frac{1}{2c_w^2}c_1^{2,\gamma W} - c_3^{2,WZ}, \qquad c_1^{2,ZZ} = \frac{1}{2c_w^2}c_6^{2,WZ} + c_3^{2,WZ}, \qquad c_3^{2,\gamma ZW} = -\frac{c_w}{s_w}c_3^{2,WZ},\\
c_1^{2,\gamma ZW} &= 2s_wc_w\left[c_1^{2,WW} - c_6^{2,WZ} - c_1^{2,\gamma W} + \frac{c_w^2-s_w^2}{2s_w^2}c_3^{2,WZ}\right], \qquad c_1^{2,\gamma ZZ} = \frac{1}{2c_w^2}c_1^{2,\gamma ZW} - \frac{c_w^2-s_w^2}{s_wc_w}c_3^{2,WZ},\\
c_1^{2,\gamma W} &= \frac{1}{2s_w^2}[2c_w^2(c_3^{2,WZ}+c_6^{2,WZ}) - (c_w^2-s_w^2)c_1^{2,WW}], \qquad c_3^{2,WW} = 2c_w^2c_2^{2,WZ} - c_2^{2,WW},\\
c_2^{2,ZZ} &= \frac{1}{2c_w^2}[(c_w^2-s_w^2)(2c_w^2c_2^{2,WZ} - c_7^{2,WZ}) + 2s_w^2c_2^{2,\gamma W}], \qquad c_2^{2,\gamma Z} = -c_2^{2,ZZ} + \frac{1}{c_w^2}(c_7^{2,WZ} + c_2^{2,\gamma W}),\\
c_5^{2,WZ} &= \frac{1}{2}\left[-c_2^{2,WZ} - c_7^{2,WZ} + \frac{s_w^2}{c_w^2}c_2^{2,\gamma W} + 2c_2^{2,WW}\right], \qquad c_4^{2,WZ} = c_5^{2,WZ} + 2(c_w^2c_2^{2,WZ} - c_2^{2,WW}),\\
c_2^{2,\gamma ZW} &= \frac{1}{2s_wc_w}[c_w^2(c_2^{2,WZ} - c_7^{2,WZ}) - s_w^2c_2^{2,\gamma W}], \qquad c_2^{2,\gamma ZZ} = \frac{1}{c_w^2}c_2^{2,\gamma ZW} - \frac{c_w^2-s_w^2}{c_ws_w}(c_5^{2,WZ} + c_4^{2,WZ}),\\
c_{4,5}^{2,\gamma ZW} &= -\frac{c_w}{2s_w}(c_5^{2,WZ} + c_4^{2,WZ}) \pm \frac{s_w}{c_w}(c_w^2c_2^{2WZ} - c_2^{2,WW}).
\end{aligned} \tag{20}
$$

IV. GENUINE QGC IN MODELS WITH A DYNAMICAL HIGGS: THE CHIRAL LAGRANGIAN

In dynamical Higgs scenarios, the Higgs particle is a composite field which happens to be a pseudo-Nambu-Goldstone boson (PNG) of a global symmetry exact at scales $\Lambda_{\rm strong}$ that corresponds to the masses of the lightest strong resonances. Because the Higgs-like particle is a PNG, the effective Lagrangian is nonlinear or "chiral," a derivative expansion [14,15,39,40] with a global $SU(2)_L \otimes SU(2)_R$ symmetry broken to the diagonal $SU(2)_c$. The effective low-energy chiral Lagrangian is entirely written in terms of the SM fermions and gauge bosons and of the physical Higgs h. In this scenario, the basic building block at low energies is a dimensionless unitary matrix transforming as a bidoublet of the global symmetry,

$$
\mathbf{U}(x) = e^{i\sigma_a\pi^a(x)/v}, \qquad \mathbf{U}(x) \to L\mathbf{U}(x)R^\dagger, \tag{21}
$$

where L and R denote $SU(2)_{L,R}$ global transformations, respectively, and π^a are the Goldstone bosons. Its covariant derivative reads

$$
\mathbf{D}_\mu\mathbf{U}(x) \equiv \partial_\mu\mathbf{U}(x) + ig\frac{\sigma^j}{2}W_\mu^i(x)\mathbf{U}(x) - \frac{ig'}{2}B_\mu(x)\mathbf{U}(x)\sigma_3. \tag{22}
$$

We define the vector chiral field and its covariant derivative as

$$
V_\mu \equiv (\mathbf{D}_\mu\mathbf{U})\mathbf{U}^\dagger, \tag{23}
$$

[3]We follow the notation in Ref. [21], but we notice that in there an additional operator $\mathcal{O}_{M,6}$ was listed, which we found to be redundant.

$$D_\mu V_\alpha = \partial_\mu V_\alpha + ig[W_\mu, V_\alpha], \tag{24}$$

and the scalar chiral field $T \equiv \mathbf{U}\sigma_3\mathbf{U}^\dagger$. These three objects transform in the adjoint of $SU(2)_L$. Moreover, the Higgs field h is a singlet under the global symmetry.

In our framework, we consider genuine QGC that appear at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ and are invariant under CP. The CP transformation properties of our building blocks can be easily obtained once we know that [15]

$$\begin{aligned} CPB_\mu(\vec{x},t)(CP)^{-1} &= -B_\mu(-\vec{x},t),\\ CPW^i_\mu(\vec{x},t)(CP)^{-1} &= \sigma^2 W^i_\mu(-\vec{x},t)\sigma^2,\\ CPT(\vec{x},t)(CP)^{-1} &= -\sigma^2 T(-\vec{x},t)\sigma^2,\\ CPV_\mu(\vec{x},t)(CP)^{-1} &= \sigma^2 V_\mu(-\vec{x},t)\sigma^2. \end{aligned} \tag{25}$$

Our choice of phases are such that $D_\mu U$ has a well-defined transformation under CP. From the above equation, we can learn that the CP conserving QGC are the ones exhibiting an even number of T's and B_μ's.

The building blocks that we use to construct genuine CP conserving QGC can be classified according the mass dimension of the operator (D) [15]. Here, we list all building block operators needed to construct up to $\mathcal{O}(p^6)$ quartic operators, as well as their expressions in the unitary gauge. There is just one operator with mass dimension 1:

$$\mathrm{Tr}[TV_\mu] = i\frac{g}{c_w}Z_\mu. \tag{26}$$

On the other hand, there are five $D=2$ building blocks; however, only four of them appear in CP-invariant quartic operators,

$$B_{\mu\nu} = c_w F_{\mu\nu} - s_w Z_{\mu\nu}, \tag{27}$$

$$\mathrm{Tr}[TD_{\mu\nu}] = i\frac{g}{c_w}Y_{\mu\nu}, \tag{28}$$

$$\mathrm{Tr}[V_\mu V_\nu] = -\frac{g^2}{2}\left(\frac{1}{c_w^2}Z_\mu Z_\nu + W^+_\mu W^-_\nu + W^-_\mu W^+_\nu\right), \tag{29}$$

$$\mathrm{Tr}[T\hat{W}_{\mu\nu}] = c_w Z_{\mu\nu} + s_w F_{\mu\nu}, \tag{30}$$

where we define the symmetric combination $D_{\mu\nu} \equiv D_\mu V_\nu + D_\nu V_\mu$.

Just two of the eight $D=3$ basic operators appear in CP-invariant quartic vertices:

$$\mathrm{Tr}[V_\mu D_{\nu\lambda}] = -\frac{g^2}{2}\left(\frac{1}{c_W^2}Z_\mu Y_{\nu\lambda} + W^+_\mu X^-_{\nu\lambda} + W^-_\mu X^+_{\nu\lambda}\right), \tag{31}$$

$$\begin{aligned} \mathrm{Tr}[V_\mu \hat{W}_{\nu\lambda}] = i\frac{g}{2}\bigg(&\frac{1}{c_w}Z_\mu(c_w Z_{\nu\lambda} + s_w F_{\nu\lambda})\\ &+ W^+_\mu W^-_{\nu\lambda} + W^-_\mu W^+_{\nu\lambda}\bigg). \end{aligned} \tag{32}$$

Of the 11 possible $D=4$ operators, just 3 of them contribute to CP-conserving quartic vertices:

$$\mathrm{Tr}[D_{\mu\nu}D_{\alpha\beta}] = -\frac{g^2}{2}\left(\frac{1}{c_w^2}Y_{\mu\nu}Y_{\alpha\beta} + X^+_{\mu\nu}X^-_{\alpha\beta} + X^-_{\mu\nu}X^+_{\alpha\beta}\right), \tag{33}$$

$$\begin{aligned} \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}_{\alpha\beta}] = \frac{1}{2}[&(c_w Z_{\mu\nu} + s_w F_{\mu\nu})(c_w Z_{\alpha\beta} + s_w F_{\alpha\beta})\\ &+ W^+_{\mu\nu}W^-_{\alpha\beta} + W^-_{\mu\nu}W^+_{\alpha\beta}], \end{aligned} \tag{34}$$

$$\begin{aligned} \mathrm{Tr}[\hat{W}_{\mu\nu}D_{\alpha\beta}] = \frac{ig}{2}\bigg[&\frac{1}{c_w}(c_w Z_{\mu\nu} + s_w F_{\mu\nu})Y_{\alpha\beta}\\ &+ W^+_{\mu\nu}X^-_{\alpha\beta} + W^-_{\mu\nu}X^+_{\alpha\beta}\bigg]. \end{aligned} \tag{35}$$

It is interesting to notice that no dimension-5 operator can give rise to p^6 CP-conserving QGC.

Using the $D = 1, 2, 3, 4$ building blocks, we construct all possible CP-conserving operators for genuine QGC, and then we remove those which can be related by total derivatives. For instance, the relation

$$\begin{aligned} &\partial_\nu\{\mathrm{Tr}[V_\mu D_\lambda V_\rho]\mathrm{Tr}[V_\alpha V_\beta]\}\\ &\quad= \mathrm{Tr}[D_\nu V_\mu D_\lambda V_\rho]\mathrm{Tr}[V_\alpha V_\beta]\\ &\qquad+ \mathrm{Tr}[V_\mu D_\nu D_\lambda V_\rho]\mathrm{Tr}[V_\alpha V_\beta]\\ &\qquad+ \mathrm{Tr}[V_\mu D_\lambda V_\rho]\mathrm{Tr}[D_\nu V_\alpha V_\beta]\\ &\qquad+ \mathrm{Tr}[V_\mu D_\lambda V_\rho]\mathrm{Tr}[V_\alpha D_\nu V_\beta] \end{aligned} \tag{36}$$

can be used to eliminate operators that contain the building block $\mathrm{Tr}[V_\mu D_\nu D_\lambda V_\rho]$. To further reduce the number of equivalent operators, we also use the relation

$$D_\mu V_\nu - D_\nu V_\mu = ig\hat{W}_{\mu\nu} - \frac{i}{2}g' B_{\mu\nu}T + [V_\mu, V_\nu]. \tag{37}$$

Moreover, we introduce a factor ig and ig' in each operator containing $\hat{W}_{\mu\nu}$ and $B_{\mu\nu}$, respectively, in order to have consistent global powers of coupling constants.

A. QGC at $\mathcal{O}(p^4)$

The lowest-order genuine quartic operators are $\mathcal{O}(p^4)$, and there are two operators which respect to the $SU(2)_C$ custodial symmetry, as well as C and P that are given by[4]

[4]We follow the notation of Refs. [4,16].

$$\begin{aligned}\mathcal{P}_6 &= \mathrm{Tr}[V^\mu V_\mu]\mathrm{Tr}[V^\nu V_\nu]\mathcal{F}_6(h)\\ &= g^4\left[\frac{1}{4c_w^4}\mathcal{O}^0_{ZZ} + \mathcal{O}^0_{WW,1} + \frac{1}{c_w^2}\mathcal{O}^0_{WZ,1}\right]\mathcal{F}_6(h),\\ \mathcal{P}_{11} &= \mathrm{Tr}[V^\mu V^\nu]\mathrm{Tr}[V_\mu V_\nu]\mathcal{F}_{11}(h)\\ &= g^4\left[\frac{1}{4c_w^4}\mathcal{O}^0_{ZZ} + \frac{1}{2}\mathcal{O}^0_{WW,1} + \frac{1}{2}\mathcal{O}^0_{WW,2}\right.\\ &\quad\left. + \frac{1}{c_w^2}\mathcal{O}^0_{WZ,2}\right]\mathcal{F}_{11}(h), \qquad (38)\end{aligned}$$

and three additional CP-conserving operators that violate $SU(2)_c$:

$$\begin{aligned}\mathcal{P}_{23} &= \mathrm{Tr}[V^\mu V_\mu](\mathrm{Tr}[TV_\nu])^2\mathcal{F}_{23}(h)\\ &= g^4\left[\frac{1}{2c_w^4}\mathcal{O}^0_{ZZ} + \frac{1}{c_w^2}\mathcal{O}^0_{WZ,1}\right]\mathcal{F}_{23}(h),\\ \mathcal{P}_{24} &= \mathrm{Tr}[V^\mu V^\nu]\mathrm{Tr}[TV_\mu]\mathrm{Tr}[TV_\nu]\mathcal{F}_{24}(h)\\ &= g^4\left[\frac{1}{2c_w^4}\mathcal{O}^0_{ZZ} + \frac{1}{c_w^2}\mathcal{O}^0_{WZ,2}\right]\mathcal{F}_{24}(h),\\ \mathcal{P}_{26} &= (\mathrm{Tr}[TV_\mu]\mathrm{Tr}[TV_\nu])^2\mathcal{F}_{26}(h) = \frac{g^4}{c_w^4}\mathcal{O}^0_{ZZ}\mathcal{F}_{26}(h). \qquad (39)\end{aligned}$$

$\mathcal{F}_i(h)$ are generic functions parametrizing the chiral-symmetry-breaking interactions of h which can be expanded as $\mathcal{F}_i(h) \equiv 1 + 2\tilde{a}_i\frac{h}{v} + \tilde{b}_i\frac{h^2}{v^2} + \cdots$. As we are looking for operators of which the lowest-order vertex contains four gauge bosons, we will be only concerned by the constant term. So, the most general Lagrangian at $\mathcal{O}(p^4)$ for genuine QGC is

$$\mathcal{L}_Q^{p=4} = \sum_{i=6,11,23,24,26} c_i^{p=4}\mathcal{P}_i. \qquad (40)$$

From Eqs. (38) and (39), we see that the above Lagrangian leads to quartic gauge couplings which do not contain photons. We also see that there are five operators matching five independent Lorentz structures that do not exhibit derivatives. In Ref. [15], we can find the p^4 QGC assuming that there is no light Higgs-like state, and this corresponds to the limit $\mathcal{F}_i \to 1$ in our framework. The correspondence between the Wilson coefficients of our framework and the ones defined in Ref. [15] is

$$\begin{aligned}&\alpha_4 = c_{11}^{p=4}, \qquad \alpha_5 = c_6^{p=4}, \qquad \alpha_6 = c_{24}^{p=4},\\ &\alpha_7 = c_{23}^{p=4}, \qquad \alpha_{10} = c_{26}^{p=4}. \qquad (41)\end{aligned}$$

B. QGC at $\mathcal{O}(p^6)$

At order $\mathcal{O}(p^6)$, there is the emergence of genuine QGC containing photons as well as only four electroweak gauge bosons with two derivatives acting on them. As in Ref. [15], we construct the p^6 operators for QGC combining the $D = 1$, 2, 3, and 4 building blocks defined above. Without loss of generality, we write the corresponding Lagrangian as

$$\mathcal{L}_Q^{p=6} = \sum_i c_i^{p=6}\mathcal{T}_i^{p=6}\mathcal{F}_i^{p=6}(h), \qquad (42)$$

where $\mathcal{T}_i^{p=6}$ are the $\mathcal{O}(p^6)$ operators constructed with the blocks defined above, and we denote by $\mathcal{F}_i^{p=6}(h)$ the corresponding arbitrary function parametrizing the h couplings. As already mentioned, we will be only concerned with the first term of its expansion $\mathcal{F}_i^{p=6}(h) = 1$.

1. $(D=1)^2(D=2)^2$ terms

There are 12 independent operators in this category:

$$\begin{aligned}&\mathcal{T}_1^{p=6} = \mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_2^{p=6} = \mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha],\\ &\mathcal{T}_3^{p=6} = \mathrm{Tr}[T\mathcal{D}^\mu_\mu]\mathrm{Tr}[T\mathcal{D}^\nu_\nu]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_4^{p=6} = \mathrm{Tr}[T\mathcal{D}^\mu_\mu]\mathrm{Tr}[T\mathcal{D}^{\nu\alpha}]\mathrm{Tr}[TV_\nu]\mathrm{Tr}[TV_\alpha],\\ &\mathcal{T}_5^{p=6} = -g^2\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\hat{W}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_6^{p=6} = -g^2\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\hat{W}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha],\\ &\mathcal{T}_7^{p=6} = ig\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_8^{p=6} = -g'^2B_{\mu\nu}B^{\mu\nu}\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha],\\ &\mathcal{T}_9^{p=6} = -g'^2B_{\mu\nu}B^{\mu\alpha}\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{10}^{p=6} = ig'B_{\mu\nu}\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha],\\ &\mathcal{T}_{11}^{p=6} = -gg'B_{\mu\nu}\mathrm{Tr}[T\hat{W}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{12}^{p=6} = -gg'B_{\mu\nu}\mathrm{Tr}[T\hat{W}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha]. \qquad (43)\end{aligned}$$

Notice that all the effective Lagrangians in this class violate $SU(2)_c$ since they contain the T field. In Appendix C we present the relations between these operators and the Lorentz structures that allow us to see that all the operators in this group contain only neutral gauge bosons.

2. $(D=1)(D=2)(D=3)$ terms

This group contains 21 operators that violate $SU(2)_c$ due to the presence of T:

$$\mathcal{T}_{13}^{p=6}=\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\alpha\mathcal{D}^{\mu\nu}],\quad \mathcal{T}_{14}^{p=6}=\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\nu\mathcal{D}^{\mu\alpha}],\quad \mathcal{T}_{15}^{p=6}=\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}],$$
$$\mathcal{T}_{16}^{p=6}=\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\mathcal{D}_\mu^\mu]\mathrm{Tr}[V^\alpha\mathcal{D}_\nu^\nu],\quad \mathcal{T}_{17}^{p=6}=\mathrm{Tr}[TV_\nu]\mathrm{Tr}[T\mathcal{D}_\mu^\mu]\mathrm{Tr}[V_\alpha\mathcal{D}^{\nu\alpha}],\quad \mathcal{T}_{18}^{p=6}=\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\mathcal{D}_{\nu\alpha}]\mathrm{Tr}[V^\alpha\mathcal{D}_\mu^\mu],$$
$$\mathcal{T}_{19}^{p=6}=ig\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\nu\hat{W}^{\mu\alpha}],\quad \mathcal{T}_{20}^{p=6}=ig\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V_\alpha\hat{W}^{\mu\alpha}],\quad \mathcal{T}_{21}^{p=6}=ig\mathrm{Tr}[TV_\nu]\mathrm{Tr}[T\mathcal{D}_\mu^\mu]\mathrm{Tr}[V_\alpha\hat{W}^{\nu\alpha}],$$
$$\mathcal{T}_{22}^{p=6}=ig\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[V^\nu\mathcal{D}^{\mu\alpha}],\quad \mathcal{T}_{23}^{p=6}=ig\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}],\quad \mathcal{T}_{24}^{p=6}=ig\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\hat{W}_{\nu\alpha}]\mathrm{Tr}[V^\alpha\mathcal{D}_\mu^\mu],$$
$$\mathcal{T}_{25}^{p=6}=-g^2\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[V^\alpha\hat{W}^{\mu\nu}],\quad \mathcal{T}_{26}^{p=6}=-g^2\mathrm{Tr}[TV_\alpha]\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[V^\nu\hat{W}^{\mu\alpha}],$$
$$\mathcal{T}_{27}^{p=6}=-g^2\mathrm{Tr}[TV^\nu]\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[V_\alpha\hat{W}^{\mu\alpha}],\quad \mathcal{T}_{28}^{p=6}=ig'\mathrm{Tr}[TV_\alpha]B_{\mu\nu}\mathrm{Tr}[V^\nu\mathcal{D}^{\mu\alpha}],\quad \mathcal{T}_{29}^{p=6}=ig'\mathrm{Tr}[TV^\nu]B_{\mu\nu}\mathrm{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}],$$
$$\mathcal{T}_{30}^{p=6}=ig'\mathrm{Tr}[TV^\nu]B_{\nu\alpha}\mathrm{Tr}[V^\alpha\mathcal{D}_\mu^\mu],\quad \mathcal{T}_{31}^{p=6}=-gg'\mathrm{Tr}[TV_\alpha]B_{\mu\nu}\mathrm{Tr}[V^\alpha\hat{W}^{\mu\nu}],\quad \mathcal{T}_{32}^{p=6}=-gg'\mathrm{Tr}[TV_\alpha]B_{\mu\nu}\mathrm{Tr}[V^\nu\hat{W}^{\mu\alpha}],$$
$$\mathcal{T}_{33}^{p=6}=-gg'\mathrm{Tr}[TV^\nu]B_{\mu\nu}\mathrm{Tr}[V_\alpha\hat{W}^{\mu\alpha}]. \tag{44}$$

From the results presented in Appendix C, we can see that the above operators give rise to W^+W^-ZZ, $ZZZZ$, γZZZ, γZW^+W^-, and $\gamma\gamma ZZ$ anomalous QGC.

3. $(D=3)^2$ terms

We find 11 operators in this class with all of them respecting the custodial symmetry:

$$\mathcal{T}_{34}^{p=6}=\mathrm{Tr}[V_\alpha\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\alpha\mathcal{D}^{\mu\nu}],\qquad \mathcal{T}_{35}^{p=6}=\mathrm{Tr}[V_\alpha\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\nu\mathcal{D}^{\mu\alpha}],\qquad \mathcal{T}_{36}^{p=6}=\mathrm{Tr}[V^\nu\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}],$$
$$\mathcal{T}_{37}^{p=6}=\mathrm{Tr}[V_\alpha\mathcal{D}_\mu^\mu]\mathrm{Tr}[V^\alpha\mathcal{D}_\nu^\nu],\qquad \mathcal{T}_{38}^{p=6}=\mathrm{Tr}[V_\nu\mathcal{D}_\mu^\mu]\mathrm{Tr}[V_\alpha\mathcal{D}^{\nu\alpha}],\qquad \mathcal{T}_{39}^{p=6}=ig\mathrm{Tr}[V_\alpha\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V^\nu\hat{W}^{\mu\alpha}],$$
$$\mathcal{T}_{40}^{p=6}=ig\mathrm{Tr}[V^\nu\mathcal{D}_{\mu\nu}]\mathrm{Tr}[V_\alpha\hat{W}^{\mu\alpha}],\qquad \mathcal{T}_{41}^{p=6}=ig\mathrm{Tr}[V_\nu\mathcal{D}_\mu^\mu]\mathrm{Tr}[V_\alpha\hat{W}^{\nu\alpha}],\qquad \mathcal{T}_{42}^{p=6}=-g^2\mathrm{Tr}[V_\alpha\hat{W}_{\mu\nu}]\mathrm{Tr}[V^\alpha\hat{W}^{\mu\nu}],$$
$$\mathcal{T}_{43}^{p=6}=-g^2\mathrm{Tr}[V_\alpha\hat{W}_{\mu\nu}]\mathrm{Tr}[V^\nu\hat{W}^{\mu\alpha}],\qquad \mathcal{T}_{44}^{p=6}=-g^2\mathrm{Tr}[V^\nu\hat{W}_{\mu\nu}]\mathrm{Tr}[V_\alpha\hat{W}^{\mu\alpha}]. \tag{45}$$

From Appendix C, we can learn that the effective Lagrangians in this class generate $W^+W^-W^+W^-$, W^+W^-ZZ, and $ZZZZ$ quartic vertices, as well as γZW^+W^-, γZZZ, and $\gamma\gamma ZZ$. Because of the custodial symmetry, the last three vertices are multiplied by s_w, therefore vanishing in the custodial-conserving limit $s_w \to 0$.

4. $(D=2)^3$ terms

There are 12 operators in this class:

$$\mathcal{T}_{45}^{p=6}=\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\nu}]\mathrm{Tr}[V^\alpha V_\alpha],\qquad \mathcal{T}_{46}^{p=6}=\mathrm{Tr}[T\mathcal{D}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha],\qquad \mathcal{T}_{47}^{p=6}=\mathrm{Tr}[T\mathcal{D}_\mu^\mu]\mathrm{Tr}[T\mathcal{D}_\nu^\nu]\mathrm{Tr}[V^\alpha V_\alpha],$$
$$\mathcal{T}_{48}^{p=6}=\mathrm{Tr}[T\mathcal{D}_\mu^\mu]\mathrm{Tr}[T\mathcal{D}^{\nu\alpha}]\mathrm{Tr}[V_\nu V_\alpha],\qquad \mathcal{T}_{49}^{p=6}=-g^2\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\hat{W}^{\mu\nu}]\mathrm{Tr}[V^\alpha V_\alpha],$$
$$\mathcal{T}_{50}^{p=6}=-g^2\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\hat{W}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha],\qquad \mathcal{T}_{51}^{p=6}=ig\mathrm{Tr}[T\hat{W}_{\mu\nu}]\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha],$$
$$\mathcal{T}_{52}^{p=6}=-g'^2B_{\mu\nu}B^{\mu\nu}\mathrm{Tr}[V^\alpha V_\alpha],\qquad \mathcal{T}_{53}^{p=6}=-g'^2B_{\mu\nu}B^{\mu\alpha}\mathrm{Tr}[V^\nu V_\alpha],\qquad \mathcal{T}_{54}^{p=6}=ig'B_{\mu\nu}\mathrm{Tr}[T\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha],$$
$$\mathcal{T}_{55}^{p=6}=-gg'B_{\mu\nu}\mathrm{Tr}[T\hat{W}^{\mu\nu}]\mathrm{Tr}[V^\alpha V_\alpha],\qquad \mathcal{T}_{56}^{p=6}=-gg'B_{\mu\nu}\mathrm{Tr}[T\hat{W}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha]. \tag{46}$$

It is interesting to notice that the operators in this group generate QGC among all electroweak gauge bosons except for $W^+W^-W^+W^-$.

5. $(D=2)(D=4)$ terms

This class contains seven operators:

$$\mathcal{T}_{57}^{p=6} = \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\nu}]\mathrm{Tr}[V^\alpha V_\alpha], \qquad \mathcal{T}_{58}^{p=6} = \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha], \qquad \mathcal{T}_{59}^{p=6} = \mathrm{Tr}[\mathcal{D}_\mu^\mu\mathcal{D}_\nu^\nu]\mathrm{Tr}[V^\alpha V_\alpha],$$
$$\mathcal{T}_{60}^{p=6} = \mathrm{Tr}[\mathcal{D}_\mu^\mu\mathcal{D}^{\nu\alpha}]\mathrm{Tr}[V_\nu V_\alpha], \qquad \mathcal{T}_{61}^{p=6} = -g^2\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[V^\alpha V_\alpha], \qquad \mathcal{T}_{62}^{p=6} = -g^2\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha],$$
$$\mathcal{T}_{63}^{p=6} = ig\mathrm{Tr}[\hat{W}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[V^\nu V_\alpha]. \tag{47}$$

These operators are $SU(2)_c$ invariant in the limit $s_w \to 0$, and this can be seen by their expression in terms of Lorentz structures presented in Appendix C. This class of Lagrangians generates the following QGC: W^+W^-ZZ, $ZZZZ$, γZW^+W^-, $\gamma\gamma W^+W^-$, γZZZ, and $\gamma\gamma ZZ$.

6. $(D=1)^2(D=4)$ terms

We find seven operators in this class that violate the custodial symmetry:

$$\mathcal{T}_{64}^{p=6} = \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{65}^{p=6} = \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{66}^{p=6} = \mathrm{Tr}[\mathcal{D}_\mu^\mu\mathcal{D}_\nu^\nu]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha],$$
$$\mathcal{T}_{67}^{p=6} = \mathrm{Tr}[\mathcal{D}_\mu^\mu\mathcal{D}^{\nu\alpha}]\mathrm{Tr}[TV_\nu]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{68}^{p=6} = -g^2\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]\mathrm{Tr}[TV^\alpha]\mathrm{Tr}[TV_\alpha],$$
$$\mathcal{T}_{69}^{p=6} = -g^2\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha], \qquad \mathcal{T}_{70}^{p=6} = ig\mathrm{Tr}[\hat{W}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^\nu]\mathrm{Tr}[TV_\alpha]. \tag{48}$$

As we can see in Appendix C, this group of effective Lagrangians gives rise to W^+W^-ZZ, $ZZZZ$, γZZZ, and $\gamma\gamma ZZ$ QGC.

Altogether we find 70 independent operators leading to genuine QGC in the chiral Lagrangian at $\mathcal{O}(p^6)$, so there are as many operators as independent Lorentz structures containing two derivatives. As mentioned above, this was also the case at $\mathcal{O}(p^4)$. This is somehow not unexpected; as is well known [41], a generic $U(1)_{\mathrm{em}}$ invariant Lagrangian, which is the only symmetry imposed in building the Lorentz structures, is also invariant under nonlinear $SU(2)_L \otimes U(1)_Y$ transformations.

V. SUMMARY

In this work, we have constructed the most general form of the QGC containing up to two derivatives acting on the electroweak gauge-boson fields. We have shown that there are 5 independent Lorentz structures that respect the $U(1)_{\mathrm{em}}$ symmetry and contain no derivatives while there are 70 structures exhibiting two derivatives.

We then derived which of these QGC are generated assuming that the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is linearly realized, as a characteristic of scenarios with a fundamental Higgs doublet. In this case, the lowest dimension that presents QGC without a TGC associated to them is 8. In these scenarios, there are only three operators that contain only massive gauge bosons and no derivative acting on them; see Eq. (16). So, because of the linear realization of the symmetry, the Wilson coefficients of the five Lorentz structures that contain no derivatives are correlated—independently of the basis of operators used; see as an example the last line in Eq. (17). In the same framework, we find seven operators containing genuine QGC with two derivatives, and they generate only 23 of the 70 possible Lorentz structures. So, again, gauge invariance in the linear realization implies correlations among the coefficients of the different Lorentz structures, as, for example, those in Eq. (20).

We also classified the quartic gauge-boson interactions, assuming that the $SU(2)_L \otimes U(1)_Y$ symmetry is realized nonlinearly with the global-symmetry breaking $SU(2)_L \otimes SU(2)_R \to SU(2)_c$, characteristic of scenarios with a light dynamical Higgs boson. At order $\mathcal{O}(p^4)$, there are five chiral operators which generate QGC without an associated TQC. They contain only $W^\pm$ and Z and no derivatives. There are 70 independent operators at order $\mathcal{O}(p^6)$, and they contain four gauge bosons and two derivatives. That is, the chiral Lagrangian for genuine QGC contains the same number of operators as independent Lorentz structures. So, no basis-independent correlation can be derived between the coefficients of the Lorentz structures in this case.

At present, the most sensitive searches for quartic gauge-boson couplings are those involving vertices with two photons. Most of the analyses carried out by the LEP [28,29], D0 [30], and LHC [24,27] collaborations used the following effective Lagrangian to study the two-photon sector [32],

$$\mathcal{L}_{\rm eff} = -\frac{\alpha_{\rm em}\pi}{2}\frac{a_0^W}{\Lambda^2}\mathcal{Q}^{\partial=2}_{\gamma W,1} - \frac{\alpha_{\rm em}\pi}{2}\frac{a_c^W}{\Lambda^2}\mathcal{Q}^{\partial=2}_{\gamma W,2} - \frac{\alpha_{\rm em}\pi}{2c_w^2}\frac{a_0^Z}{\Lambda^2}\mathcal{Q}^{\partial=2}_{\gamma Z,1} - \frac{\alpha_{\rm em}\pi}{2c_w^2}\frac{a_c^Z}{\Lambda^2}\mathcal{Q}^{\partial=2}_{\gamma Z,2}, \tag{49}$$

where $\alpha_{\rm em}$ stands for the electromagnetic fine-structure constant. In the framework of electroweak gauge invariance linearly realized, the seven operators in Eq. (18) give rise to QGC containing two photons which in the notation in Eq. (49) read

$$\begin{aligned}
a_0^W &= -\frac{M_W^2}{\pi\alpha_{\rm em}}\left[s_w^2\frac{f_{M,0}}{\Lambda^2} + 2c_w^2\frac{f_{M,2}}{\Lambda^2} + s_w c_w\frac{f_{M,4}}{\Lambda^2}\right],\\
a_c^W &= -\frac{M_W^2}{\pi\alpha_{\rm em}}\left[-s_w^2\frac{f_{M,1}}{\Lambda^2} - c_w^2\frac{f_{M,3}}{\Lambda^2} + 2s_w c_w\frac{f_{M,5}}{\Lambda^2} + \frac{s_w^2}{2}\frac{f_{M,7}}{\Lambda^2}\right],\\
a_0^Z &= -\frac{M_W^2 c_w^2}{\pi\alpha_{\rm em}}\left[\frac{s_w^2}{2c_w^2}\frac{f_{M,0}}{\Lambda^2} + \frac{f_{M,2}}{\Lambda^2} - \frac{s_w}{2c_w}\frac{f_{M,4}}{\Lambda^2}\right],\\
a_c^Z &= -\frac{M_W^2 c_w^2}{\pi\alpha_{\rm em}}\left[-\frac{s_w^2}{2c_w^2}\frac{f_{M,1}}{\Lambda^2} - \frac{1}{2}\frac{f_{M,3}}{\Lambda^2} - \frac{s_w}{c_w}\frac{f_{M,5}}{\Lambda^2} + \frac{s_w^2}{4c_w^2}\frac{f_{M,7}}{\Lambda^2}\right].
\end{aligned} \tag{50}$$

So, even in the scenario with the linear realization of the gauge invariance, these four coefficients can be fully uncorrelated. A test of the presence/absence of the correlations which can point toward an underlaying linear or chiral expansion will require the measurement with accuracy equivalent to quartic vertices involving one or zero photons, consequently requiring much more data.

At this point, it is interesting to a have an idea of the LHC potential to constrain the genuine quartic couplings. For instance, Ref. [38] studied the vector-boson fusion (VBF) production of W^+W^- and $W^\pm W^\pm$ pairs and obtained the prospective 95% C.L. limits on the p^4 QCG,

$$-0.0045 < c_6^{p=4} < 0.0055 \quad \text{and} \quad -0.0022 < c_{11}^{p=4} < 0.0027, \tag{51}$$

for a center-of-mass energy of 14 TeV and an integrated luminosity of 300 fb^{-1}. Moreover, Ref. [31] analyzed some of the p^6 QGC containing photons ($c^{p=6}_{8,9,31,32,33,42,43,44,52,53,55,56,61,62}$) through the study of the VBF production of $\gamma\gamma$ and γe^+e^-. The typical 95% C.L. limits on the modulus of these couplings are in the range 1.2–6.3 TeV^{-2} for a center-of-mass energy of 14 TeV and an integrated luminosity of 100 fb^{-1}.

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APPENDIX A: RELATIONS BETWEEN LINEAR OPERATORS AND LORENTZ STRUCTURES

In the framework where the SM gauge symmetry is realized linearly, the genuine quartic gauge couplings generated by the dimension-8 operators with two derivatives listed in Sec. III can be expressed in terms of the Lorentz structures defined in Sec. II as

$$\mathcal{O}_{M,0} = \frac{g^2v^2}{8}\left[2\mathcal{Q}^{\partial=2}_{WW,1} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,1} + c_w^2\mathcal{Q}^{\partial=2}_{WZ,6} + 2s_wc_w\mathcal{Q}^{\partial=2}_{\gamma ZW,1} + s_w^2\mathcal{Q}^{\partial=2}_{\gamma W,1} + \frac{1}{2}\mathcal{Q}^{\partial=2}_{ZZ,1} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + \frac{s_w^2}{2c_w^2}\mathcal{Q}^{\partial=2}_{\gamma Z,1}\right] \tag{A1}$$

$$\mathcal{O}_{M,1} = \frac{-g^2v^2}{8}\left[\mathcal{Q}^{\partial=2}_{WW,2} + \mathcal{Q}^{\partial=2}_{WW,3} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,2} + c_w^2\mathcal{Q}^{\partial=2}_{WZ,7} + s_wc_w\mathcal{Q}^{\partial=2}_{\gamma ZW,2} + s_w^2\mathcal{Q}^{\partial=2}_{\gamma W,2} + \frac{1}{2}\mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}^{\partial=2}_{\gamma Z,2}\right] \tag{A2}$$

$$\mathcal{O}_{M,2} = \frac{g^2v^2}{4}\left[c_w^2\mathcal{Q}^{\partial=2}_{\gamma W,1} - 2s_wc_w\mathcal{Q}^{\partial=2}_{\gamma ZW,1} + s_w^2\mathcal{Q}^{\partial=2}_{WZ,6} + \frac{1}{2}\mathcal{Q}^{\partial=2}_{\gamma Z,1} - \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + \frac{s_w^2}{2c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,1}\right] \tag{A3}$$

$$\mathcal{O}_{M,3} = \frac{-g^2v^2}{8}\left[c_w^2\mathcal{Q}^{\partial=2}_{\gamma W,2} - s_wc_w\mathcal{Q}^{\partial=2}_{\gamma ZW,2} + s_w^2\mathcal{Q}^{\partial=2}_{WZ,7} + \frac{1}{2}\mathcal{Q}^{\partial=2}_{\gamma Z,2} - \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,2}\right] \tag{A4}$$

$$\mathcal{O}_{M,4} = \frac{g^2v^2}{8}\left[(c_w^2 - s_w^2)\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,1} - \frac{1}{2c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,1}\right) - \mathcal{Q}^{\partial=2}_{\gamma ZW,3} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{WZ,3} + s_wc_w\left(\mathcal{Q}^{\partial=2}_{\gamma W,1} - \mathcal{Q}^{\partial=2}_{WZ,6} - \frac{1}{2c_w^2}\left(\mathcal{Q}^{\partial=2}_{\gamma Z,1} - \mathcal{Q}^{\partial=2}_{ZZ,1}\right)\right)\right] \tag{A5}$$

$$\mathcal{O}_{M,5} = \frac{g^2 v^2}{8}\left[(c_w^2 - s_w^2)\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,2} - \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}\right) - \mathcal{Q}^{\partial=2}_{\gamma ZW,4} - \mathcal{Q}^{\partial=2}_{\gamma ZW,5} + \frac{s_w}{c_w}(\mathcal{Q}^{\partial=2}_{WZ,4} + \mathcal{Q}^{\partial=2}_{WZ,5}) + s_w c_w (2(\mathcal{Q}^{\partial=2}_{\gamma W,2} - \mathcal{Q}^{\partial=2}_{WZ,7}) - \frac{1}{c_w^2}(\mathcal{Q}^{\partial=2}_{\gamma Z,2} - \mathcal{Q}^{\partial=2}_{ZZ,2})\right] \quad \text{(A6)}$$

$$\mathcal{O}_{M,7} = \frac{g^2 v^2}{16}\left[2\mathcal{Q}^{\partial=2}_{WW,3} + c_w^2 \mathcal{Q}^{\partial=2}_{WZ,7} + s_w c_w \mathcal{Q}^{\partial=2}_{\gamma ZW,2} + s_w^2 \mathcal{Q}^{\partial=2}_{\gamma W,2} - (\mathcal{Q}^{\partial=2}_{WZ,5} - \mathcal{Q}^{\partial=2}_{WZ,4}) - \frac{s_w}{c_w}(\mathcal{Q}^{\partial=2}_{\gamma ZW,5} - \mathcal{Q}^{\partial=2}_{\gamma ZW,4}) + \frac{1}{2}\mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}^{\partial=2}_{\gamma Z,2} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,2}\right]. \quad \text{(A7)}$$

APPENDIX B: DIMENSION-8 OPERATORS CONTAINING FOUR FIELD STRENGTH TENSORS

There are eight operators containing just field strength tensors that lead to genuine quartic anomalous couplings, which in the notation used in Ref. [21] are

$$\begin{aligned}
\mathcal{O}_{T,0} &= \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \times \mathrm{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}],\\
\mathcal{O}_{T,1} &= \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}] \times \mathrm{Tr}[\hat{W}_{\mu\beta}\hat{W}^{\alpha\nu}]\\
\mathcal{O}_{T,2} &= \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}] \times \mathrm{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\nu\alpha}],\\
\mathcal{O}_{T,5} &= \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \times B_{\alpha\beta}B^{\alpha\beta}\\
\mathcal{O}_{T,6} &= \mathrm{Tr}[\hat{W}_{\alpha\nu}\hat{W}^{\mu\beta}] \times B_{\mu\beta}B^{\alpha\nu},\\
\mathcal{O}_{T,7} &= \mathrm{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\mu\beta}] \times B_{\beta\nu}B^{\nu\alpha}\\
\mathcal{O}_{T,8} &= B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta},\\
\mathcal{O}_{T,9} &= B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha}.
\end{aligned} \quad \text{(B1)}$$

APPENDIX C: RELATIONS BETWEEN $\mathcal{O}(p^6)$ OPERATORS AND LORENTZ STRUCTURES

Expanding the 70 $\mathcal{O}(p^6)$ operators introduced in Sec. IV in terms of the Lorentz structures in Sec. II B, we find

$$\mathcal{T}_1^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,3} \quad \text{(C1)}$$

$$\mathcal{T}_2^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,4} \quad \text{(C2)}$$

$$\mathcal{T}_3^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,5} \quad \text{(C3)}$$

$$\mathcal{T}_4^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,6} \quad \text{(C4)}$$

$$\mathcal{T}_5^{p=6} = \frac{g^4}{c_w^2}[c_w^2\mathcal{Q}^{\partial=2}_{ZZ,1} + 2s_w c_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + s_w^2 \mathcal{Q}^{\partial=2}_{\gamma Z,1}] \quad \text{(C5)}$$

$$\mathcal{T}_6^{p=6} = \frac{g^4}{c_w^2}[c_w^2\mathcal{Q}^{\partial=2}_{ZZ,2} + 2s_w c_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + s_w^2 \mathcal{Q}^{\partial=2}_{\gamma Z,2}] \quad \text{(C6)}$$

$$\mathcal{T}_7^{p=6} = \frac{g^4}{c_w^3}[c_w\mathcal{Q}^{\partial=2}_{ZZ,7} + s_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,3}] \quad \text{(C7)}$$

$$\mathcal{T}_8^{p=6} = \frac{g^2 g'^2}{c_w^2}[s_w^2\mathcal{Q}^{\partial=2}_{ZZ,1} - 2s_w c_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + c_w^2 \mathcal{Q}^{\partial=2}_{\gamma Z,1}] \quad \text{(C8)}$$

$$\mathcal{T}_9^{p=6} = \frac{g^2 g'^2}{c_w^2}[s_w^2\mathcal{Q}^{\partial=2}_{ZZ,2} - 2s_w c_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + c_w^2 \mathcal{Q}^{\partial=2}_{\gamma Z,2}] \quad \text{(C9)}$$

$$\mathcal{T}_{10}^{p=6} = \frac{g^3 g'}{c_w^3}[-s_w\mathcal{Q}^{\partial=2}_{ZZ,7} + c_w \mathcal{Q}^{\partial=2}_{\gamma ZZ,3}] \quad \text{(C10)}$$

$$\mathcal{T}_{11}^{p=6} = \frac{g^3 g'}{c_w^2}[c_w s_w(-\mathcal{Q}^{\partial=2}_{ZZ,1} + \mathcal{Q}^{\partial=2}_{\gamma Z,1}) + (c_w^2 - s_w^2)\mathcal{Q}^{\partial=2}_{\gamma ZZ,1}] \quad \text{(C11)}$$

$$\mathcal{T}_{12}^{p=6} = \frac{g^3 g'}{c_w^2}[c_w s_w(-\mathcal{Q}^{\partial=2}_{ZZ,2} + \mathcal{Q}^{\partial=2}_{\gamma Z,2}) + (c_w^2 - s_w^2)\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}] \quad \text{(C12)}$$

$$\mathcal{T}_{13}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,12} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,3}\right] \quad \text{(C13)}$$

$$\mathcal{T}_{14}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,14} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,4}\right] \quad \text{(C14)}$$

$$\mathcal{T}_{15}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,13} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,4}\right] \quad \text{(C15)}$$

$$\mathcal{T}_{16}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,15} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,5}\right] \quad \text{(C16)}$$

$$\mathcal{T}_{17}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,17} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,6}\right] \quad \text{(C17)}$$

$$\mathcal{T}_{18}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,16} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,6}\right] \quad \text{(C18)}$$

$$\mathcal{T}_{19}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,23} + \mathcal{Q}^{\partial=2}_{ZZ,7} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right] \quad \text{(C19)}$$

$$\mathcal{T}_{20}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}^{\partial=2}_{WZ,24} + \mathcal{Q}^{\partial=2}_{ZZ,7} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right] \quad \text{(C20)}$$

$$\mathcal{T}_{21}^{p=6} = \frac{g^4}{2c_w^2}\,\mathcal{Q}^{\partial=2}_{WZ,25} \tag{C21}$$

$$\mathcal{T}_{22}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}^{\partial=2}_{WZ,26} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,7}\right) + s_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,7} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right)\right] \tag{C22}$$

$$\mathcal{T}_{23}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}^{\partial=2}_{WZ,27} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,7}\right) + s_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,8} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right)\right] \tag{C23}$$

$$\mathcal{T}_{24}^{p=6} = \frac{g^4}{2c_w}[c_w\mathcal{Q}^{\partial=2}_{WZ,28} + s_w\mathcal{Q}^{\partial=2}_{\gamma ZW,9}] \tag{C24}$$

$$\mathcal{T}_{25}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}^{\partial=2}_{WZ,3} + \mathcal{Q}^{\partial=2}_{ZZ,1} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,1}\right) + s_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,3} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,1}\right)\right] \tag{C25}$$

$$\mathcal{T}_{26}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}^{\partial=2}_{WZ,4} + \mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}\right) + s_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,4} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,2}\right)\right] \tag{C26}$$

$$\mathcal{T}_{27}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}^{\partial=2}_{WZ,5} + \mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}\right) + s_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,5} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,2}\right)\right] \tag{C27}$$

$$\mathcal{T}_{28}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\left(\mathcal{Q}^{\partial=2}_{WZ,26} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,7}\right) + c_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,7} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right)\right] \tag{C28}$$

$$\mathcal{T}_{29}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\left(\mathcal{Q}^{\partial=2}_{WZ,27} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{ZZ,7}\right) + c_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,8} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=2}_{\gamma ZZ,3}\right)\right] \tag{C29}$$

$$\mathcal{T}_{30}^{p=6} = \frac{g^3g'}{2c_w}[-s_w\mathcal{Q}^{\partial=2}_{WZ,28} + c_w\mathcal{Q}^{\partial=2}_{\gamma ZW,9}] \tag{C30}$$

$$\mathcal{T}_{31}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\left(\mathcal{Q}^{\partial=2}_{WZ,3} + \mathcal{Q}^{\partial=2}_{ZZ,1} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,1}\right) + c_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,3} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,1} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,1}\right)\right] \tag{C31}$$

$$\mathcal{T}_{32}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\left(\mathcal{Q}^{\partial=2}_{WZ,4} + \mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}\right) + c_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,4} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,2}\right)\right] \tag{C32}$$

$$\mathcal{T}_{33}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\left(\mathcal{Q}^{\partial=2}_{WZ,5} + \mathcal{Q}^{\partial=2}_{ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma ZZ,2}\right) + c_w\left(\mathcal{Q}^{\partial=2}_{\gamma ZW,5} + \mathcal{Q}^{\partial=2}_{\gamma ZZ,2} + \frac{s_w}{c_w}\mathcal{Q}^{\partial=2}_{\gamma Z,2}\right)\right] \tag{C33}$$

$$\mathcal{T}_{34}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}^{\partial=2}_{WW,11} + 2\mathcal{Q}^{\partial=2}_{WW,6} + \frac{2}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,12} + \frac{1}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,3}\right] \tag{C34}$$

$$\mathcal{T}_{35}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}^{\partial=2}_{WW,12} + \mathcal{Q}^{\partial=2}_{WW,7} + \mathcal{Q}^{\partial=2}_{WW,8} + \frac{2}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,14} + \frac{1}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,4}\right] \tag{C35}$$

$$\mathcal{T}_{36}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}^{\partial=2}_{WW,12} + 2\mathcal{Q}^{\partial=2}_{WW,7} + \frac{2}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,13} + \frac{1}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,4}\right] \tag{C36}$$

$$\mathcal{T}_{37}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}^{\partial=2}_{WW,13} + 2\mathcal{Q}^{\partial=2}_{WW,9} + \frac{2}{c_w^2}\mathcal{Q}^{\partial=2}_{WZ,15} + \frac{1}{c_w^4}\mathcal{Q}^{\partial=2}_{ZZ,5}\right] \tag{C37}$$

$$\mathcal{T}_{38}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,14}^{\partial=2} + \mathcal{Q}_{WW,10}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,16}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,17}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C38}$$

$$\mathcal{T}_{39}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,18}^{\partial=2} + \mathcal{Q}_{WW,16}^{\partial=2} + \mathcal{Q}_{WZ,26}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,23}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma ZW,7}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C39}$$

$$\mathcal{T}_{40}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,18}^{\partial=2} + \mathcal{Q}_{WW,15}^{\partial=2} + \mathcal{Q}_{WZ,27}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,24}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma ZW,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C40}$$

$$\mathcal{T}_{41}^{p=6} = \frac{g^4}{4}\left[-\mathcal{Q}_{WW,17}^{\partial=2} - \mathcal{Q}_{WZ,28}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,25}^{\partial=2} - \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,9}^{\partial=2}\right] \tag{C41}$$

$$\mathcal{T}_{42}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,1}^{\partial=2} + \mathcal{Q}_{WW,4}^{\partial=2} + 2\mathcal{Q}_{WZ,3}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,3}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C42}$$

$$\mathcal{T}_{43}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,5}^{\partial=2} + 2\mathcal{Q}_{WW,3}^{\partial=2} + 2\mathcal{Q}_{WZ,4}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,4}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C43}$$

$$\mathcal{T}_{44}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,5}^{\partial=2} + 2\mathcal{Q}_{WW,2}^{\partial=2} + 2\mathcal{Q}_{WZ,5}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,5}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C44}$$

$$\mathcal{T}_{45}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,18}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C45}$$

$$\mathcal{T}_{46}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,19}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C46}$$

$$\mathcal{T}_{47}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,20}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C47}$$

$$\mathcal{T}_{48}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,21}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C48}$$

$$\mathcal{T}_{49}^{p=6} = \frac{g^4}{2}\left[2c_w^2\mathcal{Q}_{WZ,6}^{\partial=2} + 4s_wc_w\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,1}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C49}$$

$$\mathcal{T}_{50}^{p=6} = \frac{g^4}{2}\left[2c_w^2\mathcal{Q}_{WZ,7}^{\partial=2} + 2s_wc_w\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,2}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C50}$$

$$\mathcal{T}_{51}^{p=6} = \frac{g^4}{2c_w}\left[c_w\mathcal{Q}_{WZ,29}^{\partial=2} + \frac{1}{c_w}\mathcal{Q}_{ZZ,7}^{\partial=2} + s_w\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{s_w}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C51}$$

$$\mathcal{T}_{52}^{p=6} = \frac{g^2g'^2}{2}\left[2s_w^2\mathcal{Q}_{WZ,6}^{\partial=2} - 4s_wc_w\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2c_w^2\mathcal{Q}_{\gamma W,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{ZZ,1}^{\partial=2} - 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C52}$$

$$\mathcal{T}_{53}^{p=6} = \frac{g^2g'^2}{2}\left[2s_w^2\mathcal{Q}_{WZ,7}^{\partial=2} - 2s_wc_w\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2c_w^2\mathcal{Q}_{\gamma W,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{ZZ,2}^{\partial=2} - 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C53}$$

$$\mathcal{T}_{54}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\mathcal{Q}_{WZ,29}^{\partial=2} - \frac{s_w}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + c_w\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{1}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C54}$$

$$\mathcal{T}_{55}^{p=6} = \frac{g^3g'}{2}\left[2(c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2s_wc_w(\mathcal{Q}_{\gamma W,1}^{\partial=2} - \mathcal{Q}_{WZ,6}^{\partial=2}) + \frac{(c_w^2 - s_w^2)}{c_w^2}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w}{c_w}(\mathcal{Q}_{\gamma Z,1}^{\partial=2} - \mathcal{Q}_{ZZ,1}^{\partial=2})\right] \tag{C55}$$

$$\mathcal{T}_{56}^{p=6} = \frac{g^3 g'}{2}\left[(c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2 s_w c_w (\mathcal{Q}_{\gamma W,2}^{\partial=2} - \mathcal{Q}_{WZ,7}^{\partial=2}) + \frac{(c_w^2 - s_w^2)}{c_w^2}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}(\mathcal{Q}_{\gamma Z,2}^{\partial=2} - \mathcal{Q}_{ZZ,2}^{\partial=2})\right] \tag{C56}$$

$$\mathcal{T}_{57}^{p=6} = \frac{g^4}{4}\left[4\mathcal{Q}_{WW,6}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,8}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,18}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C57}$$

$$\mathcal{T}_{58}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,7}^{\partial=2} + 2\mathcal{Q}_{WW,8}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,9}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,19}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C58}$$

$$\mathcal{T}_{59}^{p=6} = \frac{g^4}{4}\left[4\mathcal{Q}_{WW,9}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,10}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,20}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C59}$$

$$\mathcal{T}_{60}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,10}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,11}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,21}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C60}$$

$$\begin{aligned}\mathcal{T}_{61}^{p=6} = \frac{g^4}{4}\Big[&4\mathcal{Q}_{WW,1}^{\partial=2} + 2c_w^2\mathcal{Q}_{WZ,6}^{\partial=2} + 4 s_w c_w \mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2 s_w^2 \mathcal{Q}_{\gamma W,1}^{\partial=2} \\ &+ \frac{2}{c_w^2}\mathcal{Q}_{WZ,1}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + \frac{2s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\Big]\end{aligned} \tag{C61}$$

$$\begin{aligned}\mathcal{T}_{62}^{p=6} = \frac{g^4}{4}\Big[&2\mathcal{Q}_{WW,2}^{\partial=2} + 2\mathcal{Q}_{WW,3}^{\partial=2} + 2c_w^2\mathcal{Q}_{WZ,7}^{\partial=2} + 2 s_w c_w \mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2 s_w^2 \mathcal{Q}_{\gamma W,2}^{\partial=2} \\ &+ \frac{2}{c_w^2}\mathcal{Q}_{WZ,2}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{2s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\Big]\end{aligned} \tag{C62}$$

$$\mathcal{T}_{63}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,15}^{\partial=2} + \mathcal{Q}_{WW,16}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,22}^{\partial=2} + \mathcal{Q}_{WZ,29}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{s_w}{c_w^3}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C63}$$

$$\mathcal{T}_{64}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C64}$$

$$\mathcal{T}_{65}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,9}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C65}$$

$$\mathcal{T}_{66}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,10}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C66}$$

$$\mathcal{T}_{67}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,11}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C67}$$

$$\mathcal{T}_{68}^{p=6} = \frac{g^4}{2c_w^2}[2\mathcal{Q}_{WZ,1}^{\partial=2} + c_w^2\mathcal{Q}_{ZZ,1}^{\partial=2} + 2 s_w c_w \mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + s_w^2 \mathcal{Q}_{\gamma Z,1}^{\partial=2}] \tag{C68}$$

$$\mathcal{T}_{69}^{p=6} = \frac{g^4}{2c_w^2}[2\mathcal{Q}_{WZ,2}^{\partial=2} + c_w^2\mathcal{Q}_{ZZ,2}^{\partial=2} + 2 s_w c_w \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + s_w^2 \mathcal{Q}_{\gamma Z,2}^{\partial=2}] \tag{C69}$$

$$\mathcal{T}_{70}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,22}^{\partial=2} + \mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right]. \tag{C70}$$

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