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Graviton Kaluza-Klein modes in nonflat branes with stabilized modulus

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We consider a generalized two brane Randall-Sundrum model where the branes are endowed with nonzero cosmological constant. In this scenario, we re-examine the modulus stabilization mechanism and the nature of Kaluza-Klein (KK) graviton modes. Our result reveals that while the KK mode graviton masses may change significantly with the brane cosmological constant, the Goldberger-Wise stabilization mechanism, which assumes a negligible backreaction on the background metric, continues to hold even when the branes have a large cosmological constant. The possibility of having a global minimum for the modulus is also discussed. Our results also include an analysis for the radion mass in this nonflat brane scenario.

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I. INTRODUCTION

The gauge hierarchy problem continues to be an unsolved issue in the standard model of elementary particles despite its enormous success in describing physics up to the TeV scale. A solution to the gauge hierarchy problem was proposed by Randall and Sundrum (RS model) by considering an extra dimension compactified into a circle S^1 with Z_2 orbifolding [1]. The modulus corresponding to the radius of the extra dimension in such a model can be stabilized via the Goldberger-Wise (GW) stabilization mechanism [2]. Both RS and GW models do not invoke any intermediate scale in the theory and are robust against radiative corrections. This resulted in a large volume of work in particle phenomenology and cosmology in the backdrop of the Randall-Sundrum warped geometry scenario. In the context of collider physics, a possible role of the first Kaluza-Klein (KK) graviton mode has been extensively studied in ATLAS and CMS detectors at LHC, which have already set a stringent lower bound for the mass of the first KK graviton to be $m_{n=1} \sim 2.5$ TeV [3,4]. Various implications of this have been discussed in [5–11].

There have been several efforts to formulate some variants of the original RS model. One such effort addresses a similar warped geometry model with nonflat 3-branes in contrast to the original RS model, which assumes two flat 3-branes sitting at the two orbifold fixed points. It has been shown in [12] that one can indeed generalize the model with a nonzero cosmological constant on the visible 3-brane, i.e., on our observable Universe, and can resolve the gauge hierarchy problem concomitantly. In this generalized RS model [13,14], it has been shown that the 3-branes can be either de Sitter (dS) or anti-de Sitter (AdS) where the magnitude of the induced cosmological constant and that of the warping parameter are intimately connected. It is therefore crucially important to determine whether in such nonflat warped geometry models the Goldberger-Wise stabilization mechanism, which neglects the backreaction of the stabilizing field, can still be employed successfully to stabilize the radius of the extra dimension to the desired value $\sim M_{\rm Pl}^{-1}$. However, the fluctuation of the radius around the stable value, namely, radion dynamics [15–17] may be significantly influenced by the brane cosmological constant. Moreover, it is worth while to explore the effect of the brane cosmological constant on the masses of the graviton KK modes, which are expected to play important roles in high energy scattering processes. This work is focussed on addressing the following three questions in a generalized RS model:

(1) Can the modulus of an extra dimension be stabilized to a global minimum for the entire range of values of the cosmological constant in the context of the generalized RS model?
(2) What are the KK graviton masses for different choices of the cosmological constant?
(3) What is the effect of the brane cosmological constant on the mass of the radion in this modified brane-world scenario?

After a brief review of the original and generalized RS model in the first two sections, we focus on the modulus stabilization conditions as well as the expressions for the modified KK graviton masses and radion mass due to the presence of the nonvanishing brane cosmolgical constant.

II. RANDALL-SUNDRUM MODEL

In the RS scenario, it is predicted that there exists an extra spatial dimension in addition to the $(3+1)$ dimensional observed Universe. The corresponding five-dimensional bulk spacetime is described by a metric

$$ds^2 = \exp(-2kr|\phi|)\eta_{\mu\nu}dx^\mu dx^\nu + r^2 d\phi^2, \qquad (1)$$

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where Greek indices μ and ν run over 0, 1, 2, and 3 and refer to the four observed dimensions. The geometry of the extra dimension is S^1/Z_2 and is described by the angular coordinate "ϕ. Here, the circle S^1 has radius r, and Λ is the bulk cosmological constant, $k = \sqrt{-\Lambda/12M^3}$. The factor $\exp(-2k|y|)$ is known as the warp factor. The $\phi =$ constant slices at $\phi = 0$ and at $\phi = \pi$ are known as the hidden and the visible branes, the observable Universe being identified with the latter which has a negative brane tension as opposed to the hidden brane with a positive brane tension. It can be shown that a mass parameter m_0 of the order of Planck scale is warped to a value TeV on the visible brane following the relation $m = m_0 \exp(-kr\pi)$ for $kr \sim 11.6$. Thus, in this picture, the stability of the Higgs mass against a large radiative correction is ensured by the warped geometry of the five-dimensional spacetime. In this context, the KK graviton mass modes are determined by considering a small fluctuation around the flat metric with its KK decomposition. Some of these modes have masses $m_{n=0} = 0$, $m_{n=1} = 0.383$ TeV, and $m_{n=2} = 0.702$ TeV for $k/M_{pl} \sim 0.1$ [5]. The requirement of $k < M_{pl}$ emerges from the fact that k, which measures the bulk curvature, must be smaller than the Planck scale so that the classical solutions of Einstein's equations in the bulk can be trusted [5].

In the context of modulus stabilization, it was proposed by Goldberger and Wise that the modulus of the model (i.e., the radius of the extra dimension) can be stabilized to the desired value by introducing a massive scalar field in the bulk. Evaluating the effective modulus potential due to the massive scalar field of mass m, one gets the stabilization condition as $kr = m^2/(4\pi k^2)\ln(v_h/v_v)$, where v_h/v_v is the ratio of the vacuum expectation values (vev) of the scalar field on the hidden and visible brane. Taking $v_h/v_v \sim 1.45$ and $m/k \sim 0.2$, one gets $kr \sim 11.6$. In this analysis, it was further shown that both v_h and v_v (in Planckian units) must be smaller than unity so that the effect of the backreaction on the background metric can be ignored. Moreover, the condition of having a global minimum for the modulus potential was found to be $\delta V_v < kv_v^2$, where δV_v is a perturbation on the visible brane tension.

III. GENERALIZED RANDALL-SUNDRUM MODEL

Present cosmological observations indicate the possible existence of a four-dimensional cosmological constant ($\sim 10^{-124}$) in Planckian units. It has been demonstrated [12] that by relaxing the condition of the zero cosmological constant (i.e., flat 3-brane), it is possible to obtain a more general expression for the warp factor. Starting from a general metric ansatz,

$$ds^2 = \exp[-2A(\phi)]g_{\mu\nu}dx^\mu dx^\nu + r^2 d\phi^2, \tag{2}$$

one may solve the bulk equations for both anti-de Sitter (AdS) and de Sitter (dS) 3-branes. The corresponding warp factor for the AdS brane is

$$\exp[-A(\phi)] = \omega\cosh[\ln(\omega/c_1) + kr\phi], \tag{3}$$

with $c_1 = 1 + \sqrt{1-\omega^2}$ and $\omega^2 = -\Omega/(3k^2)$, while that for the dS brane is

$$\exp[-A(\phi)] = \omega\sinh[\ln(c_2/\omega) - kr\phi], \tag{4}$$

with $c_2 = 1 + \sqrt{1+\omega^2}$ and $\omega^2 = \Omega/(3k^2)$. Here, Ω is the brane cosmological constant and ω^2 is a dimensionless parameter. Just as in the original RS model, this generalized scenario also can address the gauge hierarchy problem for appropriate choices of the parameters, which we discuss below.

The scalar mass on the visible brane [18] gets warped through the warp factor. In order to resolve the gauge hierarchy problem, it must satisfy $\exp[-A(\pi r)] = 10^{-16} = m/m_0$. This leads to

$$\exp[-k\pi r] = (10^{-16}/c_1)\left[1 + \sqrt{1-\omega^2 10^{32}}\right] \tag{5}$$

for the AdS case and

$$\exp[-k\pi r] = (10^{-16}/c_2)\left[1 + \sqrt{1+\omega^2 10^{32}}\right] \tag{6}$$

for the dS case.

From the above two relations one can say that (1) the real solution of $k\pi r$ exists, which resolves the hierarchy problem, and (2) the warping parameter kr depends on the cosmological constant. In the following section, we employ the GW stabilization mechanism for the generalized RS model with nonflat branes to derive the new stability condition.

A. Modulus stabilization for nonflat branes

To stabilize the modulus r in the context of the generalized RS model, we adopt the method proposed by Goldberger and Wise [2]. Let us consider a massive scalar field Φ in the bulk with quartic interactions on the Planck ($\phi = 0$) and visible branes ($\phi = \pi$). The corresponding action is

$$\begin{aligned} S_5 = (1/2)\int d^4x d\phi\sqrt{-g_5}[1/r^2(\partial_\phi\Phi)^2 + m^2\Phi^2] \\ - \int d^4x d\phi\sqrt{-g_h}\lambda_h(\Phi^2 - v_h^2)^2\delta(\phi) \\ - \int d^4x d\phi\sqrt{-g_v}\lambda_v(\Phi^2 - v_v^2)^2\delta(\phi - \pi). \end{aligned} \tag{7}$$

Here, we assume that the scalar field depends only on the extra dimensional coordinate. Also, g_h and g_v are the determinants of the induced metric on the hidden and visible brane, respectively. The vacuum expectation values of the scalar field on the branes are given by v_h and v_v; λ_h and λ_v are brane tensions.

The equation of motion for the scalar field is given by

$$(1/r^2)\Phi''(\phi) - (4/r^2)A'(\phi)\Phi'(\phi) - m^2\Phi + (4/r)\lambda_v$$
$$(\Phi^2 - v_v^2)\Phi\delta(\phi-\pi) + (4/r)\lambda_h(\Phi^2 - v_h^2)\Phi\delta(\phi) = 0, \quad (8)$$

where $'$ denotes the derivative with respect to the coordinate ϕ. For large λ_h and λ_v, one obtains the following two boundary conditions:

$$\Phi(0) = v_h, \quad (9)$$

$$\Phi(\pi) = v_v. \quad (10)$$

Now, we discuss the stability mechanism for two different scenarios, i.e., AdS and dS branes separately.

1. Anti-de Sitter brane ($\Omega < 0$)

It has been shown in [12] that the magnitude of the cosmological constant on the AdS brane is constrained to have an upper bound and must lie between $-10^{-32} < \Omega < 0$. Due to this tiny value of the magnitude of the cosmological constant, we keep terms up to ω^2 order. Differentiation of both sides of Eq. (3) with respect to "y" yields,

$$A'(\phi) = kr[1 - (\omega^2/2)\exp(2kr\phi)].$$

Putting the above expression in Eq. (8), one gets the equation of motion for the scalar field in the bulk as

$$(1/r^2)\Phi''(\phi) - 4(k/r)[1 - (\omega^2/2)\exp(2kr\phi)]\Phi'(\phi)$$
$$- m^2\Phi = 0.$$

This leads to the solution

$$\Phi(\phi) = [A\exp((2+\nu)kr\phi) + B\exp((2-\nu)kr\phi)]$$
$$- \omega^2/2[A(2+\nu)/(1+\nu)\exp((4+\nu)kr\phi)$$
$$+ B(2-\nu)/(1-\nu)\exp((4-\nu)kr\phi)]. \quad (11)$$

Here, A and B are arbitrary constants, and $\nu = \sqrt{(4 + m^2/k^2)}$. An effective potential V_{eff} can be obtained by putting the above solution (11) back into the scalar field action (7) and integrating over the extra dimension. This yields an effective modulus potential at the visible brane as

$$V_{\mathrm{eff}} = [2A^2k(2+\nu)\exp(2\nu k\pi r) + 2B^2k(\nu-2)]$$
$$- \omega^2[2A^2k\exp(2\nu k\pi r) - 2B^2k(\nu-2)$$
$$- 4A^2k(2+\nu)/(1+\nu)\exp((2+2\nu)k\pi r)$$
$$- 4B^2k(2-\nu)/(1-\nu)$$
$$- 2ABk(4-\nu^2)/(1-\nu^2)\exp(2k\pi r)]. \quad (12)$$

The boundary conditions given by Eqs. (9) and (10) yield the arbitrary constants A and B in the following form:

$$A = [v_v\exp-((2+\nu)k\pi r) - v_h\exp(-2\nu k\pi r)]$$
$$+ \omega^2/2[v_v(2+\nu)/(1+\nu)\exp-\nu k\pi r)$$
$$- v_v(2-\nu)/(1-\nu)\exp-3\nu k\pi r)$$
$$+ 2v_v(\nu/1-\nu^2)\exp-((2+3\nu)k\pi r)$$
$$+ 2v_h(\nu/1-\nu^2)\exp-((2\nu-2)k\pi r)] \quad (13)$$

and

$$B = v_h[1 + \omega^2/2(2-\nu)/(1-\nu)] - A[1 + \omega^2(\nu/1-\nu^2)]. \quad (14)$$

Putting A and B in expression (12) and minimizing the modulus potential, one gets the condition

$$[v_v^2 - v_h^2\exp(-2\epsilon k\pi r)] + \omega^2/2\exp(2k\pi r)$$
$$[(v_v - v_h\exp-(\epsilon k\pi r))^2 - 8v_v^2\exp-((6+\epsilon)k\pi r)]$$
$$- \omega^2[v_v - v_h\exp(-\epsilon k\pi r)] = 0, \quad (15)$$

where we use $\nu = 2 + \epsilon$ with $\epsilon = m^2/(4k^2)$ and ignore terms proportional to ϵ. In this approximation, Eq. (15) becomes

$$k\pi r = 4(k^2/m^2)\ln(v_h/v_v)$$
$$+ (16/3)\omega^2(k^2/m^2)(v_v/v_h)^{(2+4/\epsilon)}. \quad (16)$$

Here, r is the stabilized distance between the two branes. If we now require that the same r resolves the gauge hierarchy problem as well, then the following condition holds:

$$\omega^2 = [100\ln(v_h/v_v) - 16\ln(10)]/[(1/4)10^{32}$$
$$- (400/3)(v_v/v_h)^{402}], \quad (17)$$

where we take $m/k \simeq 0.2$. This result reveals that the ratio of the vev of the scalar field at the two branes depends on the brane cosmological constant. From the above relation (17) between the brane cosmological constant and vev ratio, we obtain Fig. 1 [$\omega^2 = -\Omega/(3k^2)$] as,

Figure 1 demonstrates that for a wide range of values of the brane cosmological constant, the vev ratio varies

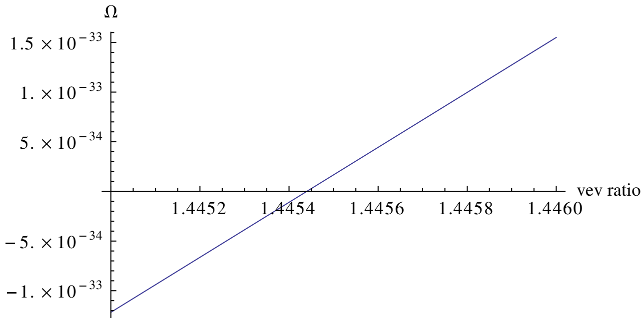

FIG. 1. (v_h/v_v) vs Ω

insignificantly and does not lead to any hierarchical values between the vevs.

2. de Sitter brane $(\Omega > 0)$

For the de Sitter brane, we split the parameter space of the cosmological constant into different regimes as follows:

(i) $0 \leq \Omega \leq 10^{-32}$: Using the dS warp factor (4), one gets the scalar field solution for this regime as

$$\begin{aligned}\Phi(\phi) = {} & [A\exp((2+\nu)kr\phi) + B\exp((2-\nu)kr\phi)] \\ & + \omega^2/2[A(2+\nu)/(1+\nu)\exp((4+\nu)kr\phi) \\ & + B(2-\nu)/(1-\nu)\exp((4-\nu)kr\phi)].\end{aligned} \tag{18}$$

Now, proceeding similarly as in the AdS case, one ends up with the relation between the brane cosmological constant and the vev ratio as

$$\begin{aligned}\omega^2 = {} & [16\ln(10) - 100\ln(v_h/v_v)]/[(1/4)10^{32} \\ & - (400/3)(v_v/v_h)^{402}].\end{aligned} \tag{19}$$

This leads to Fig. 2 $[\omega^2 = \Omega/(3k^2)]$.

(ii) $10^{-32} \leq \Omega \leq 1$:

Using the dS warp factor (4), the scalar field solution for this regime is

$$\begin{aligned}\Phi(\phi) = {} & A\exp-((\nu-2)kr\phi) \\ & \times 2F_1(2, 2-\nu; 1-\nu; (\omega^2/4)\exp(2kr\phi) \\ & + B\exp((\nu+2)kr\phi) \\ & \times 2F_1(2, 2+\nu; 1+\nu; (\omega^2/4)\exp(2kr\phi),\end{aligned}$$

where ${}_2F_1(arg)$ is the hypergeometric function. Keeping terms up to ω^2, the above solution of scalar field becomes

$$\begin{aligned}\Phi(\phi) = {} & [A\exp((2+\nu)kr\phi) + B\exp((2-\nu)kr\phi)] \\ & - \omega^2/2[A(2+\nu)/(1+\nu)\exp((4+\nu)kr\phi) \\ & + B(2-\nu)/(1-\nu)\exp((4-\nu)kr\phi)].\end{aligned}$$

Again, proceeding similarly as before, one ends up with the relation between the brane cosmological constant and the vev ratio as

$$\omega^2 = 4(v_v/v_h)^{200}. \tag{20}$$

This leads to Fig. 3 $[\omega^2 = \Omega/(3k^2)]$.

Both Figs. 2 and 3 reveal that just as in the Ads case, here also the deviation of the vev ratio of the scalar field from that of the GW value is insignificant even when the brane is endowed with a large positive cosmological constant.

B. Graviton modes

To study the graviton modes, we decompose the four-dimensional components of the metric into its Kaluza-Klein (KK) modes as

$$g_{\mu\nu}(x,y) = (1/\sqrt{r})\sum h^n_{\mu\nu}(x)\xi_n(\phi). \tag{21}$$

Here, $h^n_{\mu\nu}(x)$ is the nth KK graviton mode. Plugging back the decomposition in the action and using the appropriate gauge conditions for $h_{\mu\nu}(x)$ [19], one gets

$$S_5 = -(1/4)\int d^4xdy\sqrt{g}[\partial_M g^{ij}][\partial^M g_{ij}]. \tag{22}$$

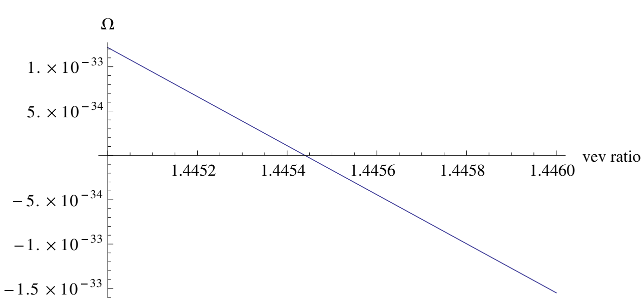

FIG. 2. (v_h/v_v) vs Ω

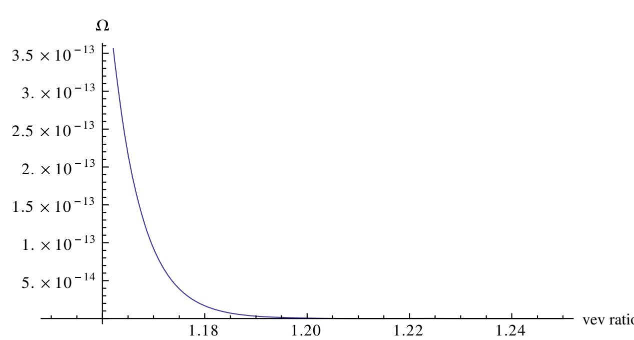

FIG. 3. (v_h/v_v) vs Ω

This modified action leads to the equation of motion

$$\eta^{\mu\nu}\partial_\mu\partial_\nu h^n_{ij}(x) = m_n^2 h^n_{ij}(x). \tag{23}$$

The above differential equation for the x-dependent part of the metric holds if

$$(1/r^2)\partial_\phi[\exp(-4A(\phi))\partial_\phi\xi^n] = -m_n^2\exp(-2A(\phi))\xi^n(\phi) \tag{24}$$

and the orthogonality condition

$$\int \exp(-2A)\chi^n(\phi)\chi^m(\phi)d\phi = \delta_{mn}$$

are simultaneously satisfied.

Here, m_n denotes the mass of nth KK graviton mode. The solution of Eq. (24) as well as graviton KK modes for AdS and dS branes are discussed in the following sections.

1. Anti-de Sitter brane ($\Omega < 0$)

As mentioned before, the range of the cosmological constant on the AdS brane is $-10^{-32} < \Omega < 0$. Once again, keeping terms up to ω^2 and using the perturbative expansion of $\xi(y)$ and m_n as

$$\begin{aligned}\xi(\phi) &= \xi_n^0(\phi) + \omega^2\xi_n^1(\phi)\\ m_n &= m_n^0 + \omega^2 m_n^1,\end{aligned} \tag{25}$$

where m_n^0 is the mass of the nth KK graviton mode on flat branes, which is the RS scenario, Eq. (24) becomes

$$\begin{aligned}&[(1/r)\partial_\phi(\exp(-4kr\phi)\xi_n^0(\phi)) + \exp(-2kr\phi)(m_n^0)^2\xi_n^0]\\ &\quad-\omega^2[(1/r)\partial_\phi(\exp(-4kr\phi)\xi_n^1(\phi))\\ &\quad-(1/r)\partial_\phi(\exp(-2kr\phi)\xi_n^0(\phi))\\ &\quad\times\exp(-2kr\phi)(m_n^0)^2\xi_n^1 - (1/2)(m_n^0)^2\xi_n^0\\ &\quad+2m_n^0 m_n^1\exp(-2kr\phi)\xi_n^0] = 0.\end{aligned} \tag{26}$$

Defining the new variable $z_n^0 = (m_n^0/k)\exp(kr\phi)$, one gets the solution of $\xi_n(\phi)$ as

$$\xi_n(\phi) = (1/N_n)\exp(2kr\phi)[J_2(z_n^0) + a_n Y_2(z_n^0) - \omega^2 f_n(z_n^0)]. \tag{27}$$

Now, demanding the continuity of the graviton wave function at two branes, we get the following boundary conditions:

$$\begin{aligned}\xi_n'(\phi=0) &= 0,\\ \xi_n'(\phi=\pi) &= 0.\end{aligned}$$

From the first boundary condition and using the approximation $(m_n/k) \ll 1$, one can conclude that the coefficient a_n is negligible. Thus, the solution (27) becomes $\xi_n(\phi) = (1/N_n)\exp(2kr\phi)[J_2(z_n^0) - \omega^2 f_n(z_n^0)]$. The other boundary condition yields

$$\begin{aligned}&\exp(k\pi r)(m_n^0/k)J_1(\exp(k\pi r)(m_n^0/k))\\ &\quad-2\omega^2 k f_n(\exp(k\pi r)(m_n^0/k))\\ &\quad-\omega^2\exp(k\pi r)(m_n^0/k)f_n'(\exp(k\pi r)(m_n^0/k)) = 0,\end{aligned} \tag{28}$$

which leads to the first-order correction of graviton KK mass modes as

$$\begin{aligned}m_{(n=1)} &= m_{n=1}^0 + 3.5 * 10^{30} * \omega^2\\ m_{(n=2)} &= m_{n=2}^0 + 2 * 10^{30} * \omega^2.\end{aligned}$$

2. de Sitter brane ($\Omega > 0$)

As before for

(i) $10^{-32} \le \Omega \le 1$ using Eq. (4), the expression for the warp factor becomes

$$\exp[-4A(\phi)] = \exp(-4kr\phi)[1 - \omega^2\exp(2kr\phi)].$$

Taking this expression of warp factor and proceeding similarly, one ends up with following graviton mass correction due to the brane cosmological constant:

$$\begin{aligned}m_{(n=1)} &= m_{n=1}^0 + (0.44),\\ m_{(n=2)} &= m_{n=2}^0 + (0.55),\end{aligned}$$

for $\Omega \sim 10^{-20}$ and the corrections are in TeV units. In a similar, way we can extend our analysis for very large values of the brane cosmological constant (Ω). We now summarize our results for different cases in Table I (for $k/M_{pl} \sim 0.1$).

From the above table, it is evident that the ratio of v_h and v_v is of the order of unity for the entire chosen range of values of the brane cosmological constant. This condition justifies the fact that the backreaction of the stabilizing scalar field on the background spacetime can be neglected even in the presence of the brane cosmological constant. Again from [12], it turns out that the perturbation of the visible brane tension due to the brane cosmological constant is given by

$$\delta V_v = 12M^3 k[\omega^2\exp(2kr\pi)]/\Big[1 + \sqrt{(1-\omega^2)} + (\omega^2/2)\exp(2kr\pi)\Big]. \tag{29}$$

Since for the AdS brane $\omega^2 = -\Omega/(3k^2)$, from Eq. (29) it is easy to see that $\delta V_v < kv_v^2$. This immediately ensures [2]

TABLE I. Warping parameter (kr), vev ratio (v_h/v_v), and graviton mass modes (m_n) for a wide range of cosmological constants (Ω).

Ω	kr	v_h/v_v	$m_{(n=1)} = (m^0_{n=1} + \Delta m_n)$(TeV)	$m_{(n=2)} = (m^0_{n=2} + \Delta m_n)$(TeV)
-10^{-32}	∼11	1.446	(0.383 + 0.03)	(0.702 + 0.02)
0	∼11	1.445 439 771	0.383	0.702
10^{-20}	∼10	1.3700	(0.383 + 0.44)	(0.702 + 0.55)
100	0.095	∼1	10.5263	21.0526
625	0.039	∼1	25.641	51.282
10^4	0.0099	∼1	101.01	202.02

that the minimum is a global one. A similar argument also holds for the dS brane.

C. Radion mass

In this section, we consider a fluctuation of branes around the stable separation (r). So, the interbrane separation can be considered as a field, and here, for simplicity, we assume that this new field depends only on the brane coordinates. The corresponding metric ansatz is

$$ds^2 = \exp[-2A(x,\phi)]g_{\mu\nu}dx^\mu dx^\nu + T^2(x)d\phi^2, \qquad (30)$$

where ϕ is the extra dimensional angular coordinate. Following the procedure adopted in [15], the warp factor for the AdS brane is given by

$$\exp[-A(x,\phi)] = \omega\cosh\left[\ln(\omega/c_1) + kT(x)\phi\right], \qquad (31)$$

while that for the dS brane is

$$\exp[-A(x,\phi)] = \omega\sinh\left[\ln(c_2/\omega) - kT(x)\phi\right]. \qquad (32)$$

From the perspective of four-dimensional effective theory, $T(x)$ is known as the radion field. In the following two subsections, we present the mass of the radion field for both AdS and dS branes.

A Kaluza-Klein reduction of the five-dimensional Einstein-Hilbert action for the anti-de Sitter warp factor (31) (in a leading order correction over the RS warp factor due to the brane cosmological constant) leads to the following kinetic part of $T(x)$:

$$S_{\rm kin}[T] = 6M^3k\pi^2\int d^4x\sqrt{-g}\exp(-2k\pi T(x))[1 + (\omega^2/3) \times k\pi T(x)\exp(2k\pi T(x))]\partial_\mu T(x)\partial^\mu T(x). \qquad (33)$$

To derive this effective action, we keep the terms only up to ω^2 order. As we see that the field $T(x)$ is not canonical, we redefine the field ($T(x) \to \Psi(x)$) by the following transformation:

$$T'(\Psi)^2 = \exp(2k\pi T(x))/(12M^3k\pi^2)[1 - (\omega^2/3) \times k\pi T(x)\exp(2k\pi T(x))], \qquad (34)$$

where $'$ denotes the derivative with respect to $\Psi(x)$. The kinetic part of the physical radion field [$\Psi(x)$] is now canonical and is given as

$$S_{\rm kin}[\Psi] = (1/2)\int d^4x\sqrt{-g}\partial_\mu\Psi\partial^\mu\Psi.$$

After obtaining the canonical radion field [$\Psi(x)$], we now find the radion mass square (m^2_Ψ) is given by

$$m^2_\Psi = [T'(\Psi)^2V''_{\rm eff}(T)]_{(<T>=r)}, \qquad (35)$$

where r is the stabilized modulus (16) and $V_{\rm eff}(T)$ is obtained from Eq. (12) by replacing r by $T(x)$. One can now easily obtain the expression of $V''_{\rm eff}(\langle T\rangle = r)$ as

$$V''_{\rm eff}(r) = 2\epsilon^2k^3v_h^2\pi^2\exp[-(4+2\epsilon)k\pi r] + (8\omega^2/3)k^3v_v^2\pi^2(4+2\epsilon)\exp[-(8+2\epsilon)k\pi r]. \qquad (36)$$

Putting the expression of $V''_{\rm eff}(\langle T\rangle)$ into Eq. (35), one ends up with the squared mass of the radion field as follows:

$$m^2_\Psi = m^2_{(0)} + (\alpha * m^2_{(1)}), \qquad (37)$$

where m^2_Ψ and $m^2_{(0)}$ are the mass square of the radion field for nonflat and flat branes, respectively, while $m^2_{(1)}$ gives the correction of the radion mass due to the nonzero cosmological constant on branes. Here, $\alpha = \omega^2\exp(2k\pi\langle T\rangle)$ and $m^2_{(0)}$, $m^2_{(1)}$ are given by the expressions

$$m^2_{(0)} = [(\epsilon^2k^2v_v^2)/(6M^3)] * \exp(-2k\pi\langle T\rangle), \qquad (38)$$

$$m^2_{(1)} = [(\epsilon^2k^2v_v^2)/(18M^3)] * (k\pi\langle T\rangle), \qquad (39)$$

where $\epsilon = m^2/(4k^2)$, m is the mass of the stabilizing scalar field, and $\langle T\rangle$ is the vev of the radion field. These expressions clearly depict how the correction to the mass term depends on the brane cosmological constant as well as on the parameters of the stabilizing scalar field. Moreover, we also note that the correction to the radion mass is always positive and is always greater than what it would be in the flat brane limit. Proceeding as before, we find that for the de Sitter brane ($\Omega > 0$) also the radion mass square is positive and is enhanced from that for a flat brane scenario.

IV. STRESS TENSOR OF STABILIZING SCALAR FIELD: CONDITION FOR NEGLIGIBLE BACKREACTION IN NONFLAT BRANE SCENARIO

Using the warp factor for the nonflat brane [Eqs. (3) and (4)], different components of the stress tensor of the stabilizing scalar field can be written as

$$T_{\phi\phi}(\Phi) = (1/4)r^2[-(1/r^2)(\partial_\phi \Phi)^2 + m^2\Phi^2]$$

and

$$T_{\mu\nu}(\Phi) = (1/4)[(1/r^2)(\partial_\phi \Phi)^2 + m^2\Phi^2]g_{\mu\nu}(x)\exp(-A(\phi)).$$

Putting the bulk scalar field solution [Eq. (11)] in the expression of $T_{\phi\phi}(\Phi)$ and $T_{\mu\nu}(\Phi)$ and using the form of A and B in terms of v_v and v_h [Eqs. (13) and (14)], one can show that the ratio of the corresponding component of the stress tensor between the bulk scalar field and the bulk cosmological constant varies as v_v^2/M^3 or v_h^2/M^3, i.e., $[T_{\phi\phi}(\Phi)/T_{\phi\phi}(\Lambda)] \sim v_v^2/M^3$ as well as $[T_{\mu\nu}(\Phi)/T_{\mu\nu}(\Lambda)] \sim v_v^2/M^3$, where $T_{\phi\phi}(\Lambda)$ and $T_{\mu\nu}(\Lambda)$ are different components of the stress tensor for the bulk cosmological constant. Thus, the stress tensor for the bulk scalar field (Φ) is less than the bulk cosmological constant (Λ) for v_v^2/M^3 and v_h^2/M^3 less than unity. Our result closely resembles that obtained in [2]. This condition allows us to neglect the backreaction of the stabilizing scalar field in comparison to the bulk cosmological constant in nonflat brane models.

It is worthwhile to mention that in Ref. [16] the warp factor and the radion mass have been estimated by considering the effects of the backreaction of the stabilizing bulk scalar field. However, in that analysis, the choice of the form of the scalar potential turns out to be crucial in getting an exact solution for the warp factor where the coefficients of the quartic term and the quadratic term (i.e., the mass term) in the scalar potential are related by a common parameter u (see [16]) such that the vanishing of one leads to the vanishing of the other as $u \to 0$. As a result, their scalar potential does not have a smooth limit that may result in the GW scalar potential, which contains only a nonvanishing quadratic mass term with coefficient m. Thus, the limit $u \to 0$ in [16] essentially amounts to $m \to 0$ in [2], which in turn makes the radion mass zero in both the cases. It is interesting to note that while the GW radion mass scales as m/k, the radion mass determined by the authors of [16] scales as u/k.

V. CONCLUSION

We now summarize the findings and the implications of our results.

(i) We have demonstrated that the extra dimensional modulus can be stabilized by the Goldberger-Wise mechanism for a wide range of values of the cosmological constant both in de Sitter and anti-de Sitter regions. It has been shown in [2] that if the vev ratio of the scalar field in the bulk is of the order ~1.46 or less than that, then one can safely ignore the backreaction of the scalar field on the background spacetime for the purpose of modulus stabilization. Now, from Table I, it is evident that since the vev ratio lies between $1 < (v_h/v_v) < 1.46$ for the entire parameter space of the cosmological constant, the backreaction can be sagely ignored even in the generalized Randall-Sundrum scenario. In this sense, the Goldberger-Wise stabilization mechanism is extremely robust against the extent of the nonflatness of our Universe. Our result also reveals that even for nonflat branes the modulus potential continues to yield a global minimum ensuring a robust modulus stabilization against perturbations. We also justify the reason for negligible backreaction of the bulk stabilizing scalar field on the background metric.

(ii) We have derived the modifications of the KK graviton mass modes due to the presence of a nonzero cosmological constant on the brane in the generalized Randall-Sundrum scenario. We found that the masses of the graviton KK modes increases with the brane cosmological constant and may deviate significantly from the values estimated in the RS scenario as the values of the brane cosmological constant increase. During this analysis, we restricted the choice of the parameters in a region so that the gauge hierarchy problem can simultaneously be resolved. In the context of the present epoch of our Universe (visible 3-brane), these results indicate that due to extreme smallness of the value of the cosmological constant (10^{-124} in Planckian units), the warped model resembles very closely the RS model with graviton KK mode masses TeV. However, this scenario will change significantly in an with a large cosmological constant. Finally, we find the dependence of the radion mass square on the brane cosmological constant for both de Sitter and anti-de Sitter space. We show that the radion mass squared continues to be positive in both cases without leading to any instability in the model.

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