Ready-to-use post-Newtonian gravitational waveforms for binary black holes with nonprecessing spins: An update

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For black-hole binaries whose spins are (anti-) aligned with respect to the orbital angular momentum of the binary, we compute the frequency-domain phasing coefficients including the quadratic-in-spin terms up to the third post-Newtonian (3PN) order, the cubic-in-spin terms at the leading order, 3.5PN, and the spin-orbit effects up to the 4PN order. In addition, we obtain the 2PN spin contributions to the amplitude of the frequency-domain gravitational waveforms for nonprecessing binaries, using recently derived expressions for the time-domain polarization amplitudes of binaries with generic spins, complete at that accuracy level. These two results are updates to [K. G. Arun, A. Buonanno, G. Faye, and E. Ochsner, Phys. Rev. D **79**, 104023 (2009).] for amplitude and [M. Wade, J. D. E. Creighton, E. Ochsner, and A. B. Nielsen, Phys. Rev. D **88**, 083002 (2013).] for phasing. They should be useful for constructing banks of templates that accurately model nonprecessing inspiraling binaries, for parameter estimation studies, and for constructing analytical template families that account for the inspiral-merger-ringdown phases of the binary.

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I. INTRODUCTION

Recently, there have been several improvements in modeling spinning binaries within the post-Newtonian formalism [1]. These developments include the computation of relative 2PN spin-orbit (SO) effects (corresponding to the 3.5PN order) in the equations of motion [2–4] as well as in the precession equations at the same relative accuracy level, and the computation of the near-zone metric at the 2PN order [5]. The work [5] also provided us with the energy function at 3.5PN order including spin-orbit (linear-in-spin) effects at the relative 2PN order, which is required to compute the phase. Further, in Ref. [6], the 2PN SO contributions were incorporated to the gravitational-wave energy flux, through the computation of relevant source multipole moments [6,7], and (time-domain) phasing at the 3.5PN order. The tail-induced SO corrections to the two latter quantities were investigated in Ref. [8] at the order 4PN, where they are the only spin-orbit effects. On the other hand, the spin-spin (quadratic-in-spin, SS) interactions were recently included at the 3PN order [9], i.e., 1PN order beyond the leading SS terms presented in Ref. [10], based on the next-to-leading-order equations of motion [11,12] and the explicit expression of the source moments [7,9]. In addition, the leading cubic-in-spin terms entering the energy and the energy flux at 3.5PN were computed in [13]. The 2PN polarizations $h_{+,\times}$ accounting for both the spin-orbit and spin-spin effects were calculated explicitly in Ref. [14], extending the earlier works of Refs. [10,15,16]. In principle, polarization waveforms can now be computed to the 2.5PN order [17]. In addition, the tail-type spin-orbit corrections entering the 3PN amplitude are also available [18]. To summarize, all spin contributions to the GW polarizations in the time-domain are known with 2PN accuracy, while the time-domain phasing is known to the 4PN, 3PN, and 3.5PN orders, for the SO, SS, and SSS effects, respectively.

Frequency-domain amplitudes for nonprecessing binaries, with spins (anti-)aligned to the orbital angular momentum vector, were first displayed to the 2PN order in Ref. [10]. Their expression complements that of the 3PN accurate polarizations for nonspinning binaries derived in [19,20]. They model the spin-orbit effects at the leading (1.5PN) order and *partial* spin-spin effects at the 2PN order. More precisely, the spin-spin contributions to the GW amplitude presented in Ref. [10] are only those that arise due to couplings involving *both* spins, i.e., of the type

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[Spin(1)-Spin(2)], as at the time this work was achieved, self-spin corrections [Spin(1)-Spin(1) and Spin(2)-Spin(2)] were not yet available. In this work we make use of the above mentioned recent time-domain results for GW polarizations with all possible spin-dependent interactions to construct their frequency-domain counterpart complete up to the 2PN order, including in particular the new 2PN SO and SS effects (besides those already present in [10]). Frequency-domain phasing with all SO contributions up to the 3.5PN order—except for those produced by the black-hole absorption at the 2.5PN order—and with all SS contributions at the 2PN order, was provided in Ref. [21]. We extend that result by adding the tail-induced spin-orbit effects at the 4PN order, as well as the quadratic and cubic spin terms contributing to the phase at the 3PN and 3.5PN orders, respectively.

This paper is organized in the following manner. We begin Sec. II by showing the form of the Fourier domain signal and defining our notations. The rest of the section is split into two parts. Section II A presents the phasing formula, which includes the spin-orbit contribution at the 4PN order, the quadratic spin terms at the 3PN order, and the cubic ones at the 3.5PN order. In Sec. II B we list our results, complementing the outcomes of Ref. [10], for the frequency-domain amplitude of the (polarization) waveforms of nonprecessing binaries in quasicircular orbits. Section III, on the other hand, contains the expressions of the spherical harmonic modes of the 2PN accurate waveform. Finally, in Sec. IV, we summarize those results and discuss their implications.

II. FREQUENCY-DOMAIN WAVEFORMS FOR NONPRECESSING BINARIES IN CIRCULAR ORBITS

Since we regard this report as an extension of [10], we basically follow the definitions and notations adopted in there. The reader must refer to that work for details. Nonetheless, we shall provide below some minimal compendium both to ensure a natural flow in the paper and to facilitate the reading. The frequency-domain amplitude of a signal $h_{\rm strain}$ produced by a gravitational wave h_{ij} can be written, truncated at some accuracy level, in the following way (see Sec. VI B of Ref. [10] for a derivation), using geometrical units where $G = c = 1$:

$$\tilde{h}_{\rm strain}(f) = \frac{M^2}{D_L}\sqrt{\frac{5\pi}{48}}\sum_{n=0}^{4}\sum_{k=1}^{6} V_k^{n-7/2} C_k^{(n)} e^{\mathrm{i}(k\Psi_{\rm SPA}(f/k)-\pi/4)}. \tag{1}$$

Here, $\tilde{h}_{\rm strain}(f)$ denotes the waveform in the frequency domain[1] as observed by the detector, while M and D_L stand for the total mass and the luminosity distance of the source, respectively. The index n indicates the PN order, whereas the index k keeps track of different harmonics of the orbital phase. Hence, the above waveform is 2PN accurate and consists of six harmonics. For the $k^{\rm th}$ harmonic, the PN parameter $v \equiv v(t)$ entering the time domain waveform has been replaced by a function V_k of the GW frequency f, defined as $V_k(f) = (2\pi M f/k)^{1/3}$. The function $\Psi_{\rm SPA}(f)$ represents essentially the phase of the first harmonic in the frequency domain as obtained under the stationary phase approximation (SPA) [22,23] (see Sec. VI B of [10] for details). Finally, the coefficients $\mathcal{C}_k^{(n)}$ depend on the intrinsic parameters of the binary, such as the masses and the spins, as well as the angular parameters specifying the binary's location and orientation.

The results of the present paper, along with those of Ref. [10], will allow one to write the amplitude corrections up to the 2PN order with all possible spin effects. As already stated, the waveform provided in [10] contains terms that describe the spin-orbit effects at the leading order (1.5PN) and *part* of the spin-spin effects (corresponding to Spin(1)-Spin(2) interactions) at the 2PN order. The coefficients $\mathcal{C}_k^{(n)}$ through which they appear are explicitly listed in Appendix D of Ref. [10]. Thus, for the brevity of presentation and the sake of avoiding repetition, we shall only show here those $\mathcal{C}_k^{(n)}$'s that are modified due to the inclusion of the spin-orbit and spin-spin effects at the 2PN order, as discussed in Sec. I. Below, we shall display our expressions for the GW phase and amplitude in two separate subsections.

A. Corrections to the phasing formula

In order to obtain the frequency-domain phasing, we follow the prescription of Ref. [24], which is based on an energy balance argument. In the case of quasicircular nonprecessing orbits, the two inputs needed for the phase derivation are the time domain center-of-mass energy E and the energy flux $\mathcal{F}$ of the binary, both given in terms of the orbital frequency. The two relations are invariant for a large class of gauge transformations.

Schematically, for the energy we can write

$$E = -\frac{\eta m}{2} v^2 [E_{\rm NS} + E_{\rm SO} + E_{\rm SS} + E_{\rm SSS}], \tag{2}$$

where $E_{\rm NS}$, $E_{\rm SO}$, $E_{\rm SS}$, and $E_{\rm SSS}$ denote the nonspinning, the spin-orbit (linear-in-spin), the spin-spin (quadratic-in-spin), and the spin-spin-spin (cubic-in-spin) contributions to the energy, while $\eta = m_1 m_2/M^2$ represents the symmetric mass ratio parameter, with m_1 and m_2 being the masses of the two companions. The nonspinning part of the energy is currently available to the 4PN approximation beyond the Newtonian order [25]. However, for the present purpose, the 3PN expression of Ref. [26], supplemented with the results of [27], is sufficient, because there cannot be any

[1]For the Fourier transform, we adopt the convention that $\tilde{h}(f) = \int \mathrm{d}t e^{2\pi \mathrm{i} f t} h(t)$.

3.5PN terms in the energy for quasicircular orbits (see [1] for a discussion). The spin-orbit (linear-in-spin) corrections to the conservative part of the dynamics, starting from the 1.5PN order, are known with a relative 2PN accuracy, i.e., at the 3.5PN order beyond the Newtonian level [2,4,5]. The same relative accuracy has been achieved for the spin-spin (quadratic-in-spin) corrections [28–30], even though it corresponds then to the 4PN order, as the leading terms of that type arise at the 2PN approximation [10]. However, since the energy flux has not been determined yet with such accuracy, it will be sufficient for us to use the spin-spin part of the energy at the 3PN order. The explicit expressions of the 3.5PN spin-orbit and the 3PN spin-spin pieces of the energy can be found in the works [5] and [9], respectively. As for the cubic-in-spin pieces, which start to contribute at the 3.5PN order, they were only computed recently [13].

Similarly, the energy flux has the following structure,

$$\mathcal{F} = \frac{32}{5}\eta^2 v^{10}[\mathcal{F}_{\rm NS} + \mathcal{F}_{\rm SO} + \mathcal{F}_{\rm SS} + \mathcal{F}_{\rm SSS}], \qquad (3)$$

where $\mathcal{F}_{\rm NS}$, $\mathcal{F}_{\rm SO}$, $\mathcal{F}_{\rm SS}$, and $\mathcal{F}_{\rm SSS}$ again denote the nonspinning, spin-orbit, spin-spin, and spin-spin-spin contributions to the energy flux. The nonspinning contributions up to the 3.5PN order beyond the leading quadrupolar flux are given in Refs. [27,31]. For the spin-orbit terms, which first appear at the 1.5PN approximation, our current knowledge extends up to the 4PN order [8]. Let us point out that the 4PN spin-orbit piece of the energy flux comes from the next-to-leading-order contributions, ignoring non-spin-orbit terms, to the so-called tail effect. This nonlinear effect can be understood as resulting from the backscattering of the linear wave on the spacetime curvature. It is hereditary in nature, which means that it depends on the past history of the binary evolution. Note that terms of this type (at the 3PN and 4PN orders) are absent from the energy [8]. Spin-spin (or quadratic-in-spin) corrections, starting from the 2PN order, can be found up to the 3PN order in Refs. [10,9]. Finally, the cubic-in-spin terms at the leading 3.5PN approximation were derived in [13].

With these time-domain expressions for the energy and the energy flux in hand, we are in the position to write the frequency-domain phasing entailed by the SPA. Like the expressions above, it has the following general structure,

$$\Psi_{\rm SPA}(f) = 2\pi f t_{\rm c} - \phi_{\rm c} + \left\{\frac{3}{128\eta v^5}[\psi_{\rm NS} + \psi_{\rm SO} + \psi_{\rm SS} + \psi_{\rm SSS}]\right\}_{v=V_1(f)}, \qquad (4)$$

where $\phi_{\rm c}$ denotes the orbital phase at the instant $t_{\rm c}$ of coalescence.

The complete 3.5PN accurate frequency-domain phasing for nonspinning binaries is presented in Refs. [24,32] while the spin-orbit corrections up to the 3.5PN accuracy level and the spin-spin corrections at the 2PN order are given in Refs. [10,21]. The contributions to the phasing we add here include (i) the tail-induced 4PN spin-orbits terms, (ii) the 3PN quadratic-in-spin terms, and (iii) the 3.5PN cubic-in-spin terms. The spin part of the phase will be decomposed as

$$\psi_{\rm Spin} \equiv \psi_{\rm SO} + \psi_{\rm SS} + \psi_{\rm SSS} = v^3[\mathcal{P}_3 + \mathcal{P}_4 v + \mathcal{P}_5 v^2 + \mathcal{P}_6 v^3 + \mathcal{P}_7 v^4 + \mathcal{P}_8 v^5 + \cdots]. \qquad (5)$$

References [10,21] list the explicit expressions for $\mathcal{P}_3$, $\mathcal{P}_4$, and $\mathcal{P}_5$ with the required accuracies. By contrast, the coefficients $\mathcal{P}_6$ and $\mathcal{P}_7$ there only contain relative 1.5PN (leading linear-in-spin tail) and relative 2PN linear-in-spin contributions, respectively. In the present work, as discussed above, we add the relative 1PN quadratic-in-spin and the leading-order cubic-in-spin corrections. In addition, we introduce a new coefficient $\mathcal{P}_8$ of order 4PN that corresponds to the tail-induced SO effect. The modified coefficients $\mathcal{P}_6$, $\mathcal{P}_7$, and the new coefficient $\mathcal{P}_8$ take the final following form:

$$\begin{aligned}\mathcal{P}_6 = {}& \pi\left[\frac{2270}{3}\delta\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N} + \left(\frac{2270}{3} - 520\eta\right)\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\right] + \left(\frac{75515}{144} - \frac{8225}{18}\eta\right)\delta(\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})(\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}) \\ & + \left(\frac{75515}{288} - \frac{263245}{252}\eta - 480\eta^2\right)(\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})^2 + \left(\frac{75515}{288} - \frac{232415}{504}\eta + \frac{1255}{9}\eta^2\right)(\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})^2,\end{aligned} \qquad (6a)$$

$$\begin{aligned}\mathcal{P}_7 = {}& \left(-\frac{25150083775}{3048192} + \frac{26804935}{6048}\eta - \frac{1985}{48}\eta^2\right)\delta\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N} \\ & + \left(-\frac{25150083775}{3048192} + \frac{10566655595}{762048}\eta - \frac{1042165}{3024}\eta^2 + \frac{5345}{36}\eta^3\right)\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N} \\ & + \left(\frac{14585}{24} - 2380\eta\right)\delta(\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})^3 + \left(\frac{14585}{24} - \frac{475}{6}\eta + \frac{100}{3}\eta^2\right)(\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})^3 \\ & + \left(\frac{14585}{8} - \frac{215}{2}\eta\right)\delta(\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})(\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})^2 + \left(\frac{14585}{8} - 7270\eta + 80\eta^2\right)(\chi_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})^2(\chi_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}),\end{aligned} \qquad (6b)$$

$$\mathcal{P}_8 = \pi\left[\left(\frac{233915}{168} - \frac{99185}{252}\eta\right)\delta\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N} + \left(\frac{233915}{168} - \frac{3970375}{2268}\eta + \frac{19655}{189}\eta^2\right)\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N}\right](1-3\ln v). \tag{6c}$$

In the above, $\boldsymbol{\chi}_\mathrm{s}$ and $\boldsymbol{\chi}_\mathrm{a}$ represent symmetric and antisymmetric combinations of the (dimensionless) spin vectors associated with the binary individual components $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$, namely,

$$\boldsymbol{\chi}_\mathrm{s} = \frac{1}{2}(\boldsymbol{\chi}_1 + \boldsymbol{\chi}_2),$$
$$\boldsymbol{\chi}_\mathrm{a} = \frac{1}{2}(\boldsymbol{\chi}_1 - \boldsymbol{\chi}_2). \tag{7}$$

The quantity $\hat{\mathbf{L}}_\mathrm{N}$ is the unit vector pointing along the Newtonian orbital angular momentum; the parameter $\delta = (m_1 - m_2)/m$ represents the difference mass ratio. Coordinate frames and parameter conventions used here are identical to the ones employed in Ref. [10]; more details can be found in Sec. II there. It should be emphasized that this result completes the SO phasing at the 4PN (relative 2.5PN) order, the SS phasing to the 3PN (relative 1PN) order, and the SSS phasing to the (leading) 3.5PN order in the frequency domain. In order to get the full 4PN phase, ignoring at this stage possible absorption effects associated with the black-hole horizons, one would still need to add: (i) the 4PN nonspinning terms, which would require to know the energy flux at that same accuracy level, and (ii) the 3.5PN and 4PN SS terms, of tail and instantaneous types, respectively. The full phasing formula including the contributions listed in previous works [10,21] is being provided, both for completeness and convenience, in a Supplemental Material [33], readable by the commercial calculus software MATHEMATICA.

B. Corrections to the amplitude: 2PN spin-orbit and spin-spin effects

In this section, we present our results for the amplitude of the GW signal emitted by nonprecessing black-hole binaries. The general structure of the waveform is given by Eq. (1). The frequency-domain amplitudes in the absence of spins up to the 2.5PN order, the spin-orbit corrections at the 1.5PN order, and part of the spin-spin corrections at the 2PN order are listed in Ref. [10]. The corresponding coefficients $\mathcal{C}_k^{(n)}$ of Eq. (1) are defined in Eqs. (6.13) and (6.14) of Ref. [10] and are listed in Appendix D there. As discussed above, we shall only provide here explicit expressions for those $\mathcal{C}_k^{(n)}$'s that are modified after including the 2PN spin-orbit and spin-spin terms computed in the time-domain in Ref. [14]. They read

$$\begin{aligned}\mathcal{C}_1^{(4)} = s_\iota\Bigg\{&F_+\Bigg[\delta\left[\frac{11\mathrm{i}}{40} + \frac{5\pi}{8} + \frac{5\mathrm{i}}{4}\log 2 + \left(\frac{7\mathrm{i}}{40} + \frac{\pi}{8} + \frac{\mathrm{i}}{4}\log 2\right)c_\iota^2\right]\\
&+\delta\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N}\left[-\frac{711}{448} + \frac{33}{16}\eta + \left(-\frac{65}{192} - \frac{23}{48}\eta\right)c_\iota^2\right]\\
&+\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N}\left[-\frac{711}{448} + \frac{173}{48}\eta + \left(-\frac{65}{192} + \frac{83}{48}\eta\right)c_\iota^2\right]\Bigg] + \mathrm{i}c_\iota F_\times\Bigg[\delta\left[\frac{9\mathrm{i}}{20} + \frac{3\pi}{4} + \frac{3\mathrm{i}}{2}\log 2\right]\\
&+\delta\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N}\left[\left(-\frac{647}{336} + \frac{41}{24}\eta\right) - \frac{\eta}{8}c_\iota^2\right] + \boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N}\left[\left(-\frac{647}{336} + \frac{125}{24}\eta\right) + \frac{\eta}{8}c_\iota^2\right]\Bigg]\Bigg\}\Theta(F_\mathrm{cut} - f),\end{aligned} \tag{8a}$$

$$\begin{aligned}\mathcal{C}_2^{(4)} = \frac{1}{\sqrt{2}}\Bigg\{&F_+\Bigg[\frac{113419241}{40642560} + \frac{152987}{16128}\eta - \frac{11099}{1152}\eta^2 + \left(\frac{165194153}{40642560} - \frac{149}{1792}\eta + \frac{6709}{1152}\eta^2\right)c_\iota^2\\
&+\left(\frac{1693}{2016} - \frac{5723}{2016}\eta + \frac{13}{12}\eta^2\right)c_\iota^4 - \left(\frac{1}{24} - \frac{5}{24}\eta + \frac{5}{24}\eta^2\right)c_\iota^6 + (1+c_\iota^2)\Bigg[\frac{49}{16}\delta(\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N})(\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N})\\
&+(\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N})^2\left(\frac{49}{32} - 6\eta\right) + (\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N})^2\left(\frac{49}{32} - \frac{\eta}{8}\right)\Bigg]\Bigg] + \mathrm{i}c_\iota F_\times\Bigg[\frac{114020009}{20321280} + \frac{133411}{8064}\eta - \frac{7499}{576}\eta^2\\
&+(\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N})^2\left(\frac{49}{16} - 12\eta\right) + \frac{49}{8}\delta(\boldsymbol{\chi}_\mathrm{a}\cdot\hat{\mathbf{L}}_\mathrm{N})(\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N}) + (\boldsymbol{\chi}_\mathrm{s}\cdot\hat{\mathbf{L}}_\mathrm{N})^2\left(\frac{49}{16} - \frac{\eta}{4}\right)\\
&+\left(\frac{5777}{2520} - \frac{5555}{504}\eta + \frac{34}{3}\eta^2\right)c_\iota^2 + \left(-\frac{1}{4} + \frac{5}{4}\eta - \frac{5}{4}\eta^2\right)c_\iota^4\Bigg]\Bigg\}\Theta(2F_\mathrm{cut} - f),\end{aligned} \tag{8b}$$

$$\mathcal{C}_3^{(4)} = \frac{s_\iota}{\sqrt{3}}\bigg\{F_+\bigg[\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}\bigg[\frac{195}{64}-\frac{141}{16}\eta+\bigg(\frac{195}{64}-\frac{249}{16}\eta\bigg)c_\iota^2\bigg]+\delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\bigg[\frac{195}{64}-\frac{39}{16}\eta+\bigg(\frac{195}{64}+\frac{69}{16}\eta\bigg)c_\iota^2\bigg]$$
$$+\delta(1+c_\iota^2)\bigg(-\frac{189\mathrm{i}}{40}+\frac{9\pi}{8}+\frac{27}{4}\mathrm{i}\log\bigg(\frac{3}{2}\bigg)\bigg)\bigg]+\mathrm{i}c_\iota F_\times\bigg[\delta\bigg(-\frac{189\mathrm{i}}{20}+\frac{9\pi}{4}+\frac{27}{2}\mathrm{i}\log\bigg(\frac{3}{2}\bigg)\bigg)$$
$$+\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}\bigg[\bigg(\frac{195}{32}-21\eta\bigg)-\frac{27}{8}\eta c_\iota^2\bigg]+\delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_N\bigg[\bigg(\frac{195}{32}-\frac{3\eta}{2}\bigg)+\frac{27}{8}\eta c_\iota^2\bigg]\bigg]\bigg\}\Theta(3F_{\rm cut}-f). \tag{8c}$$

Here, s_ι and c_ι are shorthand notations for $\sin\iota$ and $\cos\iota$, respectively, ι being the binary's inclination angle, and $\Theta(kF_{\rm cut}-f)$ are step functions to ensure that the contribution from each harmonic vanishes beyond their respective cutoff frequencies ($f > kF_{\rm cut}$). Note that, when deriving the 2PN terms in the SPA amplitude, we have taken into account all the spin contributions at the 2PN order, instead of the partial ones used in Ref. [10] to be consistent with the spin inputs. We have thus resorted to the full expression of σ displayed in Eq. (6.24) of [10] to compute the quantity $\mathcal{S}_4$ given by Eq. (6.11) there. We also provide the complete list of the $\mathcal{C}_k^{(n)}$'s that contribute to 2PN order in the same MATHEMATICA file as before, i.e., [33].

III. FOURIER TRANSFORM OF THE GW MODES

In this section, we provide the GW modes ($h_{\ell m}$) contributing to the waveform at the 2PN order. For this purpose, we must associate spherical coordinates (R,θ,ϕ) to the source in such a way that, following the conventions of [10], ϕ vanishes for an observer located on earth while θ coincides with the inclination angle ι for the same observer. As usual, the three vectors forming the standard orthogonal basis are referred to as e_r^i, e_θ^i and e_ϕ^i. The complex polarization $h\equiv h_+-\mathrm{i}h_\times\equiv -m^i m^j h_{ij}$, with $m^i=e_\theta^i-\mathrm{i}e_\phi^i$, can be conveniently expanded in terms of the spherical harmonics with spin weight -2, the set of functions ${}_{-2}Y_{\ell m}(\theta,\phi)$ whose precise definition is given by Eqs. (4.2)–(4.3) of Ref. [10]:

$$h(\theta,\phi)=\sum_{\ell=2}^{+\infty}\sum_{m=-\ell}^{\ell}h_{\ell m}\,{}_{-2}Y_{\ell m}(\theta,\phi). \tag{9}$$

The $h_{\ell m}$ modes of GW polarization have the following structure [10,20]:

$$h_{\ell m}=\frac{2M\eta}{D_L}v^2\sqrt{\frac{16\pi}{5}}\hat{h}_{\ell m}e^{-\mathrm{i}m\psi}. \tag{10}$$

Their expression for nonspinning binaries are listed in Eq. (9.4) of [20], whereas the $h_{\ell m}$'s for spinning binaries can be found in [10,14]. Fourier transforms of the individual modes (as opposed to that of the full time-domain waveform) may be useful in many studies at the interface of analytical and numerical relativity. Hence, we shall systematically provide them below. The procedure for computing them is similar to the one used by Ref. [22,23], which applied the stationary phase approximation to the individual harmonics. Following the same method, we obtain the Fourier transforms of the $h_{\ell m}$'s that are relevant for us. They have the form

$$\tilde{h}_{\ell m}(f)=\frac{M^2}{D_L}\pi\sqrt{\frac{2\eta}{3}}V_m^{-7/2}e^{-\mathrm{i}(m\Psi_{\rm SPA}(V_m)+\pi/4)}\hat{H}_{\ell m}(V_m). \tag{11}$$

Our results for $\hat{H}_{\ell m}(V_m)\equiv\hat{H}_{\ell m}$, consistently accounting for all spin effects up to the 2PN order, read

$$\hat{H}_{22}=-1+\bigg(\frac{323}{224}-\frac{451\eta}{168}\bigg)V_2^2+\bigg[-\frac{27}{8}\delta\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}+\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\bigg(-\frac{27}{8}+\frac{11}{6}\eta\bigg)\bigg]V_2^3+\bigg[\frac{27312085}{8128512}+\frac{1975055}{338688}\eta$$
$$-\frac{105271}{24192}\eta^2+(\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_N)^2\bigg(\frac{113}{32}-14\eta\bigg)+\frac{113}{16}\delta(\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N})(\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})+(\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})^2\bigg(\frac{113}{32}-\frac{\eta}{8}\bigg)\bigg]V_2^4+\mathcal{O}(5), \tag{12a}$$

$$\hat{H}_{21}=-\frac{\sqrt{2}}{3}\bigg\{\delta V_1-\frac{3}{2}(\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}+\delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})V_1^2+\delta\bigg(\frac{335}{672}+\frac{117}{56}\eta\bigg)V_1^3+\bigg[\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}\bigg(\frac{4771}{1344}-\frac{11941}{336}\eta\bigg)$$
$$+\delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\bigg(\frac{4771}{1344}-\frac{2549}{336}\eta\bigg)+\delta\bigg(-\frac{\mathrm{i}}{2}-\pi-2\mathrm{i}\log(2)\bigg)\bigg]V_1^4\bigg\}+\mathcal{O}(5), \tag{12b}$$

$$\hat{H}_{33}=-\frac{3}{4}\sqrt{\frac{5}{7}}\bigg\{\delta V_3+\delta\bigg(-\frac{1945}{672}+\frac{27}{8}\eta\bigg)V_3^3+\bigg[\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}\bigg(\frac{161}{24}-\frac{85}{3}\eta\bigg)+\delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\bigg(\frac{161}{24}-\frac{17}{3}\eta\bigg)$$
$$+\delta\bigg(-\frac{21\mathrm{i}}{5}+\pi+6\mathrm{i}\log\bigg(\frac{3}{2}\bigg)\bigg)\bigg]V_3^4\bigg\}+\mathcal{O}(5), \tag{12c}$$

$$\hat{H}_{32} = -\frac{1}{3}\sqrt{\frac{5}{7}}\left\{(1-3\eta)V_2^2 + 4\eta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}V_2^3 + \left(-\frac{10471}{10080} + \frac{12325}{2016}\eta - \frac{589}{72}\eta^2\right)V_2^4\right\} + \mathcal{O}(5), \tag{12d}$$

$$\hat{H}_{31} = -\frac{1}{12\sqrt{7}}\left\{\delta V_1 + \delta\left(-\frac{1049}{672} + \frac{17}{24}\eta\right)V_1^3 + \left[\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N}\left(\frac{161}{24} - \frac{73}{3}\eta\right) + \delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N}\left(\frac{161}{24} - \frac{29}{3}\eta\right) + \delta\left(-\frac{7{\rm i}}{5} - \pi - 2{\rm i}\log(2)\right)\right]V_1^4\right\} + \mathcal{O}(5), \tag{12e}$$

$$\hat{H}_{44} = -\frac{4}{9}\sqrt{\frac{10}{7}}\left\{(1-3\eta)V_4^2 + \left(-\frac{158383}{36960} + \frac{128221}{7392}\eta - \frac{1063}{88}\eta^2\right)V_4^4\right\} + \mathcal{O}(5), \tag{12f}$$

$$\hat{H}_{43} = -\frac{3}{4}\sqrt{\frac{3}{35}}\left\{\delta(1-2\eta)V_3^3 + \frac{5}{2}\eta(\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N} - \delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})V_3^4\right\} + \mathcal{O}(5), \tag{12g}$$

$$\hat{H}_{42} = -\frac{1}{63}\sqrt{5}\left\{(1-3\eta)V_2^2 + \left(-\frac{105967}{36960} + \frac{75805}{7392}\eta - \frac{439}{88}\eta^2\right)V_2^4\right\} + \mathcal{O}(5), \tag{12h}$$

$$\hat{H}_{41} = -\frac{1}{84\sqrt{5}}\left\{\delta(1-2\eta)V_1^3 + \frac{5}{2}\eta(\boldsymbol{\chi}_{\rm a}\cdot\hat{\mathbf{L}}_{\rm N} - \delta\boldsymbol{\chi}_{\rm s}\cdot\hat{\mathbf{L}}_{\rm N})V_1^4\right\} + \mathcal{O}(5), \tag{12i}$$

$$\hat{H}_{55} = -\frac{125}{96}\sqrt{\frac{5}{33}}\delta(1-2\eta)V_5^3 + \mathcal{O}(5), \tag{12j}$$

$$\hat{H}_{54} = -\frac{16}{9}\sqrt{\frac{2}{165}}(1-5\eta+5\eta^2)V_4^4 + \mathcal{O}(5), \tag{12k}$$

$$\hat{H}_{53} = -\frac{9}{32\sqrt{55}}\delta(1-2\eta)V_3^3 + \mathcal{O}(5), \tag{12l}$$

$$\hat{H}_{52} = -\frac{2}{27\sqrt{55}}(1-5\eta+5\eta^2)V_2^4 + \mathcal{O}(5), \tag{12m}$$

$$\hat{H}_{51} = -\frac{1}{144\sqrt{770}}\delta(1-2\eta)V_1^3 + \mathcal{O}(5), \tag{12n}$$

$$\hat{H}_{66} = -\frac{18}{5}\sqrt{\frac{3}{143}}(1-5\eta+5\eta^2)V_6^4 + \mathcal{O}(5), \tag{12o}$$

$$\hat{H}_{65} = \mathcal{O}(5), \tag{12p}$$

$$\hat{H}_{64} = -\frac{128}{495\sqrt{39}}(1-5\eta+5\eta^2)V_4^4 + \mathcal{O}(5), \tag{12q}$$

$$\hat{H}_{63} = \mathcal{O}(5), \tag{12r}$$

$$\hat{H}_{62} = -\frac{2}{297\sqrt{65}}(1-5\eta+5\eta^2)V_2^4 + \mathcal{O}(5), \tag{12s}$$

$$\hat{H}_{61} = \mathcal{O}(5). \tag{12t}$$

Let us emphasize that the source frame used to express the above polarizations (and hence the GW modes) is identical to the one used in Refs. [10,14] (for spinning black holes) but differs from that of Ref. [20] (for nonspinning black holes). The former frame is defined so that the azimuthal angle ϕ locating the earth observer vanishes, while the latter is such that $\phi = \pi/2$. From Eq. (9) and the property of the spin weighted spherical harmonics, we see that $h_{\ell m}^{([1])} = \mathrm{i}^m h_{\ell m}^{([21])}$. For the convenience of the reader, we list again all the modes contributing to the waveform at the 2PN level in Ref. [33], which we provide as Supplemental Material to our paper.

IV. CONCLUSIONS

Based on the recent developments in modeling spinning binaries [8,9,13,14], we have computed the tail-induced 4PN spin-orbit contributions, the 3PN quadratic-in-spin corrections and the 3.5PN cubic-in-spin corrections to the frequency-domain phasing of the GW signal, as well as *all* spin contributions to the amplitude of the frequency-domain waveform at the 2PN order. The 4PN phase presented here only accounts for tail-induced spin-orbit effects, which must still be supplemented by nonspinning contributions at this order. Those contributions are currently out of reach due to lack of inputs. On the other hand, some of the higher-order spin effects are still missing beyond the 3PN order. Those are (i) the instantaneous quadratic-in-spin contributions at the 4PN order (including those resulting from the interactions between the two spins, and from the effect of the spin-induced mass quadrupoles of the two black holes) and (ii) quadratic-in-spin corrections at the 3.5PN order coming from the gravitational-wave tails. Moreover, when at least one of the two companions is a spinning black hole, the imprint of the resulting absorption has yet to be incorporated to the flux at the 2.5PN order [34–36] beyond the leading quadrupole formula, with a 1.5PN relative accuracy [37]. This will generate additional terms at the 2.5PN, 3.5PN, and 4PN orders in the energy balance equation that is used to determine the orbital phase expression. Our new frequency-domain amplitude corrections involve both spin-orbit and spin-spin terms at the 2PN order. The polarizations and the spherical harmonic modes of the waveform in the frequency domain are now complete at this approximation level.

Before we conclude, it is worth recalling that these PN expressions are valid only during the inspiraling stage of the binary motion, where a slow (adiabatic) evolution may be assumed. This assumption breaks down during the late stages (i.e., close to the last stable orbit). Moreover, whether or not the stationary phase approximation (SPA) can be used to compute the Fourier transform of the signal depends crucially on the details of the amplitude and phase evolution (see Sec. VI B of [10] for details). However, it was shown in Ref. [38] that, as long as the total mass of the binary is smaller than some critical value ($\sim 12 M_\odot$), inspiral waveform models can serve in detecting compact-binary coalescences (CBCs). On the other hand, for systems heavier than this critical value, one must resort to more accurate waveform prescriptions such as the ones provided by the effective-one-body (EOB) formalism [39–41]. Interestingly, the data accumulated during the sixth science run of LIGO, and the second and third science runs of Virgo were analyzed by means of an inspiral-based template bank (constructed using 3.5PN accurate (nonspinning) TaylorF2 waveform) aiming at searching CBCs ($M < 25 M_\odot$) [42]. The choice of SPA-based inspiral waveforms over EOB waveforms, even for searching systems with masses as high as $25 M_\odot$, was motivated by the associated low computational cost for the construction of the template bank. However, the recent developments in fast-to-generate reduced-order models, first proposed in Ref. [43], which allow one to work with EOB models calibrated by means of numerical relativity simulations [44–46], along with the template placement methods developed in [47], have made it possible to construct template banks for CBCs with masses as low as $4 M_\odot$. Nevertheless, waveforms presented in this paper, because they include higher-order spin effects (quadratic/cubic-in-spin) in phasing and polarization modes, are suitable for the elaboration of inspiral-only template banks, which would indeed be useful in comparing the efficiency of the searches for low mass nonprecessing binary black holes ($M < 12 M_\odot$) with moderate spins [48,49].

Alternatively, in the past few years, there has been significant progress in building *analytical* inspiral-merger-ringdown waveform families phenomenologically. These waveforms are calibrated with the so-called *hybrid* waveforms, constructed by matching PN/EOB waveforms, which describe the early inspiral stage, to numerical relativity simulations, which describe the late inspiral, merger, and ringdown stages of binary black-hole evolution [50–53]. The SPA waveforms presented here can prove to be crucial in the construction of analytical inspiral-merger-ringdown models for nonprecessing binary black holes, including the effect of higher modes. In addition, they may be useful to investigate various tests of strong field gravity proposed in the literature [54–59] in the presence of spin.

The spin effects in the amplitude and phase of these waveforms will also help in reducing the errors associated with the parameter estimation of the spinning binary signals [21,60]. However, again, parameter estimation studies should be restricted to binary black holes with total mass smaller than $\sim 12 M_\odot$. It is important to note that, with increasing binary mass, there are fewer and fewer inspiral cycles in the detector band; hence, the contributions from merger and ringdown become more dominant. In the context of parameter estimation studies, this calls for the use of inspiral-merger-ringdown waveforms for heavy stellar/intermediate mass binary black holes [61–64].

Finally, we would like to comment on a concern raised by the authors of Ref. [65] about the relevance of inspiral waveforms, with an abrupt termination of the signal at a certain cutoff frequency, for parameter estimation. They suggest that, unless the abrupt termination is motivated by some physical arguments (such as the fact that the signal is inspiral dominated), it can cause significant bias in error estimates, and they suggest employing complete inspiral-merger-ringdown waveforms instead. However, such edge effects were studied in detail in Ref. [66] which suggests that the bias introduced in the measurement of source properties is probably negligible.

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