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Spontaneous excitation of a uniformly accelerated atom in the cosmic string spacetime

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We study, in the cosmic string spacetime, the average rate of change of energy for an atom coupled to massless scalar fields and uniformly accelerated in a direction parallel to the string in vacuum. We find that both the noninertial motion and the nontrivial global spacetime topology affect the atomic transition rates, so an accelerated atom (an Unruh detector) does feel the string contrary to claims in the literature. We demonstrate that the equivalence between the effect of uniform acceleration and that of thermal radiation on the transition rates of the atom, which is valid in the Minkowski spacetime, holds only on the string.

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I. INTRODUCTION

In recent years, a lot of work [1–11] has been done to understand the Unruh effect in terms of spontaneous excitation of atoms in an approach first proposed by Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) [12,13] and later generalized by others [1,6]. These studies reveal that ground-state atoms uniformly accelerated in vacuum would spontaneously make transitions to excited states as a result of the combined effect of vacuum fluctuations and radiation reaction, and a physically appealing interpretation of the Unruh effect has thus emerged in the sense that the spontaneous excitation of accelerated atoms gives a physically transparent illustration for why an accelerated detector clicks.

Such studies are first carried out for uniformly accelerated atoms coupled with various quantum fields ranging from scalar to Rarita-Schwinger fields in unbounded flat space [1,2,4–6] and later they are extended to bounded flat space [3,7] and even to curved space [8–11]. In unbounded spacetime, it has been demonstrated that the balance between vacuum fluctuations and radiation reaction which ensures the stability of inertial atoms in their ground state is no longer perfect for uniformly accelerated atoms so that the spontaneous excitation occurs in vacuum as if they were inertial atoms immersed in a thermal bath at the temperature $T = \frac{a}{2\pi}$ with a being the proper acceleration. However, if boundaries exist, one finds that the excitation rate of accelerated atoms differs from that of inertial ones in a thermal bath, and in this sense that the exact equivalence between acceleration and a thermal bath is lost.

Now, one may wonder what happens if atoms are accelerated in a spacetime which is locally flat but with nontrivial global topology, and one such spacetime is that of a cosmic string that may have been created in the early Universe as a consequence of phase transitions [14]. This is what we plan to do in the present paper. Though recent data of cosmic microwave observations have ruled out cosmic strings as a possible mechanism for the formation of galaxies [15], so far, they are still candidate sources for a number of important physical phenomena such as gamma ray bursts [16–19], the gravitational waves [20–23] and high-energy cosmic rays [24–26]. The simplest cosmic string spacetime is characterized by a flat metric with a deficit angle around a straight string. Previous studies on such a spacetime have shown that the nontrivial topology may induce some quantum gravitational effects near the cosmic string [27], such as a distortion in the zero-point vacuum fluctuations of the quantum fields which results in an energy density in its surrounding vacuum [28–31]. In recent years, investigations have been carried out on whether the distortion of fluctuations of quantum fields due to the deficit angle imprint on the response rate of a particle detector [27,32,33] and the radiative properties of atoms [11,34]. In particular, it was claimed in Refs. [27,33] that an Unruh detector uniformly accelerated parallel to the cosmic string does not feel the string, i.e., the response rate would be exactly the same as that in a trivial flat Minkowski spacetime. In the present paper, we will demonstrate, contrary to these claims, that the response rate of a uniformly accelerated particle detector *is* affected by the presence of a cosmic string which generates a nontrivial spacetime topology. In other words, an Unruh detector does feel the cosmic string. We do this by calculating the rate of change of energy of a uniformly accelerated atom coupled to massless scalar fields in the cosmic string spacetime based upon the DDC formalism [12,13] which separates the contributions of vacuum fluctuations and radiation reaction to the transition rates of the atom. We are also interested in how transition rates behave as compared to the counterparts in a thermal bath in a trivial Minkowski spacetime.

The paper is organized as follows. We introduce the quantum scalar field in the cosmic string spacetime in Sec. II and we generalize, in Sec. III, the DDC formalism in the cosmic string spacetime. In Sec. IV, we compute the two-point function of the massless scalar field. In Sec. V, we calculate the average rate of change of energy for an atom uniformly accelerated with the acceleration parallel to the string. We give the summary in Sec. VI.

II. QUANTUM SCALAR FIELD IN THE COSMIC STRING SPACETIME

The metric of the simplest cosmic string spacetime in which a static, straight cosmic string lies along the z-direction can be described, in the cylindrical coordinate system, by

$$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2 \tag{1}$$

where $0 \leq \theta < \frac{2\pi}{\nu}$, $\nu = (1 - 4G\mu)^{-1}$ with G and μ being Newton's constant and the mass per unit length of the string respectively. The spacetime described by this metric is locally flat with a nontrivial global topology characterized by a deficit angle.

The Klein-Gordon equation for a scalar field ϕ in this spacetime takes the form

$$\left(\partial_t^2 - \frac{1}{r}\partial_r(r\partial_r) - \frac{1}{r^2}\partial_\theta^2 - \partial_z^2\right)\phi(x) = 0. \tag{2}$$

Solving the above field equation, we get a complete set of field modes for the scalar field,

$$u_j(t,\vec{x}) = e^{-i\omega t}u_j(\vec{x}) \tag{3}$$

with

$$u_j(\vec{x}) = \frac{1}{2\pi}\sqrt{\frac{\nu}{2\omega}}e^{i\kappa z}e^{i\nu m\theta}J_{\nu|m|}(k_\perp r). \tag{4}$$

Here $j = \{\kappa, m, k_\perp\}$, $\kappa \in (-\infty, \infty)$, $m \in Z$, $k_\perp \in (0, \infty)$, and $\omega^2 = \kappa^2 + k_\perp^2$. Defining the inner product of two mode functions as

$$(u_j(t,\vec{x}), u_{j'}(t,\vec{x})) = -i\int d^3x u_j(t,\vec{x})\overleftrightarrow{\partial}_t u_{j'}^*(t,\vec{x}), \tag{5}$$

one can show that

$$(u_j(t,\vec{x}), u_{j'}(t,\vec{x})) = \delta(\kappa - \kappa')\delta_{m,m'}\frac{\delta(k_\perp - k'_\perp)}{\sqrt{k_\perp k'_\perp}} \equiv \delta_{j,j'}. \tag{6}$$

The field operator can now be expanded in terms of the complete set of field modes as

$$\phi(t,\vec{x}) = \int d\mu_j[a_j(t)u_j(\vec{x}) + a_j^\dagger(t)u_j^*(\vec{x})] \tag{7}$$

with

$$\int d\mu_j = \sum_{m=-\infty}^{\infty}\int_0^\infty dk_\perp k_\perp \int_{-\infty}^{\infty} d\kappa \tag{8}$$

and $a_j(t) = a_j(0)e^{-i\omega t}$. It is easy to verify that the creation and annihilation operators satisfy the following commutation relation:

$$[a_j(t,\vec{x}), a_{j'}^\dagger(t,\vec{x})] = \delta_{j,j'}. \tag{9}$$

Other commutators are equal to zero.

III. THE GENERALIZED DDC FORMALISM

Assume that a two-level atom interacts with a quantum scalar field in the cosmic string spacetime. The two stationary states of the atom with energies $\pm\frac{\omega_0}{2}$ are denoted by $|+\rangle$ and $|-\rangle$ respectively. The Hamiltonian that describes the evolution of the atom with respect to proper time τ is, according to Dicke [35], given by

$$H_A(\tau) = \omega_0 R_3(\tau), \tag{10}$$

where $R_3(\tau) = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$, and the Hamiltonian of the quantum scalar field reads

$$H_F(\tau) = \int d\mu_j \omega_j a_j^\dagger a_j \frac{dt}{d\tau}. \tag{11}$$

We assume that the atom is linearly coupled to the scalar field with the interaction Hamiltonian as follows:

$$H_I(\tau) = \mu R_2(\tau)\phi(x(\tau)). \tag{12}$$

Here $R_2(\tau) = \frac{i}{2}(R_-(\tau) - R_+(\tau))$, $R_-(\tau) = |-\rangle\langle+|$ and $R_+(\tau) = |+\rangle\langle-|$. The Hamiltonian that governs the evolution of the system (atom + field) can now be obtained by adding up the above three parts as

$$H(\tau) = H_A(\tau) + H_F(\tau) + H_I(\tau). \tag{13}$$

Starting from the above Hamiltonian, we can derive the Heisenberg equations of motion for the dynamical variables of the atom and the field. We can separate the solution of each equation into a "free" part which exists even in the absence of the interaction between the atom and the field, and a "source" part which is induced by the interaction,

$$R_\pm(\tau) = R_\pm^f(\tau) + R_\pm^s(\tau), \tag{14}$$

$$R_3(\tau) = R_3^f(\tau) + R_3^s(\tau), \tag{15}$$

$$a_j(t(\tau)) = a_j^f(t(\tau)) + a_j^s(t(\tau)) \tag{16}$$

with

$$\begin{cases} a_j^f(t(\tau)) = a_j(t(\tau_0))e^{-i\omega_j(t(\tau)-t(\tau_0))}, \\ a_j^s(t(\tau)) = i\mu\int_{\tau_0}^{\tau} d\tau' R_2^f(\tau')[\phi^f(x(\tau')), a_j^f(t(\tau))], \end{cases} \tag{17}$$

and

$$\begin{cases} R_\pm^f(\tau) = R_\pm^f(\tau_0)e^{\pm i\omega_0(\tau-\tau_0)}, \\ R_\pm^s(\tau) = i\mu\int_{\tau_0}^{\tau} d\tau'[R_2^f(\tau'), R_\pm^f(\tau)]\phi^f(x(\tau')), \end{cases} \tag{18}$$

$$\begin{cases} R_3^f(\tau) = R_3^f(\tau_0), \\ R_3^s(\tau) = i\mu\int_{\tau_0}^{\tau} d\tau'[R_2^f(\tau'), R_3^f(\tau)]\phi^f(x(\tau')). \end{cases} \tag{19}$$

Inserting Eq. (17) into Eq. (7), we can divide the field operator into the "free" part and the "source" part, $\phi(t,\vec{x}) = \phi^f(t,\vec{x}) + \phi^s(t,\vec{x})$, with

$$\begin{cases} \phi^f(t,\vec{x}) = \int d\mu_j[a_j^f(t)u_j(\vec{x}) + a_j^{\dagger f}(t)u_j^*(\vec{x})], \\ \phi^s(t,\vec{x}) = i\mu\int_{\tau_0}^{\tau} d\tau' R_2^f(\tau')[\phi^f(x(\tau')), \phi^f(x(\tau))]. \end{cases} \tag{20}$$

In the source parts of the above solutions, all operators on the right-hand side have been replaced by their free parts, which is correct to the first order in μ.

By resorting to the "free" part and "source" part of the dynamical variables of the atom and the field, we can now, following the DDC formalism [12,13], separate the contributions of vacuum fluctuations and radiation reaction to the evolution of the atomic observable. Writing out the Heisenberg equation of the atomic Hamiltonian,

$$\frac{dH_A(\tau)}{d\tau} = i\mu\omega_0[R_2(\tau), R_3(\tau)]\phi(x(\tau)), \tag{21}$$

separating the field operator into the free part and the source part, $\phi(x(\tau)) = \phi^f(x(\tau)) + \phi^s(x(\tau))$, and choosing a symmetric operator ordering between the variables of the atom and the field, we obtain for the rate of change of the atomic energy

$$\frac{dH_A(\tau)}{d\tau} = \left(\frac{dH_A(\tau)}{d\tau}\right)_{vf} + \left(\frac{dH_A(\tau)}{d\tau}\right)_{rr} \tag{22}$$

with

$$\left(\frac{dH_A(\tau)}{d\tau}\right)_{vf} = \frac{1}{2}i\mu\omega_0\{\phi^f(x(\tau)), [R_2(\tau), R_3(\tau)]\}, \tag{23}$$

$$\left(\frac{dH_A(\tau)}{d\tau}\right)_{rr} = \frac{1}{2}i\mu\omega_0\{\phi^s(x(\tau)), [R_2(\tau), R_3(\tau)]\}. \tag{24}$$

We assume that the field is in the vacuum state, $|0\rangle$, and the atom is initially in state $|a\rangle$. Take the average of the above two equations over the state of the system, $|0,a\rangle$, do some simplifications, and then we obtain the contributions of vacuum fluctuations and radiation reaction to the average rate of change of the atomic energy,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{vf} = 2i\mu^2\int_{\tau_0}^{\tau} d\tau' C^F(x(\tau), x(\tau'))\frac{d}{d\tau}\chi^A(\tau,\tau'), \tag{25}$$

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{rr} = 2i\mu^2\int_{\tau_0}^{\tau} d\tau' \chi^F(x(\tau), x(\tau'))\frac{d}{d\tau}C^A(\tau,\tau'), \tag{26}$$

where

$$C^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|\{\phi^f(x(\tau)), \phi^f(x(\tau'))\}|0\rangle, \tag{27}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|[\phi^f(x(\tau)), \phi^f(x(\tau'))]|0\rangle \tag{28}$$

are respectively the symmetric correlation function and the linear susceptibility function of the scalar field, and

$$C^A(\tau,\tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2(e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}), \tag{29}$$

$$\chi^A(\tau,\tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2(e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}) \tag{30}$$

are two statistical functions of the atom in state $|a\rangle$. In the above equations, $\omega_{ab} = \omega_a - \omega_b$ and the sum extends over a complete set of atomic states.

IV. GENERAL EXPRESSION FOR THE TWO-POINT FUNCTION OF THE FIELD

To calculate the average rate of change of energy of the atom, we should first find the correlation function and linear susceptibility function of the field defined in Eqs. (27) and (28). For this purpose, let us consider $\langle 0|\phi(t,\vec{x})\phi(t',\vec{x}')|0\rangle$, which can be expressed as

$$\langle 0|\phi(t,\vec{x})\phi(t',\vec{x}')|0\rangle = \frac{\nu}{8\pi^2}\sum_{m=-\infty}^{\infty}\int_0^\infty dk_\perp\int_{-\infty}^{\infty} d\kappa\frac{k_\perp}{\omega}e^{-i\omega\Delta t}e^{i\kappa\Delta z}e^{im\Delta\theta} \times J_{\nu|m|}(k_\perp r)J_{\nu|m|}(k_\perp r'), \tag{31}$$

where $\omega = \sqrt{\kappa^2 + k_\perp^2}$, $\Delta t = t - t'$, $\Delta\theta = \theta - \theta'$, and $\Delta z = z - z'$. Setting

$$\sinh\alpha = \frac{\kappa}{k_\perp}, \qquad \cosh\alpha = \sqrt{1 + \frac{\kappa^2}{k_\perp^2}},$$
$$\Delta t = s\cosh\beta, \Delta z = s\sinh\beta \tag{32}$$

for $(\Delta t)^2 > (\Delta z)^2$ (the case to be discussed in the following), the above equation is accordingly transformed to

$$\langle 0|\phi(t,\vec{x})\phi(t',\vec{x}')|0\rangle = \frac{\nu}{8\pi^2}\sum_{m=-\infty}^{\infty} e^{im\Delta\theta}\int_0^\infty dk_\perp k_\perp J_{\nu|m|}(k_\perp r)J_{\nu|m|}(k_\perp r') \times \int_{-\infty}^{\infty} d\alpha e^{-ik_\perp s\cosh(\alpha+\beta)}. \tag{33}$$

Doing the two integrations on the right-hand side of the above equation by use of formulas (3.547) and (6.522) in Ref. [36], and taking the infinite summation over m, we get

$$\langle 0|\phi(t,\vec{x})\phi(t',\vec{x}')|0\rangle = \frac{\nu}{4\pi^2 r_1 r_2}\frac{1-\Delta^{2\nu}}{1+\Delta^{2\nu}-2\Delta^\nu\cos\Delta\theta}, \tag{34}$$

where

$$r_1 = \sqrt{(r-r')^2 + (z-z')^2 - (t-t'-i\epsilon)^2}, \tag{35}$$

$$r_2 = \sqrt{(r+r')^2 + (z-z')^2 - (t-t'-i\epsilon)^2}, \tag{36}$$

$$\Delta = \frac{r_2 - r_1}{r_2 + r_1} \tag{37}$$

with ϵ being a positive infinite small real number. Thus the symmetric correlation function and the linear susceptibility function of the field are

$$C^F(x(\tau), x(\tau')) = \frac{\nu}{8\pi^2}\left(\frac{1}{r_1 r_2}\frac{1-\Delta^{2\nu}}{1+\Delta^{2\nu}-2\Delta^\nu\cos\Delta\theta} + \frac{1}{\tilde{r}_1\tilde{r}_2}\frac{1-\tilde{\Delta}^{2\nu}}{1+\tilde{\Delta}^{2\nu}-2\tilde{\Delta}^\nu\cos\Delta\theta}\right), \tag{38}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{\nu}{8\pi^2}\left(\frac{1}{r_1 r_2}\frac{1-\Delta^{2\nu}}{1+\Delta^{2\nu}-2\Delta^\nu\cos\Delta\theta} - \frac{1}{\tilde{r}_1\tilde{r}_2}\frac{1-\tilde{\Delta}^{2\nu}}{1+\tilde{\Delta}^{2\nu}-2\tilde{\Delta}^\nu\cos\Delta\theta}\right), \tag{39}$$

where $\tilde{r}_1$, $\tilde{r}_2$ and $\tilde{\Delta}$ are obtained by exchanging t and t' in r_1, r_2 and Δ.

V. RATE OF CHANGE OF ENERGY FOR A UNIFORMLY ACCELERATED ATOM

Assume that a two-level atom coupled to a quantum massless scalar field is uniformly accelerating parallel to the string, i.e., it is accelerating along the z-direction. The trajectory of the atom can be described by

$$t = \alpha\sinh\frac{\tau}{\alpha}, \qquad z = \alpha\cosh\frac{\tau}{\alpha}, \qquad r = \text{constant}, \qquad \theta = \text{constant}. \tag{40}$$

Atoms along such a trajectory are accelerating in the proper frame with acceleration α^{-1}.

Inserting the above trajectory into the general expressions of the symmetric correlation function and the linear susceptibility function of the field, Eqs. (38) and (39), we obtain

$$C^F(x(\tau), x(\tau')) = \frac{\nu}{32\pi^2 r^2}\sum_{\lambda=-,+}\frac{1}{f_\lambda(\Delta\tau, r)\sqrt{1+f_\lambda^2(\Delta\tau, r)}}\frac{1+(\sqrt{1+f_\lambda^2(\Delta\tau, r)} - f_\lambda(\Delta\tau, r))^{2\nu}}{1-(\sqrt{1+f_\lambda^2(\Delta\tau, r)} - f_\lambda(\Delta\tau, r))^{2\nu}}, \tag{41}$$

$$\chi^F(x(\tau), x(\tau')) = \frac{\nu}{32\pi^2 r^2}\sum_{\lambda=-,+}\frac{s_\lambda}{f_\lambda(\Delta\tau, r)\sqrt{1+f_\lambda^2(\Delta\tau, r)}}\frac{1+(\sqrt{1+f_\lambda^2(\Delta\tau, r)} - f_\lambda(\Delta\tau, r))^{2\nu}}{1-(\sqrt{1+f_\lambda^2(\Delta\tau, r)} - f_\lambda(\Delta\tau, r))^{2\nu}}, \tag{42}$$

where $\Delta\tau = \tau - \tau'$, $s_\pm = \mp 1$, and we have defined

$$f_\pm(\Delta\tau, r) = \mp i\frac{\alpha}{r}\sinh\left(\frac{\Delta\tau}{2\alpha} \pm i\epsilon\right). \tag{43}$$

Plugging the symmetric correlation function of the field, Eq. (41), and the antisymmetric statistical function of the atom, Eq. (30), into Eq. (25), we get, after some simplifications, the contribution of vacuum fluctuations to the average rate of change of the atomic energy,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} = -\frac{\mu^2\nu}{32\pi^2 r^2} \sum_{\lambda=-,+} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{\infty} d\Delta\tau e^{i\omega_{ab}\Delta\tau} \times \frac{1}{f_\lambda(\Delta\tau,r)\sqrt{1+f_\lambda^2(\Delta\tau,r)}} \frac{1+\left(\sqrt{1+f_\lambda^2(\Delta\tau,r)}-f_\lambda(\Delta\tau,r)\right)^{2\nu}}{1-\left(\sqrt{1+f_\lambda^2(\Delta\tau,r)}-f_\lambda(\Delta\tau,r)\right)^{2\nu}}. \tag{44}$$

In obtaining the above expression, we have assumed the time interval $\Delta\tau = \tau - \tau_0$ to be infinitely large. Similarly, the insertion of the linear susceptibility function of the field, Eq. (42), and the symmetric statistical function of the atom, Eq. (29), into Eq. (26) yields the contribution of radiation reaction to the average rate of change of the atomic energy,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} = -\frac{\mu^2\nu}{32\pi^2 r^2} \sum_{\lambda=-,+} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{\infty} d\Delta\tau e^{i\omega_{ab}\Delta\tau} \times \frac{s_\lambda}{f_\lambda(\Delta\tau,r)\sqrt{1+f_\lambda^2(\Delta\tau,r)}} \frac{1+\left(\sqrt{1+f_\lambda^2(\Delta\tau,r)}-f_\lambda(\Delta\tau,r)\right)^{2\nu}}{1-\left(\sqrt{1+f_\lambda^2(\Delta\tau,r)}-f_\lambda(\Delta\tau,r)\right)^{2\nu}}. \tag{45}$$

For an arbitrary value of the parameter ν, the integrations in the above two equations are hard to deal with. Even though these equations show that generally the contributions of vacuum fluctuations and radiation reaction are dependent on the parameter ν and the distance r between the atom and the string, so is the total rate of change of the atomic energy

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} = -\frac{\mu^2\nu}{16\pi^2 r^2} \sum_{\lambda=-,+} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{\infty} d\Delta\tau e^{i\omega_{ab}\Delta\tau} \times \frac{1}{f_-(\Delta\tau,r)\sqrt{1+f_-^2(\Delta\tau,r)}} \frac{1+(\sqrt{1+f_-^2(\Delta\tau,r)}-f_-(\Delta\tau,r))^{2\nu}}{1-(\sqrt{1+f_-^2(\Delta\tau,r)}-f_-(\Delta\tau,r))^{2\nu}}, \tag{46}$$

which is obtained by adding up Eqs. (44) and (45). It is worth noting that in Ref. [33] the response rate of an Unruh detector uniformly accelerating along such a trajectory is claimed to be completely unaffected by the presence of the cosmic string, i.e., it is exactly the same as that in a trivial Minkowski spacetime. However, we would like to point out here that the correction due to the presence of the string does not seem to be correctly estimated in Ref. [33][1] as we will explain. For the case of ν being a noninteger, the integration involved in the response rate is usually difficult to deal with as the poles are of noninteger type, while for cases with an integer ν, as the poles are of integer type, the response rate can be calculated out by the technique of contour integrations and the residuum theory. This is just what has been done in Ref. [27]. However, there the revision part was also not correctly estimated. In fact, when evaluating the integration in this case [see Eq. (2.11) in Ref. [27]], there is an infinite number of poles in and outside the contour as the function $\sinh(\frac{\Delta\tau}{2\alpha} - i\epsilon)$ becomes periodic when extended to the complex plane, and thus a nonzero correction term is expected. So an Unruh detector uniformly accelerating in a cosmic string spacetime will not be oblivious of the string, but does feel the string. For some special cases including the one with ν being an integer, accurate or approximate analytical results are obtainable. We will discuss these cases carefully in the following.

A. The case for $\nu = 1$

When $\nu = 1$, which corresponds to the case of Minkowski spacetime, the symmetric correlation function and the linear susceptibility function of the field, Eqs. (41) and (42), reduce to

$$C^F(x(\tau), x(\tau')) = -\frac{1}{32\pi^2\alpha^2}\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} - i\epsilon)} + \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} + i\epsilon)}\right), \tag{47}$$

[1] Let us note that the case of the trajectory perpendicular to the string is not correctly estimated, since the original expression of the response function of the Unruh detector should be a double integration of $\tau + \tau'$ and $\Delta\tau$, which cannot be cast into a single integration as in Eq. (16) in Ref. [33].

$$\chi^F(x(\tau),x(\tau')) = -\frac{1}{32\pi^2\alpha^2}\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} - \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right). \tag{48}$$

Accordingly, the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy become

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{vf} = \frac{\mu^2}{32\pi^2\alpha^2}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times \int_{-\infty}^{\infty} d\Delta\tau\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} + \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau}, \tag{49}$$

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{rr} = \frac{\mu^2}{32\pi^2\alpha^2}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times \int_{-\infty}^{\infty} d\Delta\tau\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} - \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau}. \tag{50}$$

By choosing appropriate contours and using the residue theory, the above two equations can be simplified to be

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{vf} = -\frac{\mu^2}{4\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{2}{e^{2\pi\omega_{ab}\alpha}-1}\right) - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{2}{e^{2\pi|\omega_{ab}|\alpha}-1}\right)\right], \tag{51}$$

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{rr} = -\frac{\mu^2}{4\pi}\left(\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 + \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\right). \tag{52}$$

The addition of the above two equations leads to the total rate of change of the atomic energy,

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\rm tot} = -\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{e^{2\pi\omega_{ab}\alpha}-1}\right) - \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{1}{e^{2\pi|\omega_{ab}|\alpha}-1}\right]. \tag{53}$$

These results are exactly what have been obtained in Ref. [1] for the rate of change of energy for an atom uniformly accelerated in a free Minkowski vacuum. So, here by taking $\nu = 1$, we recover the results in Minkowski spacetime. Equations (51)–(53) are of the same form as the rates of a static atom immersed in a thermal bath at the temperature $T = (2\pi\alpha)^{-1}$, revealing that in a free Minkowski spacetime, the effects of uniform acceleration and thermal radiation are equivalent.

B. The case for $\nu > 1$

1. The atom is uniformly accelerating on the string ($r = 0$)

We first consider a simple case in which $r = 0$, i.e., the atom is uniformly accelerated on the string. In this case, the symmetric correlation function and the linear susceptibility function of the field are simple:

$$C^F(x(\tau),x(\tau')) = -\frac{\nu}{32\pi^2\alpha^2}\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} + \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right), \tag{54}$$

$$\chi^F(x(\tau),x(\tau')) = -\frac{\nu}{32\pi^2\alpha^2}\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} - \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right). \tag{55}$$

They are exactly ν times those in Minkowski spacetime [see Eqs. (47) and (48)]. As a result, the contributions of vacuum fluctuations and radiation reaction are found to be

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} = -\frac{\mu^2 \nu}{4\pi} \left[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{2}{e^{2\pi\omega_{ab}\alpha} - 1}\right) - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{2}{e^{2\pi|\omega_{ab}|\alpha} - 1}\right)\right], \tag{56}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr} = -\frac{\mu^2 \nu}{4\pi} \left(\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \right). \tag{57}$$

And consequently the total rate of change of the atomic energy follows:

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} = -\frac{\mu^2 \nu}{2\pi} \left[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{1}{e^{2\pi\omega_{ab}\alpha} - 1}\right) - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \frac{1}{e^{2\pi|\omega_{ab}|\alpha} - 1}\right]. \tag{58}$$

Comparing Eqs. (56) and (57) with the results in a free Minkowski spacetime, Eqs. (51) and (52), we find that the contributions of vacuum fluctuations and radiation reaction to the average rate of change of the energy for an atom accelerating on the string are in the same form as those in the case of an atom uniformly accelerated in a free Minkowski vacuum, and so is the total rate of change of the atomic energy [see Eqs. (58) and (53)]. Thus spontaneous excitation (emission) occurs for a ground (excited) atom uniformly accelerated on the string as if it is immersed in a thermal bath at the temperature $T = (2\pi\alpha)^{-1}$. However, there are also some distinctions between these two cases. Notice that the results in the cosmic string spacetime, Eqs. (56)–(58), are exactly ν times those in a free Minkowski vacuum [refer to Eqs. (51)–(53)]. As $\nu > 1$, the contributions of vacuum fluctuations and radiation reaction, and thus the total rate of change of the atomic energy, are amplified due to the deficit in angle in the cosmic string spacetime. Comparing the above total rate of change of the atomic energy with that of a static atom immersed in a thermal bath in a cosmic string spacetime [34] [see Eqs. (46) and (58)], we find that the uniformly accelerated atom behaves just as if it is immersed in a thermal bath at the temperature $T = (2\pi\alpha)^{-1}$, revealing that the effects of uniform acceleration and thermal radiation are equivalent for atoms uniformly accelerated on the string.

2. *The atom is uniformly accelerated outside the string ($r \neq 0$) with $\nu \gtrsim 1$*

When the atom is uniformly accelerated outside the string, for a general value of the parameter ν, analytical results are hard to obtain. Here we consider the case $\nu \gtrsim 1$. The reason that we are interested in this case is because the parameter ν is slightly larger than unity (about 10^{-6}) for a typical cosmic string that arises in the grand unified theory.

When $\nu \gtrsim 1$, the symmetric correlation function and the linear susceptibility function of the field, Eqs. (41) and (42), can be expanded to the first order of $(\nu - 1)$ as

$$C^F(x(\tau), x(\tau')) \approx -\frac{\nu}{32\pi^2\alpha^2} \left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} - i\epsilon)} + \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} + i\epsilon)} \right) - \frac{\nu - 1}{32\pi^2 r^2} \left(\frac{\sinh^{-1}(f_-(\Delta\tau, r))}{\sqrt{1 + f_-^2(\Delta\tau, r)} f_-^3(\Delta\tau, r)} + \frac{\sinh^{-1}(f_+(\Delta\tau, r))}{\sqrt{1 + f_+^2(\Delta\tau, r)} f_+^3(\Delta\tau, r)} \right), \tag{59}$$

$$\chi^F(x(\tau), x(\tau')) \approx -\frac{\nu}{32\pi^2\alpha^2} \left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} - i\epsilon)} - \frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha} + i\epsilon)} \right) - \frac{\nu - 1}{32\pi^2 r^2} \left(\frac{\sinh^{-1}(f_-(\Delta\tau, r))}{\sqrt{1 + f_-^2(\Delta\tau, r)} f_-^3(\Delta\tau, r)} - \frac{\sinh^{-1}(f_+(\Delta\tau, r))}{\sqrt{1 + f_+^2(\Delta\tau, r)} f_+^3(\Delta\tau, r)} \right). \tag{60}$$

If we insert the above two functions of the field and the two statistical functions of the atom, Eqs. (29) and (30), into Eqs. (25) and (26) accordingly and do some simplifications, then we obtain the contributions of vacuum fluctuations and radiation reaction,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf,rr} \approx \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf,rr}^{(0)} + \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf,rr}^{(1)}, \tag{61}$$

with

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf}^{(0)} = -\frac{\mu^2 \nu}{4\pi} \left[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{2}{e^{2\pi\omega_{ab}\alpha} - 1}\right) - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{2}{e^{2\pi|\omega_{ab}|\alpha} - 1}\right)\right], \tag{62}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr}^{(0)} = -\frac{\mu^2 \nu}{4\pi} \left(\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \right), \tag{63}$$

which represent the leading parts in the contributions of vacuum fluctuations and radiation reaction, and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf}^{(1)} = \frac{\mu^2(\nu - 1)}{32\pi^2 r^2} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{\infty} d\Delta\tau e^{i\omega_{ab}\Delta\tau} \times \left(\frac{\sinh^{-1}(f_-(\Delta\tau, r))}{\sqrt{1 + f_-^2(\Delta\tau, r)} f_-^3(\Delta\tau, r)} + \frac{\sinh^{-1}(f_+(\Delta\tau, r))}{\sqrt{1 + f_+^2(\Delta\tau, r)} f_+^3(\Delta\tau, r)} \right), \tag{64}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{rr}^{(1)} = \frac{\mu^2(\nu - 1)}{32\pi^2 r^2} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \int_{-\infty}^{\infty} d\Delta\tau e^{i\omega_{ab}\Delta\tau} \times \left(\frac{\sinh^{-1}(f_-(\Delta\tau, r))}{\sqrt{1 + f_-^2(\Delta\tau, r)} f_-^3(\Delta\tau, r)} - \frac{\sinh^{-1}(f_+(\Delta\tau, r))}{\sqrt{1 + f_+^2(\Delta\tau, r)} f_+^3(\Delta\tau, r)} \right), \tag{65}$$

which stand for the correction to the leading parts. Here the correction parts are kept as integrations because they are generally difficult to deal with as the order of singularities is usually of noninteger type. Notice that the leading parts, Eqs. (62) and (63), are the same as the contributions of vacuum fluctuations and radiation reaction in the case of an atom uniformly accelerated on the string in the cosmic string spacetime with an arbitrary parameter $\nu > 1$ [see Eqs. (56) and (57)]. As ν is a bit larger than unity, $\langle \frac{dH_A(\tau)}{d\tau} \rangle_{vf,rr}$ in the cosmic string spacetime are slightly amplified due to the deficit angle, as compared to the case of an atom uniformly accelerated in a free Minkowski spacetime [see Eqs. (51) and (52)].

Adding up the contributions of vacuum fluctuations and radiation reaction, we obtain the total rate of change of the atomic energy

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} \approx \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{(0)} + \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{(1)} \tag{66}$$

with

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{(0)} = -\frac{\mu^2 \nu}{2\pi} \left[\sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{1}{e^{2\pi\omega_{ab}\alpha} - 1}\right) - \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \frac{1}{e^{2\pi|\omega_{ab}|\alpha} - 1}\right] \tag{67}$$

and

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{(1)} = -\frac{\mu^2(\nu - 1)}{16\pi^2 r^2} \sum_b \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \times \int_{-\infty}^{\infty} d\Delta\tau \frac{\arcsin(\frac{\alpha}{r}\sinh(\frac{\Delta\tau}{2\alpha} - i\epsilon)) e^{i\omega_{ab}\Delta\tau}}{\sqrt{1 - \frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha} - i\epsilon)} (\frac{\alpha}{r}\sinh(\frac{\Delta\tau}{2\alpha} - i\epsilon))^3} \tag{68}$$

representing respectively the leading part and the correction part of the average rate of change of the atomic energy. One can see that the leading part is just ν times the total rate of a uniformly accelerated atom in a free Minkowski vacuum and it is the same as the total rate of a static atom immersed in a thermal bath near the cosmic string spacetime [see Eqs. (34) and the first term in Eq. (62) in Ref. [34]], so the equivalence between the effect of uniform acceleration and thermal radiation holds up to the leading order. The correction part, which is generally expressed as an integration, can however be calculated out by using the technique of contour integration and the residuum theory, when $r = \alpha$,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot}^{(1)} = \frac{\mu^2(\nu-1)}{4\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1-\frac{1}{e^{2\pi\omega_{ab}\alpha}+1}\right)f_1(\omega_{ab},\alpha) + \sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{1}{e^{2\pi|\omega_{ab}|\alpha}+1}f_1(\omega_{ab},\alpha)\right] \tag{69}$$

with

$$f_1(\omega_{ab},\alpha) = \sinh^{-1}(1)\frac{e^{-\pi\omega_{ab}\alpha}}{\omega_{ab}\alpha}+2. \tag{70}$$

Obviously, the correction part deviates from the standard thermal form. Thus, strictly speaking, the equivalence relation between the effect of uniform acceleration and thermal radiation in terms of the transition rates of the atom, which is valid in a free Minkowski spacetime as well as for an atom uniformly accelerated on the string, does not hold for an atom uniformly accelerated outside the string.

3. The atom is uniformly accelerating parallel to the string in a spacetime with an integer ν

To better understand the influence of the nontrivial topology characterized by a deficit angle, now we consider a special case in which ν is an integer. Let us start with $\nu = 2$. In this case, the symmetric correlation function and the linear susceptibility function of the field, Eqs. (41) and (42), can be simplified to be

$$C^F(x(\tau),x(\tau')) = -\frac{1}{32\pi^2\alpha^2}\left(\frac{1-2\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}{(1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon))\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} + \frac{1-2\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}{(1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon))\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right), \tag{71}$$

$$\chi^F(x(\tau),x(\tau')) = -\frac{1}{32\pi^2\alpha^2}\left(\frac{1-2\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}{(1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon))\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)} - \frac{1-2\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}{(1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon))\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right). \tag{72}$$

Following similar steps as in the previous analysis, we can simplify the contributions of vacuum fluctuations as

$$\begin{aligned}\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{vf} &= \frac{\mu^2}{32\pi^2\alpha^2}\sum_b\omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times\int_{-\infty}^{\infty}d\Delta\tau\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}+\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau} \\ &\quad -\frac{\mu^2}{32\pi^2r^2}\sum_b\omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times\int_{-\infty}^{\infty}d\Delta\tau\left(\frac{1}{1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}+\frac{1}{1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau},\end{aligned} \tag{73}$$

and the contribution of the radiation reaction as

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{rr}=\frac{\mu^2}{32\pi^2\alpha^2}\sum_b\omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2\times\int_{-\infty}^{\infty}d\Delta\tau\left(\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}-\frac{1}{\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau}$$
$$-\frac{\mu^2}{32\pi^2r^2}\sum_b\omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2\times\int_{-\infty}^{\infty}d\Delta\tau\left(\frac{1}{1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}-i\epsilon)}-\frac{1}{1-\frac{\alpha^2}{r^2}\sinh^2(\frac{\Delta\tau}{2\alpha}+i\epsilon)}\right)e^{i\omega_{ab}\Delta\tau}. \tag{74}$$

The above equations are then calculated out to be

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{vf}=-\frac{\mu^2}{4\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{2}{e^{2\pi\omega_{ab}\alpha}-1}\right)\times(1+f_2(\omega_{ab},\alpha,r))\right.$$
$$\left.-\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{2}{e^{2\pi|\omega_{ab}|\alpha}-1}\right)\times(1+f_2(\omega_{ab},\alpha,r))\right], \tag{75}$$

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{rr}=-\frac{\mu^2}{4\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2(1+f_2(\omega_{ab},\alpha,r))\right.$$
$$\left.+\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2(1+f_2(\omega_{ab},\alpha,r))\right] \tag{76}$$

in which we have defined

$$f_2(\omega_{ab},\alpha,r)=\frac{\alpha}{2\omega_{ab}r\sqrt{\alpha^2+r^2}}\sin\left(2\omega_{ab}\alpha\cdot\sinh^{-1}\left(\frac{r}{\alpha}\right)\right). \tag{77}$$

Obviously, the contributions of vacuum fluctuations and radiation reaction are dependent on the atom-string distance, and so is the total rate of change of the atomic energy, which is given by

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\mathrm{tot}}=-\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{e^{2\pi\omega_{ab}\alpha}-1}\right)(1+f_2(\omega_{ab},\alpha,r))\right.$$
$$\left.-\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{1}{e^{2\pi|\omega_{ab}|\alpha}-1}(1+f_2(\omega_{ab},\alpha,r))\right]. \tag{78}$$

Comparing Eqs. (75), (76) and (78) with formulas (31), (34) and (35) in Ref. [3], we find that these rates are very similar to the rates for an atom uniformly accelerated parallel to a perfect reflecting plane boundary in a Minkowski vacuum. Let us note that the spacetime outside a straight cosmic string is flat everywhere but shows a nontrivial global topology characterized by the deficit angle, thus field modes propagating in this twisted spacetime are restricted as opposed to those in a free Minkowski spacetime. Our result indicates that the effect of the nontrivial global topology on the field modes in the cosmic string spacetime is very similar to that of a perfect reflecting boundary on modes traveling in a Minkowski spacetime.

When $r\sim 0$,

$$f_2(\omega_{ab},\alpha,r)\approx 1-\frac{2r^2}{3}\left(\frac{1}{\alpha^2}+\omega_{ab}^2\right), \tag{79}$$

and as a result, the rate of change of the atomic energy becomes

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{\mathrm{tot}}\approx-\frac{\mu^2}{\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{e^{2\pi\omega_{ab}\alpha}-1}\right)-\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{1}{e^{2\pi|\omega_{ab}|\alpha}-1}\right]$$
$$+\frac{\mu^2r^2}{3\pi}\left[\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\left(1+\frac{1}{e^{2\pi\omega_{ab}\alpha}-1}\right)\left(\frac{1}{\alpha^2}+\omega_{ab}^2\right)\right.$$
$$\left.-\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\frac{1}{e^{2\pi|\omega_{ab}|\alpha}-1}\left(\frac{1}{\alpha^2}+\omega_{ab}^2\right)\right]. \tag{80}$$

The first term on the right-hand side of the above equation is exactly ν times that in a free Minkowski vacuum [see Eq. (53)], and it is just the rate of an atom uniformly accelerated on the string [see Eq. (58)]. The second term which is proportional to r^2 is much smaller than the first term, and it is induced by the nontrivial topology of the cosmic string spacetime. Notice that here the second term is also not of the pure thermal form. It is worth pointing out that when taking $r \to 0$, we recover the result in the case of an atom uniformly accelerating on the string [corresponding to the result in Eq. (58) with $\nu = 2$].

When $r \to \infty$,

$$f_2(\omega_{ab}, \alpha, r) \approx \frac{\alpha}{2\omega_{ab} r^2} \sin\left(2\omega_{ab}\alpha \cdot \sinh^{-1}\left(\frac{r}{\alpha}\right)\right), \quad (81)$$

thus the rate of change of the atomic energy becomes

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} \approx -\frac{\mu^2}{2\pi}\left[\sum_{\omega_a>\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{1}{e^{2\pi\omega_{ab}\alpha} - 1}\right) - \sum_{\omega_a<\omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \frac{1}{e^{2\pi|\omega_{ab}|\alpha} - 1}\right]$$
$$- \frac{\mu^2\alpha}{4\pi r^2}\left[\sum_{\omega_a>\omega_b} \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \left(1 + \frac{1}{e^{2\pi\omega_{ab}\alpha} - 1}\right) \sin\left(2\omega_{ab}\alpha \cdot \sinh^{-1}\left(\frac{r}{\alpha}\right)\right)\right.$$
$$\left. - \sum_{\omega_a<\omega_b} \omega_{ab} |\langle a|R_2^f(0)|b\rangle|^2 \frac{1}{e^{2\pi|\omega_{ab}|\alpha} - 1} \sin\left(2\omega_{ab}\alpha \cdot \sinh^{-1}\left(\frac{r}{\alpha}\right)\right)\right]. \quad (82)$$

The first term on the right-hand side of the above equation is the same as the rate of an atom uniformly accelerated in a free Minkowski vacuum [see Eq. (53)]. The second term is much smaller than the first term. It oscillates and decreases with increasing atom-string distance, which means that the revision part can be positive or negative. As a result, the spontaneous excitation rate of an atom in the ground state and the emission rate of an atom in the excited state can be slightly larger or smaller than those in a free Minkowski spacetime.

Finally, it is worth pointing out that the above results can be easily generalized to other cases with an integer $\nu > 2$. The difference only exists in the function $f_\nu(\omega_{ab}, \alpha, r)$. For example, when $\nu = 3$,

$$f_3(\omega_{ab}, \alpha, r) = \frac{4\alpha}{\sqrt{3}\omega_{ab} r \sqrt{4\alpha^2 + 3r^2}} \times \sin\left(2\omega_{ab}\alpha \cdot \sinh^{-1}\left(\frac{\sqrt{3}r}{2\alpha}\right)\right). \quad (83)$$

Thus the properties found for the case $\nu = 2$ also hold for cases with other integer ν.

VI. CONCLUSIONS

In summary, we separately calculate the contributions of vacuum fluctuations and radiation reaction to the rate of change of energy for an atom coupled to a massless scalar field and uniformly accelerated parallel to the string in a vacuum in the cosmic string spacetime. For an atom uniformly accelerated on the string, the rate of change of the atomic energy is found to be exactly ν times that for an atom uniformly accelerated in a free Minkowski vacuum, which means that the deficit in angle in the cosmic string spacetime amplifies this rate. For an atom uniformly accelerated outside the string in a cosmic string spacetime with $\nu \gtrsim 1$, the rate is slightly revised as compared to that in a free Minkowski vacuum. We also calculate the rate of change of energy for an atom in a cosmic string spacetime with the parameter ν being an integer. Our results show that as the field modes are restricted by the special topology of the cosmic string spacetime, this rate generally depends on the atom-string distance. At infinity, the rate is composed by a leading part the same as the rate of an atom uniformly accelerated in a free Minkowski vacuum and a revision part induced by the effect of nontrivial topology of the spacetime. Finally, our results show that in the cosmic string spacetime, the equivalence between the effect of uniform acceleration and thermal radiation on the transition rates of the atom which is valid in the Minkowski spacetime holds only on the string.

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