

Dynamical fine-tuning of initial conditions for small field inflation

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Small-field inflation (SFI) is widely considered to be unnatural because an extreme fine-tuning of the initial condition is necessary for sufficiently large e -folding. In this paper, we show that the unnaturally looking initial condition can be dynamically realized without any fine-tuning if the SFI occurs after rapid oscillations of the inflaton field and particle creations by preheating. In fact, if the inflaton field ϕ is coupled to another scalar field χ through the interaction $g^2\chi^2\phi^2$ and the vacuum energy during the small field inflation is given by λM^4 , the initial value can be dynamically set at $(\sqrt{\lambda}/g)M^2/M_{\text{pl}}$, which is much smaller than the typical scale of the potential M . This solves the initial condition problem in the new inflation model or some classes of the hilltop inflation models.

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I. INTRODUCTION

The recent observation of the precise CMB data by the Planck satellite [1] gives an upper bound of the tensor to scalar ratio $r = 16\epsilon = \Delta_T^2/\Delta_S^2 < 0.12$ with 95% confidence level. Here the amplitudes of the tensor and the scalar fluctuations (in dimensionless form) are given by

$$\Delta_T^2(k) \equiv \frac{k^3}{2\pi^2} \mathcal{P}_T(k) = \frac{2\rho}{3\pi^2 M_{\text{pl}}^4}, \quad (1)$$

$$\Delta_S^2(k) \equiv \frac{k^3}{2\pi^2} \mathcal{P}_\zeta(k) = \frac{\rho}{24\pi^2 M_{\text{pl}}^4 \epsilon}. \quad (2)$$

ρ is the energy density of the universe and related to the Hubble parameter by the Einstein equation $H^2 = \rho/3M_{\text{pl}}^2$. $M_{\text{pl}} = 2.4 \times 10^{18}$ GeV is the reduced Planck scale. The scalar fluctuations (curvature perturbations) [1] are given by $\Delta_S^2 = 2.215 \times 10^{-9}$ at the pivot scale $k_{\text{CMB}} = 0.05 \text{ Mpc}^{-1}$. This constrains the energy scale of the primordial inflation $\rho^{1/4} < 1.9 \times 10^{16}$ GeV.

Large field inflation models often predict larger values of r than the observed value, and it gives a chance of revival to small field inflations (SFI). However, SFI has been known to have some drawbacks. First, SFI often predicts a smaller spectral index of the scalar perturbations compared to the observed value $n_s \sim 0.96$. The problem can be solved by introducing a tilt (a linear type potential) in the inflaton potential (see, for example, [2–5]). Another long-standing problem of SFI is that it requires very fine-tuning of the initial condition. In order to explain the sufficiently large e -folding, it is necessary to put the initial value of the SFI

very close to the top of the potential in the case of the Coleman-Weinberg type inflation.

The purpose of the paper is to show that such an unnaturally looking initial condition can be dynamically fixed by using the preheating mechanism [6,7] without introducing any fine-tunings of the initial condition. We only require that the SFI follows rapid oscillation of the inflaton field which had produced a large number of particles and modified the inflaton potential so that the inflaton field is trapped near the origin. The mechanism is similar to the moduli trapping mechanism discussed in [8], and also is discussed previously in the context of the SFI [9,10]. See also [11] for another mechanism of fine-tuning. In this paper, we revisit the problem and show that the initial value of the SFI is dynamically set at a sufficiently close point near the origin; $\phi_{\text{ini}} \sim (\sqrt{\lambda}/g)M^2/M_{\text{pl}} \ll M$ where the vacuum energy there is given by $\sim \lambda M^4$ and g is the strength of interaction $g^2\phi^2\chi^2$ between the inflaton ϕ and another scalar field χ .

The organization of the paper is as follows. In Sec. II, the fine-tuning problem of the initial condition for the SFI is explained. In Sec. III, we briefly comment on the large field inflation in the Coleman-Weinberg potential. In Sec. IV, the preheating mechanism is discussed, and we obtain the conditions for the broad parametric resonance to occur. In Sec. V, we study the effect of the created particles by preheating on the coherent motion of the inflaton and show when the broad parametric resonance ends. In Sec. VI, we show that the created particles can trap the inflaton field near the top of the hill of the potential. Finally in Sec. VII, we show that the unnaturally looking initial condition of the SFI can be dynamically set without any fine-tuning, and we summarize in Sec. VIII.

II. SMALL FIELD INFLATION AND THE FINE-TUNING PROBLEM

We particularly consider the Coleman-Weinberg (CW) type potential [12] shown in Fig. 1,

$$V(\phi) = \frac{\lambda\phi^4}{4} \left(\ln \frac{\phi^2}{M^2} - \frac{1}{2} \right) + V_0, \quad (3)$$

where $V_0 = \lambda M^4/8$. The minimum of the potential is given at $\phi = M$. In this paper, we assume $M \ll M_{\text{pl}}$.

Such potentials are studied in various situations, in particular, in the analysis of the radiative symmetry breaking of gauge theories, such as the grand unified theories (GUT) or extensions of the standard model (SM). Especially, it has attracted renewed interest recently as a simple model of physics beyond the SM satisfying the experimental constraints imposed on physics beyond the standard model (BSM) by the LHC and flavor experiments. In [13,14], based on the idea of [15], we proposed a minimal extension of the SM by introducing $B-L$ (baryon number minus lepton number) $U(1)$ gauge field Z' , an additional scalar ϕ whose vacuum expectation value (VEV) breaks the $B-L$ gauge symmetry, and the right handed neutrinos which cancel the gauge anomaly of the $B-L$ symmetry. We show that the electroweak (EW) gauge symmetry breaking is triggered by the $B-L$ breaking, which is radiatively broken by the CW mechanism. The model is a minimal extension of the SM in which radiative symmetry breaking can generate the EW scale. One of our motivations of the present analysis is to investigate the cosmological possibility of the model, but in order to make our discussions as general as possible, we do not use the specific numerical values of the coupling constants in the following.

We make use of the scalar field ϕ as an inflaton field. Since ϕ has the CW type potential as in Fig. 1, two types of inflations are possible: the large field inflation (LFI) and the SFI. The large field type can be regarded as the chaotic

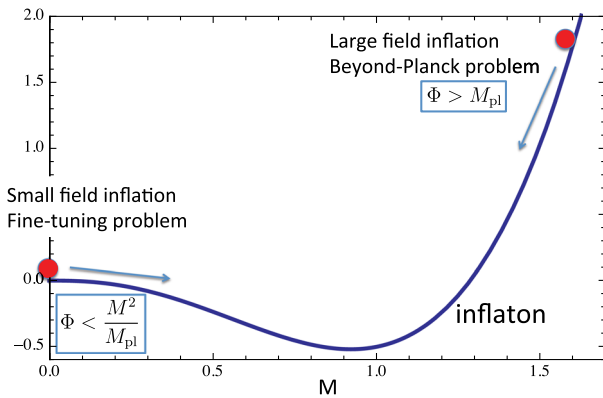


FIG. 1. Coleman-Weinberg potential has the true minimum at $\phi = M$. It is flat at the origin $\phi = 0$. Two possibilities of inflationary scenario exist, the large field inflation (chaotic inflation) and the small field inflation (new inflation). The LFI has the beyond-Planck scale problem while the SFI has the fine-tuning problem of the initial condition.

inflation with a quartic potential. On the other hand, the small field CW inflation was studied in the early 1980s in the nonsupersymmetric GUT models [16–18] and was called the new inflation. In order to realize the SFI, it is often assumed that the inflaton field is trapped at the origin due to thermal corrections to the effective potential generated in the reheating of the LFI. When the fluctuations of the field are dominated by the vacuum energy at $\phi = 0$, the SFI occurs. Then the radiation generated so far rapidly dilutes, and the inflaton field ϕ rolls down to the true minimum at $\phi = M$. The mechanism works when the reheating occurs by perturbative decay of inflaton. Since the decay process produces relativistic particles, the modification of the inflaton potential is not always sufficient to lift the true minimum to trap the field around the origin. Also, in order to explain the CMB fluctuations, the inflaton field must start from a very small value $\phi \ll M$ near the top of the potential. In the following we explain how much fine-tuning is necessary.

First we calculate the slow roll parameters to estimate the necessary initial condition in the SFI. Taking derivatives with respect to ϕ , we have

$$V' = \lambda\phi^3 \ln \frac{\phi^2}{M^2}, \quad V'' = \lambda\phi^2 \left(2 + 3 \ln \frac{\phi^2}{M^2} \right). \quad (4)$$

Mass of the scalar at the minimum is given by $m_\phi^2 = V''(M) = 2\lambda M^2$. For small values of fields ($\phi < M$), the slow roll parameters are calculated to be

$$\epsilon = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \approx 32 \left(\frac{M_{\text{Pl}}}{M} \right)^2 \left(\frac{\phi}{M} \right)^6 \left(\ln \frac{\phi^2}{M^2} \right)^2, \quad (5)$$

$$\eta = M_{\text{Pl}}^2 \left(\frac{V''}{V} \right) \approx 24 \left(\frac{M_{\text{Pl}}}{M} \right)^2 \left(\frac{\phi}{M} \right)^2 \ln \frac{\phi^2}{M^2}. \quad (6)$$

Here we used $V \approx V_0$ in the region $\phi \ll M$. The slow roll conditions $\epsilon, |\eta| < 1$ require that the field value ϕ during the SFI must be extremely smaller than M ; thus the relation $\epsilon \ll |\eta|$ follows. Inflation stops at $|\eta| = 1$ where the slow roll condition is violated.

Equation (6) can be approximately solved as

$$\phi \approx \sqrt{\frac{|\eta|}{24} \ln^{-1} \left(\frac{24 M_{\text{Pl}}^2}{|\eta| M^2} \right)} \frac{M^2}{M_{\text{pl}}}. \quad (7)$$

It requires that, in order to satisfy the slow roll condition with sufficiently large e -folding, the inflation must start from the very small initial condition,

$$\phi_{\text{ini}} \sim (10^{-3} |\eta|)^{1/2} \frac{M^2}{M_{\text{pl}}} \ll M. \quad (8)$$

This is the infamous fine-tuning problem of the initial condition of the SFI. In deriving the coefficient

numerically, we inserted $M = 10^{10}$ GeV and $|\eta| = 0.02$ but the coefficient 10^{-3} does not depend so much on the details of these values. For these values, the initial value needs to be $\phi_{\text{ini}} \sim 10^{-11}M$. Since M is the typical scale of the potential (the energy scale at the minimum of the CW potential), such a small coefficient 10^{-11} seems very unnatural as the initial condition. This is one of the reasons that large field inflations, such as the chaotic ones, are more favored than the SFI.

The slow roll parameter ϵ is much smaller than $|\eta|$ and is given by

$$\epsilon = \frac{|\eta|^3}{432 \ln(24M_{\text{Pl}}^2/|\eta|M^2)} \left(\frac{M}{M_{\text{Pl}}}\right)^4 \quad (9)$$

$$\sim 4 \times 10^{-5} |\eta|^3 \left(\frac{M}{M_{\text{Pl}}}\right)^4 \ll 1. \quad (10)$$

In order to make the amplitude of the scalar perturbation Δ_S^2 consistent with the Planck data [1], $\Delta_S^2 = 2.215 \times 10^{-9}$, the quartic coupling of the CW potential λ must be extremely small $\lambda \sim 10^{-15}$. In the following sections, we see that the smallness of the coupling becomes important to generate rapid particle creations during coherent oscillations of the inflaton field.

Here we comment on the issue of the smallness of the spectral index n_s . Those who are more interested in the initial condition problem can skip this paragraph. The e -folding number N of the SFI is related to the slow roll parameter η as

$$N = \frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi \approx \frac{3}{2} \left(\frac{1}{|\eta|} - \frac{1}{|\eta_{\text{end}}|} \right). \quad (11)$$

By putting $|\eta_{\text{end}}| = 1$, we have $\eta = -1/(2N/3 + 1)$. Since $\epsilon \ll |\eta|$, the spectral index of the scalar perturbation is given by $n_s = 1 - 6\epsilon + 2\eta \sim 1 + 2\eta$. Hence $n_s \sim 0.96$ [1] requires an e -folding number $N = 3/(1 - n_s) - 3/2 = 73.5$. On the other hand, the e -folding number at the pivot scale of CMB is given by

$$N_{\text{CMB}} = 61 + \frac{2}{3} \ln \left(\frac{V_0^{1/4}}{10^{16} \text{ GeV}} \right) + \frac{1}{3} \ln \left(\frac{T_R}{10^{16} \text{ GeV}} \right), \quad (12)$$

where we assumed that there was an epoch of the inflaton field's oscillation induced by its mass term after the inflation and then the radiation dominated epoch continues until the matter-radiation equality epoch. The smallness of the vacuum energy $V_0^{1/4} \sim 10^{-4}M \ll M_{\text{Pl}}$ suggests a smaller e -folding number than 61, which is inconsistent with the above large e -folding number $N = 73.5$. Various resolutions of the inconsistency have been proposed [2–4]. In our previous article [5], we proposed another possibility

that a linear potential is generated by the chiral condensates of quarks.

In the rest of the paper, we solve the fine-tuning problem of the initial condition given in (8) by using the dynamics of preheating during the rapid oscillation of inflaton field before the SFI starts.

III. LARGE FIELD INFLATION AND BEYOND-PLANCK PROBLEM

Before trying to solve the initial condition problem of the SFI, we remind the readers of the beyond-Planck scale problem in the LFI. If the inflaton has the potential of the CW type in Fig. 1, it is natural to think that in the early universe a coherent motion of the inflaton starts from a field value with $\phi \gg M$. Such a field value can easily be achieved due to higher scale inflationary periods before the LFI [19,20]. See also recent works [21,22] addressing the initial condition problem in LFI models. Then the potential can be approximated by the quartic one $V = \lambda\phi^4/4$. The slow roll parameters in the region $\phi \gg M$ are given by

$$\epsilon = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \approx 8 \left(\frac{M_{\text{Pl}}}{\phi} \right)^2, \quad (13)$$

$$\eta = M_{\text{Pl}} \left(\frac{V''}{V} \right) \approx 12 \left(\frac{M_{\text{Pl}}}{\phi} \right)^2 = 3\epsilon/2. \quad (14)$$

If the field value is much larger than the Planck scale M_{Pl} , the slow roll conditions are satisfied and the LFI occurs. The spectral index for the scalar perturbation is given by $n_s = 1 - 6\epsilon + 2\eta = 1 - 3\epsilon$. Hence, in order to explain the CMB data $n_s = 0.9603 \pm 0.0073$, we need $\epsilon \sim 0.013$, and accordingly the field value at the pivot scale $\phi \sim 25M_{\text{Pl}}$ is necessary. The tensor-scalar ratio is predicted as

$$r = 16\epsilon \sim 0.208, \quad (15)$$

and the e -folding is given by

$$\begin{aligned} N &= \int_{\phi_{\text{end}}}^{\phi} \frac{V}{M_{\text{Pl}}^2 V'} d\phi = \frac{1}{\sqrt{2}M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{\epsilon}} \\ &= \frac{1}{8M_{\text{Pl}}^2} (\phi^2 - \phi_{\text{end}}^2) \sim \frac{1}{\epsilon} - 1 \end{aligned} \quad (16)$$

where we put $\epsilon_{\text{end}} = 1$.

Two drawbacks are known in LFI. First it predicts a large tensor to scalar ratio (15) which seems to be inconsistent with the Planck and BICEP2/Keck Array observations [1,23]. Second the large field value $\phi \gg M_{\text{Pl}}$ may invalidate the analysis of the inflaton potential within renormalizable field theories, and higher mass-dimensional terms cannot be excluded. They are common problems of the LFI.

We call the second one the beyond-Planck-scale problem in this paper.

In the following, we show that, in the CW type potential, the SFI naturally follows the LFI. Hence, the CMB fluctuations are generated during the SFI, and the first problem is absent. Furthermore, as we show later, in order to solve the initial condition problem of the SFI (8), it is sufficient to require that, before the SFI occurs, the field starts from somewhere in the region satisfying $M < \phi$. Hence the beyond-Planck-scale problem can be avoided.

IV. PREHEATING: BROAD PARAMETRIC RESONANCE

We consider a coupled system of two scalar fields, an inflaton field ϕ and another scalar field χ . The potential of the model is

$$V(\phi, \chi) = V(\phi) + \frac{g^2}{2} \phi^2 \chi^2, \quad (17)$$

where $V(\phi)$ is given in (3). Because the initial amplitude of the oscillation of the ϕ field is much larger than M , or is close to the reduced Planck scale at the beginning of the oscillation, the ϕ field oscillates at around its origin ($\phi = 0$). The model can be considered as a toy model of the classically conformal B - L extended standard model [13]. The SM singlet scalar whose VEV breaks the B - L gauge symmetry plays the role of the inflaton ϕ , and the scalar field χ corresponds to the B - L $U(1)$ gauge field Z' . Hence the coupling g represents the B - L gauge coupling g_{B-L} . In the model [13], the Z' gauge boson is coupled to the SM particles and decays into them. In the present paper, we briefly comment on the effects of the χ decay and leave detailed (numerical) analysis for future investigations.

The strengths of the two coupling constants, λ and g , are assumed to satisfy the following inequality:

$$g^2 \gg \lambda. \quad (18)$$

It is a natural assumption since the quartic coupling of the inflaton must be very small $\lambda \sim 10^{-15}$ while the (gauge) coupling g is not necessarily so.

The equations of motion of the scalars are given by

$$\begin{aligned} \square\phi + V'(\phi) + g^2\chi^2\phi &= 0, \\ \square\chi + g^2\phi^2\chi &= 0. \end{aligned} \quad (19)$$

The d'Alembertian operator acting on modes with comoving momenta k is given by $\square = \partial_t^2 + 3H\partial_t + (k^2/a^2)$, where a is the scale factor and the Hubble constant is given by $H = \dot{a}/a$,

$$H^2 = \frac{1}{3M_{\text{pl}}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi, \chi) + \frac{\dot{\chi}^2}{2} \right]. \quad (20)$$

We divide the inflaton field into the coherent motion (zero mode) ϕ_0 and fluctuation (nonzero modes) φ as

$$\phi(t, x) = \phi_0(t) + \varphi(t, x). \quad (21)$$

For a sufficiently large initial value $\phi_0 > M_{\text{pl}}$, the coherent motion of the inflaton field realizes the LFI. The LFI ends around $\phi_0 \sim \sqrt{12}M_{\text{pl}}$ where $\eta \sim 1$, and starts falling down toward the minimum. The energy density of the universe is dominated by the coherent oscillation, and the field starts oscillation with the effective frequency $\omega_{\text{eff}} \sim \sqrt{\lambda}\Phi_0$.¹ Here Φ_0 is the amplitude of the oscillation. The effective frequency of the inflaton oscillation ω_{eff} is larger than the Hubble constant H after the oscillation starts. Hence, the expansion of the universe can be neglected during each oscillation of the ϕ field. It is also worth mentioning here that ω_{eff} largely changes in the finite density state. It is discussed in the next section.

In the coherent motion of the inflaton field, the χ and the nonzero modes of ϕ acquire time-dependent mass terms,

$$m_\chi(t)^2 = g^2\phi_0(t)^2, \quad m_\varphi(t)^2 = 3\lambda\phi_0(t)^2, \quad (22)$$

and, if the adiabaticity condition ($|\dot{\omega}|/\omega^2 < 1$) is violated, particle creations by parametric resonance occur. It is called the preheating mechanism [6]. The equation of motion for the mode with momentum k is approximated by

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \omega_\chi^2(k)\chi_k \sim 0, \quad (23)$$

where

$$\omega_\chi^2(k) = \left(g^2\Phi_0^2 \sin^2(\omega_{\text{eff}}t) + \delta m^2 + \frac{k^2}{a^2} \right). \quad (24)$$

For later convenience we added the term δm^2 which represents slowly changing mass shifts due to backreactions of the created particles. It is a finite density effect and is absent until particles are created by preheating. This term plays an important role in Sec. V.

As mentioned, the expansion of the universe is much slower than the inflaton oscillation. Then, by neglecting the Hubble term, the equation becomes identical with the Mathieu equation. The violation of the adiabaticity condition is efficient near $\phi_0 \sim 0$ and for smaller k . The equation is transformed into the standard form of the Mathieu equation by defining the new coordinate $z = \omega_{\text{eff}}t$,

¹In the quartic potential of $\lambda\phi^4/4$, the solution to the equation of motion $\ddot{\phi}_0 + \lambda\phi_0^3 = 0$ is given by $\phi_0(t) = \Phi_0 \text{cn}(\sqrt{\lambda}\Phi_0 t, 1/2)$. The Jacobi elliptic function is well approximated by the trigonometric function $\phi_0(t) \sim \Phi_0 \cos(\omega_{\text{eff}}t)$ where $\omega_{\text{eff}} = 0.8472\sqrt{\lambda}\Phi_0$.

$$\frac{\partial^2 \chi_k}{\partial z^2} + (A - 2q \cos 2z) \chi_k = 0, \quad (25)$$

$$A = \frac{1}{\omega_{\text{eff}}^2} \left(\frac{k^2}{a^2} + \delta m^2 + \frac{(g\Phi_0)^2}{2} \right), \quad (26)$$

$$q = \frac{|m_\chi|^2}{4\omega_{\text{eff}}^2} = \frac{(g\Phi_0)^2}{4\omega_{\text{eff}}^2}, \quad (27)$$

where we defined $|m_\chi| = g\Phi_0$. In the standard form, \sqrt{A} represents the ratio of the (averaged) frequency of the χ_k field to the frequency of the external force. On the other hand, q represents the strength of the external force. The Mathieu equation describes the phenomena of parametric resonances, and the solution is unstable in some regions of (A, q) parameter space. For $q < 1$, the solution is unstable only when A satisfies special conditions for the narrow resonance [6]. On the contrary, if the external force is sufficiently strong $q > 1$, the solution is unstable for a wide range of parameter space and the ratio of the frequencies A is not strongly constrained. This is called the broad parametric resonance. In a realistic model, we need to take into account backreactions from the produced particles and the redshift of momenta by the expansion of the universe. They change the narrow resonance condition for A , and hence, in order to realize the rapid increase of particles due to the Bose enhancement, the broad resonance condition $q \gg 1$ is necessary.

The particle creation is most efficient around $\phi_0 = 0$, and in order to estimate the particle creation rate, we expand the external force around $\phi_0(t) = 0$. The equation of motion (23) is then approximated by

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + \delta m^2 + |\dot{\omega}_\chi|^2 t^2 \right) \chi_k = 0, \quad (28)$$

where $|\dot{\omega}_\chi| \equiv g\Phi_0\omega_{\text{eff}}$. It is identified with the Schrödinger equation in an inverted harmonic potential, and the particle creation due to the Bogoliubov transformation around $t = 0$ is calculated as the tunneling rate in the potential. The Bogoliubov coefficient β_k is given by

$$|\beta_k|^2 = e^{-\pi\kappa_k^2}, \quad \kappa_k^2 = \frac{1}{|\dot{\omega}_\chi|} \left(\frac{k^2}{a^2} + \delta m^2 \right). \quad (29)$$

A necessary condition for the preheating is $\kappa_k \lesssim 1$. Otherwise, the adiabaticity condition is not violated, and the particle creations do not occur efficiently. This gives an upper bound of momenta of created particles.

To summarize, the particle creation due to the preheating (broad resonance) occurs only when the two conditions, $q \gg 1$ and $\kappa \lesssim 1$, are satisfied. For the sufficient particle creation of χ particles, it is necessary to satisfy

$$q_\chi = \frac{|m_\chi|^2}{4\omega_{\text{eff}}^2} = \frac{(g\Phi_0)^2}{4\omega_{\text{eff}}^2} \sim \frac{g^2}{\lambda} \gg 1, \quad (30)$$

$$\frac{k^2}{a^2} + \delta m^2 \lesssim |\dot{\omega}_\chi| = g\Phi_0\omega_{\text{eff}}. \quad (31)$$

Here we explicitly write the subscript χ in q_χ to distinguish it from the same parameter for other particles. When these conditions are satisfied together with the Bose enhancement effect, the rapid growth of the χ particles occur. Then the particle number density increases exponentially as $n_\chi \propto e^{2\mu z}$. The coefficient μ is $\mathcal{O}(0.1)$ [6]. It varies model by model, but the detailed value of μ is not essential in the following discussions.

If the created particles χ decay or annihilate into other particles, or dilute due to the rapid expansion of the universe, faster than the creation rate $\mu\omega_{\text{eff}}$, the Bose enhancement effect does not work. Hence the conditions

$$\mu\omega_{\text{eff}} > \Gamma_\chi, \quad H \quad (32)$$

need to be taken into account for the exponential growth of particle numbers to occur. The Hubble parameter $H = \sqrt{\lambda/12}(\Phi_0^2/M_{\text{pl}})$ is smaller than $\mu\omega_{\text{eff}} \sim \mu\sqrt{\lambda}\Phi_0$ for $\Phi_0 \lesssim \sqrt{12}\mu M_{\text{pl}}$, and we can neglect the effect of the expansion of the universe in the period of rapid oscillation. In the B - L model [14], χ decays into SM particles with the coupling g . Hence the decay width is given by $\Gamma_\chi \sim g^2 m_\chi \sim g^3 \Phi_0$ while the effective frequency is $\omega_{\text{eff}} \sim \sqrt{\lambda}\Phi_0$. Hence, if $\mu\sqrt{\lambda} > g^3$, the condition $\mu\omega_{\text{eff}} > \Gamma_\chi$ is satisfied. Indeed, as mentioned at the end of Sec. VII, $\lambda \sim g^4 \ll 1$ is required in the model [14], and thus the decay rate is smaller than the production rate.

Let us go back to conditions (30) and (31). The first condition (30) is satisfied in our model since λ is extremely small. When $\delta m^2 = 0$, the second condition (31) gives an upper bound of the physical momenta $p = k/a$ of the created particles,

$$p^2 < p_\star^2, \quad (33)$$

where

$$p_\star^2 \equiv |\dot{\omega}_\chi| = g\Phi_0\omega_{\text{eff}} \sim (g\Phi_0)^2/\sqrt{q_\chi}. \quad (34)$$

Since particles with lower momenta are more efficiently produced, the number distribution is far from being in thermal equilibrium. For $q_\chi \gg 1$, the mass squared $m_\chi^2(t) = (g\phi_0(t))^2$ is larger than p_\star^2 and the created χ particles behave nonrelativistically, except for short intervals of the oscillation satisfying $\phi_0(t) < q_\chi^{-1/4} g\Phi_0$. In the finite density state discussed in the next section, the left-hand side of (33) is replaced by $p^2 + \delta m^2$. Therefore, when

backreactions of created particles generate larger mass corrections $\delta m^2 > p_\star^2$, the wave equations for χ with any low momenta behave adiabatically and the particle creations stop.

In contrast to χ , φ particles (nonzero modes of ϕ) are not efficiently produced by the preheating since the first condition $q \gg 1$ is not satisfied. The parameter q for φ is given by

$$q_\varphi = \frac{|m_\varphi|^2}{4\omega_{\text{eff}}^2} = \frac{3\lambda\Phi_0^2}{4\omega_{\text{eff}}^2} \sim \mathcal{O}(1), \quad (35)$$

and it does not satisfy the broad resonance condition. Furthermore, as seen in the next section, production of χ particles leads to a larger frequency ω_{eff} , and consequently q_φ becomes much smaller than $\mathcal{O}(1)$. Hence the preheating never occurs for φ . But instead, φ particles can be rapidly produced from the χ particles by rescatterings $\chi + \phi_0 \rightarrow \chi + \varphi$ and annihilations $2\chi \rightarrow \phi_0 + \varphi$, or $2\chi \rightarrow \varphi + \varphi$. These particles have momenta $p \lesssim p_\star$. Since its mass is generated by finite density effects only, φ behaves relativistically [24]. It does hold even after the finite density effect gives a nonvanishing mass $m_\varphi^2 = \lambda\langle\varphi^2\rangle$ because $\lambda \ll g^2$ is assumed in the present analysis.

V. BACKREACTIONS AND THE END OF PREHEATING

Once the oscillation of the inflaton starts, the number densities of χ particles grow exponentially. As discussed, $\mu\omega_{\text{eff}}$ is much larger than the Hubble constant for $\Phi_0 \ll M_{\text{pl}}$, the production rate of n_χ is always larger than the expansion rate of the universe, and a precise value of μ is not important.

Soon after n_χ increases, the scattering and annihilation processes through the interaction term $g^2\chi^2\phi^2$ rapidly create nonzero modes φ , and the universe is filled with χ and φ particles.² The number density n_φ grows until it becomes equal to n_χ [24]. The χ field acquires an additional mass correction in addition to (22),

$$m_\chi^2 = g^2(\phi_0^2(t) + \langle\varphi^2\rangle). \quad (36)$$

At the beginning of preheating, the coherent part is dominant, but soon the second term becomes comparable to the first term. The 2-point function at the same space-time point can be evaluated by using the Hartree approximation as

²In [24], the evolution of occupation numbers was numerically investigated. Figure 13 in [24] shows the rapid growth of φ after χ particles are created by the preheating. It also shows that the occupation numbers of φ and χ become equal when the exponential growth stops.

$$\langle\varphi^2\rangle = \int \frac{d^3k}{(2\pi a)^3} \frac{n_{\varphi,k} + 1/2}{\omega_k} \sim \frac{n_\varphi}{p_\star}, \quad (37)$$

where ω_k is replaced by the typical momentum p_\star of φ . In the following we see that the preheating stops when both terms become comparable and indistinguishable.

The coherent motion of ϕ is also modified by the additional contribution $g^2\langle\chi^2\rangle\phi^2/2$ in the finite density state. The 2-point function for the χ field is similarly evaluated as

$$\langle\chi^2\rangle = \int \frac{d^3k}{(2\pi a)^3} \frac{n_{\chi,k} + 1/2}{\omega_k} \sim \frac{n_\chi}{g|\phi|}. \quad (38)$$

Here, by using the fact that the created particles are nonrelativistic,³ ω_k is replaced by $m_\chi = g|\phi_0(t)|$. It is justified as long as $|\phi_0| > \sqrt{\langle\varphi^2\rangle}$ in (36). Then the interaction between the inflaton and the χ field gives the induced potential which is linear [6] in ϕ ,

$$\frac{g^2}{2}\langle\chi^2\rangle\phi^2 \sim \frac{g^2}{2} \frac{n_\chi}{g|\phi|} \phi^2 = \frac{g}{2} n_\chi |\phi|. \quad (39)$$

Comparing this with the original potential $\lambda\phi^4/2$, the above term becomes more dominant when the number of created particles n_χ is larger than the number density determined by the amplitude of the inflaton oscillation; $n_\chi > (\lambda/g)\Phi_0^3$. Hence, the quartic potential of the inflaton is gradually modified by the backreaction of created particles, and when the above condition is satisfied, inflaton oscillation can be approximated by the linear-type potential $V \propto |\phi|$ for $|\phi| > \sqrt{\langle\varphi^2\rangle}$. The effective frequency of the inflaton oscillation in the linear potential is given by $\omega_{\text{eff}}^2 \sim gn_\chi/\Phi_0 \sim g^2\langle\chi^2\rangle_{\phi_0=\Phi_0}$.

When the 2-point function $\langle\varphi^2\rangle$ in (36) increases up to $\langle\varphi^2\rangle = \Phi_0^2$, $\langle\chi^2\rangle$ is replaced by

$$\langle\chi^2\rangle = \frac{n_\chi}{g\Phi_0}, \quad (40)$$

and the inflaton potential changes from the linear to the quadratic one for $\phi < \Phi_0$,⁴

$$\frac{g^2}{2}\langle\chi^2\rangle\phi^2 \sim \frac{gn_\chi}{2\Phi_0}\phi^2. \quad (41)$$

³At later stages of preheating, the energy is transferred from IR to UV momenta due to scatterings, and the χ particles become relativistic. Numerical simulations are necessary for further precise evaluations of $\langle\chi^2\rangle$. We leave it for future investigations.

⁴In [10], from the validity of the quadratic potential for $\phi > \Phi_0$ and numerical simulations, the authors concluded that the effective potential is linear for large $\phi > \Phi_0$ even after the φ production up to $\langle\varphi^2\rangle \sim \Phi_0^2$, and consequently the first order phase transition occurs.

Hence the effective frequency of the inflaton oscillation does not change and is given by $\omega_{\text{eff}}^2 = gn_\chi/\Phi_0$.

Through the preheating, the energy of the coherent motion of inflaton is transferred to the energy of the created particles. The preheating finally ends either the condition for the broad parametric resonance $q_\chi > 1$ [the value of q_χ in the zero density state is given in (30)] or the condition for the mass correction $\delta m^2 < p_\star^2$ is violated. Since q_χ , δm , p_\star are functions of g , λ , $\langle \chi^2 \rangle$, and $\langle \varphi^2 \rangle$, we need to numerically solve the evolution of 2-point functions $\langle \chi^2 \rangle$, $\langle \varphi^2 \rangle$ and the coherent mode Φ_0 . In the following we evaluate the conditions for broad parametric resonance based on the above ansatz for the 2-point functions.

The parameter q_χ gets modifications by the created particles as follows. The mass of the χ field in the finite density state is given by Eq. (36). On the other hand, the effective frequency ω_{eff} of the inflaton field is given as

$$\omega_{\text{eff}}^2 \sim \lambda(\Phi_0^2 + \langle \varphi^2 \rangle) + g^2 \langle \chi^2 \rangle, \quad (42)$$

and it receives larger corrections than the bare term $\lambda\Phi_0^2$ because $\lambda \ll g^2$. Accordingly the parameter q_χ to determine the broad parametric resonance for χ particles in the finite density state is replaced by

$$q_\chi = \frac{m_\chi^2}{4\omega_{\text{eff}}^2} \sim \frac{g^2(\Phi_0^2 + \langle \varphi^2 \rangle)}{\lambda(\Phi_0^2 + \langle \varphi^2 \rangle) + g^2 \langle \chi^2 \rangle}. \quad (43)$$

As we saw, when created particles are absent, it is $q_\chi = g^2/\lambda \gg 1$. As the χ particles are created by preheating (remember $g^2 \gg \lambda$), the last term comes to dominate the denominator and ω_{eff}^2 is approximated by $g^2 \langle \chi^2 \rangle$. The preheating stops when $\langle \chi^2 \rangle \sim \Phi_0^2$ and q_χ decreases down to $\mathcal{O}(1)$. At the time, the mass of φ becomes

$$m_\varphi = \omega_{\text{eff}} \sim g\sqrt{\langle \chi^2 \rangle} \sim g\Phi_0. \quad (44)$$

Thus, since $p_\star = \sqrt{g\Phi_0\omega_{\text{eff}}}$, we have the relation

$$m_\chi \sim m_\varphi \sim p_\star \sim g\Phi_0 \quad (45)$$

when the preheating stops. If φ are rapidly produced by scatterings, we have the chemical equilibrium $n_\varphi \sim n_\chi$. Then the relation $\langle \varphi^2 \rangle = \langle \chi^2 \rangle = \Phi_0^2$ follows (37) and (40). This is the case we study in our paper. (See also footnote 5.)

When the condition $q_\chi > 1$ is violated, the other condition $\delta m^2 < p_\star^2$ for the preheating becomes simultaneously violated in the above situations. Both sides are given by $\delta m^2 = g^2 \langle \varphi^2 \rangle$ and $p_\star^2 = g\Phi_0\omega_{\text{eff}} \sim g^2\Phi_0\sqrt{\langle \chi^2 \rangle} \sim g^2\Phi_0^2$. Hence if φ particles are rapidly produced and $\langle \varphi^2 \rangle = \langle \chi^2 \rangle = \Phi_0^2$ holds, $\delta m^2 = p_\star^2$ is satisfied and even zero-momentum particles cannot be created.

To summarize, all the fluctuations have the same amplitude $\langle \varphi^2 \rangle = \langle \chi^2 \rangle = \Phi_0^2$ when the preheating stops. And the number densities are related to the amplitude of fluctuations as

$$n_\chi(t) = n_\varphi(t) = g\Phi_0^3(t). \quad (46)$$

Initially, $g\Phi_0^3$ is larger than n_χ and n_φ , and the preheating occurs. The amplitude of the coherent oscillation Φ is reduced, and the number density n_χ increases. At the same time scatterings produce φ particles and increase n_φ . Finally, the preheating stops when the number densities become equal to $g\Phi_0^3$. In addition, the effective masses of the particles become equal, $m_\chi = m_\varphi = g\Phi_0$. These properties are consequences of the hierarchy in the coupling constants $g^2 \gg \lambda$.

The initial value $\Phi_{0,\text{start}}$ of the coherent motion of the inflaton field determines the amplitude $\Phi_{0,\text{end}}$ at which the broad parametric resonance stops. Since the particle production occurs faster than the Hubble time scale, the energy conservation gives a relation between them as follows. The potential energy of the initial inflaton configuration $\lambda\Phi_{0,\text{start}}^4$ is transferred to the energy of the χ and φ particles $\sim g^2 \langle \varphi^2 \rangle \langle \chi^2 \rangle = g^2\Phi_{0,\text{end}}^4$. Thus we have

$$\Phi_{0,\text{end}} = \left(\frac{\lambda}{g^2}\right)^{1/4} \Phi_{0,\text{start}}. \quad (47)$$

Note that the finite density state is not in the thermal equilibrium since the preheating is mostly efficient for very low momentum particles. Further studies of the process toward thermal equilibrium (i.e., turbulence flow from IR to UV momenta) are left for future investigations [25].⁵

After the preheating stops, the energy density is dominated by the χ and φ particles, and is given by $\rho \sim g^2\Phi_0^4(t)$. In the following, we use Φ to indicate the typical amplitude of the fluctuations. As the universe expands, the particle densities gradually dilute and the amplitude $\Phi(t)$ decays as well.

VI. TRAPPING THE INFLATON NEAR THE TOP OF THE POTENTIAL

The inflaton potential is modified due to the created particles, and the potential is raised, which may prevent the inflaton field from falling down to the minimum $\phi = M$ of the Coleman-Weinberg potential $V(\phi)$. Since the magnitude of the potential raise depends on the amplitude of the

⁵In [25], the authors studied a case where the created particles χ interact with themselves and its number density generates the mass for χ itself. In such cases, the exponential growth stops much earlier and the stationary turbulence (which leads to linear growth of occupation numbers) is shown to occur. In the process, the energy flows from IR modes to UV modes.

fluctuations $\langle \chi^2 \rangle = \Phi^2$, it is necessary to check whether the field is trapped without falling down to the minimum. In this section, we show that it actually happens for a wide range of parameters.

As we saw in the previous section, when the number density of created particles becomes comparable to $g\Phi_0^3$, the preheating stops. After the preheating stops, the amplitudes of fluctuations keep the relation $\langle \chi^2 \rangle = \langle \varphi^2 \rangle = \Phi^2$ since χ and ϕ particles are in chemical equilibrium and the zero mode ϕ_0 is indistinguishable from the nonzero modes φ . Keeping this relation, the amplitudes decrease as the universe expands.

When the amplitude of the fluctuations is Φ , the inflaton potential is approximated by

$$V = \frac{\lambda}{4} \phi^4 \left(\ln \frac{\phi^2}{M^2} - \frac{1}{2} \right) + \delta V, \quad (48)$$

$$\delta V = \begin{cases} \frac{g^2 \Phi^2}{2} \phi^2 & |\phi| < \Phi \\ \frac{g^2 \Phi^3}{2} |\phi| & |\phi| > \Phi. \end{cases}$$

The second term δV is the finite density contribution of created particles. The first term also receives an additional contribution in the finite density state, but since $g^2 > \lambda$, it is negligible compared to the last term. The main role of the first term is to give the minimum at $\phi = M$. Because of δV , the minimum at $\phi = M$ is lifted as in Fig. 2. The question is whether the position of the barrier between two minima is on the right of the field value $\phi = \Phi$. For $g^2 > \lambda$, it can be

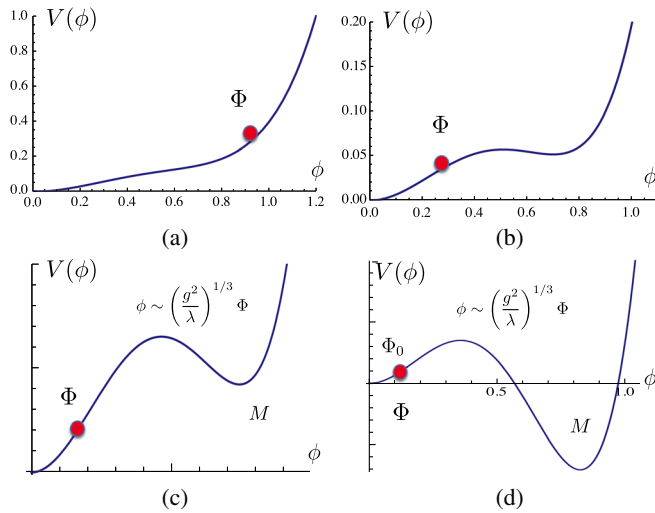


FIG. 2. Schematic figures of the inflaton potential modified by the finite density effect. When the amplitude of the fluctuations Φ is larger than $\Phi_1 \equiv (4e\lambda/3g^2)^{1/3}M$, the potential has no nontrivial minimum (a). As the amplitude becomes smaller, the nontrivial minimum appears (b), (c). For a smaller amplitude of Φ , the minimum at $\phi = M$ becomes the true minimum as in (d). The position of the potential barrier is located around $(g^2/\lambda)^{1/3}\Phi$, and it is always on the right of Φ , if $g^2 > \lambda$.

shown that the modified potential of (48) has a nontrivial minimum only when $\Phi \lesssim \Phi_1 \equiv (4e\lambda/3g^2)^{1/3}M$. Otherwise, the potential has the unique minimum at $\phi = 0$ as in Fig. 2(a). Thus the coherent motion falls toward the origin until the amplitude becomes smaller than $\Phi = \Phi_1$. For $\Phi < \Phi_1$ in Figs. 2(b), 2(c), and 2(d), the potential has two local minima. It can easily be shown that the position of the barrier between two minima is given by $\phi \sim (g^2/\lambda)^{1/3}\Phi$. Hence, for $g^2 > \lambda$, the barrier is always on the right of Φ . Even after the amplitude decreases further and the finite density effect is no longer capable to lift the minimum at $\phi = M$ above $V = 0$ [see Fig. 2(d)], the inflaton field is kept being trapped within the barrier.

The above trapping mechanism occurs only when sufficient particle creation has finished before the field falls down to the true minimum. Namely, in order to trap the field in the potential barrier, the quadratic potential must be generated before the field falls down. A sufficient condition for this is that the amplitude of the inflaton $\Phi_{0,\text{end}}$ at the end of the preheating is larger than M ; $\Phi_{0,\text{end}} > M$. Therefore, using (47), if $\Phi_{0,\text{start}}$ satisfies the condition

$$\Phi_{0,\text{start}} > (g^2/\lambda)^{1/4}M, \quad (49)$$

χ particles are sufficiently produced so that it can trap the inflaton field within the potential barrier.⁶ The field value $\Phi_{0,\text{start}}$ can be much smaller than the Planck scale, and the beyond-Planck-scale problem is absent.

Finally we comment on the effects of thermalization of χ particles on the trapping mechanism. In particular, we take the B - L model as an example. When the preheating stops, the relation $n_\chi = g\Phi^3$ holds. In the B - L model, as discussed after Eq. (32), the decay rate of χ is given by $\Gamma_\chi = g^3\Phi$. χ can also annihilate into the SM particles, whose rate is estimated as $\Gamma_{\chi,\text{annih}} \sim g^4 n_\chi / m_\chi^2 \sim g^3\Phi$. Hence, if $\Phi < g^2 M_{\text{pl}}$, these rates are larger than the Hubble parameter $H \sim g\Phi^2/M_{\text{pl}}$ and the thermal bath with the temperature $T \sim \sqrt{g}\Phi$ is produced. Furthermore, when $\Phi < g^{7/2}M_{\text{pl}}$, the B - L scattering rate satisfies $\Gamma_{\text{scatt}} \sim g^4 T > H$ and all the system including the B - L and the inflaton fields is thermalized. Then the fluctuation of fields are determined by the temperature $\Phi_T^2 \equiv \langle \varphi^2 \rangle \sim \langle \chi^2 \rangle \sim T^2 \sim g\Phi^2$. They are smaller than the original value of the fluctuations $\langle \varphi^2 \rangle \sim \langle \chi^2 \rangle = \Phi^2$. This is because thermalization transfers energy from IR to UV regions with typical momentum $p \sim T$ and reduces the large fluctuation produced by the preheating. Consequently the coefficient of the quadratic

⁶To get the actual lower bound on $\Phi_{0,\text{start}}$, accordingly the field value where the zero mode ϕ_0 starts rolling down, we need a detailed numerical calculation while being careful of the shape of the Coleman-Weinberg potential (3). It is beyond the scope of this paper and left as a future work. Depending on the initial value, new inflation with a sufficient e -folding could occur without the inflaton oscillation [26].

term in (48) is replaced by $g^2\langle\chi^2\rangle = g^2\Phi_T^2$ for $|\phi| < \Phi_T/g$. In this case, the position of the barrier is given by $(g^2/\lambda)^{1/2}\Phi_T$. For the amplitude of the fluctuation Φ_T , the barrier is always on the right of the fluctuation and the trapping mechanism similarly holds.

VII. DYNAMICAL FINE-TUNING OF THE INITIAL CONDITION

We now determine the initial value of the SFI. The amplitudes of the oscillation and fluctuations decrease in the expanding universe where the energy density is dominated by the energy of the created particles, $\rho \sim g^2\Phi^4$. But as the amplitude becomes smaller, the vacuum energy $V_0 = \lambda M^4/8$ will dominate the energy of created particles. By comparing these energies, we see that the de Sitter expansion starts when the amplitude becomes smaller than the following value:

$$\Phi \sim \left(\frac{\lambda}{8g^2}\right)^{1/4} M. \quad (50)$$

It is already small but still much larger than the necessary initial condition in Eq. (8).

During the de Sitter expansion, the inflaton continues to oscillate until the effective frequency of the inflaton $\omega_{\text{eff}} = g\Phi$ becomes smaller than the Hubble constant $H = \sqrt{\lambda M^4/24M_{\text{pl}}^2}$ of the de Sitter universe. Hence the amplitude of fluctuations continues to decay as far as the condition $\omega_{\text{eff}} > H$ is satisfied. The oscillation of the inflaton finally stops when the condition

$$\Phi \lesssim \frac{1}{g} \sqrt{\frac{\lambda}{24}} \frac{M^2}{M_{\text{pl}}} \quad (51)$$

is satisfied. After this condition is satisfied, the inequality $\omega_{\text{eff}} < H$ holds and the fluctuations of the inflaton field with lower momenta than the Hubble constant are *frozen*. Therefore, in the new inflation model with the Coleman-Weinberg potential, the amplitudes of the coherent motion and also the fluctuations are reduced to the very small value.⁷

Equation (51) solves the fine-tuning problem of the small field inflation (8). Let us estimate the coefficient in (51) numerically. The quartic coupling is determined by the amplitude of the curvature fluctuations as $\lambda \sim 10^{-15}$. Inserting the value in (51), it becomes

$$\Phi \sim 10^{-3} \left(\frac{10^{-5}}{g}\right) \frac{M^2}{M_{\text{pl}}}. \quad (52)$$

It is smaller than the upper bound of (8) if the coupling g satisfies $g \gtrsim 10^{-5}$. For general models g is a free parameter and we can take any value, but in the B - L extension of the SM [14], the β -function of the quartic coupling λ has a contribution from the gauge coupling; $\beta_\lambda = 96g^4/16\pi^2$. Hence unless $g^4 \sim \lambda$, we need a fine-tuning to keep the smallness of λ . The most natural assumption is $g \sim \lambda^{1/4} \sim 10^{-4}$. Thus, within the model [14], the initial condition problem of the SFI is naturally solved.

VIII. SUMMARY

In this paper, we proposed a mechanism to solve the fine-tuning problem of the new inflation, the small field inflation with the Coleman-Weinberg type potential. The key relation (51) to determine the initial value is obtained by comparing the effective frequency of the oscillation and the Hubble constant ($H^2 = V_0/3M_{\text{pl}}^2$). The flatness at the top of the potential is responsible for the fine-tuning problem of the SFI. Corresponding to this fact, the effective frequency should be dynamically generated by the fluctuations of created particles. Then, from the dimensional analysis, we can expect that it is given by $\omega_{\text{eff}} = g\Phi$, where g is the coupling to the field which gives the effective potential of the inflaton. Then, if the vacuum energy is given by $V_0 = \lambda M^4$, the initial value of the inflaton is given by $\Phi \sim \sqrt{\lambda}/g(M^2/M_{\text{pl}})$. Therefore, once the inflaton field is trapped by the quadratic potential generated in the preheating, the fine-tuning problem in similar models can be solved.

For hilltop inflation models with a negative curvature potential ($-\mu^2\phi^2/2$) at the origin, the dynamical fine-tuning mechanism for the small field inflation does not work since the system undergoes the first order phase transition before a sufficiently large e -folding number is gained [10]. Hence the flatness ($\mu = 0$) at the top of the hill is essential to solve the fine-tuning problem of the initial condition in SFI. It is interesting that two completely different fine-tuning problems, the Higgs mass [15] and the initial condition in SFI, can be solved dynamically by simply assuming the absence of the dimensionful parameter μ in the bare Lagrangian.

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⁷If the system is thermalized as discussed at the end of Sec. VI, the energy density of the universe ρ is given by $\sim\Phi_T^4$ and the right-hand side of (50) is replaced by $(\lambda/8)^{1/4}M$. During the de Sitter expansion, the scattering rate Γ_{scatt} becomes smaller than the Hubble constant H . Then the system is no longer in thermal equilibrium. But the amplitude of fluctuation Φ_T is redshifted and continues to reduce as a^{-1} . Since the effective frequency of inflaton is given by $\omega_{\text{eff}} = g\Phi_T$, the fluctuation is frozen at the same value of (51).

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