

Constraints on α -attractor inflation and reheatingYoshiki Ueno¹ and Kazuhiro Yamamoto^{1,2}¹*Department of Physical Science, Graduate School of Science, Hiroshima University,
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(Received 2 April 2016; published 28 April 2016)

We investigate a constraint on reheating followed by α -attractor-type inflation (the E-model and T-model) from an observation of the spectral index n_s . When the energy density of the Universe is dominated by an energy component with the cosmic equation-of-state parameter w_{re} during reheating, its e -folding number N_{re} and the reheating temperature T_{re} are bounded depending on w_{re} . When the reheating epoch consists of two phases—where the energy density of the Universe is dominated by uniform inflaton field oscillations in the first phase and by relativistic nonthermalized particles in the second phase—we find a constraint on the e -folding number of the first oscillation phase, N_{sc} , depending on the parameters of the inflaton potential. For the simplest perturbative reheating scenario, we find the lower bound for a coupling constant of inflaton decay in the E-model and T-model depending on the model parameters. We also find a constraint on the α parameter, $\alpha \gtrsim 0.01$, for the T-model and E-model when we assume a broad resonance reheating scenario.

DOI: [10.1103/PhysRevD.93.083524](https://doi.org/10.1103/PhysRevD.93.083524)**I. INTRODUCTION**

Inflation is a key to exploring the beginning of the Universe. There are various inflation models. However, recent precise observations of the cosmic microwave background and the large-scale structure of galaxies impose useful constraints on inflation models [1]. The combination of constraints on the spectral index n_s and the scalar-tensor ratio r excludes the simplest single power-law potential models. A class of inflation models that is consistent with observations is the α -attractor-type models, which were recently proposed in a unified manner [2–10]; they include the Starobinsky model [11,12] (cf., Refs. [13–16]) and the Higgs inflation model [17–21].

Reheating after inflation is important for the inflation model itself as a mechanism to realize the hot big bang Universe. The energy of an inflaton field is converted to thermal radiation during a reheating epoch by processes that may include the physics of particle creation and nonequilibrium phenomena. Reheating processes have been investigated in many studies (e.g., Refs. [22–29]), in which successful scenarios of preheating and subsequent thermalization processes were discussed; however, many uncertainties still remain (see, e.g., Refs. [30,31] for a review).

Some authors recently investigated a constraint on the reheating epoch [32–34] that uses a recent precise constraint on the spectral index n_s [1]. The authors of Refs. [32–34] investigated constraints on the e -folding number and reheating temperature depending on the effective equation-of-state parameter of the reheating epoch. In this paper, we investigate the constraint on the

reheating epoch of the α -attractor-type inflation models. The authors of Ref. [34] investigated the constraint on the reheating epoch in the Higgs inflation model; however, our investigations focus on a wider class of α -attractor-type models, the E-model and T-model [35,36], which are consistent with the observations. Some aspects of reheating followed by the E-model and T-model were investigated in Ref. [37] by introducing a phase diagram, but we examine this problem from a different perspective.

In our investigation, our approach to a constraint on reheating differs from those of Refs. [32–34,37]. These previous works assumed that the Universe is dominated by an energy density with a constant equation-of-state parameter w_{re} . In this work, we consider a reheating epoch consisting of two phases. The first phase is an epoch in which the energy of the Universe is dominated by uniform inflaton field oscillations (the oscillation phase), and the second phase is an epoch in which the Universe is dominated by relativistic but nonthermalized particles produced by decay of the inflaton field (the thermalization phase). Our analysis constrains the e -folding number of the oscillation phase using an observation of the spectral index n_s , which we use to discuss constraints on a parameter of the inflaton potential and a coupling constant for inflaton decay depending on two reheating scenarios.

This paper is organized as follows. In Sec. II, we briefly review how to constrain the e -folding number of reheating and the reheating temperature using an observational constraint on the spectral index n_s . In Sec. III, we investigate a constraint on the reheating epoch in a single-field α -attractor model, assuming that the reheating epoch is dominated by an energy component with the

equation-of-state parameter w_{re} . We demonstrate that our result is consistent with previous results. In Sec. IV, we consider a reheating epoch consisting of the two phases, i.e., the scalar field oscillation phase and the thermalization epoch. In Sec. V, we discuss the impacts of our results on two reheating scenarios. Section VI presents a summary and conclusions. We adopt the convention $M_{\text{pl}}^2 = 1/8\pi G$, where G is the gravitational constant.

II. CONSTRAINT ON REHEATING

We briefly review how to constrain the e -folding number of reheating N_{re} and the reheating temperature T_{re} using an observational constraint on the spectral index n_s [32–34,37]. We consider a single-field inflation model with a potential $V(\phi)$, which obeys

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \quad (1)$$

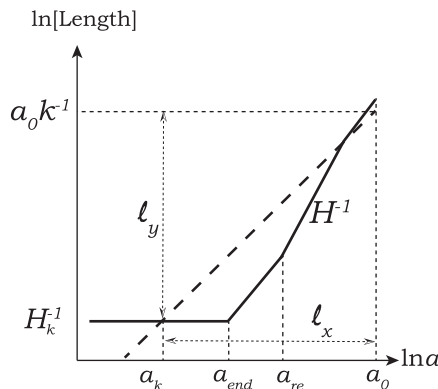
where the dot indicates differentiation with respect to cosmic time, and a is the scale factor determined by the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\text{pl}}^2} \left(\frac{\dot{\phi}^2}{2} + V(\phi)\right). \quad (2)$$

Adopting the slow-roll approximation during inflation, the above equations are approximated as

$$3H\dot{\phi} + V'(\phi) = 0, \quad (3)$$

$$H^2 = \frac{V(\phi)}{3M_{\text{pl}}^2}, \quad (4)$$



where the prime denotes differentiation with respect to ϕ , and $H = \dot{a}/a$ is the Hubble parameter. Introducing the slow-roll parameters,

$$\epsilon = \frac{1}{2} \left(\frac{M_{\text{pl}} V'}{V} \right)^2, \quad (5)$$

$$\eta = \frac{M_{\text{pl}}^2 V''}{V}, \quad (6)$$

we may write the spectral index and tensor-to-scalar ratio as

$$n_s = 1 - 6\epsilon + 2\eta, \quad (7)$$

$$r = 16\epsilon, \quad (8)$$

and the energy density during the inflation epoch is written as $\rho = (1 + \epsilon/3)V$. We define the end of inflation as $\epsilon = 1$, at which the energy density of the Universe can be written as

$$\rho_{\text{end}} = \frac{4}{3}V(\phi_{\text{end}}) = \frac{4}{3}V_{\text{end}}, \quad (9)$$

where ϕ_{end} is the value of the scalar field at the end of inflation. The e -folding number between horizon crossing of a perturbation of wave number k and the end of inflation is estimated as

$$N_k = \ln\left(\frac{a_{\text{end}}}{a_k}\right) = -\frac{1}{M_{\text{pl}}^2} \int_{\phi}^{\phi_{\text{end}}} \frac{V}{V'} d\phi, \quad (10)$$

where a_k and a_{end} are the scale factors at horizon crossing of a perturbation of wave number k and at the end of inflation, respectively (see Fig. 1).

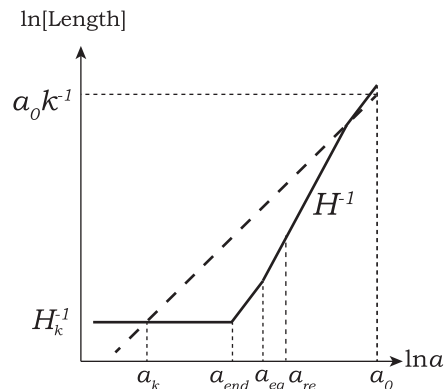


FIG. 1. Sketch of the evolution of the Hubble horizon distance H^{-1} (solid curve) from the inflation epoch to the present epoch as a function of the scale factor a . The long dashed line shows the evolution of the physical wavelength of a perturbation with the comoving wave number k . Here a logarithmic scale is adopted for both axes. a_k , a_{end} , a_{eq} , a_{re} , and a_0 are the scale factors at horizon crossing during inflation, at the end of inflation, at the equal time during reheating, at the end of reheating, and at the present epoch, respectively. The left and right panels illustrate the assumptions in Secs. III and IV, respectively. In the present paper, we adopt $k = 0.05 \text{ Mpc}^{-1}$.

Following previous works [32–34,37], we first assume that during the reheating epoch, the Universe is dominated by an energy component with an effective equation-of-state parameter w_{re} . At the end of the reheating epoch, we assume that the energy density of the Universe is written as

$$\rho_{\text{re}} = \frac{\pi^2 g_{\text{re}}}{30} T_{\text{re}}^4, \quad (11)$$

where T_{re} is the reheating temperature, and g_{re} is the number of internal degrees of freedom of relativistic particles at the end of reheating, which we assume to be $g_{\text{re}} = \mathcal{O}(100)$. Defining the scale factor at the end of reheating, a_{re} , then, we can write the e -folding number of the reheating epoch,

$$N_{\text{re}} = \ln\left(\frac{a_{\text{re}}}{a_{\text{end}}}\right) = -\frac{1}{3(1+w_{\text{re}})} \ln\left(\frac{\rho_{\text{re}}}{\rho_{\text{end}}}\right), \quad (12)$$

where a_{re} is the scale factor at the end of reheating.

Using an observational constraint on the spectral index of the initial curvature perturbations, we can constrain the e -folding number N_{re} and the effective equation-of-state parameter w_{re} of the reheating epoch. The horizon crossing of a perturbation with the wave number k occurs at $a_k H_k = k$, where a_k and H_k are the scale factor and Hubble parameter, respectively, at horizon crossing during inflation. Then, we can write

$$0 = \ln\left(\frac{k}{a_k H_k}\right) = \ln\left(\frac{a_{\text{end}} a_{\text{re}} a_0}{a_k a_{\text{end}} a_{\text{re}} a_0} \frac{k}{H_k}\right), \quad (13)$$

where a_0 is the scale factor at the present epoch. Using the definitions (10) and (12), Eq. (13) yields

$$N_k + N_{\text{re}} + \ln\left(\frac{a_0}{a_{\text{re}}}\right) + \ln\left(\frac{k}{a_0 H_k}\right) = 0. \quad (14)$$

The geometrical meaning of Eq. (14) is the equality in the lengths $\ell_x = \ell_y$ in the left panel of Fig. 1.

From the conservation of entropy, we may write

$$\frac{a_{\text{re}}}{a_0} = \left(\frac{43}{11g_{\text{re}}}\right)^{1/3} \frac{T_0}{T_{\text{re}}}, \quad (15)$$

where $T_0 = 2.725$ K is the temperature of the Universe at the present epoch. Using Eq. (11), Eq. (15) is rewritten as

$$\frac{a_{\text{re}}}{a_0} = \left(\frac{43}{11g_{\text{re}}}\right)^{1/3} T_0 \left(\frac{\pi^2 g_{\text{re}}}{30\rho_{\text{re}}}\right)^{1/4}. \quad (16)$$

Furthermore, using Eqs. (9) and (12), we have

$$\rho_{\text{re}} = \frac{4}{3} V_{\text{end}} \left(\frac{a_{\text{re}}}{a_{\text{end}}}\right)^{-3(1+w_{\text{re}})} = \frac{4}{3} V_{\text{end}} e^{-N_{\text{re}} 3(1+w_{\text{re}})}. \quad (17)$$

Then, the logarithm of Eq. (16) yields the following expression in terms of N_{re} :

$$\ln\left(\frac{a_{\text{re}}}{a_0}\right) = \frac{1}{3} \ln\left(\frac{43}{11g_{\text{re}}}\right) + \frac{1}{4} \ln\left(\frac{\pi^2 g_{\text{re}}}{30}\right) + \frac{1}{4} \ln\left(\frac{3T_0^4}{4V_{\text{end}}}\right) + \frac{3N_{\text{re}}(1+w_{\text{re}})}{4}. \quad (18)$$

Using the amplitude of the scalar perturbations, $A_s = H^4/(4\pi^2 \dot{\phi}^2)$, and the slow-roll approximation, we may write

$$H_k = \frac{\pi M_{\text{pl}} \sqrt{r A_s}}{\sqrt{2}}. \quad (19)$$

Inserting Eqs. (18) and (19) into Eq. (14), we finally have

$$N_{\text{re}} = \frac{4}{1-3w_{\text{re}}} \left[-N_k - \ln\left(\frac{k}{a_0 T_0}\right) - \frac{1}{4} \ln\left(\frac{40}{\pi^2 g_{\text{re}}}\right) - \frac{1}{3} \ln\left(\frac{11g_{\text{re}}}{43}\right) + \frac{1}{2} \ln\left(\frac{\pi^2 M_{\text{pl}}^2 r A_s}{2V_{\text{end}}^{1/2}}\right) \right]. \quad (20)$$

In our analysis, we adopt the amplitude of the scalar perturbation at the pivot scale A_s given by $10^{10} A_s = e^{3.064}$ (Table 4 of Ref. [38]) and $k = 0.05$ Mpc $^{-1}$ as a pivot wave number. Combining Eqs. (9), (12), and (15), we also have

$$T_{\text{re}} = \exp\left[-\frac{3}{4}(1+w_{\text{re}})N_{\text{re}}\right] \left(\frac{2V_{\text{end}}}{5\pi^2}\right)^{1/4}. \quad (21)$$

Because the wave number k and n_s are related implicitly through the scalar field ϕ with $H_k a_k = k$ [Eqs. (7) and (10)], one can write N_{re} and T_{re} as functions of the spectral index n_s .

III. SINGLE-FIELD α ATTRACTORS

In this paper, we focus on a class of single-field inflation models of the α -attractors in a unified manner [7–9], which includes Starobinsky's R^2 inflation model [11,12] and the Higgs inflation model [17–21]. This class of inflation models can be generated by spontaneously breaking the conformal symmetry [2,4,35,36]. In this paper, we consider the E-model and T-model as generalized models of α attractors, which are specified by the potentials (23) and (27), respectively. Starobinsky's model corresponds to the E-model with $\alpha = 1$ and $n = 1$ in Eq. (23). Single power-law inflation models are reproduced as the limit of large α .

The authors of Ref. [1] demonstrated that the α -attractor models are consistent with observations of the cosmic microwave background anisotropies. For rough consistency with the observed tensor-to-scalar ratio, we require that the parameter α is less than $\mathcal{O}(100)$. In Figs. 4–6, the shaded region in each panel is excluded from the constraint

on the scalar-tensor ratio. We first investigate constraints on reheating after an inflation of the E-model and T-model by following previous works [32–34,37]. To this end, we adopt

$$n_s = 0.9667 \pm 0.0040 \quad (22)$$

(see Table 4 in Ref. [38]).

A. E-model

The E-model is specified by the potential [35,36]

$$V = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}}\right)^{2n}, \quad (23)$$

where Λ , n , and α are the parameters. Using the slow-roll approximation, we find the expressions for the spectral index and the tensor-to-scalar ratio,

$$n_s = 1 - \frac{8n \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}} + n \right)}{3\alpha \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}} - 1 \right)^2}, \quad (24)$$

$$r = \frac{64n^2}{3\alpha \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}} - 1 \right)^2}, \quad (25)$$

and for the e -foldings as functions of ϕ from Eq. (10),

$$N_k = -\frac{3\alpha}{4n} \left[e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_{\text{end}}}{M_{\text{pl}}}} - e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}} + \sqrt{\frac{2}{3\alpha}} \frac{\phi - \phi_{\text{end}}}{M_{\text{pl}}} \right]. \quad (26)$$

Thus, we can write T_{re} and N_{re} as functions of n_s , regarding ϕ as an implicit parameter.

Figure 2 plots N_{re} (upper panels) and T_{re} (lower panels) as functions of n_s , where we fix $n = 1$. The left, central, and right panels adopt $\alpha = 0.1, 1$, and 5 , respectively. The curves in each panel represent different equation-of-state parameters w_{re} : $-1/3$ (red curve), 0 (blue curve), $1/6$ (orange curve), and $2/3$ (green curve). Our result for $n = 1$ is the same as that in Ref. [34].

The yellow region shows the observational constraint on n_s , Eq. (22). Note that for a set of the parameters w_{re} , n , and α , there appears the maximum value $N_{\text{re}}^{(\text{max})}$ so that the curve of N_{re} is consistent with the observational constraint on n_s . For example, in the central panels, which assume $n = 1$ and $\alpha = 1$, the maximum values are $N_{\text{re}} = 8, 15, 30$, and 40 , respectively, for $w_{\text{re}} = -1/3, 0, 1/6$, and $2/3$.

B. T-model

The T-model is specified by the potential [4,35,36]

$$V = \Lambda^4 \tanh^{2n} \left(\frac{\phi}{\sqrt{6\alpha} M_{\text{pl}}} \right), \quad (27)$$

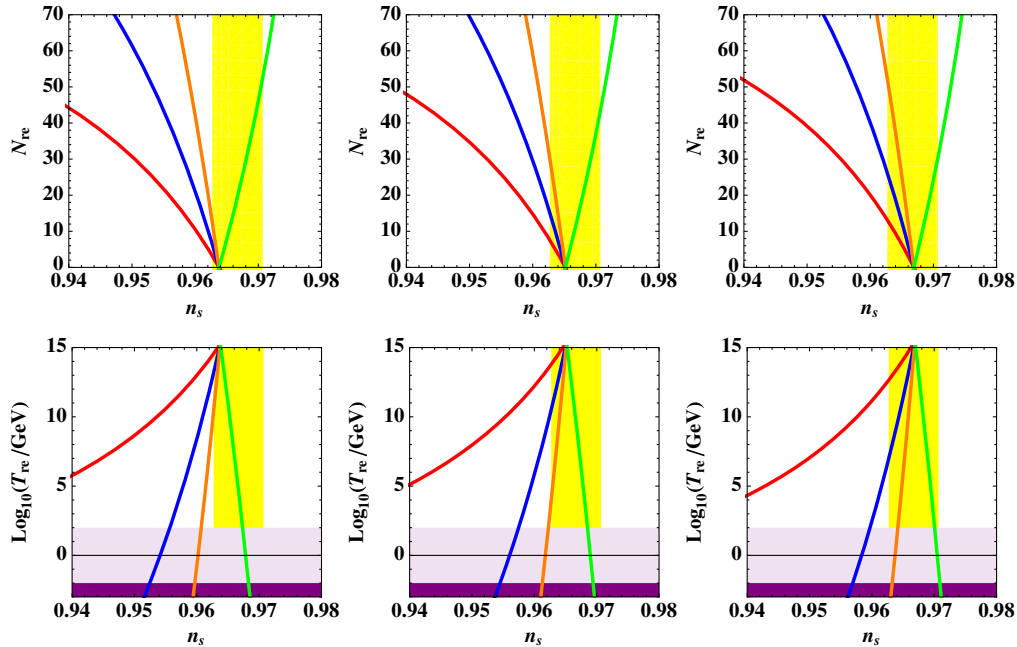


FIG. 2. N_{re} (upper panels) and T_{re} (lower panels) as functions of n_s for the E-model with $n = 1$ and $\alpha = 0.1$ (left panels), $\alpha = 1$ (central panels), and $\alpha = 5$ (right panels). In each panel, the curves represent different equation-of-state parameters w_{re} : $-1/3$ (red curve), 0 (blue curve), $1/6$ (orange curve), and $2/3$ (green curve). Yellow regions indicate the observational constraint, Eq. (22). In each panel, the point at which the four curves intersect—which corresponds to instant reheating—gradually moves from left to right as the value of α increases. The light purple and dark purple regions in the lower panels show temperatures below the electroweak scale, $T < 100$ GeV, and the big bang nucleosynthesis scale, $T < 10$ MeV, respectively. For consistency with big bang nucleosynthesis, $T_{\text{re}} \gtrsim 10$ MeV.

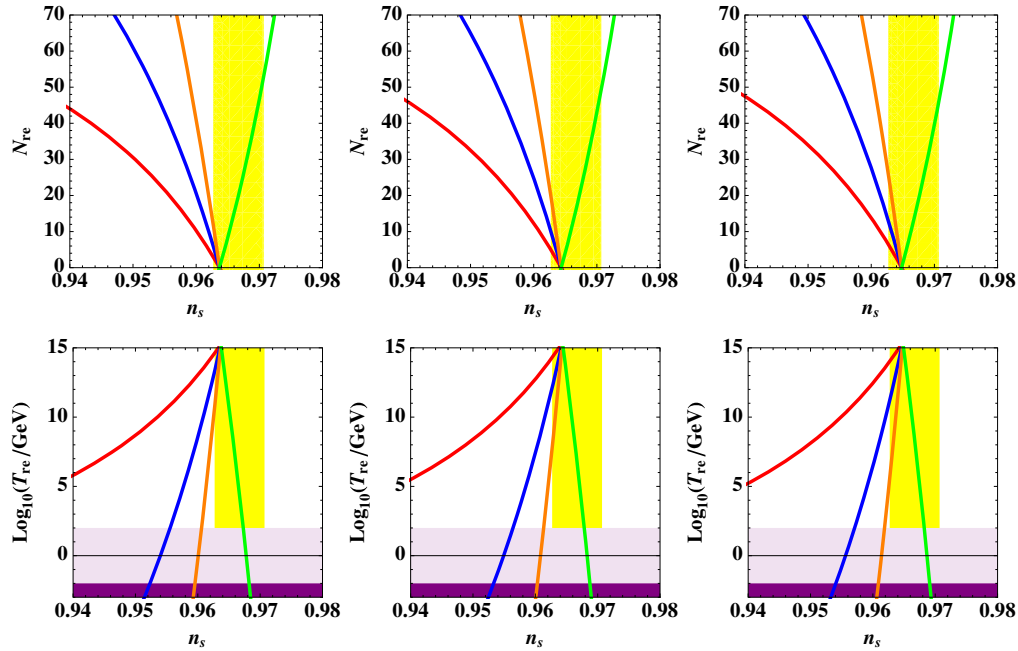


FIG. 3. Same as Fig. 2 but for the T-model. We fix $n = 1$ and $\alpha = 0.1$ (left panels), $\alpha = 1$ (central panels), and $\alpha = 5$ (right panels). In each panel, the curves represent different equation-of-state parameters w_{re} : $-1/3$ (red curve), 0 (blue curve), $1/6$ (orange curve), and $2/3$ (green curve).

where Λ , n , and α are the parameters. Within the slow-roll approximation, we find expressions for the spectral index, tensor-to-scalar ratio, and e -foldings as functions of ϕ :

$$n_s = 1 - \frac{1}{3\alpha} \left[8n(1+n) \text{csch}^2 \sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}} + 4n \text{sech}^2 \sqrt{\frac{1}{6\alpha}} \frac{\phi}{M_{\text{pl}}} \right], \quad (28)$$

$$r = \frac{64n^2 \text{csch}^2 \sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}}}{3\alpha}, \quad (29)$$

$$N_k = -\frac{3\alpha}{4n} \left[\cosh \sqrt{\frac{2}{3\alpha}} \frac{\phi_{\text{end}}}{M_{\text{pl}}} - \cosh \sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\text{pl}}} \right]. \quad (30)$$

Figure 3 is the same as Fig. 2 but for the T-model; N_{re} (upper panels) and T_{re} (lower panels) are plotted as functions of n_s , where we fix $n = 1$. In the left, central, and right panels, $\alpha = 0.1$, 1 , and 5 , respectively. In each panel, the curves represent different equation-of-state parameters w_{re} : $-1/3$ (red curve), 0 (blue curve), $1/6$ (orange curve), and $2/3$ (green curve).

IV. TWO-PHASE REHEATING MODEL

In this section, we consider a simple scenario of reheating that consists of two phases. The first is an epoch in which the energy density of the Universe is dominated by uniform oscillations of the inflaton field (the oscillation

phase), and the second is an epoch in which the Universe is dominated by relativistic but nonthermalized particles produced by decay of the inflaton field (the thermalization phase). Figure 1 illustrates the difference between the assumption of this section and that of the previous section. In the oscillation phase, the scalar field and scale factor follow Eqs. (1) and (2), respectively. When the scalar field oscillates around the minimum, which is approximated as

$$V = \Lambda^4 \left(\frac{2}{3\alpha M_{\text{pl}}^2} \phi^2 \right)^n \quad (31)$$

and

$$V = \Lambda^4 \left(\frac{1}{6\alpha M_{\text{pl}}^2} \phi^2 \right)^n \quad (32)$$

for the E-model and T-model, respectively, the equation-of-state parameter of the scalar field is expressed in terms of the parameter n . When the time scale of oscillation about the minimum is small, the virial theorem predicts that the energy density of the oscillating scalar field has the equation-of-state parameter

$$w_{\text{sc}} = \frac{n-1}{n+1} \quad (33)$$

for a potential $V \propto \phi^{2n}$ around the minimum [29]. One can check the validity of this formula by numerical solutions for an expanding universe. This is because the period of

oscillation is small compared to the Hubble time. Thus, the first oscillation phase of reheating is characterized by coherent oscillations of inflaton, in which the energy density is specified by the equation-of-state parameter w_{sc} in Eq. (33).

During the oscillation phase, light relativistic particles are produced gradually by a certain mechanism. We assume that the energy density of the oscillating field and the energy density of relativistic particles become equal at the scale factor a_{eq} and that the relativistic particle component dominates the energy density of the Universe after a_{eq} . However, the thermalization process might not be completed quickly. Then, the second phase of reheating is for thermalization. We assume that the thermalization phase continues until the scale factor becomes a_{re} , at which the temperature of the Universe is T_{re} and the energy density is given by Eq. (11). Then, the e -folding number of the reheating epoch is written as a combination of the two phases:

$$N_{\text{re}} = \ln\left(\frac{a_{\text{re}}}{a_{\text{end}}}\right) = \ln\left(\frac{a_{\text{re}} a_{\text{eq}}}{a_{\text{eq}} a_{\text{end}}}\right) = N_{\text{sc}} + N_{\text{th}}, \quad (34)$$

where we defined

$$N_{\text{sc}} = \ln\left(\frac{a_{\text{eq}}}{a_{\text{end}}}\right) = -\frac{1}{3(1+w_{\text{sc}})} \ln\left(\frac{\rho_{\text{eq}}}{\rho_{\text{end}}}\right), \quad (35)$$

$$N_{\text{th}} = \ln\left(\frac{a_{\text{re}}}{a_{\text{eq}}}\right) = -\frac{1}{4} \ln\left(\frac{\rho_{\text{re}}}{\rho_{\text{eq}}}\right), \quad (36)$$

and N_{sc} and N_{th} are the e -folding numbers for the oscillation phase and thermalization phase, respectively.

On the basis of this assumption, we repeat the computation in Sec. II, which yields the following expressions instead of Eqs. (20) and (21):

$$N_{\text{sc}} = \frac{4}{1-3w_{\text{sc}}} \left[-N_k - \ln\left(\frac{k}{a_0 T_0}\right) - \frac{1}{4} \ln\left(\frac{40}{\pi^2 g_{\text{re}}}\right) - \frac{1}{3} \ln\left(\frac{11g_{\text{re}}}{43}\right) + \frac{1}{2} \ln\left(\frac{\pi^2 M_{\text{pl}}^2 r A_s}{2V_{\text{end}}^{1/2}}\right) \right], \quad (37)$$

$$T_{\text{re}} e^{N_{\text{th}}} = \exp\left[-\frac{3}{4}(1+w_{\text{sc}})N_{\text{sc}}\right] \left(\frac{2V_{\text{end}}}{5\pi^2}\right)^{\frac{1}{4}}. \quad (38)$$

Note that the expression for N_{sc} is equivalent to N_{re} in Eq. (20) and that the reheating temperature is modified by the e -folding number of thermalization N_{th} , but $T_{\text{re}} e^{N_{\text{th}}}$ is the same as the right-hand side of Eq. (21).

As we described in the previous section, a maximum e -folding number appears for consistency with the constraint on n_s . From Eq. (37), for the two-phase reheating model, we obtain the maximum e -folding number for N_{sc} , which is the same as that for N_{re} , when we fix α , n , and w_{sc} instead of w_{re} . In addition, from Eq. (38), we obtain the

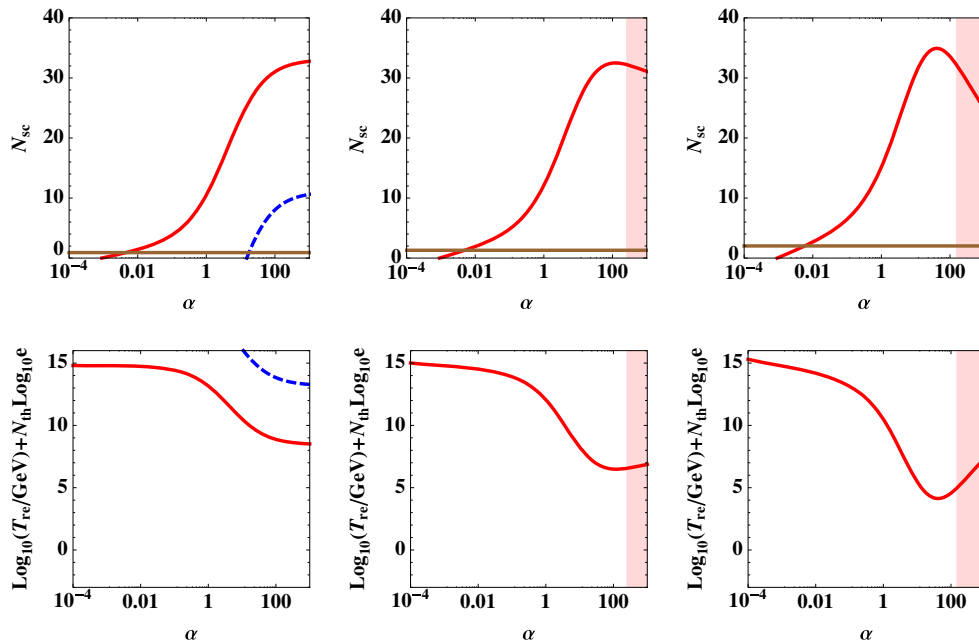


FIG. 4. The red solid curve is the maximum value of N_{sc} (upper panels) and the minimum value of $\log_{10}(T_{\text{re}}/\text{GeV}) + N_{\text{th}} \log_{10} e$ (lower panels) as a function of α for the E-model with $n = 1/2$ (left panel), $n = 3/4$ (central panel), and $n = 1$ (right panel). The dashed curve in the left panels shows the minimum value of N_{sc} and the maximum value of $\log_{10}(T_{\text{re}}/\text{GeV}) + N_{\text{th}} \log_{10} e$. The brown line in the upper panels shows the e -folding number for $\mathcal{N}_{\text{osc}} = 20$ oscillations, which is required for broad resonance preheating. The light shaded region in the central and right panels is excluded from the observational constraint on the tensor-to-scalar ratio r .

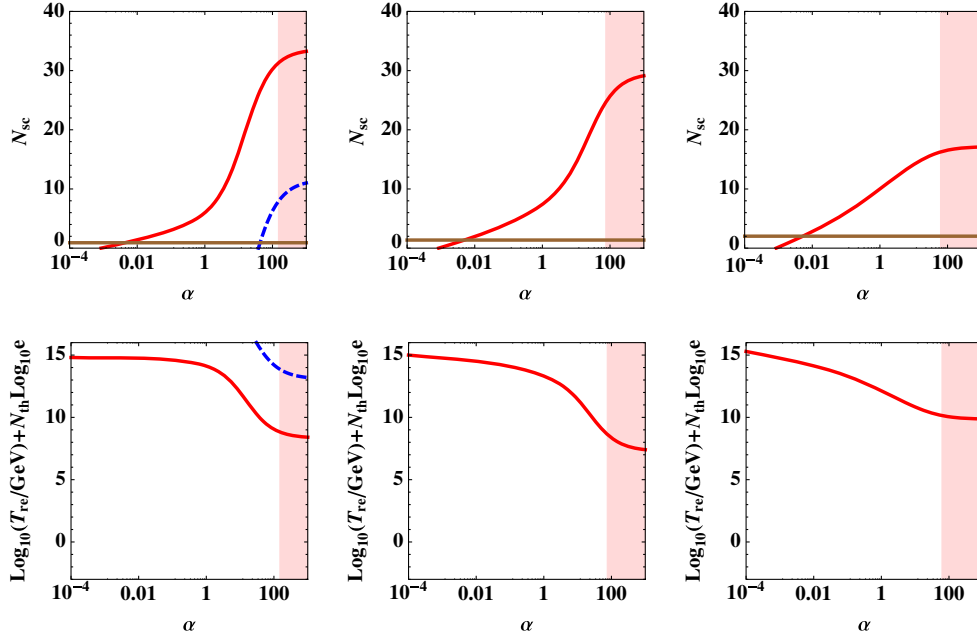


FIG. 5. Same as Fig. 4 but for the T-model.

minimum reheating temperature. Note that $T_{\text{re}} e^{N_{\text{th}}}$ in the two-reheating-phase model is the same as the right-hand side of Eq. (21); therefore, we obtain the minimum reheating temperature, $\log_{10} T_{\text{re}} + N_{\text{th}} \log_{10} e$, in this case.

Figure 4 shows the maximum value of N_{sc} (upper panels) and $\log_{10}(T_{\text{re}}/\text{GeV}) + N_{\text{th}} \log_{10} e$ (lower panels) as functions of α with $n = 1/2$ (left panels), $n = 3/4$ (central panels), and $n = 1$ (right panels) for the E-model. Figure 5 is the same as Fig. 4 but for the T-model.

V. IMPACT ON REHEATING SCENARIOS

In this section, we discuss the impacts of the results in the previous section on reheating scenarios by comparing the results with theoretical predictions. We may consider two types of interaction between an inflaton field ϕ and a light scalar field χ ,

$$L_I^{(4)} = -\frac{1}{2} \tilde{g}^2 \phi^2 \chi^2 \quad (39)$$

and

$$L_I^{(3)} = -g \phi \chi^2, \quad (40)$$

which describe χ -particle production through the processes $\phi + \phi \rightarrow \chi + \chi$ and $\phi \rightarrow \chi + \chi$, respectively, where \tilde{g} and g are their respective coupling constants.

A. Perturbative reheating

We first consider a scenario in which effective resonant particle creation does not occur. We consider the perturbative reheating scenario as an elementary reheating scenario

in which inflatons decay perturbatively through the interaction (40). In this case, the evolution of the number density of inflatons is described by

$$\frac{d(a^3 n_\phi)}{dt} = -\Gamma_{\phi \rightarrow \chi\chi} (a^3 n_\phi), \quad (41)$$

where the decay rate $\Gamma_{\phi \rightarrow \chi\chi}$, described through the interaction in Eq. (40), is

$$\Gamma_{\phi \rightarrow \chi\chi} = \frac{g^2}{8\pi m_\phi}, \quad (42)$$

where m_ϕ is the inflaton's mass. Assuming that the background Universe is dominated by the energy density of inflaton oscillation, which might be treated as a fluid with the equation-of-state parameter w_{sc} in Eq. (33), the Friedmann equation is

$$\frac{\dot{a}^2}{a^2} = \frac{\rho_{\text{end}}}{3M_{\text{pl}}^2} \left(\frac{a}{a_{\text{end}}} \right)^{-6n/(1+n)}. \quad (43)$$

The above e -folding of perturbative reheating is simply understood as follows. We may estimate the epoch of χ -particle decay as

$$H = \Gamma_{\phi \rightarrow \chi\chi}, \quad (44)$$

which yields

$$\left(\frac{a_{\text{eq}}}{a_{\text{end}}} \right)^{3n/(1+n)} = \frac{8\pi m_\phi}{g^2 M_{\text{pl}}^2} \sqrt{\frac{\rho_{\text{end}}}{3}} = \frac{16\pi m_\phi}{3g^2 M_{\text{pl}}} V_{\text{end}}^{1/2}, \quad (45)$$

where we used Eq. (9) in the second equality. We may write $V_{\text{end}} \sim \Lambda^4(2/3\alpha)^n$ and $V_{\text{end}} \sim \Lambda^4(1/6\alpha)^n$ for the E-model and T-model, respectively; then, we have the following expressions for the e -folding number, defined by $e^{N_{\text{sc}}} = a_{\text{eq}}/a_{\text{end}}$:

$$N_{\text{sc}} = -\frac{n+1}{3n} \ln \left[\frac{3M_{\text{pl}}}{16\pi m_\phi} \left(\frac{g}{\Lambda} \right)^2 \left(\frac{3\alpha}{2} \right)^{n/2} \right], \quad (46)$$

$$N_{\text{sc}} = -\frac{n+1}{3n} \ln \left[\frac{3M_{\text{pl}}}{16\pi m_\phi} \left(\frac{g}{\Lambda} \right)^2 (6\alpha)^{n/2} \right] \quad (47)$$

for the E-model and T-model, respectively. This puts a useful constraint on the coupling constant g for a successful perturbative reheating scenario that is consistent with the observational constraint obtained in the previous section. When we choose $m_\phi = 10^{13}$ GeV and define

$$\frac{g}{\Lambda} = 10^{\beta-15}, \quad (48)$$

we have

$$\beta \gtrsim 12.9 - n\gamma - 0.65 \frac{n}{n+1} N_{\text{sc}} - \frac{n}{4} \log_{10} \alpha, \quad (49)$$

where $\gamma = 0.044$ and $\gamma = 0.19$ for the E-model and T-model, respectively. Figure 6 shows the minimum value of β as a function of α for the E-model (left panel) and T-model (right panel) with $n = 1$. For example, $\beta > 7.9$ for the E-model with $\alpha = 1$ and $\Lambda = 10^{15}$ GeV. For the T-model with $\alpha = 1$ and $\Lambda = 10^{15}$ GeV, for successful perturbative reheating, $\beta > 9.4$ is imposed.

B. Broad resonance preheating

After the end of slow-roll inflation, the inflaton field ϕ oscillates around a potential minimum, which is assumed to be approximated by Eqs. (31) and (32) for the E-model and

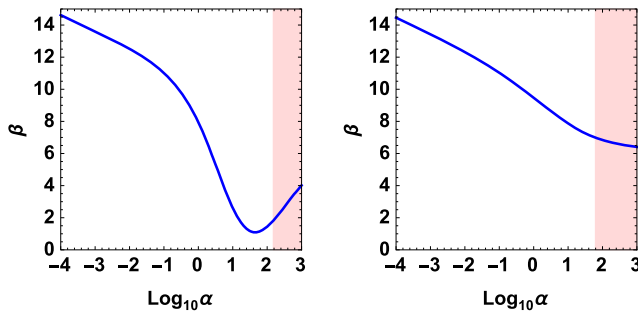


FIG. 6. The blue curve is the minimum value of β for the coupling constant $g/\Lambda = 10^{\beta-15}$ as a function of α for the E-model (left panel) and the T-model (right panel). Here we adopted $n = 1$ and $m_\phi = 10^{13}$ GeV. Note that the coupling constant is defined so as to be $g = 10^\beta$ GeV when we choose $\Lambda = 10^{15}$ GeV.

T-model, respectively. When $n = 1$, these potentials are the harmonic potential, and we may assume that the oscillation of $\phi(t)$ is approximated by

$$\phi(t) \simeq \Phi \sin m_\phi t, \quad (50)$$

where Φ is the amplitude of the oscillation, and m_ϕ is understood as $m_\phi = 2\Lambda^2/\sqrt{3\alpha}M_{\text{pl}}$ and $m_\phi = \Lambda^2/\sqrt{3\alpha}M_{\text{pl}}$ for the E-model and T-model, respectively. When $n \neq 1$, the oscillation of $\phi(t)$ is not approximated by such a simple function.

We here consider a resonant particle production scenario that was intensively investigated by Kofman *et al.* [26] (see also Ref. [27]). The equations of motion for a Fourier mode of the χ field are

$$\ddot{\chi}_k(t) + 3\frac{\dot{a}}{a}\dot{\chi}_k(t) + \left(\frac{k^2}{a^2} + m_\chi^2 + \tilde{g}^2\phi^2(t) \right) \chi_k = 0 \quad (51)$$

and

$$\ddot{\chi}_k(t) + 3\frac{\dot{a}}{a}\dot{\chi}_k(t) + \left(\frac{k^2}{a^2} + m_\chi^2 + 2g\phi(t) \right) \chi_k = 0 \quad (52)$$

for the interactions in Eqs. (39) and (40), respectively.

This scenario of reheating relies on resonant particle creation due to the periodic time-dependent background at the earlier stage of reheating, which is called preheating. Particle creation effectively occurs when the Wentzel-Kramers-Brillouin approximation breaks down, which occurs at $\cos 2m_\phi t \sim 0$ or $\sin m_\phi t \sim 0$, depending on the interaction. We follow this scenario (see Ref. [12] for a review). We first consider the four-point interaction. After \mathcal{N}_{osc} oscillations of the inflaton field around the minimum, the ratio of the number density of χ particles to that during inflation is estimated as

$$\frac{n_\chi}{n_\phi} \sim \frac{k_*^3 n_k(\mathcal{N}_{\text{osc}})}{\frac{1}{2} m_\phi \Phi_0^2} \sim m_\phi^{1/2} \tilde{g}^{3/2} 3^{\mathcal{N}_{\text{osc}}} \Phi_0^{-1/2}, \quad (53)$$

where we choose $k_* = m_\phi^3 (\tilde{g}\Phi_0/m_\phi)^{3/2}$, and Φ_0 is the inflaton's oscillation amplitude, which we take to be $\Phi_0 \sim M_{\text{pl}}$. Using this relation, we can estimate the ratio of the energy density of χ particles to that during inflation as

$$\frac{\epsilon_\chi}{\epsilon_\phi} \sim \frac{m_\chi n_\chi}{m_\phi n_\phi} \sim \tilde{g}^{5/2} m_\phi^{-1/2} \mathcal{N}_{\text{osc}}^{-1} 3^{\mathcal{N}_{\text{osc}}} \Phi_0^{1/2}, \quad (54)$$

where we assumed $m_\chi = \mathcal{O}(m_\phi) = \mathcal{O}(\tilde{g}\Phi) \sim \tilde{g}\Phi_0/\mathcal{N}_{\text{osc}}$. Then $\epsilon_\chi \simeq \epsilon_\phi$ appears after \mathcal{N}_{osc} oscillations,

$$\mathcal{N}_{\text{osc}} \simeq 12\text{--}30, \quad (55)$$

for a wide range of $10^{-5} < \tilde{g} < 10^{-3}$, where we assumed $m_\phi = 10^{13}$ GeV. Here we compute the e -foldings to realize

$\mathcal{N}_{\text{osc}} = 20$ inflaton oscillations; this yields the minimum duration required for successful preheating. The e -folding number for $\mathcal{N}_{\text{osc}} = 20$ oscillations is of order $\mathcal{O}(1 \sim 2)$. Then we may write $N_{\text{sc}} \gtrsim \mathcal{O}(1-2)$. A more explicit value is obtained by solving Eqs. (1) and (2). χ particles can decay into other lighter particles quickly, which do not directly couple to the inflaton [39]. The brown line in the upper panels of Figs. 4 and 5 shows the value of N_{sc} for a broad resonance preheating scenario with $\mathcal{N}_{\text{osc}} = 20$. For the interaction in Eq. (40), the estimation is essentially the same as the above estimation of the interaction in Eq. (39).

When we also consider the constraint in the previous section, for consistency with the broad resonance preheating scenario, we need the rough condition

$$\alpha \gtrsim 0.01. \quad (56)$$

VI. SUMMARY AND CONCLUSIONS

We investigated a constraint on the reheating epoch using an observational constraint on the spectral index n_s , in which we assumed the E-model and T-model as generalized α -attractor models of inflation. When the reheating epoch is dominated by an energy component of the cosmic equation-of-state parameter w_{re} , the e -folding number for reheating, N_{re} , is bounded depending on w_{re} ,

which also limits the reheating temperature T_{re} . Assuming that the reheating consists of two phases, an oscillation phase and a thermalization phase, we investigated the e -folding number of the oscillation phase N_{sc} and the reheating temperature T_{re} , depending on the equation-of-state parameter w_{sc} , which is determined by the potential. N_{sc} is constrained by the observational constraint on n_s , and the allowed regions of N_{sc} and T_{re} were obtained in Sec. IV. For example, we found $N_{\text{sc}} \lesssim 16$ and $T_{\text{re}} e^{N_{\text{th}}} \gtrsim 10^{10}$ GeV for the E-model with $n = 1$ and $\alpha = 1$, whereas $N_{\text{sc}} \lesssim 10$ and $T_{\text{re}} e^{N_{\text{th}}} \gtrsim 10^{12}$ GeV for the T-model with $n = 1$ and $\alpha = 1$. We discussed the implications of our results for two simple reheating scenarios. For the simplest perturbative reheating scenario, the ratio of the coupling constant g for a decay to the mass scale of the potential of inflation Λ should be $g \gtrsim 10^{7.9} (\Lambda/10^{15} \text{ GeV}) \text{ GeV}$ for the E-model and $g \gtrsim 10^{9.4} (\Lambda/10^{15} \text{ GeV}) \text{ GeV}$ for the T-model for $n = 1$ and $\alpha = 1$. Along a broad resonance preheating scenario, the α parameter is roughly constrained, $\alpha \gtrsim 0.01$, for the T-model and E-model.

ACKNOWLEDGMENTS

This work was supported by MEXT/JSPS KAKENHI Grant No. 15H05895.

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