

Quark stars admixed with dark matterPayel Mukhopadhyay^{1,2,*} and Jürgen Schaffner-Bielich^{2,†}¹*Indian Institute of Science Education and Research, Kolkata, Mohanpur, 741252 Nadia, West Bengal, India*²*Institut für Theoretische Physik, J. W. Goethe Universität, Max Von Laue-Straße 1, D-60438 Frankfurt Am Main, Germany*

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Compact stars consisting of massless quark matter and fermionic dark matter are studied by solving the Tolman-Oppenheimer-Volkoff equations for two fluids separately. Dark matter is further investigated by incorporating interfermionic interactions among the dark matter particles. The properties of stars made of quark matter particles and self-interacting and free dark matter particles are explored by obtaining their mass-radius relations. The regions of stability for such a compact star are determined, and it is demonstrated that the maximum stable total mass of such a star decreases approximately linearly with an increasing dark matter fraction.

DOI: [10.1103/PhysRevD.93.083009](https://doi.org/10.1103/PhysRevD.93.083009)**I. INTRODUCTION**

A quark star is a hypothetical compact star and consists of self-bound strange quark matter [1–5]. The existence of quark stars is controversial, and their equation of state is also uncertain.

One of the popular models for the equation of state of a quark star is the so called MIT bag model [6]. The model is often used for describing cold and massless (strange) quark matter [7,8]. Standard values for the MIT bag constant are around $B^{1/4} = 145$ MeV as follows from fits to hadron masses [7], which results in maximum masses of about $2.0 M_{\odot}$ at a radius of about 11 km [1,7], which are actually very close to the ones of realistic neutron star models.

Fermionic or bosonic particles are equally considered for modelling dark matter in current models. For a study of compact fermionic stars, we refer to the papers [9–11] and references therein. Boson stars with self-interactions have been considered in Ref. [12] and references therein. More recent work can be found in Ref. [13] and most recently in Ref. [14] and references therein. Dark matter stars are modelled in our work as a free or self-interacting fermion gas at zero temperature. The possible candidates for dark matter particles are a type of fermion predicted in extensions of the standard model including supersymmetric particles, the neutralino, the gravitino, and the axino [15]. In our discussion, we consider dark matter to be made of fermionic particles with a mass of 100 GeV, the classical weakly interacting massive particle (WIMP) mass scale. We assume that the dark matter particles cannot self-annihilate as in asymmetric dark matter [16,17].

Self-annihilating WIMP dark matter with masses above a few GeV accreted onto neutron stars may trigger a

conversion of the star into a strange star [18], or the accreted dark matter may significantly affect the kinematical properties of the compact star [19]. Constraints on the properties of dark matter candidates can be obtained from stars on which asymmetric dark matter can accrete in its lifetime and collapse into a neutron star [20]. Constraints on the mass of dark matter candidates can also be obtained by the possible collapse of compact stars due to dark matter accretion [21,22]. The cooling process of compact objects can be affected by the capture of dark matter which can annihilate the star [23]. Recent studies have been done to explore compact stars with non-self-annihilating dark matter to analyze the gravitational effects of dark matter on the stellar matter under intense conditions [24–27]. In these studies, masses of dark matter in the GeV range have been assumed. Studies have also been performed to investigate the compact objects formed due the admixture of neutron star matter and dark matter [28] leading to the possibilities of new stable solutions of compact stars with planetlike masses. Therefore, it is of great interest to analyze the effects of dark matter on compact stellar objects. If quark stars do exist in nature, they can accumulate dark matter which will lead to various changes in the mass-radius relations which is studied in this work.

This paper is organized as follows. In Sec. II, we shortly discuss the two-fluid Tolman Oppenheimer Volkoff (TOV) equations. In Sec. III, we discuss the equations of state for both quark matter and fermionic dark matter. We also discuss the general scaling relations of stellar objects in this section. In Sec. IV, we present the numerically obtained results (mass-radius relations) for quark matter stars by solving the TOV equations. In Sec. V, TOV equations are solved for dark matter composed of both strongly self-interacting and free fermions, and their corresponding mass-radius relations are obtained. Section VI is dedicated to the numerical solutions of two-fluid TOV equations,

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namely, dark matter and quark matter, which are coupled together only by gravity. We demonstrate that the maximum mass of the quark star admixed with dark matter reduces due to the presence of dark matter and decreases in a linear fashion in case of strongly self-interacting dark matter fermions, while for free dark matter, the maximum mass remains almost unaffected. Finally, in Sec. VII, we summarize our findings and discuss our results.

Throughout the paper, we use natural units where $c = \hbar = 1$, c being the speed of light and \hbar being the reduced Planck constant.

II. TWO FLUID TOLMANN-OPPENHEIMER-VOLKOFF EQUATIONS

Since our aim is to see the properties of a quark star admixed with dark matter, we need the TOV equations for two fluids admixed with each other. There will be a hydrostatic equilibrium condition for each of the two fluids, and the fact that there is only gravitational interaction between them will be encoded in the metric describing the system. The two-fluid TOV equations that we use here are [26,28]

$$\begin{aligned} \frac{dp_1}{dr} = & -\frac{GM(r)\rho_1(r)}{r^2} \left(1 + \frac{p_1(r)}{\rho_1(r)}\right) \\ & \times \left(1 + 4\pi r^3 \frac{(p_1(r) + p_2(r))}{M(r)}\right) \left(1 - 2G \frac{M(r)}{r}\right)^{-1} \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{dp_2}{dr} = & -\frac{GM(r)\rho_2(r)}{r^2} \left(1 + \frac{p_2(r)}{\rho_2(r)}\right) \\ & \times \left(1 + 4\pi r^3 \frac{(p_1(r) + p_2(r))}{M(r)}\right) \left(1 - 2G \frac{M(r)}{r}\right)^{-1} \end{aligned} \quad (2)$$

$$\frac{dM_1}{dr} = 4\pi r^2 \rho_1(r) \quad (3)$$

$$\frac{dM_2}{dr} = 4\pi r^2 \rho_2(r) \quad (4)$$

$$M(r) = M_1(r) + M_2(r). \quad (5)$$

Here, $M(r)$ represents the total mass at radius r ; p_1 , p_2 , ρ_1 , and ρ_2 are the pressures and densities of the fluids 1 and 2, respectively. We could separate out the hydrostatic equilibrium condition for the two fluids into Eqs. (1) and (2) because the interaction acts only through gravity and nothing else. The gravitational interaction is taken into account because $M(r)$ is the total mass of both the fluids at radius r which means that each fluid attracts the other gravitationally. The equations for the conservation of mass

of the two fluids remain the same as that for individual fluids.

For solving the two-fluid TOV equations, we need proper boundary conditions. $M_1(0)$ and $M_2(0)$ must be equal to zero at $r = 0$. Central pressures for the two fluids are calculated from the central densities given as the initial condition using the respective equations of state of the two fluids. Then, the two-fluid TOV equations are solved together simultaneously, and we obtain either R_1 or R_2 as the radius of the complete star depending on which fluid ends up having a larger radius. The radius of the individual fluids occurs at those points where the individual pressures drop down to zero.

III. EQUATIONS OF STATE FOR QUARK MATTER AND FOR FREE AND SELF-INTERACTING DARK MATTER

The equations of state (EOS) for quark matter are discussed using the MIT bag model. The EOS for free dark matter particles along with strongly self-interacting dark matter particles is briefly described using statistical mechanics of free and self-interacting fermions. Scaling relations for quark stars and dark matter stars is also discussed.

A. Equation of state for quark matter

The MIT Bag equation of state [7,8] is taken as the equation of state for quark matter in our work. In this model, the quarks are assumed to be made of free fermions constrained within a bag with a vacuum pressure that keeps the particles within the bag. The MIT bag equation of state is

$$p = \frac{1}{3}(\epsilon - 4B). \quad (6)$$

Here, p denotes the pressure, ϵ denotes the energy density, and B is the bag constant of which the standard accepted values are around $B^{1/4} = 145$ MeV or $B^{1/4} = 200$ MeV [7]. Note that the equation of state for a cold gas of interacting massless quarks within perturbative quantum chromodynamics can be approximated by a similar form of the equation of state as the MIT bag model [29].

B. Equation of state for free and self-interacting dark matter fermions

In our work, we assume dark matter to be made of fermions with a mass of 100 GeV. We consider cold dark matter so that the temperature of dark matter is much smaller than the mass of the dark matter particles. Furthermore, the temperature usually seen for compact stars is in the range of 10^6 K which corresponds to about the 100 eV mass scale. This scale is much smaller than the

mass of dark matter considered in our investigation. The range of Fermi energies of dark matter used in our calculations is not much smaller than the mass of dark matter particles, so temperature effects can be safely ignored. The equation of state for a gas of free fermions can be calculated via explicit expressions for energy density (ϵ) and pressure (p) [30],

$$\begin{aligned} \epsilon &= \frac{1}{\pi^2} \int_0^{k_F} k^2 \sqrt{m_f^2 + k^2} dk \\ &= \frac{m_f^4}{8\pi^2} \left[(2z^3 + z) \sqrt{1 + z^2} - \sinh^{-1}(z) \right] \end{aligned} \quad (7)$$

$$\begin{aligned} p &= \frac{1}{3\pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{m_f^2 + k^2}} dk \\ &= \frac{m_f^4}{24\pi^2} \left[(2z^3 - 3z) \sqrt{1 + z^2} + 3\sinh^{-1}(z) \right], \end{aligned} \quad (8)$$

where $z = k_F/m_f$ is the dimensionless Fermi momentum.

Similarly, the interactions between the fermions are modelled by considering the simplest two-body interactions. Self-repulsions between the fermions constituting the dark matter star are modelled by considering the interaction energy density to be proportional to n^2 [12,31,32] to the lowest order approximation, where n is the number density of fermions. The resulting equation of state has been calculated in Ref. [31],

$$\begin{aligned} \frac{\epsilon}{m_f^4} &= \frac{1}{8\pi^2} \left[(2z^3 + z) \sqrt{1 + z^2} - \sinh^{-1}(z) \right] \\ &+ \left[\left(\frac{1}{3\pi^2} \right)^2 y^2 z^6 \right] \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{p}{m_f^4} &= \frac{1}{24\pi^2} \left[(2z^3 - 3z) \sqrt{1 + z^2} + 3\sinh^{-1}(z) \right] \\ &+ \left[\left(\frac{1}{3\pi^2} \right)^2 y^2 z^6 \right], \end{aligned} \quad (10)$$

where $z = k_F/m_f$ is again the dimensionless Fermi momentum and y is the dimensionless interaction strength parameter. The strength parameter $y = m_f/m_I$ is defined as the ratio between the fermion mass m_f and the interaction mass scale m_I [31]. For strong interactions, the typical interaction mass scale is $m_I \sim 100$ MeV (typical scale of interactions by chiral effective models of QCD given by the pion decay constant). For weak interactions, $m_I \sim 300$ GeV (mediated by W and Z bosons). Accordingly, WIMP dark matter candidates in the mass range of 100 GeV [33] can have weak self-interactions with $y \sim 0.1$, or they may be strongly self-interacting with $y \sim 10^3$ where the strong interaction scale corresponds to $\Lambda_{\text{QCD}} \approx 100$ MeV. In our discussion, we focus on two

situations, one for free fermionic dark matter with $y = 0$ and the other for strongly self-interacting dark matter with $y = 10^3$.

C. Scaling relations for quark matter and dark matter

We generally scale dimensional quantities to dimensionless ones in order to represent any arbitrary mass configuration of a star in a single graph. From Eq. (6), it is clear that if we scale the energy density and pressure values by $1/4B$ the EOS reduces to a dimensionless form [1,7], and the corresponding total mass and radius of the star would then be scaled by $1/\sqrt{4B}$. Similarly, for the dark matter fermionic particles, it is a natural choice to scale the pressure and energy density by $1/m_f^4$, which will again make the equations dimensionless. The scaling relations for quark matter are $\epsilon'_{\text{quark}} = \epsilon_{\text{quark}}/(4B)$, $p'_{\text{quark}} = p_{\text{quark}}/(4B)$, $M'_{\text{quark}} = M_{\text{quark}}/(2\sqrt{B})$, and $R'_{\text{quark}} = R_{\text{quark}}/(2\sqrt{B})$. The corresponding relations for fermions are $\epsilon'_f = \epsilon_f/m_f^4$, $p'_f = p_f/m_f^4$, $M'_f = M_f/a$, and $R'_f = R_f/b$ where $a = M_p^3/m_f^2$ and $b = M_p/m_f^2$ where M_p is the Planck mass ($G = M_p^{-2}$). For detailed derivations of the scaling relations, we refer to Ref. [12].

IV. SOLVING TOV EQUATIONS FOR QUARK MATTER STAR

Numerical solutions of the mass-radius relations of quark stars can be found in the literature [34]. In our nomenclature, M_{quark} and R_{quark} represent the mass and radius of the quark star, respectively. The curve is shown in Fig. 1 for two different bag values.

Up to a certain point, the mass increases with the radius, reaching a maximum value of mass at a certain value of the radius, after which the mass starts decreasing, and the star

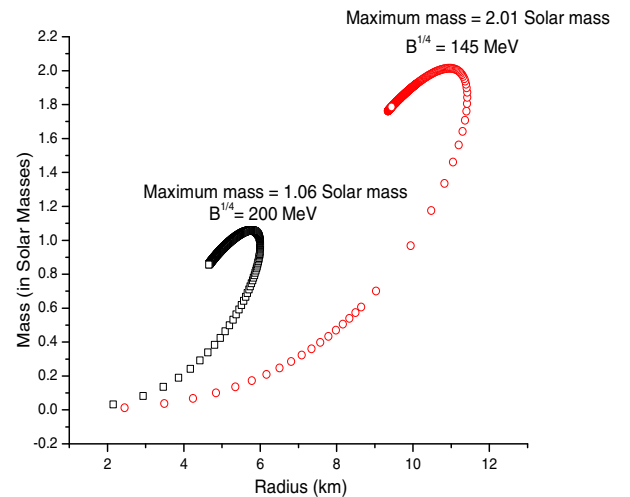


FIG. 1. Mass (M_{quark}) vs radius (R_{quark}) curve for quark stars for two different bag values.

starts becoming unstable from this point. The maximum stable mass for $B^{1/4} = 145$ MeV is about $2.01 M_{\odot}$, and the corresponding radius is around 11 km, while $B^{1/4} = 200$ MeV gives a maximum mass of about $1.06 M_{\odot}$ with a radius of 5.8 km. Quark stars are incompressible stars and form a self-bound system [2,5].

V. SOLVING TOV EQUATION NUMERICALLY FOR FREE AND STRONGLY SELF-INTERACTING DARK MATTER PARTICLES

A. Solutions for free fermionic dark matter

We first consider dark matter made of free fermionic particles with 100 GeV mass. Single fluid TOV equations are solved taking (7) and (8) as the equation of state. M_{dark} and R_{dark} represents the mass and radius of the dark matter star composed of free fermions, respectively. The resulting mass-radius curve is plotted in Fig. 2.

From the graph, we see that the mass at first increases with a decrease in radius for increasing central energy density values, reaches a maximum, and then starts decreasing. Stellar configurations to the right side of the maximum mass are stable, whereas those on the left side are unstable. The maximum stable mass for the dark matter star made of free fermions comes out to be $6.27 \times 10^{-5} M_{\odot}$ with a radius of 0.81 m.

B. Solutions for strongly self-interacting dark matter

The TOV equations are solved for strongly self-interacting dark matter particles ($\gamma = 10^3$) of mass 100 GeV. The equations of state used are (9) and (10). M_{int} and R_{int}

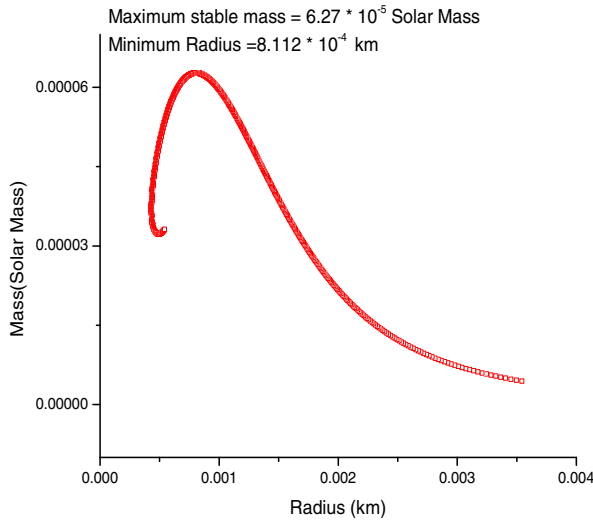


FIG. 2. Plot of the mass (M_{dark}) vs radius (R_{dark}) for a free gas of fermions of mass 100 GeV at zero temperature. This graph corresponds to the mass-radius curve of the dark matter composed of free fermionic particles.

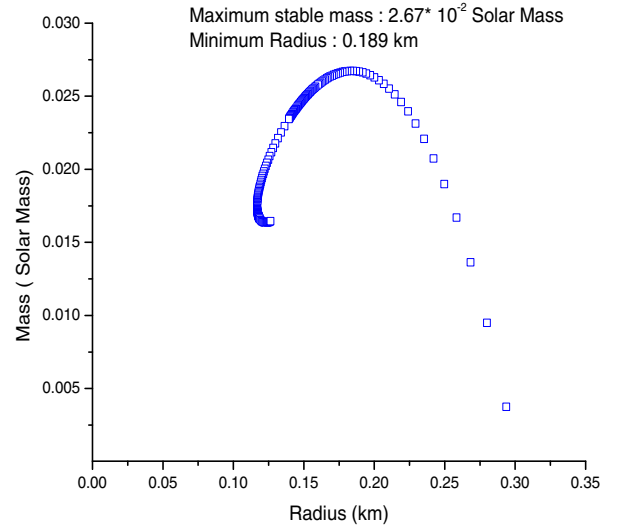


FIG. 3. Plot of mass M_{int} vs radius R_{int} for strongly self-interacting dark matter fermionic particles ($\gamma = 10^3$).

represent the mass and radius of the dark matter star composed of strongly self-interacting fermions, respectively. The resulting mass-radius curve is plotted in Fig. 3.

For strong self-interaction, the mass and radius are much larger compared to free fermions, and the maximum mass and the minimum radius are about 1000 times larger. From the curve, it is observed that for very low central densities of dark matter particles, i.e., the tail of the graph, the rate of increase of the mass with the decreasing radius is much higher compared to the free dark matter particle case discussed in the previous subsection. The maximum mass and the minimum radius for the self-interacting dark matter star turn out to be $2.67 \times 10^{-2} M_{\odot}$ and 0.189 km, respectively, larger than for the noninteracting case due to repulsive forces between the dark matter particles.

VI. SOLUTION OF TOV EQUATION FOR AN ADMIXTURE OF QUARK MATTER AND DARK MATTER

The nomenclature used is M_{quark} and R_{quark} for the mass and radius of quark matter, M_{dark} and R_{dark} for the mass and radius of the star composed of free dark matter particles, and M_{int} and R_{int} for a strongly self-interacting dark matter star. $\epsilon_{0,\text{quark}}$, $\epsilon_{0,\text{dark}}$, and $\epsilon_{0,\text{intdark}}$ represent the central energy densities of quark matter, dark matter made of free fermions, and dark matter made of strongly self-interacting fermions, respectively.

A. Solution for combination of quark matter and free dark matter particles

The two-fluid TOV equations (1), (2), (3), and (4) are solved for a mixture of quark matter with the MIT bag model by taking the bag value to be $B^{1/4} = 145$ MeV and dark matter composed of free fermionic particles of mass

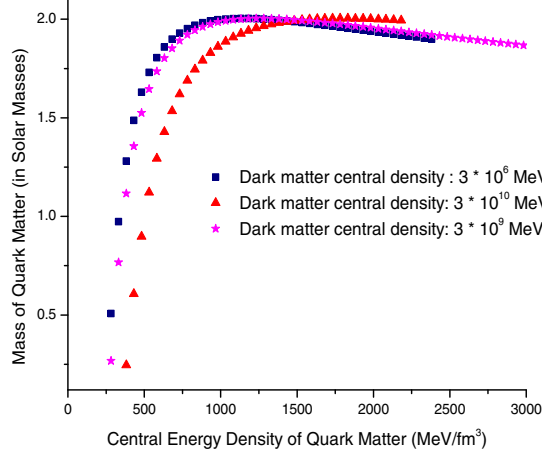


FIG. 4. Plot of mass (M_{quark}) vs central energy density ($\epsilon_{0,\text{quark}}$) of quark matter for three different central energy densities of dark matter ($\epsilon_{0,\text{dark}}$).

100 GeV. We start with initial central energy densities for the two components and compute the corresponding central pressures using the EOS for the respective fluids [Eq. (6) for quark matter and (7) and (8) for dark matter].

Mass (M_{quark}) vs the central energy density of quark matter ($\epsilon_{0,\text{quark}}$) is plotted for three different values of the central density of dark matter ($\epsilon_{0,\text{dark}}$), each kept constant at a time (see Fig. 4). From the plot, it is clear that as the central density of dark matter is increased in the mixture the maximum mass of the quark matter still reaches $2.005 M_{\odot}$, but now at higher central densities ($\epsilon_{0,\text{quark}}$) of quark matter. This behavior can be explained via the fact that as ($\epsilon_{0,\text{dark}}$) increases then, within the stable branch of dark matter, the allowed mass of dark matter inside the quark star also increases, which contributes to a greater gravitational pull, so a much higher central quark energy density ($\epsilon_{0,\text{quark}}$) is needed to support the maximum possible mass against the greater gravitational pull. The maximum stable mass of the quark component ($M_{\text{max,quark}}$) is almost the same as a pure quark star ($2.01 M_{\odot}$) because the maximum possible value of the dark matter mass is $6.27 \times 10^{-5} M_{\odot}$ (Sec. VA), which is much less than $2.01 M_{\odot}$, to cause a notable reduction in the quark matter mass. Figure 5 shows the plot for the radius (R_{quark}) vs the central energy density ($\epsilon_{0,\text{quark}}$) of quark matter for different values of $\epsilon_{0,\text{dark}}$. The figure shows that the maximum stable radius of the quark matter is independent of the amount of free fermionic dark matter present in the admixed star.

After observing that the maximum possible mass of quark matter is hardly reduced in the presence of free fermionic dark matter particles of various central densities, it is essential to determine which configurations of the admixed star are stable. The plots for the profile of dark matter component are obtained by keeping $\epsilon_{0,\text{quark}}$ fixed and slowly varying $\epsilon_{0,\text{dark}}$ (Fig. 6 and 7). It is seen that the dark matter masses and radii are the same for varying $\epsilon_{0,\text{quark}}$,

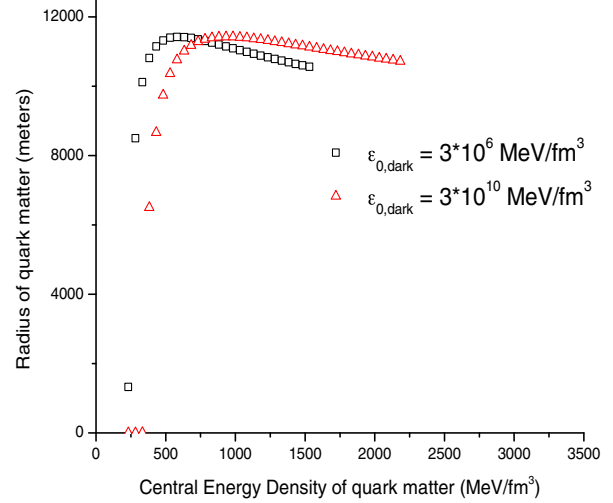


FIG. 5. Plot of radius (R_{quark}) vs central energy density ($\epsilon_{0,\text{quark}}$) of quark matter for different central energy densities of dark matter ($\epsilon_{0,\text{dark}}$).

which is expected since dark matter is more compact than quark matter and is not affected much by the presence of quarks.

Figures 4, 5, 6, and 7 allow us to analyze the stability of the entire configuration. Since we realize from Figs. 6 and 7 that $\epsilon_{0,\text{dark}}$, for which dark matter mass hits a maximum, is the same for all $\epsilon_{0,\text{quark}}$, we at first mark those points where the quark matter becomes unstable, i.e., hits the maximum mass by doing the plots done in Figs. 4 and 5 for different $\epsilon_{0,\text{dark}}$. As we slowly increase $\epsilon_{0,\text{dark}}$, the dark matter content inside the admixed star keeps on increasing, and the radius of the dark matter keeps on decreasing. After a sufficiently large $\epsilon_{0,\text{dark}}$, the dark matter mass content hits a maximum, after which the dark matter mass decreases with a further increase in $\epsilon_{0,\text{dark}}$. This is then the unstable branch for the

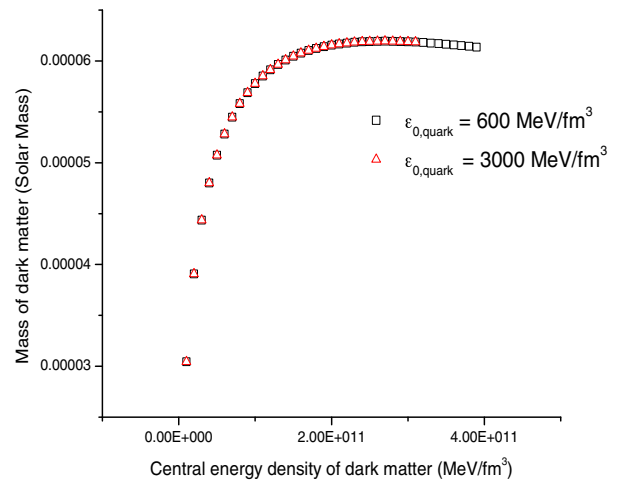


FIG. 6. Plot of mass (M_{dark}) vs central energy density ($\epsilon_{0,\text{dark}}$) of dark matter for different central energy densities of quark matter ($\epsilon_{0,\text{quark}}$).

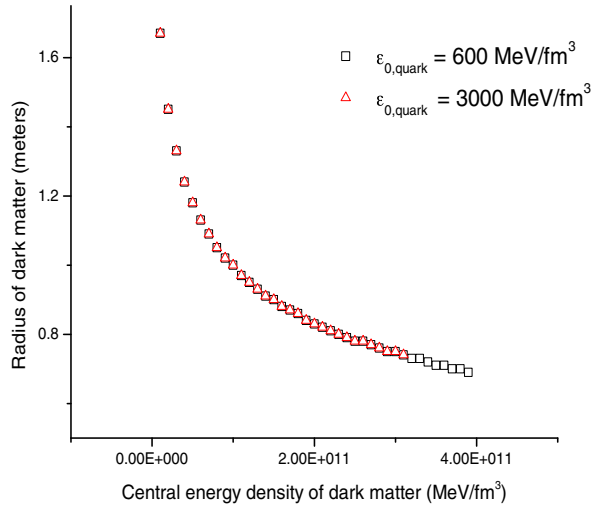


FIG. 7. Plot of radius (R_{dark}) vs central energy density ($\epsilon_{0,\text{dark}}$) of dark matter for different central energy densities of quark matter ($\epsilon_{0,\text{quark}}$).

dark matter. After this critical value of $\epsilon_{0,\text{dark}}$, no dark matter configurations are stable, and hence quark matter and dark matter cannot exist together since the dark matter would collapse into a black hole. Hence, we expect that up to a certain value of $\epsilon_{0,\text{dark}}$ the quark matter stable mass increases to reach $2.005 M_{\odot}$ for sufficiently high $\epsilon_{0,\text{quark}}$, and after a critical $\epsilon_{0,\text{dark}}$, dark matter itself becomes unstable, which leads to an instability of the admixture of the dark matter and quark matter.

Next, we study the configurations in the $\epsilon_{0,\text{quark}}-\epsilon_{0,\text{dark}}$ plane. At first, $\epsilon_{0,\text{dark}}$ is kept fixed, and $\epsilon_{0,\text{quark}}$ is slowly increased. The stable boundary is marked in the contour plot (Fig. 8) by marking the maximum stable quark matter

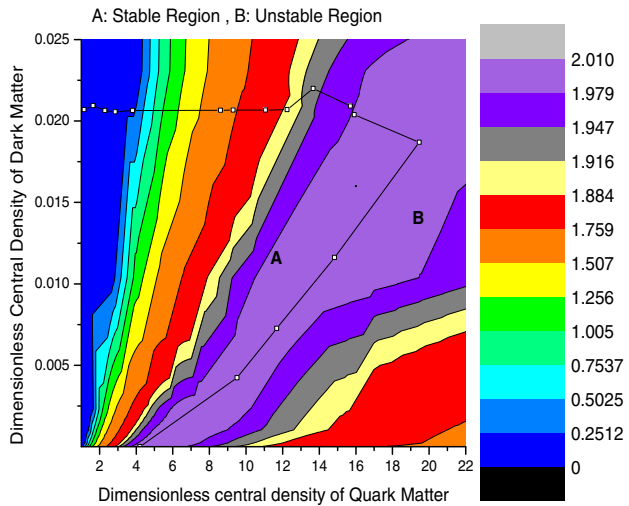


FIG. 8. Contour plot with $[\frac{\epsilon_{0,\text{quark}}}{4B}, \frac{\epsilon_{0,\text{dark}}}{m_s^2}, \text{ and } M_{\text{total}} \text{ (in } M_{\odot})]$ as the x , y , and z axes, respectively. The region marked as A represents the stable region for the formation of a quark star admixed with dark matter, while all the points in region B are unstable for such a formation.

mass for increasing $\epsilon_{0,\text{dark}}$, which is the line inclined at an angle in the contour plot. The sequence continues up to the point where the dark matter mass reaches its maxima. Above this value of $\epsilon_{0,\text{dark}}$, all configurations become unstable since dark matter itself becomes unstable. This leads to the boundary line that is almost parallel to the x axis. For a quark star admixed with dark matter made of free gas of fermionic particles, the maximum possible mass of the stable configuration is approximately $M_{\text{total}} \sim 2.01 M_{\odot}$ with a dark matter content of around $0.63 \times 10^{-4} M_{\odot}$, which has a small radius of about 0.80 m while the quark matter extends much farther to a radius of around 11 km.

B. Solution for combination of quark matter and strongly self-interacting dark matter

The two-fluid TOV equations [Eqs. (1), (2), (3), and (4)] are solved for massive dark matter fermions taken to be strongly self-interacting using the model discussed in Sec. III B. The interaction strength γ is taken to be 10^3 . The presence of self-interaction causes the maximum stable mass of a dark star to be increased from about $10^{-4} M_{\odot}$ for the free fermionic case to about $10^{-2} M_{\odot}$ for the strongly self-interacting case (Secs. VA and VB).

The plot for the mass of the quark component (M_{quark}) vs the central energy density $\epsilon_{0,\text{quark}}$ of the quark component for different values of $\epsilon_{0,\text{intdark}}$ (Fig. 9) reveals that the maximum stable mass of the quark matter decreases with increasing central energy density of dark matter ($\epsilon_{0,\text{intdark}}$) within the stable branch of dark matter, though the decrease is very moderate. The maximum stable mass of the quark component at $\epsilon_{0,\text{intdark}} = 10^5 \text{ MeV/fm}^3$ is $1.995 M_{\odot}$, and this mass reduces to $1.937 M_{\odot}$ at

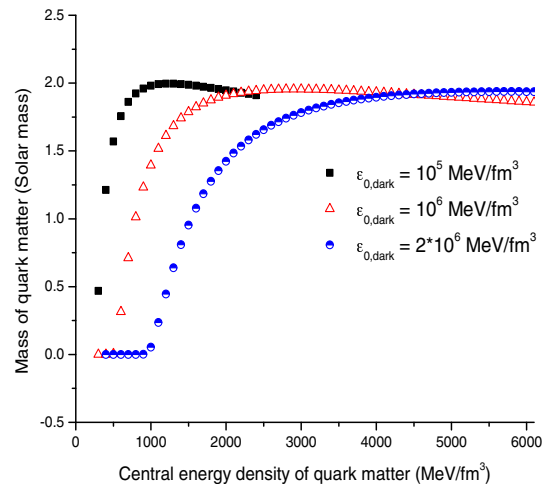


FIG. 9. Plot for the mass of the quark component (M_{quark}) vs the quark central density $\epsilon_{0,\text{quark}}$ for three different dark matter central densities ($\epsilon_{0,\text{intdark}}$). It is visible that as $\epsilon_{0,\text{intdark}}$ is increased the maximum stable quark mass ($M_{\text{quark,max}}$) is reduced and is now attained at a higher $\epsilon_{0,\text{quark}}$.

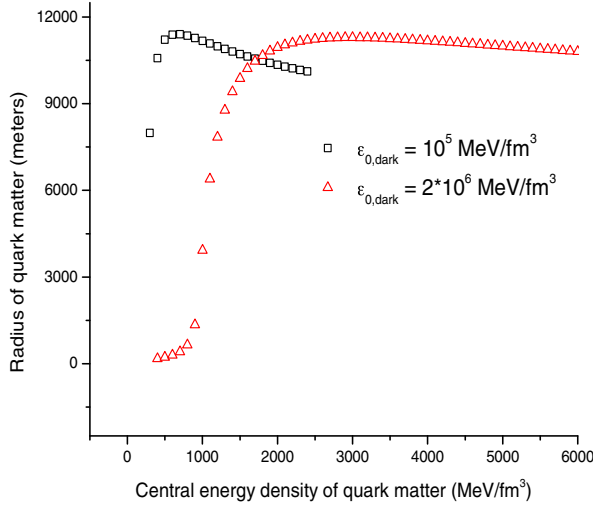


FIG. 10. Plot for the radius of the quark component (R_{quark}) vs the quark central density $\epsilon_{0,\text{quark}}$ for different dark matter central densities ($\epsilon_{0,\text{intdark}}$).

$\epsilon_{0,\text{intdark}} = 2 \times 10^6 \text{ MeV/fm}^3$. It is also observed that the maximum quark mass ($M_{\text{quark,max}}$) is attained at a much higher value of $\epsilon_{0,\text{quark}}$, the reason being the same as described in the free fermion case. The corresponding plot for the radius of the quark component vs $\epsilon_{0,\text{quark}}$ for two different $\epsilon_{0,\text{intdark}}$ is shown in Fig. 10, which shows that the maximum radius of quark component also decreases with an increase in $\epsilon_{0,\text{intdark}}$.

Keeping $\epsilon_{0,\text{quark}}$ constant and obtaining M_{int} vs $\epsilon_{0,\text{intdark}}$ and R_{int} vs $\epsilon_{0,\text{intdark}}$ gives the profile for dark matter present in the admixture (Figs. 11 and 12). It is evident from Figs. 11 and 12 that the dark matter mass and radius profile

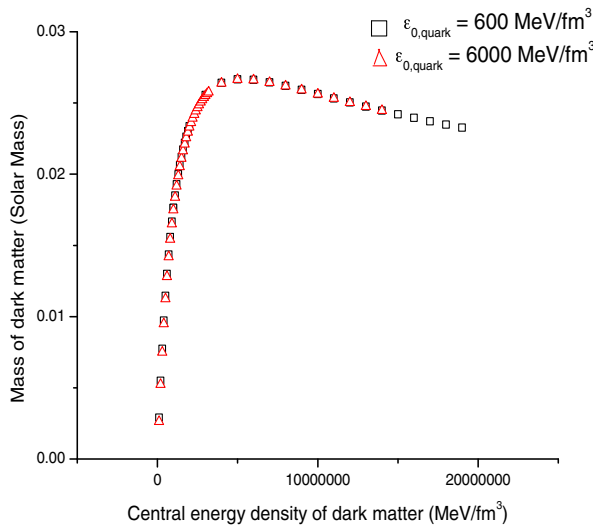


FIG. 11. Plot for the mass of the dark component (M_{int}) vs the dark central density $\epsilon_{0,\text{intdark}}$ of different quark matter central densities ($\epsilon_{0,\text{quark}}$). It is visible that the dark matter mass profile is not altered very much with changing $\epsilon_{0,\text{quark}}$.

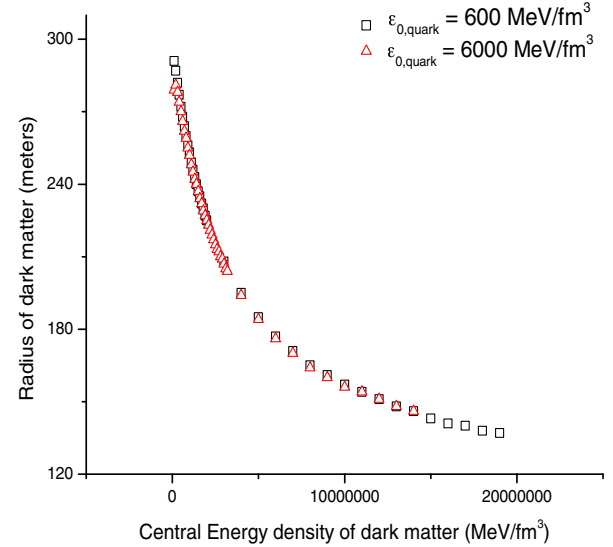


FIG. 12. Plot for the radius of the dark component (R_{int}) vs the dark central density $\epsilon_{0,\text{intdark}}$ for different quark matter central densities ($\epsilon_{0,\text{quark}}$).

do not change much with increasing $\epsilon_{0,\text{quark}}$ since dark matter is much more compact than quark matter, and its particles are also much more massive to be significantly affected by quark matter particles. Figures 9, 10, 11, and 12 allow us to determine the stability of the quark matter star admixed with self-interacting dark matter. The first two plots showing the dependence of the mass and radius of the quark matter vs $\epsilon_{0,\text{quark}}$ tell us up to which point the quark star configuration would remain stable by noting the point of maxima of the mass and the radius. The next two plots, Figs. 11 and 12, allow us to determine up to which point the dark matter remains stable for varying $\epsilon_{0,\text{quark}}$. It is evident from the graphs that the dark matter parameter profile is almost independent of $\epsilon_{0,\text{quark}}$. So, the $\epsilon_{0,\text{intdark}}$ at which the dark matter becomes unstable is the same for all $\epsilon_{0,\text{quark}}$.

The contour plot showing the dependence of the total mass of the entire star (M_{total}) on the dimensionless central energy densities of the two fluids (Fig. 13) reflects the decrease in the maximum stable total mass with an increase in $\epsilon_{0,\text{intdark}}$.

The region of stability is marked in the contour diagram. The shape of the boundary is similar to the free case discussed before. The upper branch of the boundary line is an indicator of the $\epsilon_{0,\text{intdark}}$, after which the dark matter component becomes unstable for a given $\epsilon_{0,\text{quark}}$. As a quick check, in the contour diagram, the plot converges to the appropriate mass limit for low $\epsilon_{0,\text{intdark}}$. For low $\epsilon_{0,\text{intdark}}$ say, 10^5 MeV/fm^3 , the maximum stable mass is $\sim 2.0 M_{\odot}$ at a radius of about 11 km showing the convergence to the pure quark star limit.

Figure 14 shows the maximum stable total mass vs the fraction of dark matter which is fitted by a linear fit. The slope of the fit comes out to be about -3.62 .

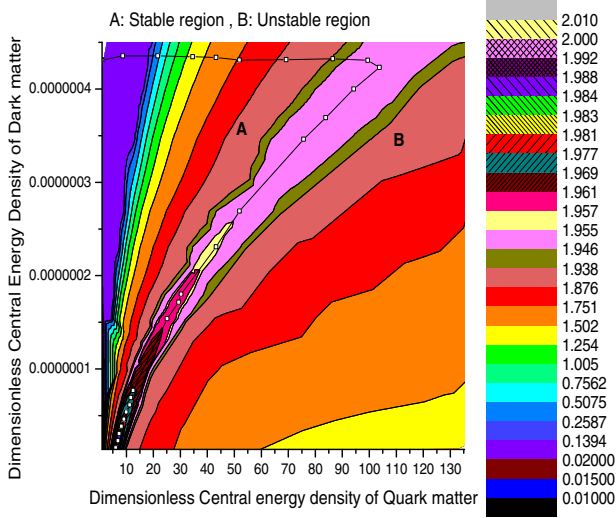


FIG. 13. Contour plot with $[\frac{\epsilon_{0,quark}}{4B}, \frac{\epsilon_{0,intdark}}{m_f^4}, M_{total}$ (in M_\odot)] as the x , y , and z axes, respectively. The region marked as A represents the stable region for the formation of a quark star admixed with dark matter, while all the points in region B are unstable to such a formation. $\frac{\epsilon_{0,quark}}{4B}$ starts from 1.0 in the plot because the energy density of quark matter cannot be zero according to the equation of state (6).

The reason for the decrease of the maximum total mass with increasing dark matter content is the increased gravitational force due to extra dark matter content which causes an earlier collapse of the star. Using the linear fit in Fig. 14, the equation for the dependence of the maximum total stable mass of the admixed star on the dark matter fraction present is given as

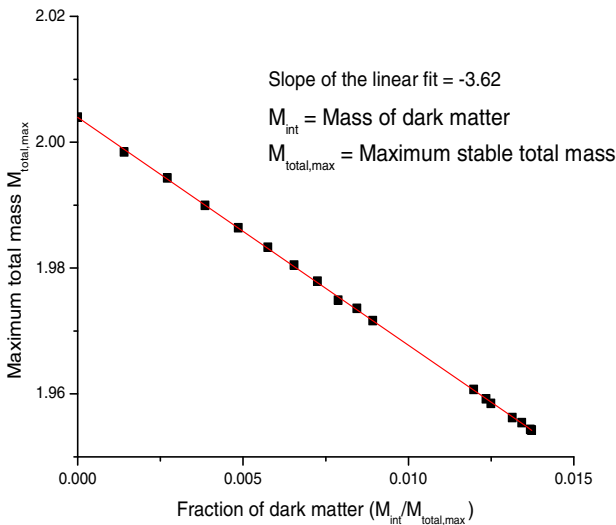


FIG. 14. Plot showing the dependence of the maximum total mass vs the dark matter content for a quark star admixed with dark matter. The slope of the plot is -3.62 . It shows that the maximum total mass decreases with increasing dark matter content in the star. The red line shows the linear fit.

$$\frac{M_{tot,max}}{M_\odot} = 2.004 - 3.62 \frac{M_{int}}{M_{tot,max}} \quad (11)$$

or

$$\frac{M_{tot,max}}{M_\odot} = 2.004 - 3.62f, \quad (12)$$

where f is the fraction of self-interacting dark matter in the admixed star at the maximum stable total mass. This linear plot gives the maximum stable total mass of the star for a given fraction of dark matter. The linear relation between the maximum possible stable mass and the dark matter content implies that the rate of decrease of the maximum mass is a constant and hence the decrease is slower as compared to other possible kinds of fall like an exponential fall, which would have led to a much steeper fall of the maximum mass with an increasing dark matter fraction. So, the maximum mass decreases slowly with increasing dark matter content. Since the equations have been numerically solved using two-fluid TOV equations including gravitational interaction between the constituents and with a full general relativistic treatment, the linear relation obtained is quite robust. However, since only gravitational interaction has been included and all other possible interactions between the particles have been ignored, some deviations from this linear relation are expected depending on the nature of those interactions. For masses above the maximum stable dark matter and quark matter masses, the linear relation will no longer hold. The linear relation is strictly valid only for the range of parameters that we investigated. We cannot predict beforehand a linear relation of this form for any arbitrary set of parameters. The maximum allowed strongly self-interacting dark matter content is about $2.64 \times 10^{-2} M_\odot$ at a maximum total stable mass of about $1.95 M_\odot$, which gives a maximum limit on the possible dark matter fraction $f_{max} \approx 0.014$. Since the maximum mass falls slowly with increasing dark matter, observing drastic changes in the maximum total quark star masses (if they exist) is not expected. Also, since the maximum possible dark matter fraction is very small ($\sim 1.4\%$), it would be difficult to detect dark matter in quark stars in the $2.0 M_\odot$ mass range. Radio timing observations of the pulsar J0348 + 0432 and phase-resolved optical spectroscopy of its white-dwarf companion lead to a precise pulsar mass measurement of $2.01 \pm 0.04 M_\odot$, which is by far the highest yet measured with this precision [35,36]. The maximum stable dark admixed quark star mass ($1.95 M_\odot$) falls slightly lower than the error limit of the highest measured pulsar mass.

VII. SUMMARY AND DISCUSSIONS

Pure quark matter is studied by using the MIT bag model [8]. A dark matter star is studied by considering the dark matter to be made up of fermionic particles of mass 100 GeV [16,17] with the assumption that these particles

do not self-annihilate. The maximum stable mass of the dark matter star composed of strongly self-interacting particles is about $2.7 \times 10^{-2} M_{\odot}$ at a radius of about 0.19 km, while for free fermions, the mass is about $6.0 \times 10^{-5} M_{\odot}$ at a radius of around 1 m.

The complete dimensional two-fluid TOV equations are solved to study the behavior of quark matter admixed with dark matter. First, the equations are solved for a mixture of quark matter and free dark matter. The maximum stable mass of the admixed star is almost the same as that for a pure quark star for an increasing dark matter fraction within the star. As the content of dark matter is gradually increased in the admixed star, the dark matter reaches its maximum stable configuration, after which no admixed star configuration remains stable since the dark matter component collapses to form a black hole. For a quark star admixed with dark matter made of free gas of particles, the maximum possible mass of the stable configuration is approximately $M_{\text{total}} \sim 2.01 M_{\odot}$ with a dark matter content of around $0.63 \times 10^{-4} M_{\odot}$. A reduction in the maximum stable total mass is noted in case of a quark star admixed with a dark matter star composed of strongly self-interacting fermions. The decrease is from $2.01 M_{\odot}$ for

zero dark matter content inside the star to about $1.95 M_{\odot}$ for the maximum allowed mass of strongly self-interacting dark matter in the star. The maximum dark matter content is around $2.64 \times 10^{-2} M_{\odot}$ at a maximum stable total mass of about $1.95 M_{\odot}$. The maximum stable total mass in case of strongly self-interacting dark matter is seen to reduce linearly with an increasing dark matter fraction in the star. The maximum accretion rate of dark matter by the quark star can be estimated to be about $\frac{M_{\text{int,max}}}{\tau} \sim 2.03 \times 10^{-12} M_{\odot}$ per year, where $\tau \sim 1.3 \times 10^{10}$ years is the estimate for the age of the Universe and $M_{\text{int,max}}$ is the maximum possible self-interacting dark matter content in the quark star. If the accretion rate is higher than this, the quark star will collapse.

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