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Electroweak naturalness and deflected mirage mediation

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We investigate the question of electroweak naturalness within the deflected mirage mediation (DMM) framework for supersymmetry breaking in the minimal supersymmetric standard model. The class of DMM models considered are nine-parameter theories that fall within the general classification of the 19-parameter phenomenological minimal supersymmetric standard model. Our results show that these DMM models have regions of parameter space with very low electroweak fine-tuning, at levels comparable to the phenomenological minimal supersymmetric standard model. These parameter regions should be probed extensively in the current LHC run.

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I. INTRODUCTION

Theories with TeV-scale supersymmetry, such as the minimal supersymmetric standard model (MSSM) and its extensions, have long been considered to be leading candidates for new physics that can elucidate the origin of electroweak symmetry breaking and address the gauge hierarchy problem associated with the Standard Model Higgs boson. With the null result to date of searches for superpartners, and with the very recent turn-on of the LHC which will probe TeV-scale energies at an unprecedented level, the issue of theoretical "naturalness" is a key question for this class of theories.

Of course, the question of how "fine-tuned" a specific model is depends on the criteria used to gauge it. One general method, which has become a standard approach, is to evaluate the sensitivity to observables such as the Z boson mass m_Z to changes in the input parameters at a high scale, for example by the Barbieri-Giudice fine-tuning measure [1],

$$\Delta_{\rm BG} = \max_i \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|, \qquad (1)$$

in which the a_i represent the parameters at the theory. This measure quantifies the extent to which electroweak to TeV-scale observables are sensitive to variations in the high-scale parameters and as such is a gauge of the naturalness of the theory.

However, it has recently been emphasized (see, e.g., Ref. [2]) that to address the specific question associated with naturalness in light of the nonobservation of supersymmetry at the LHC, which is how the observed value of m_Z emerges when the superpartners must generically have masses that far exceed this value, fine-tuning measures other than $\Delta_{\rm BG}$ may yield valuable information. One specific fine-tuning measure of this type is known as the electroweak fine-tuning measure $\Delta_{\rm EW}$, [2–14], which assesses the extent to which cancellations occur in the prediction of the Z boson mass as a function of the model parameters. In practical terms, fine-tuning by this measure is a reflection of the degree to which the model parameters that enter in the expression for m_Z are of order m_Z themselves at the electroweak scale (low fine-tuning) or are much larger (high fine-tuning). It has been emphasized previously that the naturalness measure Δ^{-1} can serve as a Bayesian prior and as a likelihood estimate [15,16].

More precisely, in the MSSM, the Z boson mass is given at one loop by the following well-known relation,

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2, \qquad (2)$$

in which the $\Sigma_{u,d}^{u,d}$ are the one-loop corrections for down-type and up-type quarks, respectively (explicit expressions can be found in Ref. [4]). The expression for $\Delta_{\rm EW}$ then takes the form

$$\Delta_{\rm EW} = \max_i |C_i|/(m_Z^2/2), \qquad (3)$$

in which the C_i are the terms in Eq. (2), for example $-m_{H_u}^2\tan^2\beta/(\tan^2\beta - 1)$. As each of the C_i are defined at the electroweak scale, each is determined purely by the supersymmetric spectrum, independent of the high-scale dynamics and renormalization group running effects that yield that spectrum. For this reason, this fine-tuning assessment is often referred to as a determination of the degree of "electroweak naturalness" of a given model.

In studies of electroweak naturalness for various models of the MSSM soft terms, it has been noted that several general conditions at the electroweak scale result in low values of $\Delta_{\rm EW}$ and hence a small degree of fine-tuning. These conditions include (i) $|\mu| \sim 100$ GeV, which results

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in light Higgsino-like neutralinos; (ii) $m_{H_u}^2(m_Z) \sim -m_Z^2/2$, as easily seen from Eq. (2); and (iii) large A_t, which is needed to raise the Higgs mass without requiring heavy stops. These conditions are not easily met within certain classes of models but can be achieved in others. In the 19-parameter phenomenological MSSM (pMSSM) [17], it is to be expected that there are regions of parameter space that meet these criteria and hence yield low values for $\Delta_{\rm EW}$, which was shown explicitly in a recent study [14]. Similarly, this can also be achieved in the 19-parameter supergravity mode (SUGRA19) [7].

However, in models with fewer free parameters, clearly these conditions are more difficult to achieve. For example, minimal gauge mediation has difficulty because the A-terms are generated at two loops, and $m_{H_u}^2$ tends to run large and negative. Models of mirage mediation (the mixed moduli-anomaly mediation scenario [18–21] based on the Kachru-Kallosh-Linde-Trivedi (KKLT) construction [22]) also have difficulties, tending to have either large values of μ, for similar reasons as minimal gauge mediation, or small values of μ and fail constraints on B meson decays [10] (see Refs. [23–25] for studies of the phenomenology of mirage mediation). However, the variation on minimal supergravity known as NUHM2 [26], a nonuniversal Higgs model that has six free parameters, can satisfy all of these criteria, yielding results for $\Delta_{\rm EW}$ as low as $\sim$5–10 [8,10,14]. The low fine-tuning allowed in this scenario is quite striking given that the NUHM2 is only a six-parameter model. As such, it has been dubbed "radiative natural supersymmetry" (RNS), wherein the MSSM and electroweak symmetry breaking arise naturally as the low energy limit of an underlying supersymmetry (SUSY) grand unified theory, and its phenomenological implications at the LHC and for dark matter physics have been thoroughly explored [2,4–6,9,11–13].

The purpose of this paper is to explore the question of electroweak naturalness within a class of supersymmetric models known as deflected mirage mediation (DMM) models. This framework is a natural extension of mirage mediation to include additional contributions from gauge mediation [27,28]. In deflected mirage mediation, the gauge-mediated contributions to the soft supersymmetry breaking parameters can be comparable to the gravity-mediation and anomaly-mediated contributions at the grand unified theory (GUT) scale, which in turn produces distinct phenomenology and a rich theory space for exploring current and future LHC supersymmetry searches, including examples of both simplified and compressed supersymmetric spectra [29–31]. It is worth noting that the question of fine-tuning using high-scale fine-tuning measures such as $\Delta_{\rm BG}$ within the DMM framework has been explored [31], particularly in light of the Higgs mass measurement at the LHC [32–34], though there is no prior fine-tuning study of DMM using the electroweak naturalness criterion.

As will be discussed in more detail shortly, the deflected mirage mediation framework, in its most general form, has a rich parameter space that can include regions that are outside the realm of the pMSSM (e.g., that predicts nonuniversal scalar masses for the first and second generations). However, for phenomenological reasons, it is useful to consider only the subspace of DMM theory space that falls within the pMSSM guidelines. Hence, we consider this restricted DMM parameter region within this paper. This will allow for a straightforward comparison with the pMSSM. We will demonstrate that within DMM models of this type there are regions of parameter space with $\Delta_{\rm EW}$ as low as $\sim$3.7; i.e., it is roughly equivalent to the best-case scenarios in SUGRA19 and slightly better than the best-case scenarios in the NUHM2/RNS scenario. (Here, we note that direct comparisons with the pMSSM scan done in Ref. [11] are difficult as they likely have not sampled enough of the space to capture the low fine-tuned regions that SUGRA19, NUMH2, and DMM explore, which are all embeddable at low energy in the pMSSM.)

The paper is organized as follows. In the next section, we quickly review the soft terms in DMM and discuss the parameter space for the subspace of DMM models of interest in this paper. In Sec. III, we investigate the question of electroweak naturalness for this class of models and show that there is a region of parameter space with extremely low values of the fine-tuning measure $\Delta_{\rm EW}$. We then summarize and conclude in Sec. IV.

II. OVERVIEW OF DEFLECTED MIRAGE MEDIATION MODELS

Deflected mirage mediation models are characterized by three classes of contributions to the soft supersymmetry breaking parameters. As is the case in mirage models, there is a KKLT-like contribution to the soft masses that consists of tree-level supergravity contributions associated with a modulus field, as well as comparable anomaly mediation terms at a high scale, which is taken for simplicity to be the GUT-scale $M_{\rm G}$. Deflected mirage mediation scenarios also include gauge-mediated contributions, which take the form of a deflection of the soft terms at some messenger scale $M_{\rm mess}$. The messenger fields associated with the gauge mediation terms are typically taken to be N vectorlike pairs of fundamental representations of $SU(5)$. In these scenarios, the MSSM matter and Higgs fields are also each characterized by a modular weight label n_i that appears in the respective Kähler potential terms for each of these fields. [To be more rigorous, the Kähler potential for the MSSM matter fields is taken to be diagonal in family space. For the matter and Higgs fields, it generically takes the form $K \sim \sum_i \tilde{K}_i \overline{\Phi}_i \Phi_i$, with $\tilde{K}_i = (T + \overline{T})^{-n_i}$].

More explicitly, the high-scale soft terms at M_G take the form

$$M_a(\mu = M_G) = M_0\left[1 + \frac{g_0^2}{16\pi^2} b'_a \alpha_m \ln\frac{M_P}{m_{3/2}}\right], \quad (4)$$

$$A_i(\mu = M_G) = M_0\left[(1 - n_i) - \frac{\gamma_i}{16\pi^2}\alpha_m \ln\frac{M_P}{m_{3/2}}\right], \quad (5)$$

$$m_i^2(\mu = M_G) = M_0^2\left[(1 - n_i) - \frac{\theta'_i}{16\pi^2}\alpha_m \ln\frac{M_P}{m_{3/2}} - \frac{\dot{\gamma}'_i}{(16\pi^2)^2}\left(\alpha_m \ln\frac{M_P}{m_{3/2}}\right)^2\right], \quad (6)$$

in which $m_{3/2}$ is the gravitino mass ($m_{3/2}$ generically exceeds M_0 by about a loop factor in magnitude and thus is typically of order 10–100 TeV). Note that the physical trilinear terms are $A_{ijk}y_{ijk}$, in which $A_{ijk} = A_i + A_j + A_k$.

In the above expressions, $b'_a = b_a + N$, in which b_a are the one-loop beta functions for the gauge couplings [$b_{1,2,3} = (33/5, 1, -3)$ in the MSSM]. The anomalous dimensions $\gamma'_i = \gamma_i$ are given by $\gamma_i = 2\sum_a g_a^2 c_a(\Phi_i) - (1/2)\sum_{lm}|y_{ilm}|^2$, in which the y_{ijk} are the normalized MSSM Yukawa couplings and the c_a are the quadratic Casimirs. The $\dot{\gamma}_i$'s are given by $\dot{\gamma}_i = 2\sum_a g_a^4 b_a c_a(\Phi_i) - \sum_{lm}|y_{ilm}|^2 b_{yilm}$, in which b_{yilm} is the beta function of the Yukawa coupling y_{ilm}. The quantities $\theta'_i = \theta_i$ are given by $\theta_i = 4\sum_a g_a^2 c_a(Q_i) - \sum_{ijk}|y_{ijk}|^2(3 - n_i - n_j - n_k)$. (Explicit expressions for these quantities can be found, for example, in Appendix A of Ref. [30].)

The threshold contributions due to gauge mediation at M_{mess} take the form

$$\Delta M_a(\mu = M_{\text{mess}}) = -M_0 N \frac{g_a^2(M_{\text{mess}})}{16\pi^2}\alpha_m(1 + \alpha_g)\ln\frac{M_P}{m_{3/2}}, \quad (7)$$

$$\Delta m_i^2(\mu = M_{\text{mess}}) = M_0^2 \sum_a 2c_a N \frac{g_a^4(M_{\text{mess}})}{(16\pi^2)^2} \times \left[\alpha_m(1 + \alpha_g)\ln\frac{M_P}{m_{3/2}}\right]^2. \quad (8)$$

Note that the trilinear terms do not receive threshold contributions at one-loop order, and hence these contributions are negligible.

From Eqs. (4)–(8), we see that the model parameters for a general DMM scenario thus include (i) an overall mass scale M_0 associated with the tree-level supergravity mediation; (ii) a dimensionless parameter α_m, which denotes the relative importance of anomaly mediation with respect to the tree-level gravity mediation (the KKLT scenario predicts $\alpha_m = 1$); (iii) the number of messenger pairs N; (iv) the messenger scale M_{mess}; (v) the dimensionless parameter α_g, which denotes the relative importance of gauge mediation with respect to anomaly mediation; (vi) the modular weights n_i; (vii) the ratio of electroweak Higgs vacuum expectation values; and (viii) the sign of μ. Here, the standard procedure has been followed in which the model-dependent Higgs parameters μ and $b = B\mu$ are replaced by $\tan\beta$, m_Z, and the sign of μ.

In a general DMM model of this type, the soft scalar mass-squared parameters have generation-dependent labels given by the possibility of family-dependent n_i values, as well as the presence of Yukawa couplings in the $\theta_i{}'$ and $\dot{\gamma}'_i$ quantities. While the contributions to the anomalous dimensions, etc., are typically negligible for all practical purposes for the first and second generations due to the hierarchical standard model fermion masses, a general assignment of the modular weights n_i can yield a sizable nonuniversal contribution. For simplicity as well as phenomenological reasons, we thus will always restrict ourselves to a subspace of DMM parameter space in which the matter fields all carry a universal modular weight n_M. In addition, we will assume that the two electroweak Higgs fields also carry an independent modular weight n_H, which introduces an amount of nonuniversality.

The DMM scenarios studied here thus have nine independent parameters (two masses, six dimensionless parameters, and one sign): the mass scales M_0 and M_{mess}, the dimensionless quantities α_m and α_g, the number of $SU(5)$ messenger pairs N, the modular weights n_M and n_H, $\tan\beta$, and signμ. We note that with this assumption regarding the modular weights these scenarios represent a subset of the full 19-parameter pMSSM.

III. ELECTROWEAK NATURALNESS IN DMM MODELS

In our analysis of electroweak naturalness in this class of DMM models, we use a subset of the data set as studied in Ref. [30]. This data set was determined as follows: for a randomly chosen mirage mediation point in the region $M_0 \in [1, 5]$ TeV, $\tan\beta \in [5, 50]$, and $\alpha_m \in [0, 2]$, we build a three-dimensional scan in the DMM parameter, scanning $\alpha_g \in [-1, 1]$ in steps of 0.05, $\log_{10}[M_{\text{mess}}/\text{GeV}] \in [5, 14]$ in unit steps, and $N \in [1, 5]$ in unit steps. The modular weights n_M and n_H for the matter and Higgs fields, respectively, are allowed to vary independently between 0 and 1 in half-integer steps. The renormalization group (RG) equations were solved using a version of the package SOFTSUSY 3.3.9 [35] that has been modified to account for the gauge mediation contributions [27,28,30].

The phenomenological constraints applied to these model points are as follows. At the electroweak scale, points with negative mass squares or that do not result in electroweak symmetry breaking or that do not have a neutralino lightest supersymmetric particle (LSP) are excluded. The surviving points are then cut according to an upper bound on the relic density, $\Omega_\chi h^2 \le 0.128$, taken from Ref. [36], as calculated by MicrOMEGAs 2.2 [37],

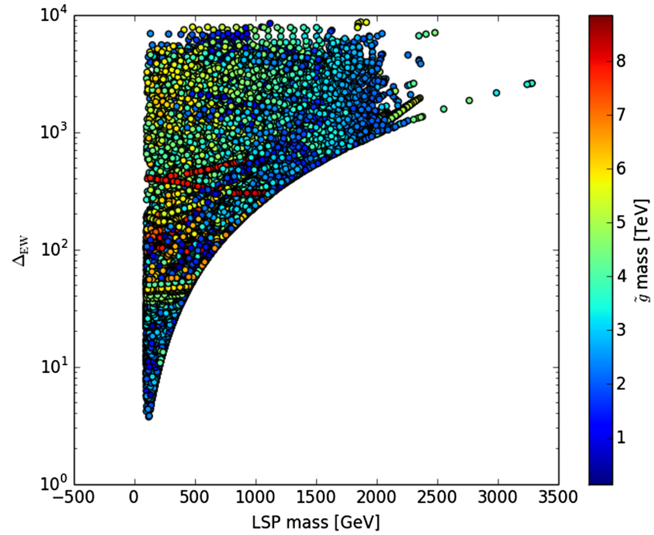

FIG. 1. Δ_{EW} as a function of the LSP mass for the full data set without a cut on the gluino mass and shaded by the gluino mass in TeV.

and a (conservative) Higgs mass bound 123 GeV $\le m_h \le$ 127 GeV [32–34]. Finally, we apply constraints from $B_s \to \mu^+\mu^-$ and $b \to s\gamma$, with the values of $\mathrm{Br}(B_s \to \mu^+\mu^-)$ taken within $(1.5 - 4.3) \times 10^{-11}$ [38–40] and a value of $\mathrm{Br}(b \to s\gamma)$ within $(3.03 - 4.08) \times 10^{-4}$ [41]. These cuts follow the ranges used in the previous work on Δ_{EW} [10,14] to facilitate comparisons with these studies. In total, we use a 2 million point subset and after application of all phenomenological constraints, leading to slightly more than 200,000 viable DMM model points.

We now turn to the determination of Δ_{EW} for these model points. Figure 1 shows the results for Δ_{EW} from the entire scan described previously and used in Ref. [30]. The results of the figure do not change if we require $m_{\tilde{g}} > 1.3$ TeV, consistent with current generic bounds from the LHC [42], because the cut on $\mathrm{Br}(b \to s\gamma)$ removes some of the low mass gluinos. We use this bound on the gluino henceforth.

In Fig. 1 and the left panel of Fig. 2, we see that there is a large region that is less than 1% fine-tuned. The minimum fine-tuning is of order 27%, $\Delta_{EM} \approx 3.7$. These are smaller than the minimum values for the NUHM2 model of $\Delta_{EM} \approx 7$ [8,14] or ≈ 10 [10], slightly larger than the values found in SUGRA19 [7]. Furthermore, we see that if we want less than 1% electroweak fine-tuning, then gluino masses less than about 8 TeV are allowed in DMM.

The right panel of Fig. 2 shows the relic density as a function of the LSP mass. The points with the smallest fine-tuning typically have $O(150\ \mathrm{GeV})$ LSPs and do not fulfill the relic density constraint, and another nonthermal species such as an axion is needed [12]. This does not need to be the case if there are coannihilations between a heavier Higgsino-like LSP and the right-handed sleptons. An example of this sort of spectra is shown in the right panel of Fig. 3, in which less than 0.5% fine-tuning can be achieved in a corner of parameter space with large M_0 and α_m and both modular weights equal to 1. Removing the upper bound on the amount of fine-tuning admits spectra where the proper relic density can be achieved through coannihilation with a stop or gluino as well. Over much of the Higgsino-like parameter space, the difference between $\tilde{\chi}_2^0 - \tilde{\chi}_1^0$ is typically < 10 GeV and often less than < 5 GeV, leading to very soft and likely hard to detect signals at LHC13 [9].

The region with Δ_{EW} less than $\sim$100 is made up of points with mass spectra similar to the two example spectra shown in Fig. 3. These two points share a light, highly mixed stop and very light pure Higgsino LSP, but they also share a near degeneracy among the next heaviest particles after the Higgsino-like neutralinos and charginos. In the left panel,

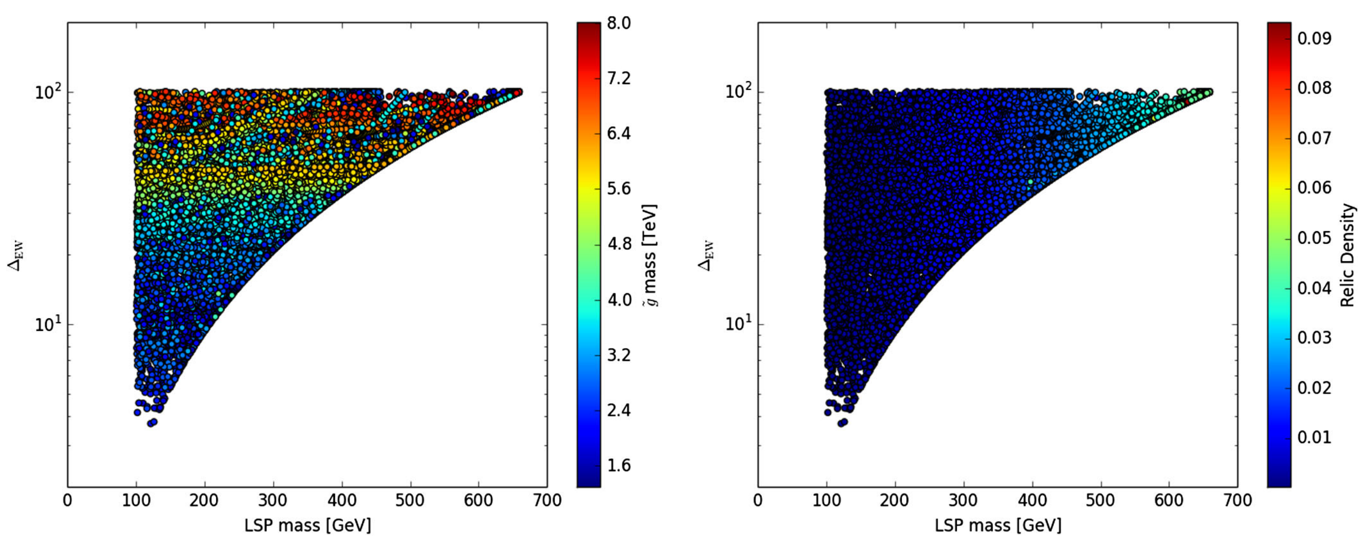

FIG. 2. A plot of Δ_{EW} as a function of the LSP mass for points with $\Delta_{EW} < 100$, (left) shaded by the gluino mass in TeV, and (right) shaded by the relic density.

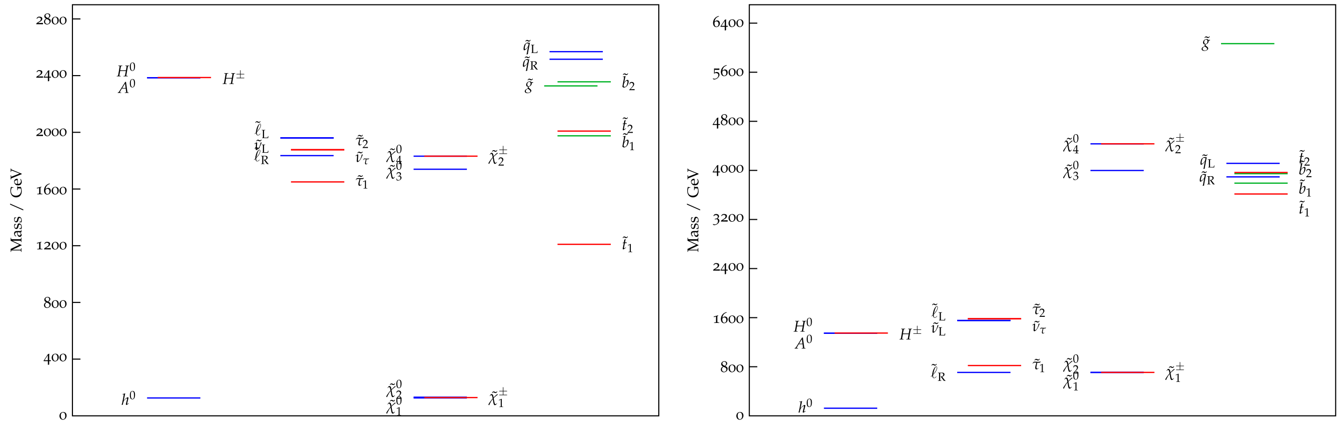

FIG. 3. Examples of the Higgs and superpartner spectra for two representative points with small(ish) values of $\Delta_{\rm EW}$. The left panel is a point with $\Delta_{\rm EW} < 4$ and a low value of the relic density ($\Omega_\chi h^2 = 2.4 \times 10^{-3}$), for which $M_0 = 2600$ GeV, $\alpha_m = 1.21$, $\tan\beta = 22$, $(n_M, n_H) = (0.5, 0)$, $\alpha_g = 0.1$, $M_{\rm Mess} = 10^9$ GeV, and $N = 2$. The right panel is a point with $\Delta_{\rm EW} < 120$ and a large value of the relic density ($\Omega_\chi h^2 = 0.113$), for which $M_0 = 4800$ GeV, $\alpha_m = 1.36$, $\tan\beta = 38$, $(n_M, n_H) = (1, 1)$, $\alpha_g = -0.4$, $M_{\rm Mess} = 10^{14}$ GeV, and $N = 1$.

we have a near degenerate gluino and stop, and in the right, we have a near degenerate stau, smuon, and selectron. As mentioned earlier, coannihillation with the stau allows the spectrum to generate a value of the dark matter relic density near the measured value from the Planck experiment [36]. The left-hand spectra in Fig. 3 are similar to the spectra found in points with low fine-tuning in the NUHM2 model. In this model, and similarly in the SUGRA19 model, the least tuned points tend to have a ∼1 TeV, highly mixed stop and a plethora of particles above ∼1.5 TeV, including a ∼2 TeV gluino. Unlike NUMH2, DMM does not have universal gaugino masses, and so M_1 and M_2 can and tend to be much larger.

If we further use a tighter bound of 3.3% fine-tuning, we can compare the bounds in the NUHM2 model from Ref. [14]. In most respects, the two models agree. In DMM, the gluino mass is capped at about 5 TeV, larger than the upper bound in Ref. [14]. Similarly, the bounds on M_1 and M_2 of 900 and 1700 GeV, respectively, from Ref. [14] are instead 4.35 and 4.15 TeV in DMM. Nonuniversal gaugino masses, like those in DMM Eq. (4), relax the limits in minimal supergravity-like models. Other bounds, like those on the lightest stop, are similar to NUHM2 and below the upper bounds from the pMSSM or SUGRA19 from Refs. [7,14]. The results indicate that as we increase the number of degrees of freedom the bounds on particle masses, other than the lightest stop, and parameters, other than μ and $m^2_{H_u}$, weaken but are still within the range of future colliders and Higgs factorylike experiments [43].

Wino- and binolike LSPs are typically more fine-tuned than Higgsino-like points. The minimum point with a winolike LSP has $\Delta_{\rm EW} \approx 60$, and the minimum binolike point in the sample has $\Delta_{\rm EW} \approx 176$. These are both more fine-tuned than the values explored in the natural supersymmetry phenomenological study in the NUHM2 model of Ref. [11] because they allow for other hierarchies in the soft terms other than the $M_1 > M_2 > M_3$ in DMM at the GUT scale. The messenger-scale contribution Eq. (8) preserves the same hierarchy in gaugino masses, although the hierarchy in terms of the absolute value of the gaugino masses may change. This allows for winolike LSPs while there were none in mirage mediation. For winolike points, the deflection must be large, $N \geq 3$ and $\alpha_g \sim 1$, and at a low scale. The deflection to the Higgs masses for these points are large, positive, and leads to large values of $m^2_{H_u}$ at $M_{\rm SUSY}$, to compensate we will need a larger value of μ or of the messenger-scale corrections, however either solution leads to large fine-tuning. If we were to allow a lower messenger scale or larger α_g, we would likely admit winolike DMM points with smaller fine-tuning. Running may also modify this hierarchy, opening up the possibility of regions with binolike LSPs. Light, pure-bino dark matter tends to be overproduced, setting a lower bound on M_1 and μ, leading to increased fine-tuning in the binolike sample.

An investigation of the Higgs mass as a function of the stop mass and the stop mixing parameter X_t, as shown in Fig. 4, demonstrates that points with low fine-tuning are typically those that are near maximally mixed $|X_t/\tilde{t}_1| \sim 2.5$ and have TeV-scale stops [44,45]. In the left panel, we see that in the $X_t/m_{\tilde{t}_1}$ vs $\Delta_{\rm EW}$ plane a lighter Higgs will typically allow points with lower fine-tuning. The notch in the lower left corner of both distributions corresponds to a region that is excluded by the constraint on ${\rm Br}(b \to s\gamma)$.

In Fig. 5, we see that there are two distinctive regimes for the μ parameter. In one regime, $m^2_{H_u}$ is negative at $M_{\rm SUSY}$, and in the other, $m^2_{H_u}$ is positive and runs to negative values below $M_{\rm SUSY}$. For Higgsino-like points, the latter set forms a tight band, while the former has a spread. For the same value of μ, points where $m^2_{H_u}$ is already negative are less

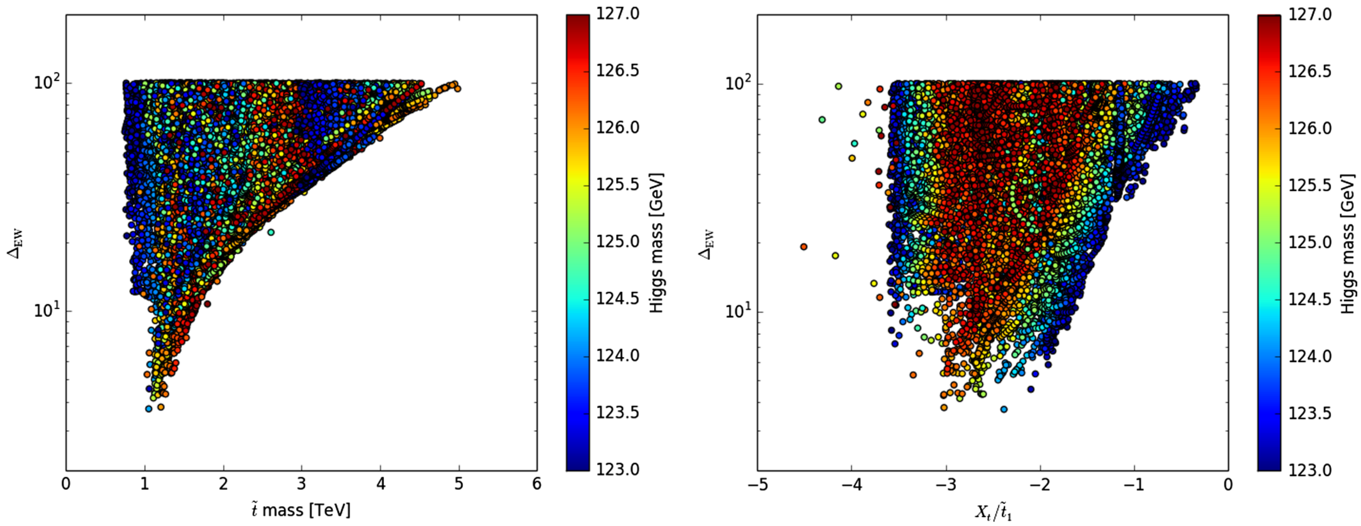

FIG. 4. A plot of Δ_{EW} as a function of the stop mass (left) and Δ_{EW} vs X_t divided by the stop mass (right), with $\Delta_{EW} < 100$ and a 1.3 TeV cut on the gluino mass. The notches in the lower left corners correspond to regions that are excluded by $\mathrm{Br}(b \to s\gamma)$ and $\mathrm{Br}(B_s \to \mu^+\mu^-)$.

fine-tuned than points where it is positive, but the positive points reach lower overall values of μ and of Δ_{EW}. The gap between the two branches comes from the dearth of points where $m_{H_u}^2 \sim 0$ at values of $\mu \gtrsim 1$ TeV, above which electroweak symmetry breaking is difficult to achieve. For bino-like points, all have negative $m_{H_u}^2$, with lower fine-tuning occuring for points with larger values of the up-type Higgs mass. Wino-like points arise from all values of $m_{H_u}^2$, with positive values typically having a smaller μ parameter and lower fine-tuning.

If we break the sample down by modular weights as shown in Fig. 6, we see that the majority of points with

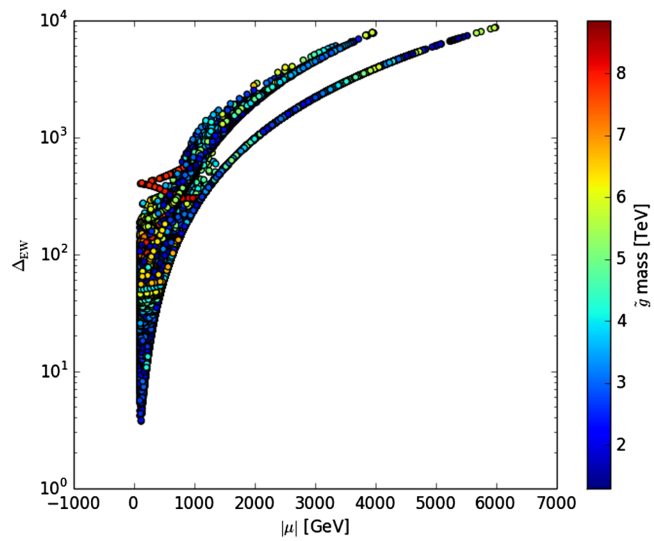

FIG. 5. A plot of Δ_{EW} vs. the μ parameter, colored by the gluino mass. The two tails correspond to points in which $m_{H_u}^2$ is positive (top) and negative (bottom) at M_{SUSY}.

extremely low fine-tuning have $n_H = 0$. $n_H = 0$ implies that the tree-level supergravity contribution to the Higgs soft mass-squared parameters is maximized [see Eq. (6)], leading to small masses at the GUT scale, since the anomaly contribution has the opposite sign. In mirage mediation models, $m_{H_u}^2$ will typically run to large and negative, leading to large values of Δ_{EW}. In DMM, the addition of the messenger fields deflects the soft masses upward, leading typically to shallower values at M_{SUSY} compared to mirage mediation. This is the radiatively natural scenario explored in Refs. [4–6,9,11–13]. Since the corrections can be large, there are regions in DMM, where for $\Delta_{EW} \gtrsim 7$, $n_H = 1/2$ can lead to low fine-tuning as well.

Our results show that in DMM all values of n_M studied can result in low electroweak fine-tuning, as opposed to the case of mirage models, which single out $n_M = 1$ [10]. That being said, the best results in DMM tend to occur for $n_M = 1$ or $1/2$. These DMM points tend to have soft mass-squared parameters that are negative at the GUT scale but are positive at M_{SUSY} through RG evolution and the positive messenger-scale deflection due to the gauge mediation terms. The addition of messengers in DMM leads to larger values for the gauge couplings at the GUT scale, causing the anomaly mediation contribution to become increasingly large and negative, and so it may require a nonzero tree-level gravity contribution, $n_M \neq 1$, to moderate the soft mass-squared parameters to get light, $\mathcal{O}(1\ \mathrm{TeV})$ stops. In mirage models, the GUT-scale value of the couplings is smaller, leading to a small positive anomaly mediation contribution at the GUT scale, which then runs to give us light stops. If $n_M \neq 1$ in these models, the addition of moduli would lead to a heavy stop, and the model would be fine-tuned.

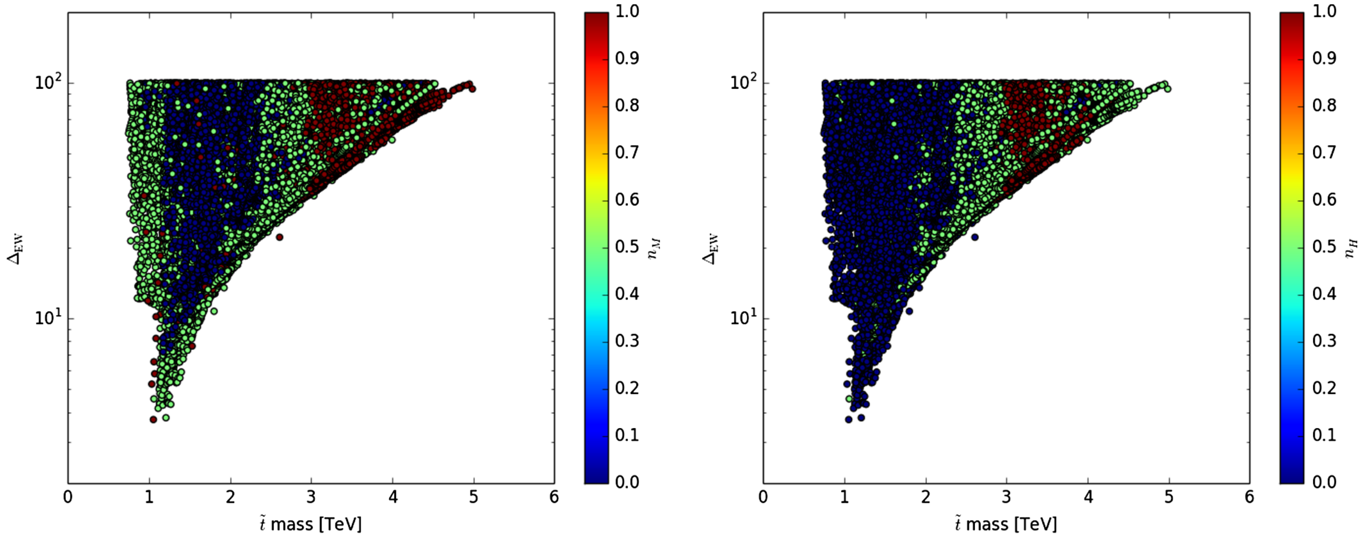

FIG. 6. $\Delta_{\rm EW}$ vs the stop mass, with the values of the matter modular weight n_M indicated on the left and of the Higgs modular weight n_H on the right, for points with $\Delta_{\rm EW} < 100$.

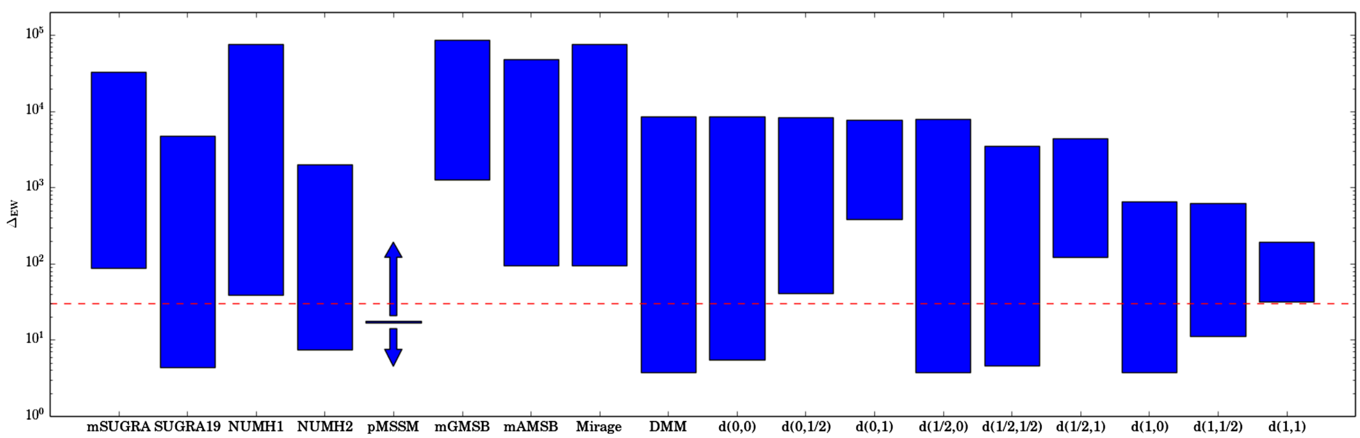

FIG. 7. A comparison of our results with the fine-tuning ranges found in many SUSY models as taken from Refs. [7,10,14]. The line for the pMSSM denotes the lower bound determined in Ref. [14]; the arrows denote that the fine-tuning ranges may go below as well as above this line if a more comprehensive scan is performed. The DMM points are further broken down by the modular weights n_M and n_H for the matter and Higgs fields; these points are denoted by $d(n_M, n_H)$. The dashed line represents $\Delta_{\rm EW} < 30$, which is considered not fine-tuned.

Breaking the results down by other parameters reveals a few other trends in the fine-tuning results. DMM points with low fine-tuning tend to have one or two messengers which integrate out at a possibly high scale with potentially any value of α_g. Large values of N also tend to deflect $m^2_{H_u}$ too much to run to a shallow minimum over much of the parameter space, giving large fine-tuning. Furthermore, M_0 must be greater than roughly 1.5 TeV, to get mixed stops to bolster the Higgs mass, which leads to gluino masses between 2 and 4 TeV that are accessible at the LHC.

IV. CONCLUSIONS

In this paper, we have shown that deflected mirage mediation, a nine-parameter scenario in which gravity, anomaly, and gauge mediation all can contribute comparably to the soft supersymmetry breaking parameters of the MSSM, admits spectra with low electroweak fine-tuning. A comparison of the electroweak fine-tuning ranges in DMM with many standard models for the soft terms of the MSSM is shown in Fig. 7. Here, we note that an upper bound was not determined in Ref. [14] for the pMSSM and hence we quote the lowest $\Delta_{\rm EW}$ presented (the line) and use the arrows to denote that the range of fine-tuning likely goes far up and down, past models that can be embedded in the pMSSM like minimal gauge mediated supersymmetry breaking, and at least as low as models like DMM. Given the large parameter space of the pMSSM, we suspect that a more thorough scan of this scenario would lead to points that are at least as, or less fine-tuned than DMM or SUGRA19.

The results show that DMM does better in general than these standard scenarios and is comparable to high-scale models with many more parameters such as SUGRA19. We saw that in DMM models with low numbers of messengers can lead to values with low fine-tuning. The other parameters that enter into the deflection, α_g and M_{mess}, do not have any preferred values. For the parameters that enter into the GUT-scale masses, we notice $\alpha_m > 1$, with some points near $\alpha_m = 2$, $M_0 > 1500$ GeV, and with any value of $\tan\beta$. Similarly, $n_H = 1$ almost exclusively leads to high fine-tuning, but other combinations of modular weights lead to acceptable values of Δ_{EW}. Hence, we see that in DMM models the combination of gravity mediation, anomaly mediation, and gauge mediation opens up new avenues with lower fine-tuning and should motivate us to look at models beyond the minimal set, where correlations between parameters can lead to unexpected results.

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