

# Genesis of electroweak and dark matter scales from a bilinear scalar condensate

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The condensation of scalar bilinear in a classically scale invariant strongly interacting hidden sector is used to generate the electroweak scale, where the excitation of the condensate is identified as dark matter. We formulate an effective theory for the condensation of the scalar bilinear and find in the self-consistent mean field approximation that the dark matter mass is of  $O(1)$  TeV with the spin-independent elastic cross section off the nucleon slightly below the LUX upper bound.

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## I. INTRODUCTION

Can we explain the origin of “mass without mass” [1]? Yes, a large portion of the baryon mass can be produced by dynamical chiral symmetry breaking (D $\chi$ SB) “from nothing” [2,3]. This nonperturbative mechanism, instead of the Brout-Englert-Higgs mechanism, can also be applied to trigger electroweak symmetry breaking [4,5]. After the discovery of the Higgs particle [6,7], however, it is a fair assumption that can exist. Since the Higgs mass term is the only term in the standard model (SM) that breaks scale invariance at the classical level, we can thus ask where the Higgs mass term comes from. Even the Higgs mass term, too, may have its origin in a nonperturbative effect. In fact D $\chi$ SB in a QCD-like hidden sector has been recently used to induce the Higgs mass term in a classically scale invariant extension of the SM [8–11].

In this paper we focus on another nonperturbative effect, the condensation of the scalar bilinear (CSB) [12,13] (see also [14,15]) in a strongly interacting hidden sector, to generate directly the Higgs mass term via the Higgs portal [16]. The main difference between two classes of models, apart from how a scale is dynamically generated, is that in the first class of models (with D $\chi$ SB) the scale generated in a hidden sector has to be transmitted to the SM via a mediator, e.g. a SM singlet scalar in the model considered in [8–11], while such a mediator is not needed in the second class of models (with CSB). This will be an important difference if two classes should be experimentally distinguished. Another important difference is that the DM particles of the first class are  $CP$ -odd scalars, while they are  $CP$ -even scalars in the second class, as we will see.

Our interest in the second class of models is twofold: first, because the discussion on how the Higgs mass is generated in [16] is rather qualitative, we here formulate an effective theory to nonperturbative breaking of scale invariance by the CSB. This enables us to perform an approximate but

quantitative treatment. Second, since only one flavor for the strongly interacting scalar field  $S$  is considered in [16] so that there is no dark matter (DM) candidate, we introduce  $N_f$  flavors and investigate whether we can obtain realistic candidates of DM. The DM candidates in our scenario are scalar-antiscalar bound states, which are introduced as the excitation of the condensate in the self-consistent mean field approximation (SCMF) [17,18]. Their interactions with the SM can be obtained by integrating out the “constituent” scalars. In this approximation we can constrain the parameter space of the effective theory in which realistic DM candidates are present.

## II. THE MODEL AND ITS EFFECTIVE LAGRANGIAN

We start by considering a hidden sector described by an  $SU(N_c)$  gauge theory with the scalar fields  $S_i^a$  ( $a=1, \dots, N_c, i=1, \dots, N_f$ ) in the fundamental representation of  $SU(N_c)$ . The Lagrangian of the hidden sector is

$$\mathcal{L}_H = -\frac{1}{2} \text{tr} F^2 + ([D^\mu S_i]^\dagger D_\mu S_i) - \hat{\lambda}_S (S_i^\dagger S_i) (S_j^\dagger S_j) - \hat{\lambda}'_S (S_i^\dagger S_j) (S_j^\dagger S_i) + \hat{\lambda}_{HS} (S_i^\dagger S_i) H^\dagger H, \quad (1)$$

where  $D_\mu S_i = \partial_\mu S_i - ig_H G_\mu S_i$ ,  $G_\mu$  is the matrix-valued gauge field, the trace is taken over the color indices, and the parentheses in Eq. (1) stands for an  $SU(N_c)$  invariant product. The SM Higgs doublet field is denoted by  $H$ . The total Lagrangian is  $\mathcal{L}_T = \mathcal{L}_H + \mathcal{L}_{SM}$ , and the SM part,  $\mathcal{L}_{SM}$ , contains the SM gauge and Yukawa interactions along with the scalar potential  $V_{SM} = \lambda_H (H^\dagger H)^2$  without the Higgs mass term.

We assume that below a certain energy the gauge coupling  $g_H$  becomes so strong that the  $SU(N_c)$  invariant scalar bilinear forms a  $U(N_f)$  invariant condensate [12,13]

$$\langle\langle (S_i^\dagger S_j) \rangle\rangle = \left\langle \sum_{a=1}^{N_c} S_i^{a\dagger} S_j^a \right\rangle \propto \delta_{ij}. \quad (2)$$

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This nonperturbative condensate breaks scale invariance, but it is not an order parameter, because scale invariance is broken by scale anomaly [19]. The breaking by anomaly is hard but only logarithmic, which means basically that the coupling constants depend on the energy scale [19]. Moreover, we should note that the mass term is not generated by the anomaly since the beta function of the mass is proportional to the mass itself; see e.g. [20].<sup>1</sup> The creation of the mass term from nothing can happen only by a nonperturbative effect, i.e. after the condensate (2) has taken place.<sup>2</sup> Therefore, the nonperturbative breaking due to the condensation may be assumed to be dominant, so that we can ignore the breaking by scale anomaly in the lowest order approximation to the breaking of scale invariance.

Under this assumption the condensate is a good order parameter, and we would like to formulate an effective theory, which is an analog of the Nambu–Jona-Lasinio (NJL) theory [3] to  $D\chi$ SB. The Lagrangian of the effective theory will not contain the  $SU(N_c)$  gauge fields, because they are integrated out, while it contains the “constituent” scalar fields  $S_i^a$ , for which we use the same symbol as the original scalar fields. Since the effective theory should describe the symmetry breaking dynamically, the effective Lagrangian has to be invariant under the symmetry transformation in question:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^\dagger S_i)(S_j^\dagger S_j) - \lambda'_S (S_i^\dagger S_j)(S_j^\dagger S_i) \\ & + \lambda_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2, \end{aligned} \quad (3)$$

with all positive  $\lambda$ 's. This is the most general form which is consistent with the  $SU(N_c) \times U(N_f)$  symmetry and the classical scale invariance, where we have not included the kinetic term for  $H$  in  $\mathcal{L}_{\text{eff}}$ , because it does not play any significant role as far as the effective theory for the CSB is concerned.<sup>3</sup> Note that the couplings  $\hat{\lambda}_S$ ,  $\hat{\lambda}'_S$  and  $\hat{\lambda}_{HS}$  in  $\mathcal{L}_H$  of (1) are not the same as  $\lambda_S$ ,  $\lambda'_S$  and  $\lambda_{HS}$  in  $\mathcal{L}_{\text{eff}}$ , respectively. We emphasize that the effective Lagrangian (3) is scaleless and is defined slightly above the confinement scale; thus, the scalar condensate has not taken place yet. Therefore, the mixing of multiple scales discussed in [21] does not appear in this level.<sup>4</sup> Using the effective Lagrangian (3), we attempt to approximately describe the nonperturbative genesis of scale in the original gauge theory (1) by the

<sup>1</sup>In the viewpoint of the Wilsonian renormalization group, the classical scale invariance means that the bare mass is exactly put on the critical surface [21]. Once this tuning is made, the renormalized mass keeps vanishing under the renormalization group transformation.

<sup>2</sup>Once the mass is dynamically generated, the scale anomaly contributes to the mass running.

<sup>3</sup>Quantum field theory defined by (3) with the kinetic term for  $H$  is renormalizable in perturbation theory [22].

<sup>4</sup>In contrast, the NJL Lagrangian has a scale at which the Lagrangian is given, although the NJL model is also defined before the dynamical breaking of chiral symmetry.

dimensional transmutation, *à la* Coleman–Weinberg in the effective theory. In the following, we present our formalism by considering first the  $N_f = 1$  case, and explicitly demonstrate this mechanism.

### A. $N_f = 1$ (with $\lambda'_S = 0$ )

In the SCMF approximation, which has proved to be a successful approximation for the NJL theory [17], the perturbative vacuum is Bogoliubov–Valatin (BV) transformed to  $|0_B\rangle$ , such that  $\langle 0_B | (S^\dagger S) | 0_B \rangle = f$ , where  $f$  has to be determined in a self-consistent way. One first splits up the effective Lagrangian (3) into the sum  $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MFA}} + \mathcal{L}_I$ , where  $\mathcal{L}_I$  is normal ordered (i.e.  $\langle 0_B | \mathcal{L}_I | 0_B \rangle = 0$ ), and  $\mathcal{L}_{\text{MFA}}$  contains at most the bilinears of  $S$  which are not normal ordered. Using the Wick theorem  $(S^\dagger S) = : (S^\dagger S) : + f$ ,  $(S^\dagger S)^2 = : (S^\dagger S)^2 : + 2f(S^\dagger S) - f^2$ , etc., we find

$$\mathcal{L}_{\text{MFA}} = (\partial^\mu S^\dagger \partial_\mu S) - M^2 (S^\dagger S) - \lambda_H (H^\dagger H)^2 + \lambda_S f^2,$$

where  $M^2 = 2\lambda_S f - \lambda_{HS} H^\dagger H$ . To the lowest order in the SCMF approximation the “interacting” part  $\mathcal{L}_I$  does not contribute to the amplitudes without external  $S$ 's (the mean field vacuum amplitudes). We emphasize that, in applying the Wick theorem, only the  $SU(N_c)$  invariant bilinear product  $(S^\dagger S) = \sum_a^{N_c} S^{a\dagger} S^a$  has a nonzero (BV transformed) vacuum expectation value. To compute loop corrections we employ the  $\overline{\text{MS}}$  scheme, because dimensional regularization does not break scale invariance. To the lowest order the divergences can be removed by renormalization of  $\lambda_I(I = H, S, HS)$ , i.e.  $\lambda_I \rightarrow (\mu^2)^\epsilon (\lambda_I + \delta\lambda_I)$ , and also by the shift  $f \rightarrow f + \delta f$ , where  $\epsilon = (4 - D)/2$ , and  $\mu$  is the scale introduced in dimensional regularization. The effective potential for  $\mathcal{L}_{\text{MFA}}$  can be straightforwardly computed:

$$V_{\text{MFA}} = M^2 (S^\dagger S) + \lambda_H (H^\dagger H)^2 - \lambda_S f^2 + \frac{N_c}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda_H^2}, \quad (4)$$

where  $\Lambda_H = \mu \exp(3/4)$  is chosen such that the loop correction vanishes at  $M^2 = \Lambda_H^2$ . ( $V_{\text{MFA}}$  with a term linear in  $f$  included but without the Higgs doublet  $H$  has been discussed in [23–25]. The classical scale invariance forbids the presence of this linear term.) Note here that the scale  $\Lambda_H$  is generated by the nonperturbative loop effect. To find the minimum of  $V_{\text{MFA}}$  we look for the solutions of

$$0 = \frac{\partial}{\partial S^a} V_{\text{MFA}} = \frac{\partial}{\partial f} V_{\text{MFA}} = \frac{\partial}{\partial H_l} V_{\text{MFA}} (l = 1, 2). \quad (5)$$

The first equation gives  $0 = (S^a)^\dagger M^2 = (S^a)^\dagger (2\lambda_S f - \lambda_{HS} H^\dagger H)$ , which has three solutions: (i)  $\langle S^a \rangle \neq 0$  and  $\langle M^2 \rangle = 0$ , (ii)  $\langle S^a \rangle = 0$  and  $\langle M^2 \rangle = 0$ , and (iii)  $\langle S^a \rangle = 0$  and  $\langle M^2 \rangle \neq 0$ .

The effective potential  $V_{\text{MFA}}$  in the solution (i) has a flat direction, which corresponds to the end-point contribution discussed in [26]. In the flat direction (i.e.  $f = H = 0$ ),  $V_{\text{MFA}} = 0$  for any value of  $S^a$ , so that the  $SU(N_c)$  symmetry is spontaneously broken. If all the extremum conditions (5) are imposed for the solution (i), we obtain  $\langle f \rangle = \langle ([S^a]^\dagger S^a) \rangle = (2\lambda_H/\lambda_{HS})\langle H^\dagger H \rangle$  along with  $C = 0$  and  $\langle V_{\text{MFA}} \rangle = 0$  [23].<sup>5</sup> Next we consider (ii) and find that  $\langle S^a \rangle = \langle f \rangle = \langle H \rangle = 0$  with  $\langle V_{\text{MFA}} \rangle = 0$ . The third solution (iii) can exist if

$$C = 4\lambda_H\lambda_S - \lambda_{HS}^2 > 0 \quad (6)$$

is satisfied, and we find

$$|\langle H \rangle|^2 = v_h^2/2 = \frac{\lambda_{HS}}{C} \Lambda_H^2 \exp\left(\frac{32\pi^2\lambda_H}{N_c C} - \frac{1}{2}\right),$$

$$\langle f \rangle = \frac{2\lambda_H}{\lambda_{HS}} |\langle H \rangle|^2,$$

$$\langle V_{\text{MFA}} \rangle = -\frac{N_c}{64\pi^2} \Lambda_H^4 \exp\left(\frac{64\pi^2\lambda_H}{N_c C} - 1\right) < 0.$$

Consequently, the solution (iii) presents the true potential minimum if (6) is satisfied [in the energy region where (3) should serve as the effective Lagrangian]. Self-consistency means that  $f = \langle 0_{\text{B}} | (S^\dagger S) | 0_{\text{B}} \rangle$  is equal to  $\langle f \rangle$  at the potential minimum in the mean field approximation. The Higgs mass at this level of approximation becomes

$$m_{h_0}^2 = \frac{\lambda_{HS}\Lambda_H^2}{C} \left( \frac{16\lambda_H^2\lambda_S}{C} + \frac{N_c\lambda_{HS}^2}{8\pi^2} \right) \exp\left(\frac{32\pi^2\lambda_H}{N_c C} - \frac{1}{2}\right). \quad (7)$$

In the small  $\lambda_{HS}$  limit we obtain  $m_{h_0}^2 \simeq 4\lambda_H |\langle H \rangle|^2 = 2\lambda_{HS} \langle f \rangle$ , where the first equation is the SM expression, and the second one is simply assumed in [16]. So the Higgs mass (7) contains the backreaction. The analysis above shows that the scale created in the hidden sector can be desirably transmitted to the SM sector. The reason that  $\langle V_{\text{MFA}} \rangle < 0$  for the solution (iii) is the absence of a mass term in the effective Lagrangian (3); the classical scale invariance does not allow the mass term. A mass term in (3) would generate a term linear in  $f$  in  $V_{\text{MFA}}$ , which can lift the  $\langle V_{\text{MFA}} \rangle$  into a positive direction [24,25], while  $V_{\text{MFA}} = 0$  remains in the flat direction [26].

At this stage we would like to mention that Bardeen and Moshe [26] (and also others) pointed out the intrinsic instability inherent in (3) (which is related to its triviality) if one regards (3) as a fundamental Lagrangian. We however discard this fundamental problem, because we assume that

<sup>5</sup>Due to  $\langle M^2 \rangle = 0$  there exists a tachyonic state, because the inequality of [24],  $16\pi^2/(2N_c\lambda_S) - \ln[\langle M^2 \rangle/\Lambda_H^2 \exp(-3/2)] < 0$ , cannot be satisfied for a finite  $\Lambda_H$  and a positive  $\lambda_S$ .

such a problem is absent in the original theory described by (1).

## B. $N_f > 1$ and dark matter

Here we consider the case with  $N_f > 1$  and take into account the excitations of the condensate,  $\sigma$  and  $\phi^\alpha$  ( $\alpha = 1, \dots, N_f^2 - 1$ ), which are introduced as

$$\langle 0_{\text{B}} | (S_i^\dagger S_j) | 0_{\text{B}} \rangle = f_{ij} = \langle f_{ij} \rangle + Z_\sigma^{1/2} \delta_{ij} \sigma + Z_\phi^{1/2} t_{ji}^\alpha \phi^\alpha. \quad (8)$$

Here  $t^\alpha$  are the  $SU(N_f)$  generators in the Hermitian matrix representation, and  $Z_\sigma$  and  $Z_\phi$  are the wave function renormalization constants of a canonical dimension 2. The unbroken  $U(N_f)$  flavor symmetry implies  $\langle f_{ij} \rangle = \delta_{ij} f_0$  and  $\langle \phi^\alpha \rangle = 0$ , where  $\langle \sigma \rangle$  can be absorbed into  $f_0$ , so that we can always assume  $\langle \sigma \rangle = 0$ . Furthermore, the flavor symmetry ensures the stability of  $\phi^\alpha$ , i.e. they can be good DM candidates, because they are electrically neutral and their interactions with the SM sector are loop suppressed, as we will see. Note that  $\sigma$  and  $\phi^\alpha$  in (8) are introduced as c-numbers without kinetic terms. However, their kinetic terms will be generated through  $S^a$  loop effects, and consequently we will reinterpret them as quantum fields describing physical degrees of freedom. The investigation of the vacuum structure is basically the same as in the  $N_f = 1$  case. We are interested in the solution of type (iii) of the previous case, i.e.  $f_0 \neq 0$ ,  $|\langle H \rangle| = v_h/2 \neq 0$ , which is the true potential minimum if

$$G = 4N_f\lambda_H\lambda_S - N_f\lambda_{HS}^2 + 4\lambda_H\lambda'_S > 0 \quad (9)$$

is satisfied. Similar calculations as in the previous case yield among other things

$$m_{h_0}^2 = \frac{\lambda_{HS}N_f\Lambda_H^2}{G} \left( \frac{16\lambda_H^2(N_f\lambda_S + \lambda'_S)}{G} + \frac{N_cN_f\lambda_{HS}^2}{8\pi^2} \right) \times \exp\left(\frac{32\pi^2\lambda_H}{N_c G} - \frac{1}{2}\right). \quad (10)$$

The SCMF Lagrangian  $\mathcal{L}'_{\text{MFA}}$  involving  $\sigma$  and  $\phi^\alpha$  can now be written as

$$\begin{aligned} \mathcal{L}'_{\text{MFA}} = & (\partial^\mu S_i^\dagger \partial_\mu S_i) - M_0^2 (S_i^\dagger S_i) \\ & + N_f(N_f\lambda_S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda'_S}{2} Z_\phi \phi^\alpha \phi^\alpha \\ & - 2(N_f\lambda_S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger t_{ij}^\alpha \phi^\alpha S_j) \\ & + \frac{\lambda_{HS}}{2} (S^\dagger S) h (2v_h + h) - \frac{\lambda_H}{4} h^2 (6v_h^2 + 4v_h h + h^2), \end{aligned} \quad (11)$$

where  $M_0^2 = 2(N_f\lambda_S + \lambda'_S)f_0 - \lambda_{HS}v_h^2/2$ , and  $\text{Tr}(t^\alpha t^\beta) = \delta^{\alpha\beta}/2$ . Further,  $h$  is the Higgs field contained in the Higgs

doublet as  $H^T = (H^+, (v_h + h + i\chi)/\sqrt{2})$ , where  $H^+$  and  $\chi$  are the would-be Nambu-Goldstone fields. Linear terms in  $\sigma$  and  $h$  are suppressed in (11), because they will be canceled against the corresponding tadpole corrections.

Using (11) and integrating out the constituent scalars  $S_i^a$ , we can obtain effective interactions among  $\sigma$ ,  $\phi$  and the Higgs  $h$ . We first compute their inverse propagators, up to and including one-loop order, to obtain their masses and the wave function renormalization constants:

$$\begin{aligned}\Gamma_\phi^{\alpha\beta}(p^2) &= Z_\phi \delta^{\alpha\beta} \lambda'_S \Gamma_\phi(p^2) = Z_\phi \delta^{\alpha\beta} \lambda'_S [1 + 2\lambda'_S N_c \Gamma(p^2)], \\ \Gamma_\sigma(p^2) &= 2Z_\sigma N_f (N_f \lambda_S + \lambda'_S) [1 + 2N_c (N_f \lambda_S + \lambda'_S) \Gamma(p^2)], \\ \Gamma_{h\sigma}(p^2) &= -2Z_\sigma^{1/2} v_h \lambda_{HS} (N_f \lambda_S + \lambda'_S) N_f N_c \Gamma(p^2), \\ \Gamma_h(p^2) &= p^2 - m_{h0}^2 + (v_h \lambda_{HS})^2 N_f N_c (\Gamma(p^2) - \Gamma(0)),\end{aligned}\quad (12)$$

where  $m_{h0}^2$  is given in (10), the canonical kinetic term for  $H$  is included, and

$$\Gamma(p^2) = -\frac{1}{16\pi^2} \int_0^1 dx \ln \left[ \frac{-x(1-x)p^2 + M_0^2}{\Lambda_H^2 \exp(-3/2)} \right].$$

We have included neither the wave function renormalization constant for  $h$  (which is approximately equal to 1 within the approximation here) nor the corrections to  $\Gamma_h$  coming from the SM sector (which will only slightly influence our result).

The DM mass is the zero of the inverse propagator, i.e.

$$\Gamma_\phi^{\alpha\beta}(p^2 = m_{\text{DM}}^2) = 0, \quad (13)$$

and  $Z_\phi$  (which has a canonical dimension 2) can be obtained from  $Z_\phi^{-1} = 2(\lambda'_S)^2 N_c (d\Gamma/dp^2)|_{p^2=m_{\text{DM}}^2}$ . The  $\sigma$  and Higgs masses are obtained from the zero eigenvalues of the  $h - \sigma$  mixing matrix. Strictly speaking, this mixing has to be taken into account in determining the renormalization constants (matrix) for  $\sigma$  and  $h$ . However, the mixing is less than 1% in a realistic parameter space so that we ignore the mixing for the renormalization constants. As we can see from (12), the radiative correction to the inverse propagator is proportional to  $2\lambda'_S N_c / 16\pi^2$ , so that the solution of (13) for a real positive  $p^2$  can exist if  $\lambda'_S N_c$  is sufficiently large. Therefore, if an upper limit of  $\lambda'_S$  is set, there will be a minimum value of  $N_c$ . It turns out that the minimum  $N_c$  is 3 for  $\Gamma_\phi(p^2)$  with  $N_f = 2$  to have a zero if  $0 < \lambda'_S < 2\pi$ . For a larger  $N_f$  we need a larger  $N_c$ : the minimum  $N_c$  is 4 for  $N_f = 3$  for instance.

The link of  $\phi$  to the SM model is established through the interaction with the Higgs, which is generated at one loop as shown in Fig. 1. We use the s-channel momenta  $p = p' = (m_{\text{DM}}, \mathbf{0})$  for DM annihilation, because we restrict ourselves to the s-wave part of the velocity-averaged

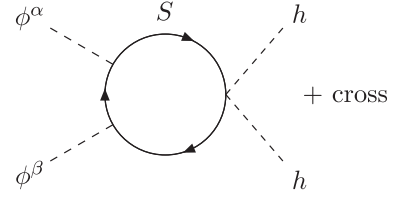


FIG. 1. The interaction between DM and the Higgs  $h$  arises at the one-loop level. Diagrams  $\propto \lambda_{HS}^2 (v_h/M_0)^2$  are ignored, because  $\lambda_{HS}^2 (v_h/M_0)^2 \ll \lambda_{HS}$ .

annihilation cross section  $\langle v\sigma \rangle$ . For the spin-independent elastic cross section off the nucleon  $\sigma_{SI}$  we use the t-channel momenta  $p = -p' = (m_{\text{DM}}, \mathbf{0})$ . In these approximations the diagrams of Fig. 1 yield the effective couplings

$$\kappa_{s(t)} \delta^{\alpha\beta} = \delta^{\alpha\beta} \Gamma_{\phi^2 h^2}(M_0, m_{\text{DM}}, \epsilon = 1(-1)), \quad (14)$$

where

$$\begin{aligned}\Gamma_{\phi^2 h^2}(M_0, m_{\text{DM}}, \epsilon) &= \frac{Z_\phi N_c (\lambda'_S)^2 \lambda_{HS}}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \\ &\times [M_0^2 + m_{\text{DM}}^2 (x(x-1) + y(y-1) - 2\epsilon xy)]^{-1},\end{aligned}$$

and we consider only the parameter space with  $m_{\text{DM}}, m_\sigma < 2M_0$ , because beyond that our SCMF approximation will break down. Then we obtain

$$\langle v\sigma \rangle = \frac{1}{32\pi m_{\text{DM}}^3} \sum_{I=W,Z,t,h} (m_{\text{DM}}^2 - m_I^2)^{1/2} a_I + \mathcal{O}(v^2),$$

where  $m_{W,Z,t,h}$  are the  $W$ ,  $Z$ , top quark and Higgs masses, respectively, and

$$\begin{aligned}a_{W(Z)} &= 4(2)\kappa_s^2 \Delta_h^2 m_{W(Z)}^4 \left( 3 + 4 \frac{m_{\text{DM}}^4}{m_{W(Z)}^4} - 4 \frac{m_{\text{DM}}^2}{m_{W(Z)}^2} \right), \\ a_t &= 24\kappa_s^2 \Delta_h^2 m_t^2 (m_{\text{DM}}^2 - m_t^2), \\ a_h &= \kappa_s^2 \left( 1 + 24\lambda_H \Delta_h \frac{m_W^2}{g^2} \right)^2\end{aligned}$$

with  $\Delta_h = (4m_{\text{DM}}^2 - m_h^2)^{-1}$  [ $m_h$  is the corrected Higgs mass which should be compared with  $m_{h0}$  of (10).] The DM relic abundance is  $\Omega_{\text{DM}} \hat{h}^2 = (N_f^2 - 1) \times (Y_\infty s_0 m_{\text{DM}}) / (\rho_c / \hat{h}^2)$ , where  $Y_\infty$  is the asymptotic value of the ratio  $n_{\text{DM}}/s$ ;  $s_0 = 2890/\text{cm}^3$  is the entropy density at present;  $\rho_c = 3H^2/8\pi G = 1.05 \times 10^{-5} \hat{h}^2 \text{ GeV}/\text{cm}^3$  is the critical density;  $\hat{h}$  is the dimensionless Hubble parameter;  $M_{pl} = 1.22 \times 10^{19} \text{ GeV}$  is the Planck energy; and  $g_* = 106.75 + N_f^2 - 1$  is the number of the effectively massless degrees of freedom at the freeze-out temperature. To obtain  $Y_\infty$  we solve the Boltzmann equation



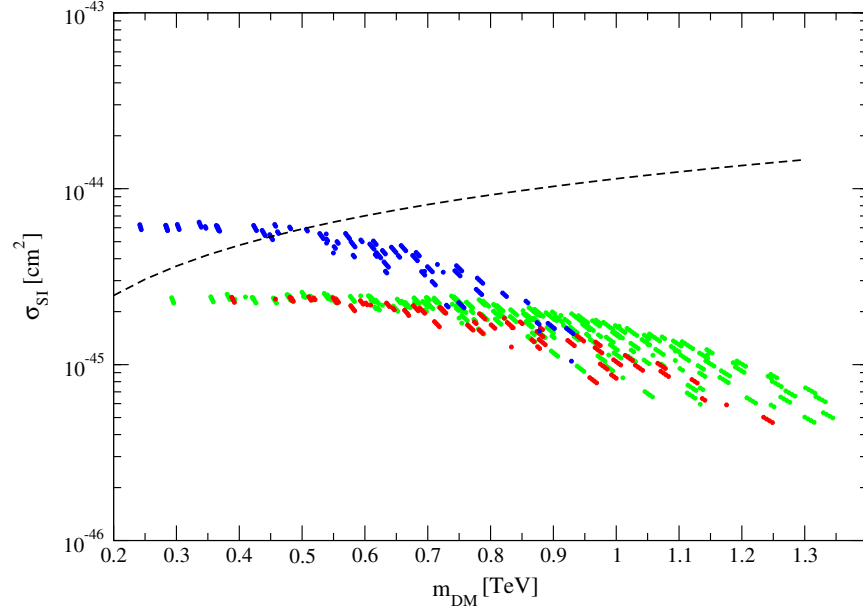


FIG. 2. The spin-independent elastic cross section  $\sigma_{SI}$  of DM off the nucleon as a function of  $m_{DM}$  for  $N_f = 2$ ,  $N_c = 5$  (red) and 8 (green) and for  $N_f = 3x$ ,  $N_c = 5$  (blue), where  $\Omega\hat{h}^2$  is required to be consistent with the PLANCK experiment at  $2\sigma$  level [33]. The black dashed line stands for the central value of the LUX upper bound [30].

$$\frac{dY}{dx} = -0.264g_*^{1/2} \left( \frac{m_{DM}M_{PL}}{x^2} \right) \langle v\sigma \rangle (Y^2 - \bar{Y}^2)$$

numerically, where  $x$  is the inverse temperature  $m_{DM}/T$ , and  $\bar{Y}$  is  $Y$  in thermal equilibrium. The spin-independent elastic cross section off the nucleon  $\sigma_{SI}$  can be obtained from [27]

$$\sigma_{SI} = \frac{1}{4\pi} \left( \frac{\kappa_t \hat{f} m_N^2}{m_{DM} m_h^2} \right)^2 \left( \frac{m_{DM}}{m_N + m_{DM}} \right)^2,$$

where  $\kappa_t$  is given in (14),  $m_N$  is the nucleon mass, and  $\hat{f} \sim 0.3$  stems from the nucleonic matrix element [28].<sup>6</sup>

Before we scan the parameter space, we consider a representative point in the four-dimensional parameter space of the scalar couplings with  $N_f = 2$  and  $N_c = 5$ :

$$\begin{aligned} \lambda_S &= 1.20, & \lambda'_S &= 5.38, \\ \lambda_{HS} &= 0.0525, & \lambda_H &= 0.130, \end{aligned}$$

which give  $f_0 = 0.0749$  TeV<sup>2</sup>,  $M_0 = 1.08$  TeV,  $m_{DM} = 0.801$  TeV,  $m_\sigma = 1.98$  TeV,  $\Lambda_H = 0.501$  TeV,  $\Omega\hat{h}^2 = 0.121$ ,  $\sigma_{SI} = 1.68 \times 10^{-45}$  cm<sup>2</sup>,  $\kappa_s = 0.3988$ ,  $\kappa_t = 0.3089$ . In Fig. 2 we show in the  $m_{DM} - \sigma_{SI}$  plane the predicted area for various  $N_f$  and  $N_c$ . The predicted values of  $\sigma_{SI}$  are just below the LUX upper bound (black dashed line) [30] and can be tested by XENON1T, whose sensitivity is

<sup>6</sup>If the value  $\hat{f}$  improved by the recent lattice simulation [29] is used, we obtain slightly smaller values (about 20%).

$O(10^{-47})$  cm<sup>2</sup> [31,32]. If we increase  $N_f$ , we have to suppress  $Y_\infty$ , because  $\Omega\hat{h}^2 \propto (N_f^2 - 1)Y_\infty$ , which requires a larger  $\langle v\sigma \rangle$ , leading to a larger  $\sigma_{SI}$ .

### III. SUMMARY

We have assumed that the SM without the Higgs mass term is coupled through a Higgs portal term with a classically scale invariant gauge sector, which contains  $N_f$  scalar fields. Due to the strong confining force the gauge invariant scalar bilinear forms a condensate, thereby violating scale invariance. The Higgs portal term is responsible for the transmission of the scale to the SM sector, realizing electroweak scalegenesis. We have formulated an effective theory for the condensation of the scalar bilinear. The excitation of the condensate is identified as DM, where its scale is dynamically generated in the hidden gauge sector. Our formalism is simple and its application will be multifold. We have found that the DM mass is of  $O(1)$  TeV and the predicted spin-independent elastic cross section off the nucleon is slightly below the LUX upper bound and could be tested by the XENON1T experiment.

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