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Generalized focus point and mass spectra comparison of highly natural SUGRA GUT models

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Supergravity grand unified models (SUGRA GUTs) are highly motivated and allow for a high degree of electroweak naturalness when the superpotential parameter $\mu \sim 100$–300 GeV (preferring values closer to 100 GeV). We first illustrate that models with radiatively driven naturalness enjoy a generalized focus-point behavior wherein *all* soft terms are correlated instead of just scalar masses. Next, we generate spectra from four SUGRA GUT archetypes: 1. $SO(10)$ models where the Higgs doublets live in different ten-dimensional irreducible representations (irreps), 2. models based on $SO(10)$ where the Higgs multiplets live in a single ten-dimensional irrep but with D-term scalar mass splitting, 3. models based on $SU(5)$, and 4. a more general SUGRA model with 12 independent parameters. Electroweak naturalness implies for all models a spectrum of light Higgsinos with $m_{\tilde{W}_1,\tilde{Z}_{1,2}} \lesssim 300$ GeV and gluinos with $m_{\tilde{g}} \lesssim 2$–4 TeV. However, masses and mixing in the third generation sfermion sector differ distinctly between the models. These latter differences would be most easily tested at a linear e^+e^- collider with $\sqrt{s}\sim$ multi-TeV scale but measurements at a 50–100 TeV hadron collider are also possible.

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I. INTRODUCTION

Grand unified theories (GUTs) based on the gauge groups $SU(5)$ [1] and $SO(10)$ [2] present an impressive picture of both gauge group unification and matter unification and predict the quantization of electric charge. However, the problem of gauge hierarchy stabilization in GUT theories was noted early on. The gauge hierarchy problem was solved via the introduction of supersymmetry (SUSY) [3] into the overall construct [4]. SUSY added the additional unification of Fermi and Bose degrees of freedom and received some impressive experimental support from the measured strength of gauge forces at LEP which were found to unify under renormalization group (RG) evolution within the minimal supersymmetric standard model (MSSM) but not within the SM [5]. SUSY is also supported by the recently discovered Higgs scalar with $m_h \simeq 125$ GeV [6,7] which falls squarely within the predicted MSSM window [8,9]. Unification within *local SUSY* or supergravity grand unification [10] brought gravity into the picture and offered new successes such as a mechanism for the uplifting of the soft SUSY breaking terms. In supergravity (SUGRA) models, also known as gravity mediated SUSY breaking, local SUSY is broken in a hidden sector via the super-Higgs mechanism [11]: the gravitino field absorbs the would-be Goldstino leading to a massive gravitino with value $m_{3/2}$. For a well-defined hidden sector, the various MSSM soft breaking terms are then all calculable as multiples of the gravitino mass [12] which is anticipated phenomenologically to exist somewhere around the TeV scale.

This impressive construct fell into some disrepute on the experimental side via the failure to observe flavor- and CP-violating processes, by proton decay, and more recently by the failure to discover the predicted weak scale superpartners at LHC [13,14]. On the theory side, four-dimensional SUSY GUTs require rather large Higgs multiplets to implement the GUT symmetry breaking, and these seem to be inconsistent with the larger picture where the SUGRA GUT theory might emerge from string theory [15]. The awkward role of Higgs multiplets was further exacerbated by the traditional doublet-triplet splitting problem: the MSSM Higgs doublets are associated with weak scale physics while the required remnant Higgs multiplets must reside up near $Q \simeq m_{\rm GUT} \sim 2 \times 10^{16}$ GeV.

Solutions to these several Higgs-related problems were found in the formulation of extradimensional GUT models. Initial models were formulated with the $SU(5)$ or $SO(10)$ GUT symmetry in five [16–19] or six [20] spacetime dimensions. Orbifold compactification of the extra spacetime dimensions could be used as an alternative to symmetry breaking via the Higgs mechanism as a means to break the grand unified symmetry. Such models could dispense with the large Higgs representations and also offer means to suppress or forbid proton decay and to solve the doublet-triplet splitting problem [17].

More recently, the rather large value of light Higgs mass $m_h \simeq 125$ GeV [6,7] and the lack of superpartners in LHC8 [13,14] have called into question the *naturalness* of SUSY GUT models. These two disparate measurements require, in the first case, highly mixed TeV-scale top squarks to

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bolster the Higgs mass [8] and, in the second case, multi-TeV values for the gluinos and first/second generation squarks. Such heavy masses seem inconsistent with many calculations of upper bounds on sparticle masses from the naturalness principle [21–24] which naively requires sparticle masses around the 100 GeV scale.

However, naturalness calculations using the Barbieri-Giudice (BG) measure [21,25] $\Delta_{\rm BG} = \max_i |\partial \log m_Z^2 / \partial \log p_i|$ (where the p_i are fundamental parameters of the theory) were challenged [26,27] in that they were applied to multiparameter effective theories rather than the underlying SUGRA theory where all the soft terms arise as multiples of the gravitino mass $m_{3/2}$. Such a misapplication of BG fine-tuning leads to *overestimates* of $\Delta_{\rm BG}$ and obscures a knowledge of which SUSY particle masses ought to lie at the 100 GeV scale. In SUGRA theories, the appropriate parameter choices p_i should be the gravitino mass $m_{3/2}$ and the superpotential μ parameter. Reevaluation of $\Delta_{\rm BG}$ in terms of these parameters implies that it is only the Higgsinos which must lie in the 100 GeV regime while other sparticle masses are comparable to $m_{3/2}$ which can lie comfortably in the multi-TeV regime [28]: this latter choice is consistent with LHC8 sparticle and Higgs mass limits and in fact was already presaged by the cosmological gravitino problem [29] and a decoupling solution to the SUSY flavor and CP problems [30].

A different naturalness measure [31,32] $\Delta_{HS} \equiv \delta m_h^2/(m_h^2/2)$ which seemed to require several sub-TeV scale third generation squarks [33] was challenged as leading to overestimates of fine-tuning on the basis of neglecting other *dependent* contributions to m_h^2 which can lead to large cancellations [26,27,34]. Regions of parameter space of the two extraparameter nonuniversal Higgs model (NUHM2) were identified where light Higgsinos ~100–300 GeV could coexist with $m_h \sim 125$ GeV and LHC8 sparticle mass limits where rather mild electroweak fine-tuning at the 5%–20% level was allowed.

The questions then emerge: what is the GUT basis of the NUHM2 model and are there other possibilities for SUGRA GUT models which allow for a high degree of electroweak (EW) naturalness? Some previous work was reported which explored whether naturalness could coexist with $b-\tau$ or $t-b-\tau$ Yukawa unified models. To allow for $t-b-\tau$ unification, a rather large MSSM threshold correction to m_b is required where [8,35–37]

$$\Delta m_b/m_b \simeq \frac{\alpha_3 \mu m_{\tilde g} \tan\beta}{m_{\tilde b}^2} + \frac{f_t^2 \mu A_t \tan\beta}{m_{\tilde t}^2}. \qquad (1)$$

The required small value of μ seems to preclude Yukawa-unified natural SUSY for $t-b-\tau$ unification better than ~30% and also disfavors b-tau unification. However, it is conceivable that GUT scale threshold corrections along with effects from compactification may evade these requirements.

In this paper, we explore several aspects of naturalness in SUGRA GUT models. First, in Sec. II we show that models with radiatively driven naturalness exhibit a generalized focus point behavior where weak scale contributions to m_Z^2 are rather insensitive to $m_{3/2}$ for correlated choices of parameters. In Sec. III we list three SUSY GUT archetype models which are examined for consistency with electroweak naturalness.[1] We define these several SUGRA GUT archetype models and their associated parameter space. These three models include: 1. $SO(10)$ based models where the two Higgs doublets live in different ten-dimensional irreducible representations (irreps) (the NUHM2 model), 2. $SO(10)$ SUSY GUT models where the two Higgs doublets live in the same ten-dimensional irrep (the D-term splitting model, DT). and 3. a generic $SU(5)$ SUSY GUT model where $H_u \in \mathbf{5}$ and $H_d \in \mathbf{5}^*$. We will compare these results against a more general SUGRA model with 12 independent parameters defined at the GUT scale. In Sec. IV, we present results from a scan over each model parameter space where we identify regions of natural SUSY parameter space. We find all four constructs allow for highly natural SUSY. In these regions of high SUSY naturalness, we find common amongst all four models that light Higgsinos with mass $m(\text{Higgsinos}) \lesssim 200$–$300$ GeV should exist and that gluinos with mass $m_{\tilde g} \lesssim 4$–6 TeV should occur. In contrast, we find the third generation squark and slepton mixing can be very different amongst the four models. To test such mixing, probably very high energy e^+e^- colliders with $\sqrt{s} > 2m$(squark, slepton) are needed. Some tests might be done at a much higher energy pp collider with $\sqrt{s} \sim 50$–100 TeV. A summary and conclusions are presented in Sec. V.

II. RADIATIVELY DRIVEN NATURALNESS AS GENERALIZED FOCUS POINT BEHAVIOR

To understand SUSY models with low fine-tuning, we begin with the Ellis-Enqvist-Nanopoulos-Zwirner/BG fine-tuning measure [21,25]

$$\Delta_{\rm BG} = \max_i c_i = \max_i \left| \frac{\partial \log m_Z^2}{\partial \log p_i} \right|, \qquad (2)$$

where the p_i are fundamental parameters of the theory labeled by index i. To evaluate $\Delta_{\rm BG}$, we first express m_Z^2 in terms of weak scale SUSY parameters via the well-known scalar potential minimization conditions in the MSSM

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{(\tan^2\beta - 1)} - \mu^2 \simeq -m_{H_u}^2 - \mu^2, \qquad (3)$$

where the latter partial equality holds for $\tan\beta \gtrsim 3$. Next, using semianalytical solutions to the renormalization group equations for μ and $m_{H_u}^2$, we may express these weak scale quantities in terms of GUT scale quantities. It is found for example with $\tan\beta = 10$ that [42–44]

[1] For some recent related work, see, e.g., [38–41].

$$\begin{aligned} m_Z^2 = &-2.18\mu^2 + 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ &+ 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ &- 0.025M_1A_t + 0.22A_t^2 + 0.004M_3A_b - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ &+ 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ &+ 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ &+ 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned} \tag{4}$$

where the quantities on the right-hand side are all GUT scale parameters. If we evaluate the $i = Q_3$ sensitivity coefficient $\Delta_{BG}(m_{Q_3}^2) = 0.73m_{Q_3}^2/m_Z^2$ and take $m_{Q_3} \gtrsim 1$ TeV in accord with LHC sparticle limits and Higgs mass measurement, then we expect $\Delta_{BG} > 90$ or already about 1% fine-tuning.

It was observed long ago by Feng *et al.* [45] that if instead we assume scalar mass unification, with $m_{H_u} = m_{Q_3} = m_{U_3} \equiv m_0$ at the GUT scale, then we can combine the contributions from lines 4 and 5 of Eq. (4) so that $m_Z^2 \sim 0.017m_0^2$. The coefficient of the squared scalar mass term has dropped by a factor of 43: what appeared highly fine-tuned using $m_{Q_3}^2$ as a fundamental parameter is in fact low fine-tuned when unification conditions are imposed due to cancellations between various contributions to m_Z^2. This is the *focus point* (FP) scenario wherein large third generation scalar masses can be quite consistent with low fine-tuning. A related manifestation of FP SUSY is that for a wide range of m_0^2 values, then $m_{H_u}^2$ runs to nearly the same value at $Q = m_{\rm weak}$ (the RG trajectory is focused at the weak scale) so that the value of m_Z^2 is relatively insensitive to variation in the high scale parameter m_0.

While the FP behavior reduces the fine-tuning expected in the scalar sector, there remains possible large fine-tuning contributions to m_Z^2 due to the gaugino terms in Eq. (4). Current limits from LHC13 imply $m_{\tilde g} \simeq M_3 \gtrsim 1.5$–$1.8$ TeV [46]. Thus, we might expect large fine-tuning from the second term of line 1 of Eq. (4) as such: $\Delta_{BG} \geq c_{M_3} \gtrsim 3.84M_3^2/m_Z^2 \gtrsim 1000$ so that SUSY appears again fine-tuned at the 0.1% level.

At this point—following Refs. [26,27]—we recall that the soft parameters entering Eq. (4) are only taken as independent parameters in the low energy effective theory which is expected to arise from some more fundamental SUGRA or string theory. In the SUGRA theory, SUSY breaking occurs in the hidden sector of the model and the gravitino gains a mass $m_{3/2}$ via the super-Higgs mechanism [12]. The soft SUSY breaking terms arise from non-renormalizable terms in the SUGRA Lagrangian and are obtained by taking the Planck mass limit $M_P \to \infty$ while keeping $m_{3/2}$ fixed. For any particular hidden sector, the soft terms are all calculable as multiples of $m_{3/2}$ so that in reality they are all *dependent* terms. The soft terms are usually taken as independent terms in the low energy effective theory only in order to parametrize the effects of a wide range of hidden sector possibilities. By writing each soft term properly as a multiple of $m_{3/2}$ and then combining dependent terms on the right-hand side of Eq. (4), then we arrive at the simpler expression

$$m_Z^2 \simeq -2\mu^2 + a \cdot m_{3/2}^2, \tag{5}$$

where a depends on the particular spectrum which is generated. BG naturalness then requires $\mu^2(\text{GUT}) \sim m_Z^2$ and $am_{3/2}^2 \sim m_Z^2$. Since μ hardly evolves, then equating $m_Z^2 \simeq -2\mu^2 - 2m_{H_u}^2$ as a weak scale relation to Eq. (5) we find that $am_{3/2}^2 \simeq m_{H_u}^2(\text{weak})$ so that BG naturalness requires the same as tree-level EW naturalness [47], namely $|m_{H_u}^2(\text{weak})| \sim m_Z^2$.

The *generalized focus point* behavior is merely the observation that for certain relations amongst *all* the soft parameters, a wide range of high scale input parameters $m_{H_u}^2$ can be driven to nearly the same weak scale values.[2] As an example, imagine a hidden sector which produces the following soft terms:

$$m_0^2 = m_{3/2}^2, \tag{6}$$

$$A_0 = -1.6m_{3/2}, \tag{7}$$

$$m_{1/2} = m_{3/2}/5, \tag{8}$$

$$m_{H_d}^2 = m_{3/2}^2/2. \tag{9}$$

Here, we take as usual m_0 to be a common *matter scalar* soft mass which is not in general equal to the Higgs sector soft masses m_{H_u} or m_{H_d}. We also anticipate μ to arise via some mechanism such as radiative Peccei-Quinn symmetry breaking [49] where we take $\mu(\text{weak}) = 156.6$ GeV so that $\mu(\text{GUT}) = 150$ GeV. Then, to accommodate the measured value of $m_Z = 91.2$ GeV, we would find that the GUT scale value of $m_{H_u}^2$ is required to be

[2]General conditions for the focusing of $m_{H_u}^2$ at the weak scale were previously discussed in Refs. [48]. We thank C. Wagner for bringing these papers to our attention.

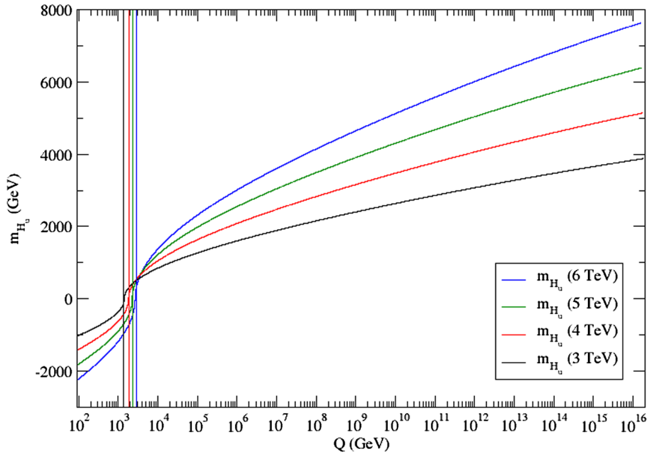

FIG. 1. Plot of $\mathrm{sign}(m_{H_u}^2) \cdot \sqrt{|m_{H_u}^2|}$ vs Q for four different values of gravitino mass $m_{3/2} = 3$–6 TeV.

$$m_{H_u}^2(\mathrm{GUT}) = 1.8 m_{3/2}^2 - (212.52\ \mathrm{GeV})^2. \tag{10}$$

As we vary $m_{3/2}$ over some large range, we expect to generate values of $m_{H_u}^2(\mathrm{weak})$ at nearly the same values: i.e., $|m_{H_u}^2(\mathrm{weak})|$ is focused to modest values $\sim m_Z^2$ at the weak scale.

While the above argument makes use of the semianalytic 1-loop RG solution for m_Z^2 in Eq. (4), this behavior should be revealed for the usual spectrum generator codes such as ISAJET and others which make use of full 2-loop renormalization group equations (RGEs) and radiatively corrected sparticle masses and scalar potential. As an example, we show the running of $m_{H_u}^2$ versus scale Q in Fig. 1 for four choices of $m_{3/2}$: 3, 4, 5, and 6 TeV. The locus of the $Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ value at which the parameters are extracted for optimized minimization of the scalar potential are shown as vertical lines. We see that indeed the value of $m_{H_u}^2(\mathrm{weak})$ exhibits focus point behavior for the correlated soft terms as given in Eqs. (9) and (10).

III. THREE UNIFIED SUGRA GUT ARCHETYPE MODELS AND ONE NONUNIFIED MODEL

For all four SUSY GUT models, we assume that nature is symmetric under the GUT gauge symmetry at energy scales $Q > m_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV and that below m_{GUT} nature is described by the MSSM augmented by three gauge singlet right-hand neutrino superfields N_i^c, $i = 1$–3 which are in turn integrated out at their respective mass scales M_{N_i}. It is possible that the GUT theory is a four-dimensional quantum field theory with GUT symmetry breaking via Higgs multiplets [50], or that nature is described by a $d > 4$ dimensional GUT theory at $Q > m_{\mathrm{GUT}}$ where the GUT symmetry is broken via compactification of the extra dimensions via (perhaps) orbifolding [16,17]. A theory of the latter type which can give rise to SUSY with radiatively driven naturalness has recently been presented in Ref. [51]. For our numerical study, we will feign ignorance as to the GUT symmetry breaking mechanism.

A. General $SO(10)$ model with each Higgs in a separate 10: NUHM2

For the general $SO(10)$ SUSY GUT model, we require all matter superfields to lie in the 16-dimensional spinor representation so that matter scalar masses are unified to $m_{16}(= m_0)$. In this model, we assume the two MSSM Higgs doublets live in different ten-dimensional $SO(10)$ Higgs irreps so that the GUT scale Higgs soft masses $m_{H_u}^2$ and $m_{H_d}^2$ are independent parameters. Also, in this model one might expect under the simplest conditions to have $b-\tau$ Yukawa coupling unification but not $t-b-\tau$ Yukawa unification. For ease of computing within the restrictions of natural models, we trade the GUT scale inputs $m_{H_u}^2$ and $m_{H_d}^2$ in lieu of weak scale parameters μ and m_A. For this model, then, the relevant parameter space is that of the well-known two-extra-parameter nonuniversal Higgs mass model also known as NUHM2 [52],

$$m_0,\quad m_{1/2},\quad A_0,\quad \tan\beta,\quad \mu,\quad m_A(\mathrm{NUHM2}). \tag{11}$$

We scan over the following parameters:

$$\begin{aligned} m_0&:\ 0\text{–}20\ \mathrm{TeV},\\ m_{1/2}&:\ 0.2\text{–}3\ \mathrm{TeV},\\ -3 &< A_0/m_0 < 3,\\ \mu&:\ 0.1\text{–}0.5\ \mathrm{TeV},\\ m_A&:\ 0.15\text{–}20\ \mathrm{TeV},\\ \tan\beta&:\ 3\text{–}60. \end{aligned} \tag{12}$$

We take the various generations of scalar soft terms to be degenerate as is suggested by the degeneracy solution to the SUSY flavor and CP problems. We require of our solutions that

(i) electroweak symmetry be radiatively broken (REWSB),
(ii) the neutralino $\tilde{Z}_1$ is the lightest MSSM particle,
(iii) the light chargino mass obeys the model independent LEP2 limit, $m_{\tilde{W}_1} > 103.5$ GeV [53],
(vi) LHC8 search bounds on $m_{\tilde{g}}$ and $m_{\tilde{q}}$ from the m_0 vs $m_{1/2}$ plane [13] are respected,
(vii) $m_h = 125 \pm 2$ GeV.

The calculational framework allowing weak scale μ and m_A inputs in lieu of $m_{H_u}^2$ and $m_{H_d}^2$ is encoded in ISAJET/ISASUGRA versions ≥ 7.72 [52]. For the spectra calculations presented here, we use ISAJET 7.85 [54]. Here we do not enforce $b-\tau$ Yukawa coupling unification, thus allowing for GUT scale threshold effects which may modify this relation.

The m_0 vs $m_{1/2}$ parameter space plane of NUHM2 is shown in Fig. 2 for $\tan\beta = 10$, $A_0 = -1.6 m_0$ with $\mu = 150$ GeV, and $m_A = 1$ TeV. We also show contours of Higgs mass (red curves), gluino mass (blue curves), and

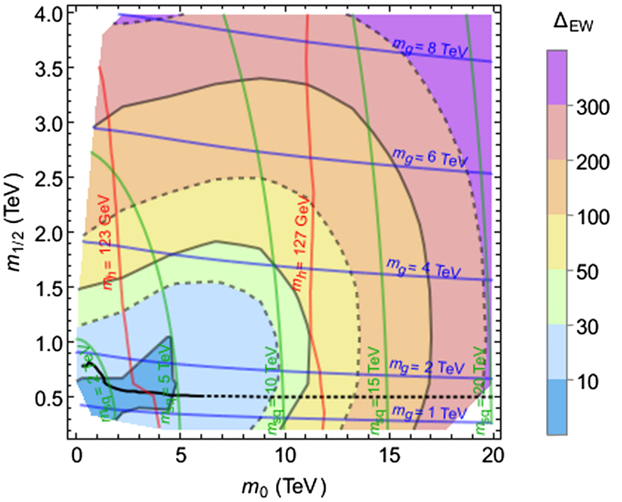

FIG. 2. Plot of contours of $\Delta_{\rm EW}$ in the m_0 vs $m_{1/2}$ plane for $\tan\beta = 10$ and $A_0 = -1.6m_0$ with $\mu = 150$ GeV and $m_A = 1$ TeV. We also show contours of $m_{\tilde g}$, average squark mass $m_{\tilde q} = 2$, 5, 10, 15, and 20 TeV and Higgs mass m_h. We show the LHC8 Atlas excluded region below the black contour.

average first generation squark mass (green curves). The color-coded regions show $\Delta_{\rm EW} < 10$ (blue) in the lower left and $\Delta_{\rm EW} < 30$ (light blue). These highly natural regions can lie well beyond the current reach limits from LHC8 and also beyond the ultimate reach of LHC14 with 300–1000 fb^{-1} of integrated luminosity. As one moves to larger values of m_0 and $m_{1/2}$, the model becomes increasingly fine-tuned and unnatural.

B. *SO*(10) model with Higgs in a single 10: *D*-term splitting

For this model, we again assume that the matter superfields live in the 16-dimensional spinorial irrep of $SO(10)$ so that matter is unified as well as forces. But now we will assume that there is a single **10** of Higgs $\phi(10)$ (which contains both a **5** and a **5***) of $SU(5)$ Higgses and that the MSSM Higgs doublets are both elements of the same ten-dimension GUT Higgs rep. We will assume in this case that the GUT scale Higgs mass splitting arises from D-term contributions to scalar masses which arise from the $SO(10)$ breaking. The D-term splitting also gives a well-defined pattern of matter scalar mass splittings, and moreover these splittings are correlated with the Higgs splitting,

$$\begin{aligned} m_Q^2 &= m_E^2 = m_U^2 = m_{16}^2 + M_D^2, \\ m_D^2 &= m_L^2 = m_{16}^2 - 3M_D^2, \\ m_{H_{u,d}}^2 &= m_{10}^2 \mp 2M_D^2, \\ m_N^2 &= m_{16}^2 + 5M_D^2, \end{aligned} \tag{13}$$

where M_D^2 parametrizes the magnitude of the D-term splitting. The value of M_D^2 can be taken as a free parameter of order the weak scale owing to our ignorance of the gauge symmetry breaking mechanism. It can take either positive or negative values. Thus, the *DT* model is initially characterized by the following six free parameters:

$$m_{16}, \quad m_{10}, \quad M_D^2, \quad m_{1/2}, \quad A_0, \quad \mathrm{sign}(\mu), \quad \tan\beta. \tag{14}$$

Here, the GUT scale soft breaking Higgs masses $m_{H_u}^2$ and $m_{H_d}^2$ are determined by m_{10} and M_D^2. These input parameters are rather awkward for generating SUSY models with electroweak naturalness where μ is required to be small. The main problem is that electroweak symmetry is *barely broken* in radiative natural SUSY [47] $[|m_{H_u}^2(\mathrm{weak})| \sim m_Z^2]$ and since we use an iterative up-down running approach to the RG solution, EW symmetry must be properly broken on each iteration in order to generate a viable mass spectrum. In barely broken electroweak symmetry breaking, frequently EW symmetry will fail to be broken on some iteration so then the whole calculation fails.

A better scheme for natural SUSY is to use μ and m_A as input parameters which then determine m_{H_u} and m_{H_d} at the weak scale. The values of $m_{H_u}^2$ and $m_{H_d}^2$ are then run from $m_{\rm weak}$ to $m_{\rm GUT}$ using the RGEs to determine their GUT scale values. At $Q = m_{\rm GUT}$, the required values of m_{10} and M_D^2 can be determined as outputs instead of inputs. To implement this scheme, we have programmed this new model into ISAJET 7.85 as model choice No. 11: the NUHM *D*-term splitting model. The DT model functions similarly to the NUHM2 model except that now the matter scalars are split according to Eq. (13) at the GUT scale. Thus, for the DT model, we will adopt the parameter space

$$m_0, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad \mu, \quad m_A(\mathrm{DT}), \tag{15}$$

where the first three are GUT scale inputs while the latter three are weak scale inputs and where we take $m_{16} \equiv m_0$. In this case, M_D^2 and m_{10} are outputs of the code. While the parameter space is the same as the NUHM2 model, the spectrum is quite different since now there is matter scalar splitting which is correlated with the GUT scale Higgs soft term splitting.

In this simple model, a high degree of $t - b - \tau$ Yukawa coupling unification would be expected in the simplest models. However, previous investigations find this difficult to reconcile with natural SUSY [55] due to a suppression by the small μ parameter of the needed weak scale threshold effects.

For the DT model, we will scan the same range of parameters as in the NUHM2 case.

C. *SU*(5) model

For simplicity, we assume that the MSSM + right-hand-neutrino model is the correct effective field theory below $Q = m_{\rm GUT}$ but that the MSSM boundary conditions at

$Q = m_{\rm GUT}$ respect the $SU(5)$ symmetry. Thus, the parameter space of the model is given by

$$m_5,\quad m_{10},\quad m_{1/2},\quad A_t,\quad A_b = A_\tau,\quad \tan\beta,\quad \mu,\quad m_A[SU(5)], \tag{16}$$

where as usual the L_i and D_i superfields live in a $\mathbf{5}^*$ ψ^j and the Q_i, U_i, and E_i live in a 10 ϕ_{jk} of $SU(5)$. The index i is a generation index while j, k are $SU(5)$ indices. One Higgs doublet H_u lives in a $\mathbf{5}$ of Higgs while the H_d lives in a $\mathbf{5}^*$ Higgs irrep. Here as usual we have traded the two GUT scale Higgs doublet soft masses $m_{H_u}^2$ and $m_{H_d}^2$ in favor of the weak scale parameters μ and m_A. Since we use μ and m_A as input parameters, we may use Eq. (3) to compute the required weak scale values of $m_{H_u}^2$ and $m_{H_d}^2$ so as to enforce the measured value of m_Z. The values of $m_{H_u}^2$ and $m_{H_d}^2$ are then run from $Q = m_{\rm weak}$ to $Q = m_{\rm GUT}$ according to their RGEs resulting in nonuniversal GUT scale scalar masses. Since the MSSM Higgs doublets are required to occur in separate $\mathbf{5}$ and $\mathbf{5}^*$ reps of $SU(5)$, this scheme is in accord with $SU(5)$ gauge symmetry.

For our parameter space scans, we will scan the $SU(5)$ model over the following ranges:

$$\begin{aligned} m_{5,10}&: 0.1\text{–}20\ \text{TeV},\\ m_{1/2}&: 0.2\text{–}3\ \text{TeV},\\ -40 < A_{t,b} &< 40\ \text{TeV},\\ \mu&: 0.1\text{–}0.5\ \text{TeV},\\ m_A&: 0.15\text{–}20\ \text{TeV},\\ \tan\beta&: 3\text{–}60. \end{aligned} \tag{17}$$

D. SUGRA model with 12 free parameters: SUGRA12

For purposes of comparison, we will contrast the above results with those of a model which includes RGE running but where the GUT scale soft scalar masses are unrelated. This is in accord with assuming that the SM gauge symmetry is valid at $Q > m_{\rm GUT}$ although we do still maintain gaugino mass unification (gaugino mass nonuniversality for highly natural SUSY models is explored in Ref. [56]). We will again trade the GUT scale values of m_{H_u} and m_{H_d} in lieu of weak scale values μ and m_A. This is the 12-free-parameter SUGRA model[3] with parameter space given by

$$m_{Q,U,D,L,E},\quad m_{1/2},\quad A_t,\quad A_b,\quad A_\tau,\quad \mu,\quad m_A,\quad \tan\beta(\text{SUGRA12}), \tag{18}$$

where we assume all three generations of matter scalars are degenerate in accord with a degeneracy solution to the SUSY flavor and CP problems [58,59]. This model is susceptible to large contributions to unnaturalness from electroweak D-term contributions to scalar masses [60].

For the SUGRA12 model, we scan over the following ranges:

$$\begin{aligned} m_{Q,U,D,L,E}&: 0.1\text{–}20\ \text{TeV},\\ m_{1/2}&: 0.2\text{–}3\ \text{TeV},\\ -40 < A_{t,b,\tau} &< 40\ \text{TeV},\\ \mu&: 0.1\text{–}0.5\ \text{TeV},\\ m_A&: 0.15\text{–}20\ \text{TeV},\\ \tan\beta&: 3\text{–}60. \end{aligned} \tag{19}$$

E. $b-\tau$ Yukawa unification

As a first examination, we compute the degree of $b-\tau$ Yukawa coupling unification vs $\Delta_{\rm EW}$ from each of the four models. We quantify the degree of Yukawa coupling unification via

$$R_{b\tau} = \max(f_b, f_\tau)/\min(f_b, f_\tau), \tag{20}$$

where the Yukawa couplings f_b and f_τ are understood to be GUT scale values.

In Fig. 3, our results are shown for the four models with color coded points corresponding to $\tan\beta < 15$ (green), $15 < \tan\beta < 30$ (blue), and $\tan\beta > 30$ (red). Points with $R_{b\tau} = 1$ would have exact $b-\tau$ unification at $Q = m_{\rm GUT}$.

The first point of emphasis is that low $\Delta_{\rm EW}$ ranging as low as 10 ($\Delta_{\rm EW}^{-1} = 10\%$ electroweak fine-tuning) solutions can be found for all four models. For a second point, from frame (a) we see that in the NUHM2 model $R_{b\tau} \simeq 1$ does occur for several solutions but with rather high $\Delta_{\rm EW} > 100$. For very natural models with $\Delta_{\rm EW} < 30$, then $b-\tau$ Yukawa couplings unify at the $R_{b\tau} \sim 1.2$–1.5 level. Generally, to allow for $b-\tau$ unification, one needs a large 1-loop b-quark threshold correction [see Eq. (1)] but with μ small for low $\Delta_{\rm EW}$ solutions, and this is never large. These results appear uniform across all four models although for $SU(5)$ we did find some $b-\tau$ unified solutions with $\Delta_{\rm EW}$ as low as ~50.

IV. NATURALNESS IN SUGRA GUT MODELS: NUMERICAL RESULTS

A. Gluino, wino, and bino masses

For our numerical mass results from a scan over the four SUGRA GUT models, we show in Fig. 4 the value of $m_{\tilde g}$ vs $\Delta_{\rm EW}$ for each case. In frame (a), we find, as shown earlier in Refs. [47,61] that for $\Delta_{\rm EW} < 30$ then $m_{\tilde g} \lesssim 4$ TeV in the NUHM2 model. This bound arises due to the contribution of the running $SU(3)$ gaugino mass M_3 on the values of $m_{\tilde t_{1,2}}$; these latter values enter $\Delta_{\rm EW}$ via the $\Sigma_u^u(\tilde t_{1,2})$ terms.

[3]A subset of the 19 free parameter SUGRA model [57] where gaugino masses are unified and generations are unified.

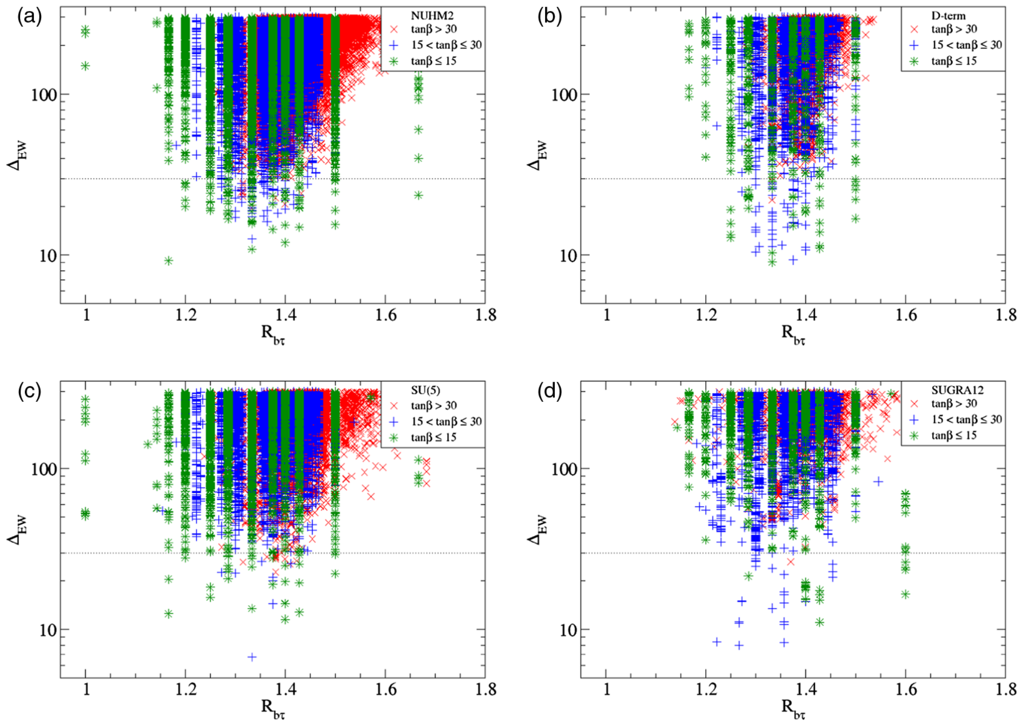

FIG. 3. Plot of $\Delta_{\rm EW}$ vs $R_{b\tau}$ for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

In frame (b) for the DT model, the upper bound on $m_{\tilde g}$ is comparable if not slightly stronger: $m_{\tilde g} \lesssim 3.5$ TeV.

In contrast, the less constrained $SU(5)$ and SUGRA12 models shown in frames (c) and (d) allow a weaker bound on $m_{\tilde g} \lesssim 6$ TeV. These bounds are slightly stronger than the corresponding bounds from the pMSSM model (with no RG running) shown in Ref. [61] where $m_{\tilde g} \lesssim 7$ TeV due to 2-loop contributions to the scalar potential [62]. In

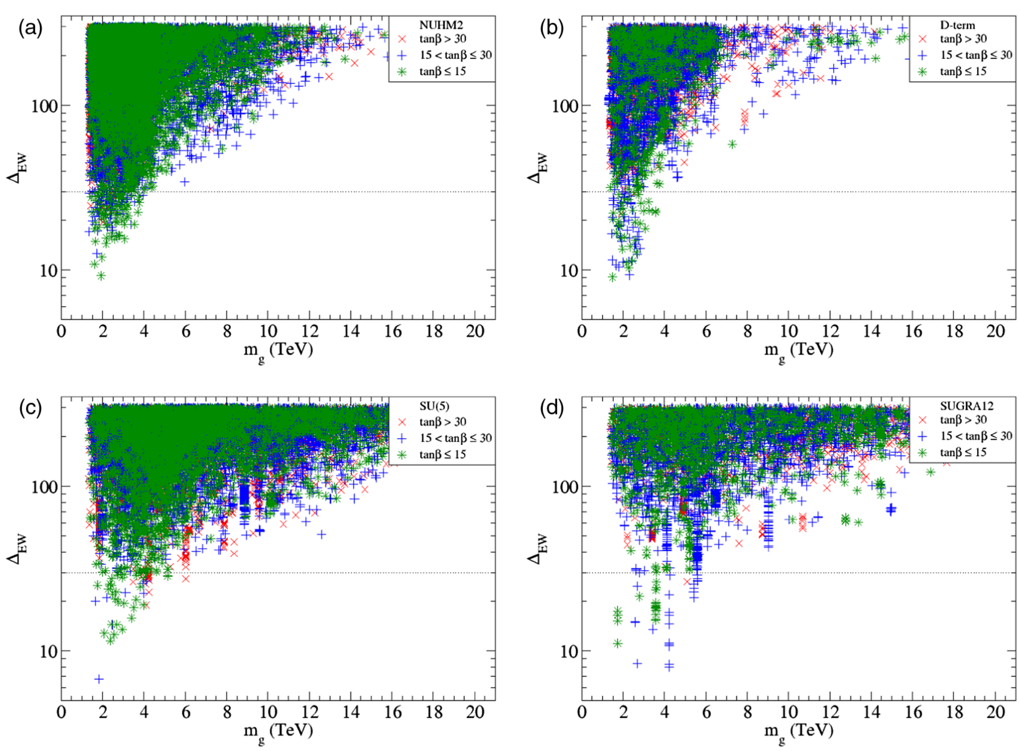

FIG. 4. Plot of $\Delta_{\rm EW}$ vs $m_{\tilde g}$ for (a) the NUHM2 model, (b) the DT model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

comparison with these mass bounds, we remark that the 5σ reach of LHC14 for gluino pair production extends to about $m_{\tilde{g}} \sim 2$ TeV for 300–1000 fb^{-1} of integrated luminosity [63]. Thus, LHC14 will be able to probe only the lower range of $m_{\tilde{g}}$ allowed by natural SUSY.

In the models presented here, we always assume gaugino mass unification $M_1 = M_2 = M_3$ at the GUT scale. RG evolution then leads to $7M_1 \sim 3.5M_2 \sim M_3$ at the weak scale for the bino, wino, and gluino masses, respectively. As gaugino mass bounds for $\Delta_{\rm EW} < 30$, we find that the bino mass $M_1 \lesssim 600$ GeV for NUHM2 and the DT model, but $M_1 \lesssim 900$ GeV for $SU(5)$ and SUGRA12. Likewise, we find that the wino mass $M_2 \lesssim 1200$ GeV for NUHM2 and DT models but $M_2 \lesssim 1800$ GeV for $SU(5)$ and SUGRA12 models.

B. μ parameter

The magnitude of the superpotential μ parameter is highly restricted by Eq. (3) to lie not too far from m_Z or m_h. Indeed, from Fig. 5 we see that for $\Delta_{\rm EW} < 30$ then $\mu \lesssim 350$ GeV for all cases since the μ parameter enters $\Delta_{\rm EW}$ at tree level. This is the most robust prediction of electroweak naturalness for SUSY models. It leads to the presence of four light Higgsino-like charginos and neutralinos $\tilde{W}_1^{\pm}$, $\tilde{Z}_{1,2}$ with mass $\sim$100–350 GeV. The mass splittings amongst the Higgsinos $m_{\tilde{W}_1} - m_{\tilde{Z}_1}$ and $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ are governed by how heavy the binos and winos are, and as seen from the last section these are also restricted by naturalness. Thus, typically from natural SUSY we obtain mass splittings $\sim$10–30 GeV. Tinier mass splittings require a larger gaugino-Higgsino mass gap but this splitting cannot get arbitrarily large according to the last subsection. Larger mass splittings can be obtained from models with gaugino mass nonuniversality [56]. The expected small mass splittings mean that Higgsino pair production at LHC results in events with very soft tracks which are difficult to trigger on much less than distinguish from SM background processes. The light Higgsinos should easily be observed in the clean environment of an e^+e^- collider with $\sqrt{s} > 2m(\text{Higgsino})$ [64].

C. Third generation sfermion masses and mixing

In Fig. 6 we show the lightest top squark mass $m_{\tilde{t}_1}$ vs $\Delta_{\rm EW}$ for each of four models. The top squark masses have sharp upper bounds due to the $\Sigma_u^u(\tilde{t}_{1,2})$ terms in Eq. (3). The precise contributions are listed in Ref. [47]. For the NUHM2, $SU(5)$, and SUGRA12 models we find $m_{\tilde{t}_1} \lesssim 3$ TeV for $\Delta_{\rm EW} < 30$. For the DT model, this bound seems tightened slightly to $m_{\tilde{t}_1} \lesssim 2$ TeV. These upper bounds are much higher than expected from old natural SUSY models [33] where three third generation squarks with mass $\lesssim$500 GeV were expected. For comparison, the reach of LHC14 in terms of $m_{\tilde{t}_1}$ is to the 1 TeV vicinity for various simplified models. Thus, as in the case of the gluino, natural SUSY can easily evade LHC stop searches with stops in the 1–3 TeV region.

One aspect of the stop sector which may distinguish among the four models is listed in Fig. 7 where we plot the

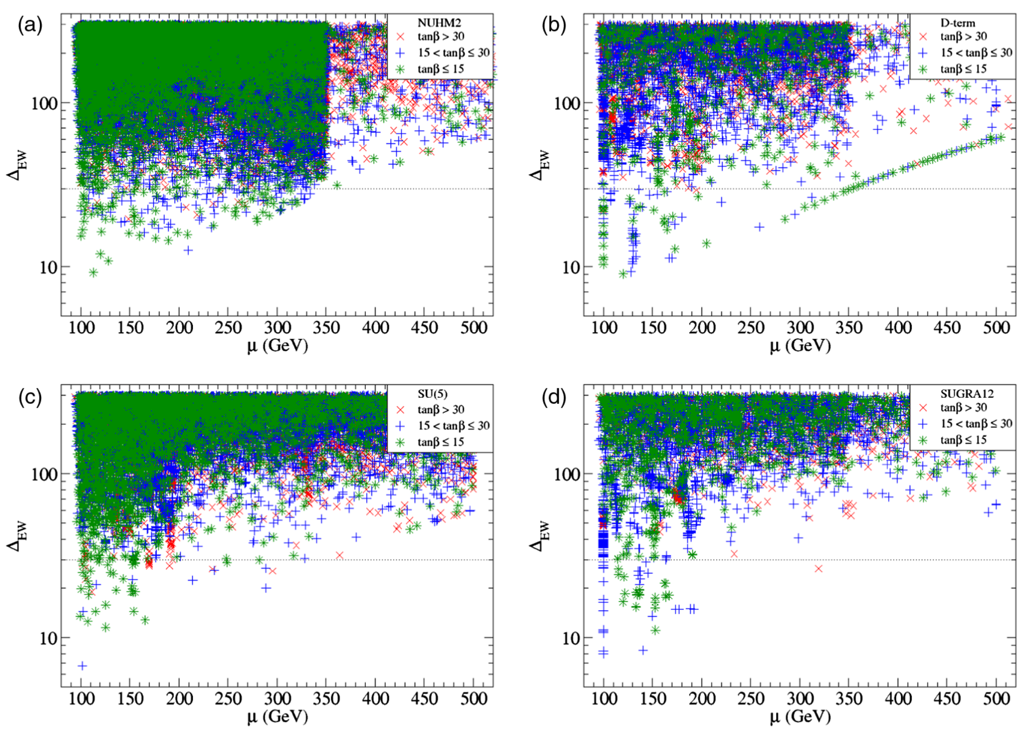

FIG. 5. Plot of $\Delta_{\rm EW}$ vs μ for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

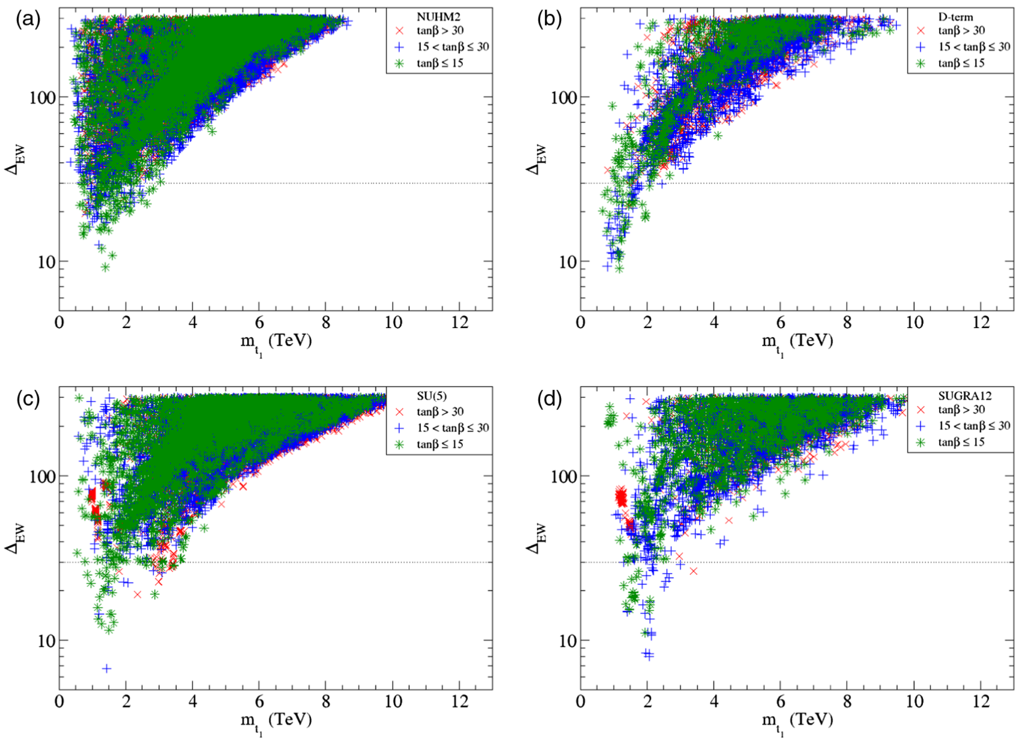

FIG. 6. Plot of $\Delta_{\rm EW}$ vs $m_{\tilde{t}_1}$ for (a) the NUHM2 model, (b) the *D*-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

stop mixing angle θ_t vs $\Delta_{\rm EW}$. Here we follow the notation of Ref. [65] where $\tilde{t}_1 = \cos\theta_t \tilde{t}_L - \sin\theta_t \tilde{t}_R$. Thus, $\cos\theta_t \sim 0$ leads to a $\tilde{t}_1$ which is mainly a right state. From Fig. 7 we see that for low $\Delta_{\rm EW} < 30$, then the NUHM2, DT, and $SU(5)$ models all require a mainly right $\tilde{t}_1$. In contrast, the greater parameter freedom of the SUGRA12 model allows for low $\Delta_{\rm EW}$ solutions with both left and right $\tilde{t}_1$ states. If an e^+e^- collider such as CLIC ($\sqrt{s}$ up to 3 TeV) is built with

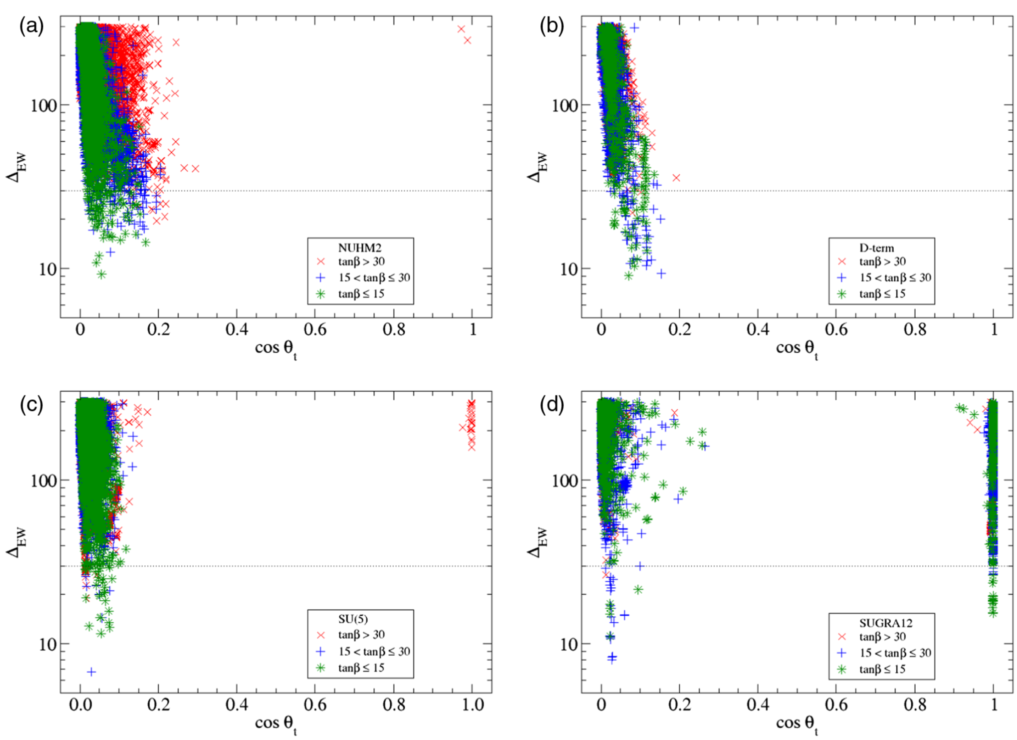

FIG. 7. Plot of $\Delta_{\rm EW}$ vs $\theta_{\tilde{t}}$ for (a) the NUHM2 model, (b) the *D*-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

$\sqrt{s} > 2m_{\tilde{t}_1}$, then the production cross sections for various beam polarizations will depend on the handedness of the stops being produced. Also, the left stops decay largely into charginos while the right stops mainly decay only to neutralinos. Such branching fraction measurements from an e^+e^- collider could help to distinguish these cases.

In the case of $\tilde{b}$-squarks, we list the corresponding mixing angle θ_b vs Δ_{EW} for the four models in Fig. 8. Here again, $\tilde{b}_1 = \cos\theta_b \tilde{b}_L - \sin\theta_b \tilde{b}_R$. From the plots, we see that for natural solutions with $\Delta_{EW} < 30$ in the NUHM2 model, then $\tilde{b}_1$ is expected to only occur as a left squark. In the other three models, natural solutions exist where $\tilde{b}_1$ can occur as either left or right squarks. This can be understood in the NUHM2 model as a consequence of GUT scale universality: $m_{Q_3} = m_{D_3}$ where m_{Q_3} is driven smaller than m_{D_3} by the large top quark Yukawa coupling. For the other models where m_{Q_3} may be greater than m_{D_3} at $Q = m_{GUT}$, then the lighter sbottom $\tilde{b}_1$ may be either left or right.

In Fig. 9 we show the stau mixing angle $\cos\theta_\tau$ vs Δ_{EW} where $\tilde{\tau}_1 = \cos\theta_\tau \tilde{\tau}_L - \sin\theta_\tau \tilde{\tau}_R$. In contrast to the stop and sbottom cases, we find that natural solutions with either right or left staus can occur for all four models. Thus, measuring the handedness of the lighter staus is unlikely to distinguish between models. Whereas in models like mSUGRA one always expects the lightest stau to be a right state, in models with nonuniversality (at least in the Higgs sector) means that a large S term contribution ($S = 0$ in models with scalar mass universality) to RG running can reverse this situation and the lightest stau may in fact be a left state.

D. Squark and slepton masses

To a very good approximation, the masses of the first generation of sfermions are given by

$$m^2_{\tilde{u}_L} = m^2_{Q_1} + m^2_u + M^2_Z \cos 2\beta\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right), \tag{21}$$

$$m^2_{\tilde{d}_L} = m^2_{Q_1} + m^2_d + M^2_Z \cos 2\beta\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right), \tag{22}$$

$$m^2_{\tilde{u}_R} = m^2_U + m^2_u + M^2_Z \cos 2\beta\left(\frac{2}{3}\sin^2\theta_W\right), \tag{23}$$

$$m^2_{\tilde{d}_R} = m^2_D + m^2_d + M^2_Z \cos 2\beta\left(-\frac{1}{3}\sin^2\theta_W\right), \tag{24}$$

$$m^2_{\tilde{e}_L} = m^2_{L_1} + m^2_e + M^2_Z \cos 2\beta\left(-\frac{1}{2} + \sin^2\theta_W\right), \tag{25}$$

$$m^2_{\tilde{\nu}_e} = m^2_{L_1} + M^2_Z \cos 2\beta\left(\frac{1}{2}\right), \tag{26}$$

$$m^2_{\tilde{e}_R} = m^2_E + m^2_e + M^2_Z \cos 2\beta(-\sin^2\theta_W), \tag{27}$$

where the first terms on the right-hand side of these expressions are the weak scale soft SUSY breaking masses

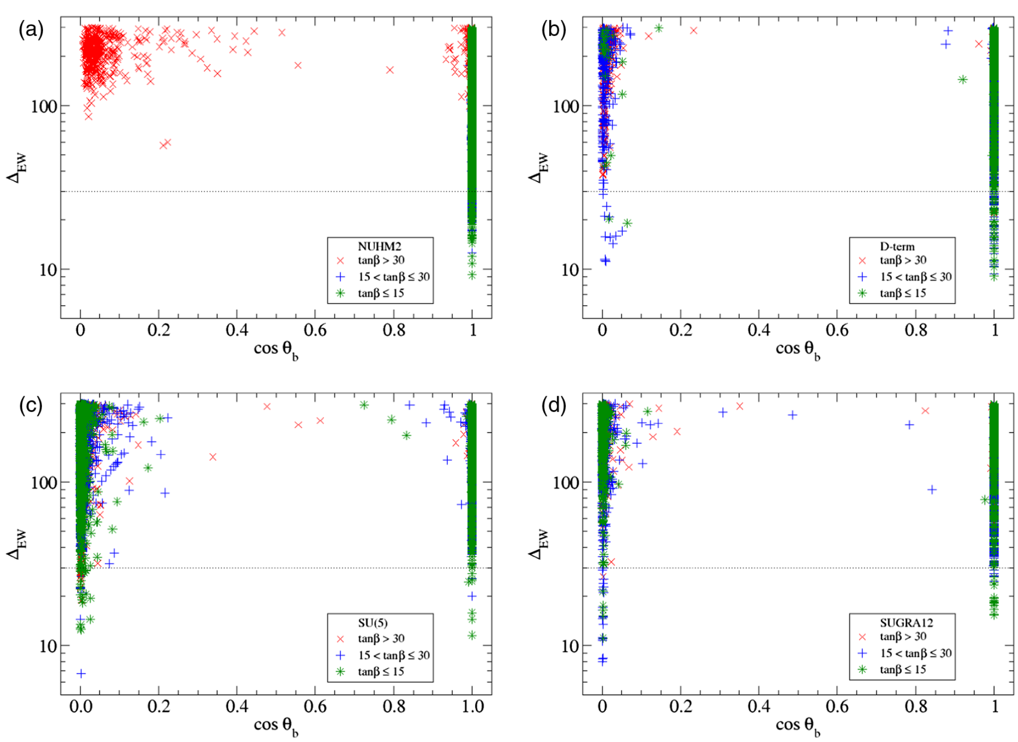

FIG. 8. Plot of Δ_{EW} vs $\theta_{\tilde{b}}$ for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

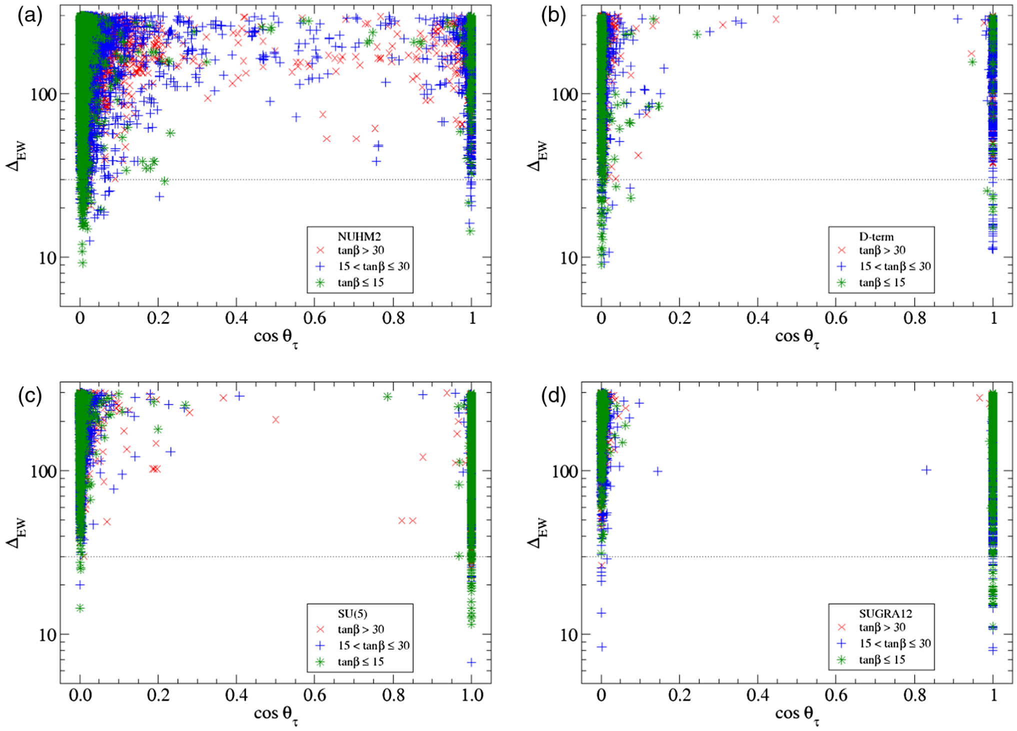

FIG. 9. Plot of $\Delta_{\rm EW}$ vs $\theta_{\tilde\tau}$ for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

for the first generation of sfermions. There are analogous expressions for second generation masses. It seems from a lack of signal from squark/slepton searches at LHC that sfermion masses are likely in the multi-TeV region. In that case, the D-term contributions to sfermion masses (those proportional to M_Z^2) are likely suppressed compared to the soft term contributions, and hence the measured sfermion masses would very nearly provide the weak scale soft term masses. The weak scale first/second generation soft terms have simpler RG running solutions so that a precise

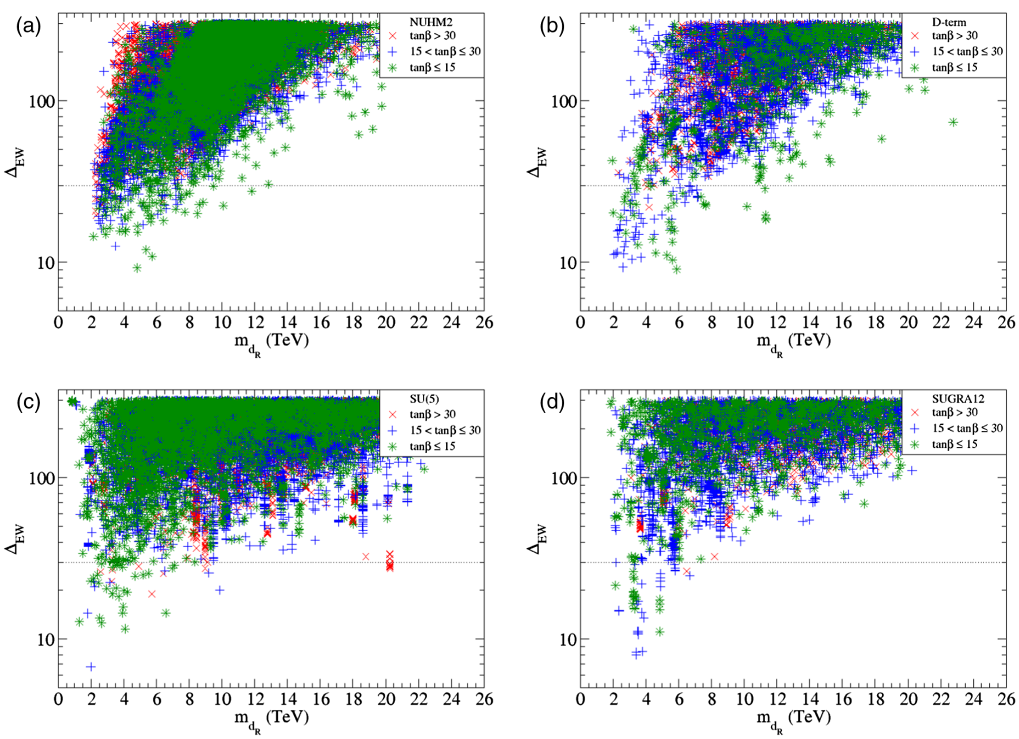

FIG. 10. Plot of $\Delta_{\rm EW}$ vs $m_{\tilde d_R}$ for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

measurement of weak scale sfermion masses could yield the GUT scale soft terms [66], especially if the gaugino masses are measured. A knowledge of the GUT scale soft terms could then reveal whether the sfermions arrange themselves into GUT multiplets which would reflect a mass organization according to one (or none) of the models considered.

In Fig. 10, we show for example the $\tilde{d}_R$ squark masses vs Δ_{EW}. While these squark masses may be as low as ~2 TeV for natural solutions, they can also range up to the vicinity of 10 TeV (and even up to 20 TeV for nonuniversal generations [47]). Thus, an e^+e^- collider with $\sqrt{s} > 2m(\text{sfermion})$ would likely be required for such squark mass determinations. Typically the $\sqrt{s}$ values needed would be beyond any sort of ILC projections and perhaps even beyond suggested CLIC energies. It is also possible such measurements could be made at a 50–100 TeV pp collider as suggested in Ref. [67].

E. Heavy Higgs masses

Mass limits on heavy Higgs bosons have been shown previously for the NUHM2 model in Ref. [68]. As confirmed in Fig. 11(a), the value of m_A is bounded by about 8–10 TeV for this model. Similar mass bounds are found for the DT model in frame (b) and the $SU(5)$ model [frame (c)]. For the SUGRA12 model in frame (d), the mass bound appears lower since now D-term contributions from first/second generation scalar masses come into play in the Σ_u^u terms in Eq. (3) and lead to unnaturalness for nondegenerate squarks and sleptons in the multi-TeV vicinity [60]. Thus, the apparent tighter mass bound on m_A in frame (d) is likely due to difficulty sampling at very high scalar masses.

F. Four SUGRA GUT benchmark models

In Table I we list four benchmark models, one for each model considered in the text. Each model has $m_{1/2} = 800$ GeV, $A_0 = -5700$ GeV, $\tan\beta = 10$, $\mu = 150$ GeV, and $m_A = 3000$ GeV. The first case, NUHM2, has degenerate matter scalars with mass $m_0 = 4$ TeV but with split Higgs mass soft terms. This model has low $\Delta_{EW} = 23.7$ or 4% EW fine-tuning. The gluino mass is $m_{\tilde{g}} = 1972$ GeV which is somewhat above current limits from LHC13. The Higgsinos $\tilde{W}_1^\pm$ and $\tilde{Z}_{1,2}$ are clustered around 150 GeV. The $b-\tau$ Yukawa unification occurs at about the 33% level.

The DT model is listed next with input parameters μ and m_A as listed. These values determine $m_{H_u}^2(\text{weak})$ and $m_{H_d}^2(\text{weak})$ which are then run up to $Q = m_{GUT}$ to determine the required D-term splitting. The matter scalars are split according to Eq. (13) leading to $m_{Q,U,E} = 3597$ GeV and $m_{D,L} = 5019$ GeV. In spite of the different sfermion mass splitting, the value of Δ_{EW} remains at 23.4. The low energy spectrum of gluinos and Higgsinos (and binos/winos) should ultimately be accessible to a combination of LHC14 and ILC measurements. Since *all* four models have a similar spectrum of gauginos and Higgsinos, they will all look rather similar to LHC14 and ILC.

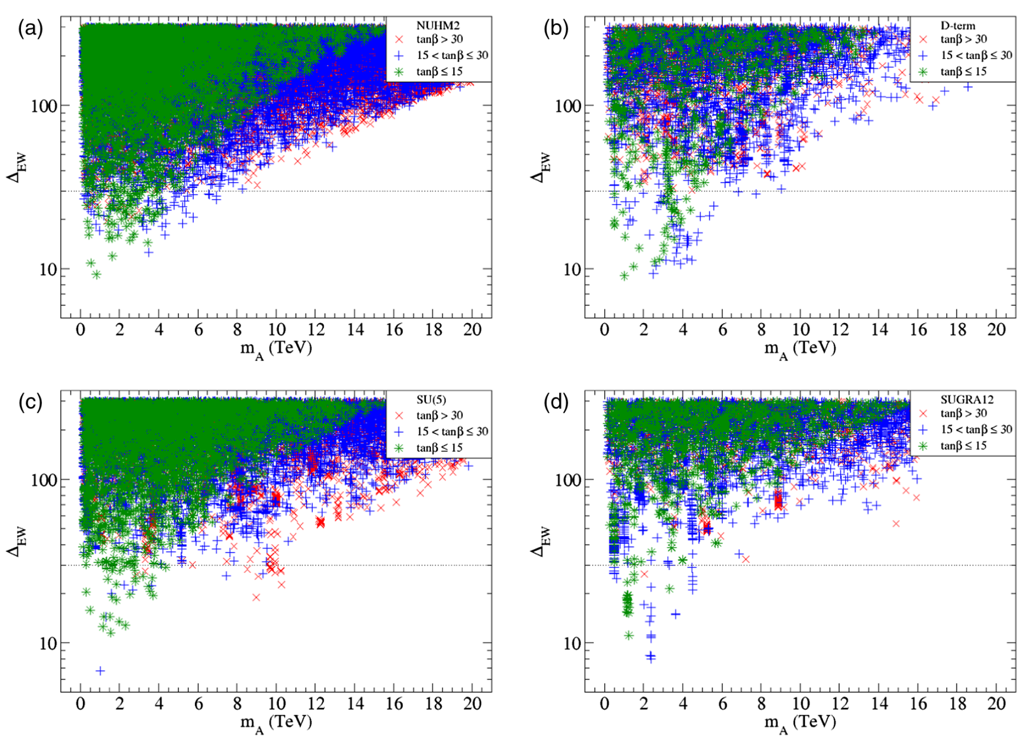

FIG. 11. Plot of Δ_{EW} vs m_A for (a) the NUHM2 model, (b) the D-term model, (c) the $SU(5)$ model, and (d) the SUGRA12 model.

TABLE I. Input parameters and masses in GeV units for the four *radiatively driven natural SUSY* benchmark points from 1. the NUHM2 model, 2. the D-term model, 3. the $SU(5)$ model, and 4. the SUG12 model. For all four cases, we take $m_{1/2} = 800$ GeV, $A_0 = -5700$ GeV, $\tan\beta = 10$, $\mu = 150$ GeV, and $m_A = 3000$ GeV. We also take $m_t = 173.2$ GeV.

Parameter	NUHM2	D-term	$SU(5)$	SUG12
m_Q	4000	3597	5000	5000
m_U	4000	3597	5000	5000
m_E	4000	3597	5000	3000
m_D	4000	5019	3000	3000
m_L	4000	5019	3000	5000
m_{H_u}	4970	4648	5797	5468
m_{H_d}	3043	3063	3022	3421
$m_{\tilde{g}}$	1972.4	1965.1	1993.6	1989.9
$m_{\tilde{u}_L}$	4250.3	3869.2	5194.8	5273.1
$m_{\tilde{u}_R}$	4317.3	3928.1	5287.0	4949.6
$m_{\tilde{d}_R}$	4226.1	5220.2	3230.7	3485.9
$m_{\tilde{e}_L}$	4074.3	5074.1	3120.3	4819.7
$m_{\tilde{e}_R}$	3910.8	3517.3	4885.8	3596.4
$m_{\tilde{t}_1}$	1536.2	1060.0	2393.4	1798.9
$m_{\tilde{t}_2}$	3122.8	2758.1	3980.7	4103.6
$m_{\tilde{b}_1}$	3146.4	2789.4	3163.6	3412.2
$m_{\tilde{b}_2}$	4155.4	5147.7	3991.1	4137.6
$m_{\tilde{\tau}_1}$	3851.1	3445.3	3084.3	3528.8
$m_{\tilde{\tau}_2}$	4045.9	5044.8	4837.5	4795.1
$m_{\tilde{\nu}_\tau}$	4049.8	5054.1	3082.0	4797.5
$m_{\tilde{W}_2}$	684.7	687.1	681.5	685.9
$m_{\tilde{W}_1}$	154.8	154.4	155.9	155.9
$m_{\tilde{Z}_4}$	695.5	695.5	696.9	701.0
$m_{\tilde{Z}_3}$	359.7	359.8	360.0	360.8
$m_{\tilde{Z}_2}$	158.0	157.7	158.4	158.3
$m_{\tilde{Z}_1}$	142.0	141.7	142.5	142.4
m_h	122.7	123.7	122.0	122.0
$\Omega^{std}_{\tilde{Z}_1} h^2$	0.008	0.008	0.008	0.008
$BF(b \to s\gamma) \times 10^4$	3.0	2.9	3.1	3.1
$BF(B_s \to \mu^+\mu^-) \times 10^9$	3.8	3.8	3.8	3.8
$\sigma^{SI}(\tilde{Z}_1 p)$ (pb)	4.2×10^{-9}	4.1×10^{-9}	4.3×10^{-9}	4.2×10^{-9}
$R_{b\tau}$	1.33	1.35	1.36	1.33
$\Delta_{\rm EW}$	23.7	23.4	54.0	37.6
θ_t	1.51	1.50	1.53	1.54
θ_b	0.0035	0.0012	1.57	1.56
θ_τ	1.56	1.57	0.0015	1.57

Higher energy colliders such as CLIC ($\sqrt{s} \sim 3$ TeV) or a 100 TeV pp collider $pp(100)$ will be required to distinguish the very massive sfermions. For the case of the DT model, measurements of $m_{\tilde{u}_L,\tilde{u}_R,\tilde{e}_R}$ vs $m_{\tilde{d}_R,\tilde{e}_L}$ would distinguish the split rather than degenerate matter scalars. A measurement of m_A could help determine $m^2_{H_d}$(weak) which may then be run to $m_{\rm GUT}$ to determine m_{H_d}(GUT). If knowledge of $m^2_{H_u}$(GUT) can be extracted, then it might be possible to determine if the D-term splitting in the matter scalars is in accord with the Higgs soft mass splitting as in the DT model, or as in the $SU(5)$ model where $m_{Q,U,E}$ are split from $m_{D,L}$ in a manner quite different from the DT case. Note also that the $SU(5)$ model has a different pattern of stop-sbottom-stau mixing from the NUHM2 or DT case where now the $\tilde{b}_1$ is mainly a right squark. The SUG12 model has a more arbitrary form of sfermion mass splitting. In this case, measurements that $m_{\tilde{d}_R} \simeq m_{\tilde{e}_R}$ and $m_{\tilde{u}_L} \sim m_{\tilde{u}_R} \sim m_{\tilde{e}_L}$ would signal that the various matter sfermions do not live in GUT multiplets. In this latter case, there are incomplete cancellations of contributions to $\Delta_{\rm EW}$ from the matter scalars [60] which may lift the calculated value of $\Delta_{\rm EW}$ beyond what is otherwise expected.

V. CONCLUSIONS

In this paper we have examined two topics: generalized focus point behavior of SUSY GUT models with radiatively driven naturalness and a comparison of mass spectra expected from four different SUSY GUT models. A crucial insight into naturalness was gleaned in Ref. [45] where it was demonstrated that for universal GUT scale boundary conditions on soft breaking scalar masses, large cancellations in the Higgs and squark contributions to the Z boson mass allowed for very heavy, TeV-scale third generation squarks while respecting naturalness. In our discussion of generalized focus-point behavior in Sec. II, we emphasized (as in Ref. [27]) that in more fundamental SUSY theories (such as supergravity GUT theories) *all* the soft terms are calculable as multiples of the gravitino mass $m_{3/2}$ (or Λ in gauge mediated supersymmetry breaking models) so that *all* the soft term contributions to m_Z^2 should be combined. In this situation, the BG naturalness measure agrees with tree-level low electroweak fine-tuning as expressed by the $\Delta_{\rm EW}$ measure. We demonstrate for a hypothetical set of soft term relationships which link all the soft terms to $m_{3/2}$ that the weak scale value of $m_{H_u}^2$ is indeed focused to values $\sim m_Z^2$ for a wide range of gravitino mass values.

In the remainder of this paper we examined four scenarios expected from highly natural SUSY GUT models with gaugino mass unification but not scalar mass universality. The first task was to verify that all could generate low values of $\Delta_{\rm EW} \lesssim 30$. The next task was to examine how compatible $b-\tau$ Yukawa unification is with electroweak naturalness and low μ: we found them compatible to $R_{b\tau} \sim$ 1.2–1.5 or 20%–50% $b-\tau$ Yukawa unification. The third task was to examine the spectra from the four cases NUHM2, DT, $SU(5)$, and SUGRA12 to examine if the models could be experimentally differentiable. In fact, all four models look rather alike for colliders like LHC14 and ILC. For these cases, we expect the gluino mass to be bounded by about 4–5 TeV which may or may not be detectable at LHC. Also, a spectrum of light Higgsinos with mass $\lesssim$200–300 GeV are expected which should be detectable at ILC. To differentiate the models, a very high energy hadron collider such as a 100 TeV pp machine will be needed for robust squark pair production or a very high energy e^+e^- machine will be needed for sfermion pair production. In such a case, it may be possible to distinguish if the sfermions have nearby masses as expected in models like NUHM2 with matter scalar (but not Higgs) universality, or whether the spectrum is more spread out as expected in models with D-term splitting or where the sfermions come in independent 10s and $\mathbf{5}^*$s of $SU(5)$. High energy e^+e^- or pp colliders may also be able to differentiate the decay modes of third generation squarks to determine their handedness, and determine if that agrees with expectations from various highly natural SUSY GUT models.

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