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Dynamically emergent flavor in a confining theory with unbroken chiral symmetry

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I discuss "truncated" QCD studied recently by Glozman *et al.* through numerical simulations. For two flavors it was observed that truncation restores the full chiral $U(2)\times U(2)$ symmetry of the Lagrangian. Moreover, additional enhancement of the above symmetry connecting representations with distinct Lorentz spins was observed. I argue that the chiral symmetry restoration in a confining theory could entail emergent (extra) dynamical flavors which would show up in the spectrum of color-singlet particles, provided their mass $\neq 0$. As an example, I consider truncated QCD with a *single* massless Dirac quark. Assuming the validity of the above observations, I demonstrate how a dynamical $SU(2)_{\rm fl}$ symmetry could emerge for *massive* spin-1 mesons without contradicting general principles.

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I. INTRODUCTION

Spontaneous chiral symmetry breaking is a crucial feature of QCD. Since it occurs at strong coupling, so far there is no full understanding of the underlying dynamics. It was conjectured [1] that the chiral symmetry restoration could happen for highly excited (mesonic and baryonic) states, but this conjecture did not hold [2].

The standard QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2} G^{\mu\nu\,a} G^a_{\mu\nu} + \sum_f \bar{\Psi}\, iD_\mu \gamma^\mu \Psi \qquad (1)$$

can include one, two or more distinct Dirac fermions (flavors) in the fundamental representation. Denoting this number as N_f we can say that, with the vanishing quark masses, Lagrangian (1) possesses a chiral symmetry

$$U(N_f)\times U(N_f). \qquad (2)$$

After spontaneous breaking of the chiral symmetry (χSB), and taking account of the axial anomaly, the flavor symmetry of conventional QCD is

$$SU(N_f)_{\rm diag}\times U(1)_V. \qquad (3)$$

If $N_f = 1$, i.e., with a single flavor, the conventional symmetry is just $U(1)_V$, the fermion charge.

The Banks-Casher formula [3] tells us that the quark condensate responsible for χSB does not develop, provided the fermion Dirac operator modes do not condense near zero [i.e., the density of the eigenvalues $\rho(0) = 0$.] Thus, the near-zero modes play a special role in the chiral symmetry breaking.

A conjecture was put forward in [4] that in lattice calculations with dynamical quarks in the vacuum but with *truncated* fermion Green functions, the chiral symmetry will be restored [5,6]. By "truncated" I mean that a (fixed) number of the lowest-lying eigenmodes of the Dirac operator are eliminated in the quark propagator "by hand." This framework will be referred to as "truncated QCD."

This conjecture was verified in recent calculations [4–6].[1] It was found indeed that the chiral symmetry is restored in the spectrum; e.g., vector and axial vector mesons are degenerate. Moreover, an additional observation was that the effects due to the anomaly in the flavor-singlet axial $U(1)$ current disappear. Thus, the first surprising "experimental" finding of [4–6] is that the symmetry of the spectrum is at least $U(2)\times U(2)$ in truncated $SU(3)_{\rm c}$ QCD with two flavors.[2]

It is clear that the above truncation introduces nonlocal deformations in QCD. Their impact is unclear. For instance, it is not *a priori* certain that causality remains undamaged. The local Lagrangian description in the "truncation framework" does not exist, and, moreover, no nonlocal Lagrangian is known. We can be certain, however, that truncation does not break the Lorentz invariance. This is important.

The second surprising finding of truncation is as follows. The numerical results mentioned above exhibit an unexpected enhancement of the spectral symmetry, at least in the spin-1 sector—the enhancement that goes beyond the expected $U(2)\times U(2)$. A dynamical $U(4)$ symmetry was discussed in the literature [7,8] [in truncated $SU(3)_{\rm c}$ QCD with two flavors] in connection with this enhancement. The above $U(4)$ symmetry entangles geometric (dotted and undotted) spinorial quark indices with isospin, which would contradict the Coleman-Mandula theorem [9],

[1]There are certain reservations, however; for a discussion see Sec. II.

[2]Here and below it will be assumed that all quark masses vanish.

generally speaking. Below, I discuss how a phenomenon that looks like a "flavor doubling" could emerge in the space of color-singlet mesons with mass $\neq 0$ in a hypothetical QCD with no χSB.

II. PRELIMINARIES

Let me try to concisely summarize the main results of Glozman *et al.*

Each massless Dirac fermion field in the fundamental representation of $SU(3)_{\rm c}$ is built from one dotted and one undotted Weyl spinor, χ^i_α and $\bar\eta^{\dot\alpha i}$, respectively,

$$\Psi = \begin{pmatrix} \chi^i_\alpha \\ \bar\eta^{\dot\alpha i} \end{pmatrix}, \tag{4}$$

where i is the color index (usually omitted in what follows). The standard definition of flavor implies that in the quark sector we have N_f Dirac spinors (4). The unbroken chiral $U(N_f)\times U(N_f)$ symmetry implies the following:

(i) If the flavor indices f and g are introduced as χ^f and η_g $(f,g=1,2,\ldots,N_f)$, then all $2N_f$ fermion numbers, corresponding to the currents

$$(j_{\dot\alpha\alpha})^f_f = \bar\chi_{\dot\alpha f}\chi^f_\alpha, \qquad \eta_{\alpha f}\bar\eta^f_{\dot\alpha}, \\ f=1,2,\ldots,N_f;\ \text{no summation over } f!, \tag{5}$$

are conserved separately (see [4–6] in which $N_f=2$). In other words, there are $2N_f$ conserved quark charges. This follows from conservation of *all diagonal* vector and axial-vector currents, including the axial flavor singlet.

(ii) Conventional spin representations can be generalized to chiral spin: The hadronic states can be classified with regards to dotted and undotted indices separately; for instance, the state $\bar\chi^f_{\dot\alpha}\chi^g_\alpha$ has spin $S=(\frac12,\frac12)$ while $\chi^f_\alpha\eta^\alpha_g$ has $S=(0,0)$.

(iii) Confinement of quarks in color-singlet states is not damaged.

(iv) An additional degeneracy of the spectrum of the color-singlet mesons connects chiral multiplets with interchanged dotted and undotted indices, for instance,

$$\chi^i_\alpha\eta_{\beta i} \leftrightarrow \bar\eta^{\dot\alpha i}\eta_{\beta i}. \tag{6}$$

(v) The least straightforward result of Glozman *et al.* is as follows. Despite the fact that the parity degeneracy in the spin-1 sector is restored, which would normally imply that the χSB does not occur upon truncation (the biquark condensate does not develop), the falloff of the correlation functions of the scalar operators $\chi_\alpha\eta^\alpha$ and $\bar\chi_{\dot\alpha}\bar\eta^{\dot\alpha}$ is not exponential but rather powerlike, implying that the spin-zero mesons are massless, as if they were Goldstones. In other channels (i.e., $J=1,2,\ldots$) the corresponding mesons are massive.

Needless to say, all of the above "experimental" results should be checked by independent group(s) before their status can be elevated to "firmly established." A special emphasis should be put on clarification of the controversial point (v).

For the time being, let us imagine, however, that a consistent truncated version of QCD can be worked out and address the question of whether an enhanced flavor symmetry can appear in the particle spectra upon the chiral symmetry restoration. Since this question can be raised even for one Dirac spinor[3]; (i.e., $N_f=1$), I will discuss, namely, this situation because of simplifying indices.

III. WHAT HAPPENS IF CHIRAL SYMMETRY IS UNBROKEN

First, let us note that two distinct "diagonal" 2-point functions

$$\langle\bar\chi_{\dot\alpha}\chi_\alpha(x)(\bar\chi_{\dot\alpha}\chi_\alpha(0))^\dagger\rangle \quad\text{and}\quad \langle\chi_\alpha\eta_\beta(x)(\chi_\alpha\eta_\beta(x)(0))^\dagger\rangle \tag{7}$$

cannot be saturated by the same mesons. This is because a "cross" correlator

$$\langle\bar\chi_{\dot\alpha}\chi_\alpha(x)(\chi_\alpha\eta_\beta(x)(0))^\dagger\rangle \tag{8}$$

vanishes identically which, in turn, follows from separate conservation of both the vector and axial current, i.e., the χ and η quark numbers. Thus, the correlation functions in (7) are saturated by different massive mesons. In the rest frame both have conventional spin 1. However, the chiral spin structure (which can be used for classification in the case of unbroken χSB) is different. I also use the fact that truncated QCD [4–6] does not violate Lorentz (in fact, Poincaré) invariance. The additional degeneracy of the spectrum of the color-singlet mesons detected by Glozman *et al.* connects the following multiplets with interchanged dotted and undotted indices, for instance,

$$\chi_{\{\alpha}\eta_{\beta\}} \leftrightarrow \bar\eta^{\dot\alpha}\eta_\beta \tag{9}$$

where the braces indicate symmetrization (three spin states) and

$$\partial^\beta_{\dot\alpha}(\bar\eta^{\dot\alpha}\eta_\beta)=0, \tag{10}$$

so that the operator on the right-hand side of (9) also produces three spin states (conventional spin 1). The scalar-pseudoscalar state $\chi_{[\alpha}\eta_{\beta]}$ has no partner because of (10), and the same is true for the χ current. Since their "experimental" interpretation is not yet clear (see above), I will not discuss them, and I focus on two different spin-1 states in (9).[4]

[3] In this case the conjecture of [4–6] would imply $U(2)$ flavor symmetry instead of conventional $U(1)\times U(1)$.

[4] In fact, each of the two operators produces two parity degenerate mesons.

In conventional QCD the symmetry of the theory under consideration is just vectorial $U(1)$. It is implemented trivially since all mesons have the corresponding charge zero.

Restoration of the chiral symmetry in truncated QCD would lead to $U(1)\times U(1)$. Enhancement of symmetry needed for (9) was not expected *a priori*. We need something like an $SU(2)$ converting χ into $\bar\eta$. If we do so literally, we would break the Lorentz symmetry by rotating an undotted spinor index into the dotted one. We must act in a more subtle way.

IV. EMERGENT DYNAMICAL FLAVOR FOR SPIN-1 MESONS

In our simplified example there are four color-singlet massive mesons (if I choose interpolating fields without derivatives),

$$\chi^i_\alpha\eta_{\beta,i},\qquad \bar\chi_{\dot\alpha i}\bar\eta^i_{\dot\beta},\qquad \bar\eta^{\dot\alpha i}\eta_{\beta i},\qquad \chi^i_\alpha\bar\chi^{\dot\beta}_i. \tag{11}$$

There are four interpolating fields; each one produces, generally speaking, four chiral spin states. More exactly, we must focus only on the states with conventional spin 1, as was discussed above. This means that we must symmetrize with respect to α and β in the first pair and take into account the fact that, due to transversality, (pseudo)scalar states are not produced by the second pair. Then, each operator will produce three spin states.

A crucial point is that for $M\neq 0$, where M is the meson mass, each state [the Lorentz spins $S=(\frac12,\frac12)$ and $S=(1,0)+(0,1)$], being distinctly different, can be described by one and the same formalism.[5] Dotted indices can be converted into undotted and vice versa by applying the energy-momentum operator $P_{\alpha\dot\alpha}$ or $P^{\dot\alpha\alpha}$ which are invertible because $P^2=M^2\neq 0$. Then, instead of the four operators (11) with distinct Lorentz structure, we can introduce[6]

$$\begin{aligned}&\chi_\alpha\eta_\beta,\qquad \bar\chi_{\dot\alpha}\bar\eta_{\dot\beta},\\ &(P_{\alpha\dot\alpha}M^{-1})\bar\eta^{\dot\alpha}\eta_\beta,\qquad (P^{\dot\alpha\alpha}M^{-1})\chi_\alpha\bar\chi^{\dot\beta}.\end{aligned} \tag{12}$$

I omitted the color indices as well as the symmetrization braces. Now, the operators in (12) carry either both dotted or both undotted indices. It is important that $P_{\alpha\dot\alpha}$ is invertible. $P_{\alpha\dot\alpha}P^{\dot\alpha\beta}=\delta^\beta_\alpha P^2$ and $P^2>0$. Because of this fact, instead of (12) one could also represent all operators in the $S=(\frac12,\frac12)$ form.

All four states in (12) can now be related by standard $U(2)_{\rm fl}$ transformations. Two diagonal transformations are equivalent to charge conservation for χ's and η' separately, while off-diagonal ones are (see the Appendix)

$$\begin{aligned}\delta(\chi^i_\alpha\eta_{\beta,i})&=\frac{i}{\sqrt{P^2}}[P_{\alpha\dot\alpha}(\bar\eta^{\dot\alpha i}\eta_{\beta,i})+P_{\beta\dot\beta}(\chi^i_\alpha\bar\chi^{\dot\beta}_i)]\bar\varepsilon,\\ \delta(\bar\eta^{\dot\alpha i}\bar\chi^{\dot\beta}_i)&=\frac{i}{\sqrt{P^2}}[P^{\dot\alpha\beta}(\chi^i_\beta\bar\chi^{\dot\beta}_i)+P^{\dot\beta\beta}(\bar\eta^{\dot\alpha i}\eta_{\beta,i})]\varepsilon,\\ \delta(\bar\eta^{\dot\alpha i}\eta_{\beta i})&=\frac{i}{\sqrt{P^2}}[P^{\dot\alpha\alpha}(\chi^i_\alpha\eta_{\beta i})\varepsilon+P_{\beta\dot\beta}(\bar\eta^{\dot\alpha i}\bar\chi^{\dot\beta}_i)\bar\varepsilon],\\ \delta(\chi^i_\alpha\bar\chi^{\dot\beta}_i)&=\frac{i}{\sqrt{P^2}}[P_{\alpha\dot\alpha}(\bar\eta^{\dot\alpha i}\bar\chi^{\dot\beta}_i)\bar\varepsilon+P^{\dot\beta\beta}(\chi^i_\alpha\eta_{\beta i})\varepsilon],\end{aligned} \tag{13}$$

where ε is a complex parameter of the off-diagonal transformations of $SU(2)_{\rm fl}$, and $\sqrt{P^2}=M$ when acting on a representation with a given mass M. It is obvious that (13) does not contradict the Coleman-Mandula theorem. Convolution of two dotted or two undotted indices on the left-hand side of the first or the second line will produce current divergences on the right-hand side, as I have already mentioned.

It is obvious that the multiplet (12) is closed and irreducible, and all mesons in this multiplet must have degenerate masses. Altogether we have 12 distinct degenerate spin-1 states (including triple spin degeneracy), instead of two nondegenerate (real) sextets (including triple spin degeneracy) that would be present in the spectrum if the symmetry of the problem was just $U(1)\times U(1)$. In conventional QCD we would have four distinct real spin-1 triplets.

V. $J=0$ MESONS

In the theory with one Dirac spinor under consideration, spin-zero mesons are produced by the operators $\chi_\alpha\eta^\alpha$ and $\bar\chi_{\dot\alpha}\bar\eta^{\dot\alpha}$. In terms of real fields we have two degenerate mesons: one scalar and one pseudoscalar. The parity degeneracy is due to the chiral symmetry restoration. Unlike spin-1 states, no extra dynamical symmetry can emerge in this channel.

VI. TWO DIRAC SPINORS

In principle, it is not difficult to generalize to two (or more) flavors. All enhanced degeneracies found by Glozman *et al.* can be explained by $U(2)_{\rm fl}$, as discussed in Sec. IV, times $SU(2)\times SU(2)$ chiral symmetry of the theory with two Dirac spinors. Whether or not full $U(4)$ emerges will become clear only after the dynamical reason is fully understood.

VII. POSSIBLE DYNAMICAL REASON

The symmetry as in (13) could emerge in truncated QCD, provided truncation suppresses quark spin interactions in the background gluon field (to be integrated over in color-singlet two-point functions). In other words, in this

[5]This was emphasized by Arkady Vainshtein.

[6]In truncated QCD, unlike QCD *per se* the action of the operator $P_{\alpha\dot\alpha}$ on the quark field (e.g. $iD_{\alpha\dot\alpha}\bar\eta^{\dot\alpha}$) does not produce zero.

(and only in this) section, I will use the Dirac rather than the Weyl notation—it turns out to be more economic in the case at hand.

Let us have a closer look at the propagator of the Dirac fermion in the operator form (see, e.g., Ref. [10]),

$$G(x,y) = \left\langle y \middle| \mathcal{P}\left(\mathcal{P}^2 + \frac{i}{2}G_{\mu\nu}\sigma^{\mu\nu}\right)^{-1} \middle| x \right\rangle \qquad (14)$$

where $G_{\mu\nu}$ is the background gluon field, and $\mathcal{P}_\mu$ is the momentum operator in this background field. While $\mathcal{P}^2$ does not carry spinor indices, both $\mathcal{P}$ and $\sigma^{\mu\nu}$ have them. In particular,

$$G_{\mu\nu}\sigma^{\mu\nu} = \begin{pmatrix} \vec{\sigma}\,\vec{E} + i\vec{\sigma}\,\vec{B} & 0 \\ 0 & -\vec{\sigma}\,\vec{E} + i\vec{\sigma}\,\vec{B} \end{pmatrix}, \qquad (15)$$

where $\vec{E}$ and $\vec{B}$ are chromoelectric and chromomagnetic fields, respectively.[7] The upper left corner is associated with propagation of χ and the lower right corner with $\bar{\eta}$. They do not mix in the massless theory. Usually a mass term is needed for infrared regularization of the Green function. However, the truncation procedure discards the zero and low-lying modes and, therefore, automatically provides an infrared regularization.

Assume that truncation somehow suppressed the spin terms in (14), so one can replace (14) by

$$G(x,y) \to \langle y|\mathcal{P}\mathcal{P}^{-2}|x\rangle. \qquad (16)$$

Accepting (16) as a working hypothesis will result in a symmetry enhancement. In particular, the two-point functions in the channels $(\frac{1}{2},\frac{1}{2})$ and (1,0),(0,1) become proportional. Namely,

$$i\int d^4x e^{iqx}\langle \bar{\Psi}(x)\Gamma\Psi(x), \bar{\Psi}(0)\Gamma\Psi(0)\rangle \qquad (17)$$

where $\Gamma = \gamma^\mu$ and $\Gamma = \sigma^{\mu\nu}$ can be shown to be equal up to trivial kinematical structures.

Thus the assumption (16) would explain the degeneracy enhancement detected by Glozman *et al.* The problem is that the very same assumption predicts even further degeneracy, between spin 1 and spin 0 (e.g., the correlation functions, say, for $\Gamma = \gamma^\mu$ and $\Gamma = \gamma^5$ are the same). As was mentioned above the issue with scalar and pseudoscalar channels, as they are observed now in the numerical calculations of Golzman *et al.*, remains open since the corresponding correlator cannot be fitted by exponentials. So far, the exponential fit occurs only in the case of the spin-1, spin-2, etc. channels [4–6]. This situation is controversial.

[7] I use gamma matrices in the spinorial representation.

VIII. A CURIOSITY: TRADING COLOR FOR LORENTZ INDICES

This section is not directly related to the previous sections. It presents an observation in passing that was seemingly overlooked in the past.

A. $SU(4)_c$ and two-index antisymmetric quarks

This example was not discussed in [11], although the two-index antisymmetric representation of $SU(4)_c$ is quasireal and should have been included in the analysis. Since the spinors $\chi_\alpha^{[ij]}$, $\eta_{\alpha[km]}\varepsilon^{kmij}$ transform in the same manner under color and Lorentz transformation, they can be rotated into each other. Thus, the flavor symmetry in this case is $SU(2N_f)\times U(1)$. Since the Levi-Civita tensor ε^{kmij} is symmetric under the interchange $[ij] \leftrightarrow [km]$, the pattern of the chiral symmetry breaking in (untruncated) QCD will be the same as for the adjoint quarks, namely,

$$SU(2N_f) \to SO(2N_f), \qquad (18)$$

with $2N_f^2 + N_f - 1$ Goldstone bosons (pions). For a single Dirac flavor we have two Goldstones, and for two Dirac flavors we have nine pions in the symmetric two-index representation of $O(4)$. If the idea of [4–6]—the quark condensate suppression—is correct, then QCD truncation would eliminate all Goldstones and restore the full $SU(2N_f)$ flavor symmetry.

For $SU(3)_c$, the two-index antisymmetric quark is identical to fundamental quarks. The advantage of two-index antisymmetric quarks becomes obvious [12] at large N.

B. $SU(2)_c$ and fundamental quarks

This example was analyzed long ago [11]. In $SU(2)_c$ all representations are either real or quasireal. The fundamental representation is quasireal. The extended chiral symmetry at the Lagrangian level is $SU(2N_f)\times U(1)$, where N_f is the number of *Dirac* flavors. The pattern of the chiral symmetry breaking in (untruncated) QCD is

$$SU(2N_f) \to Sp(2N_f), \qquad (19)$$

with $2N_f^2 - N_f - 1$ Goldstone bosons (pions). In the simplest example $N_f = 1$, the chiral symmetry is unbroken since $Sp(2)$ is isomorphic to $O(3)$ and to $SU(2)$. No Goldstones emerge. In the case $N_f = 2$ considered in [4–6] we have $SU(4)\to Sp(4)\sim O(5)$, with five Goldstones. Suppressing the gluon condensate as in [4–6] one can conclude that in truncated QCD the full $U(4)_{\rm fl}$ is restored.

C. Reducing to $D=2$ and 3

Starting from the four-dimensional Dirac spinor (4) in $SU(3)_{\rm c}$ QCD, we can dimensionally reduce the theory

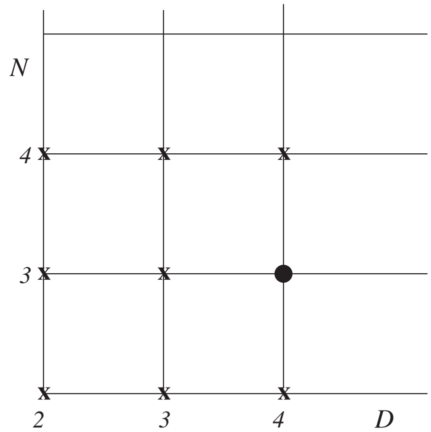

FIG. 1. Chiral symmetry vs number of dimensions and colors. The closed circle shows actual QCD. Crosses denote theories with naturally enhanced chiral symmetry.

down to $D = 3$ or even $D = 2$. More exactly, we discard one (or two) spacial dimensions leaving everything else intact.

If $D = 3$ the number of Lorentz rotations reduces to three: two boosts and one spacial rotation. This implies that the Lorentz group is just a single $SU(2)$, and the distinction between dotted and undotted spinors disappears.

The Dirac spinor in Eq. (4) is composed of two two-component spinors; with two flavors we get four two-component spinors. Then the flavor symmetry is obviously $U(4)$.

The Lorentz group in two dimensions includes just a single boost. Needless to say, the Dirac spinor (4) can be decomposed into two Dirac 2D spinors, again implying that the flavor symmetry is $U(4)$.

Thus, if four-dimensional truncated QCD dynamically "selects" three-dimensional geometry at least with regards to spin degrees of freedom, then one would expect the spectrum of the theory to be (approximately) $U(4)$ symmetric. By the same token, in the general case of N_f flavors, we would get $U(2N_f)$. This is not in contradiction with the Coleman-Mandula theorem.

The above observations are summarized in Fig. 1.

IX. CAVEATS

Truncated QCD with the vanishing quark condensate (and presumed confinement) contradicts the Casher argument [13] that confinement necessarily leads to chiral symmetry breaking. Although Casher's argument is imprecise, it still tells us that the procedure of discarding near-zero modes from the quark propagators needs more theoretical understanding.

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APPENDIX

Here I describe a mnemonic procedure (two generators) acting on quarks, although it is ill defined for quarks because of their masslessnes. This appendix is only for mnemonics. The actual physical context making it well defined is explained in Sec. IV. It is important to understand that the quark mass term or a nonvanishing quark condensate would ruin the transformations presented below. However, we consider a massless quark in combination with a vanishing biquark condensate in truncated QCD [1].

The energy-momentum operator $P_{\alpha\dot\beta}$ generating spacetime shifts carries one dotted and one undotted index. Therefore, the transition $\alpha \leftrightarrow \dot\alpha$ can be achieved by combining $P_{\alpha\dot\beta}$ with the generators of $SU(2) \times SU(2)$ Lorentz rotations (boosts). The meaning of the operator $(P^2)^{-1/2}$ is explained in Sec. IV. The quark transformation "laws" are

$$\delta\chi_\alpha = \frac{i}{\sqrt{P^2}} P_{\alpha\dot\alpha}\bar\eta^{\dot\alpha}\bar\varepsilon, \qquad \delta\bar\eta^{\dot\alpha} = \frac{i}{\sqrt{P^2}} P^{\dot\alpha\alpha}\chi_\alpha\varepsilon \tag{A1}$$

(and, of course, the Hermitian conjugate of the above). Here ε is a complex transformation parameter. The 4×4 generator matrices (analogs of the Pauli matrices) can be written as

$$\Sigma_1 = \frac{1}{\sqrt{P^2}} \begin{pmatrix} 0 & P_{\alpha\dot\alpha} \\ P^{\dot\alpha\alpha} & 0 \end{pmatrix}, \qquad \Sigma_2 = \frac{1}{\sqrt{P^2}} \begin{pmatrix} 0 & -iP_{\alpha\dot\alpha} \\ iP^{\dot\alpha\alpha} & 0 \end{pmatrix}. \tag{A2}$$

Moreover, the matrices

$$\Sigma_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad I = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \tag{A3}$$

where I is the 2×2 unit matrix, being diagonal, are responsible for independent phase rotations. We keep the above conservation laws in mind. The commutation relations for the generators (A2) are exactly the same as for the Pauli matrices.

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