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Radial Regge trajectories and leptonic widths of the isovector mesons

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It is shown that two physical phenomena are important for high excitations: (i) the screening of the universal gluon-exchange potential and (ii) the flattening of the confining potential owing to creation of quark loops, and both effects are determined quantitatively. Taking the first effect into account, we predict the masses of the ground states with $l = 0$, 1, 2 in agreement with experiment. The flattening effect ensures the observed linear behavior of the radial Regge trajectories $M^2(n) = m_0^2 + n_r\mu^2$ GeV^2, where the slope μ^2 is very sensitive to the parameter γ, which determines the weakening of the string tension $\sigma(r)$ at large distances. For the ρ trajectory the linear behavior starts with $n_r = 1$ and the values $\mu^2 = 1.40(2)$ GeV^2 for $\gamma = 0.40$ and $\mu^2 = 1.34(1)$ GeV^2 for $\gamma = 0.45$ are obtained. For the excited states the leptonic widths $\Gamma_{ee}(\rho(775)) = 7.0(3)$ keV, $\Gamma_{ee}(\rho(1450)) = 1.7(1)$ keV, $\Gamma_{ee}(\rho(1900)) = 1.0(1)$ keV, $\Gamma_{ee}(\rho(2150)) = 0.7(1)$ keV, and $\Gamma_{ee}(1^3D_1) = 0.26(5)$ keV are calculated, if these states are considered as purely $q\bar{q}$ states. The width $\Gamma_{ee}(\rho(1700))$ increases if $\rho(1700)$ is mixed with the 2^3S_1 state, giving for a mixing angle $\theta = 21°$ almost equal widths: $\Gamma_{ee}(\rho(1700)) = 0.75(6)$ keV and $\Gamma_{ee}(1450) = 1.0(1)$ keV.

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I. INTRODUCTION

Meson spectroscopy continues to be an important issue both for experimentalists and theoreticians. More precise experimental data have appeared in the last years [1–9] and a large number of theoretical works are devoted to light-meson properties [10–20]. The important idea that mesons have universal properties, from light-light to heavy quarkonia, is supported in many studies [21–28] and a kind of universal $q\bar{q}$ potential was used for all mesons in different models [9,10,12–16,22–24]. The detailed analysis of meson spectra was done in the relativized potential model (RPM), introducing a phenomenological (universal) potential [9]. A convenient systematics of radial excitations was suggested in Ref. [11], where it was assumed that the slope of the radial Regge trajectories (RTs) has a universal value (with a good accuracy) for all mesons. However, up to now the discussions continue about the true value of the slopes of the radial RTs [24–27], and even the linearity of the radial RTs is disputed [28]. However, the physical effects which are responsible for the observed universality remain unclear up to now.

Here we use the relativistic string Hamiltonian (RSH) [12,13], derived in the framework of the field correlator method [29,30], which allows for expressing the meson properties via two fundamental parameters: the string tension and the QCD constant Λ. In principle, the RSH contains both perturbative and nonperturbative dynamics, and yields also the spin-dependent interactions, so that, in general, all possible dynamical regimes in the $q\bar{q}$ systems can be addressed. It is the main purpose of our work to start a general analysis of the light-meson dynamics both in radial and orbital excitations. However, in the present paper we confine ourselves to the case of the radial excitations of the vector mesons $\rho(nS)$ and the ground states with $l = 0$, 1, 2, where the physical picture is more simple and transparent. Thus, our analysis can be considered as the first step towards the overall picture, which may be more complicated.

We show that in light mesons the dynamics is more complicated than in heavy quarkonia, which manifests itself in two effects: the so-called screening of the gluon-exchange (GE) interaction and the flattening of the linear confining potential, which is especially important for high excitations. These phenomena occur for extended objects owing to $q\bar{q}$ holes (loops), which are created inside the film subtended by the Wilson loop. These two effects can be described by the RSH and will be the main subject of our analysis. As was shown in Ref. [12], the RSH defines two regimes: the string regime, valid for the states with large l, $l \geq 3$, and the potential-like regime, taking place for low-lying states. For the ground states (with large l) the mass formula $M^2(l, n_r = 0) = 2\pi\sigma\sqrt{l(l+1)}$ was derived, which explicitly shows that the slope of the leading RT is equal to $2\pi\sigma = (1.13 \pm 0.02)$ GeV^2 with great accuracy (e.g. for $l = 3$ the accuracy is 0.8%). To derive this expression it was assumed that in RSH the centrifugal term (the rotation of the string) gives a large contribution,

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while the corrections to the mass from the GE, the spin-dependent potentials, and the self-energy term are considered to be small and may be neglected. However, for low-lying states these terms in the $q\bar{q}$ potential are not small [22,23] and therefore the question arises how to match the mass relations, valid for the states with large l, and those with $l = 0$, 1, 2. First, we discuss the well-established features of the universal $q\bar{q}$ interaction.

In the RSH approach the $q\bar{q}$ potential is defined in a gauge-invariant way via the Wilson loop [29,30] and the confining potential is shown to be scalar and linear (if no quark loops occur): $V_C(r) = \sigma_0 r$, with the string tension $\sigma_0 = 0.18(2)$ GeV^2 fixed by the slope of the leading RT (with $j = l + s$) [31,32]. In the static potential, the confining and GE potentials enter as a sum to satisfy the Casimir scaling, observed on the lattice with very good accuracy [33,34]. A very important point is that the parameters of the GE potential cannot be taken arbitrarily but have to be determined in full correspondence with the existing information from pQCD [35]. From high energy experiments the QCD constant $\Lambda_{\overline{\rm MS}}(n_f = 5)$ is now well established, while the QCD constants for $n_f = 3$, 4 are defined by matching the coupling at the quark mass thresholds [35–37]; it gives the value $\Lambda_{\overline{\rm MS}}(n_f = 3) = (339 \pm 10)$ MeV, if $\alpha_s(M_Z) = 0.1184(7)$ is used [35], or a bit smaller $\Lambda_{\overline{\rm MS}}(n_f = 3) = (327 \pm 12)$ MeV is obtained for the new world average $\alpha_s(M_Z) = 0.1177(13)$ [37]. Knowledge of $\Lambda_{\overline{\rm MS}}(n_f = 3)$ is very important, because its value determines the "vector" constant $\Lambda_V(n_f = 3)$, entering the vector coupling in the GE potential: $\Lambda_V(n_f = 3) = 1.4753\Lambda_{\overline{\rm MS}}(n_f = 3) = (485 \pm 25)$ MeV [36]. Besides, as shown recently in Ref. [38], the infrared regulator (IR) M_B is not an extra parameter, but can be expressed via the string tension: $M_B^2 = 2\pi\sigma = 1.13(11)$ GeV^2 (the accuracy of calculations is estimated to be ∼10%). Then taking the central values, $\Lambda_V = 0.485$ GeV and $M_B = 1.13$ GeV, one defines the two-loop freezing coupling (called *critical*), $\alpha_{\rm crit} = \alpha_V(q^2 = 0) = 0.6065$. Surprisingly, this value of the two-loop $\alpha_{\rm crit}$ with a large $\Lambda_V \sim 480$ MeV coincides with the one-loop phenomenological $\alpha_{\rm crit}$ from Ref. [9], where a very small (unrealistic) $\Lambda(n_f = 3) = 200$ MeV is used. Knowledge of this freezing constant $\alpha_{\rm crit} = 0.60 \pm 0.04$ is crucially important for heavy quarkonia, where the GE interaction remains important up to high excitations. Unfortunately, the role of the GE interaction in light mesons is not fully understood and here we pay special attention to the correct definition of the universal potential to distinguish between true dynamical effects and artifacts coming from different fitting parameters, including the constituent masses. Many features of the light-meson dynamics become more transparent if one studies the S-wave isovector mesons, which are more simple from the theoretical point of view, since they are not subject to chiral effects (with exception of the π-meson) and for them there does not exist a complicated centrifugal term in the RSH.

We now pay special attention to the radial RTs with the systematics, suggested in Ref. [11], assuming that the radial RTs are linear in the (n_r, M^2) plane (J^{PC} is fixed),

$$M^2(n_r) = M^2(n_r = 0) + n_r\mu^2. \tag{1}$$

Here $M(0)$ is the mass of the lowest-lying meson on the RT and μ^2 is the slope parameter. According to Refs. [8,11] the slope μ^2 is approximately the same for all radial RT trajectories, $\mu^2 = 1.25 \pm 0.15$ GeV^2. This strong statement cannot be checked in many cases, since no sufficient experimental information is available about high radial excitations, with exception of the ρ family, and in the literature there are also other predictions for μ^2 [18,25], and even the linear behavior is disputed [28]. The question is whether this slope is universal or not. Notice, that in different RPMs [9,16,23] a much larger mass difference $\mu_1^2 = M^2(\rho(1450)) - M^2(\rho(775)) = (1.54 \pm 0.04)$ GeV^2 is obtained and this value agrees with experimental $\mu_1^2(\exp)$, if the central values of the mass, $M(\rho(775))$ and $M(\rho'(1450))$ from the Partical Data Group [1] are taken. An even larger value $\mu_1^2(\exp) = 1.63(2)$ GeV^2 corresponds to the recent *BABAR* data for $M(\rho') = (1493 \pm 15)$ MeV [5]. In the present paper we will show that this large mass difference is not accidental and occurs because the $\rho(775)$ mass is "too small" due to large GE and self-energy contributions. For that reason (in contrast to other RTs) the linear behavior of the radial ρ trajectory starts with the first excitation $n_r = 1$.

Our calculations here are done in closed-channel approximation, neglecting the widths and hadronic shifts, while in a strict sense light mesons have to be studied as many-channel systems, taking into account a contribution of every channel to the meson wave function (WF). But such many-channel calculations form a very difficult task, which needs individual consideration of every meson and a complete theory of meson decays, which does not exist now. Therefore, calculations in closed-channel approximation continue to be very important: they allow for a separation of the conventional $q\bar{q}$ mesons from multiquark systems of a different nature [7]. Moreover, the influence of open channels can be effectively taken into account, introducing the string tension $\sigma(r)$ depending on the separation r [23]. This effect occurs owing to the creation of virtual quark loops in the Wilson loop, causing the string tension to decrease and depend on r, and this effect is very important for higher radial excitations, while the ground states are not affected by this *flattening* effect, since they have relatively small sizes.

An important point is that one can introduce the critical value of the string tension, $\sigma_{\rm crit}$, when the breaking of the $q\bar{q}$ string takes place. If the string tension is taken as in Ref. [23]: $\sigma(r) = \sigma_0(1 - \gamma f(r))$ [with $\sigma_0 = 0.18(2)$ GeV^2], then at not too large distances, $r \leq 1.2$ fm the string tension is almost constant, $\sigma(r) \approx \sigma_0$, while at larger distances the function $f(r) \to 1$ and the critical value is

$$\sigma_{\rm lim} = \sigma_0(1-\gamma). \tag{2}$$

The calculations show that a good description of the radial excitations is reached if the parameter $\gamma = 0.43 \pm 0.03$ is used as a fitting parameter. Moreover, the value of γ strongly affects the slope of the radial RT and therefore it can be extracted from this slope, if there are good experimental data for the mass of the excitations with $n_r \geq 2$. In particular, more precise data on the masses of $\rho(1900)$ and $\rho(2150)$ could allow for distinguishing between the value $\mu^2 = 1.43(13)$ GeV^2, suggested in Ref. [18], and $\mu^2 = 1.365(108)$ obtained in Refs. [8,11] from the analysis of the Crystal Barrel data [2].

Here we also calculate the leptonic widths of the $\rho(n^3S_1)$ and $\rho(n^3D_1)$ states. However, the accuracy of these calculations is limited by the fact that they are done in closed-channel approximation, where the norm of the $q\bar{q}$ component of the WF at the origin remains undetermined, e.g. for the states with $J^{PC} = 1^{--}$ the WF can be schematically written as

$$\psi_{\rm S}(r) = C_{q\bar{q}}(\cos\theta\psi_{\rm S}(r) - \sin\theta\psi_{\rm D}(r)) + C_{\rm cont}(S)\psi_{\rm CS},$$
$$\psi_{\rm D}(r) = C_{q\bar{q}}(\cos\theta\psi_{\rm D}(r) + \sin\theta\psi_{\rm S}(r)) + C_{\rm cont}(D)\psi_{\rm CD}, \tag{3}$$

assuming that the $q\bar{q}$ components of the S- and D-wave WFs have equal (or close) values and allowing for $S-D$ mixing. Fortunately, knowledge of the continuum component is not important for the leptonic widths, since a multiquark component of the WF, even if it is large, gives a small contribution to the WF at the origin [39]. Thus the weight $C^2_{q\bar{q}}$ remains as the relevant unknown parameter in the closed-channel approximation which produces an uncertainty in the theoretical predictions of the leptonic widths. Here, in our calculations of the leptonic widths of $\rho(nS), \rho(nD)$ with $J^{PC} = 1^{--}$ we take $C_{q\bar{q}} = 1$.

II. THE STRING REGIME

In the RSH of light mesons, the quark mass $m_q = 0$ and all spin-dependent potentials are considered as a perturbation; then the RSH is given by the expression [12,22]:

$$H(\omega,\nu) = \omega + \frac{p_r^2}{\omega} + \frac{l(l+1)}{\omega + \int_0^1 d\beta\nu(\beta)(1-\frac{\beta^2}{2})^2} + \frac{\sigma^2 r^2}{2}\int_0^1 d\beta \frac{1}{\nu(\beta)} + \frac{1}{2}\int_0^1 d\beta\nu(\beta). \tag{4}$$

This Hamiltonian contains two variables $\omega, \nu(\beta)$, which are defined from the extremum conditions. The variable $\nu(\beta)$ is shown to be different for the states with large $l \geq 3$ (in the so-called the string regime) and for small $l \leq 2$ (the potential-like regime) in order to provide the minimal value of the mass [12,13]. In the string regime the centrifugal term and the term proportional to $\sigma^2 r^2$ dominate and thus the ground state masses $M_{\rm str}(l, n_r = 0)$ were obtained neglecting the contributions from the GE and the fine-structure potentials. In that approximation the masses of all members of the multiplet are equal to the centroid mass, which for the ground state ($n_r = 0$) with large l is

$$M^2_{\rm str}(l, n_r = 0) = 2\pi\sigma\sqrt{l(l+1)}. \tag{5}$$

From this formula one can see that the mass difference, $\beta_l = M^2_{\rm str}(l+1) - M^2_{\rm str}(l)$, is practically equal to $2\pi\sigma = 1.13(1)$ GeV^2 [$\sigma = 0.18(2)$ GeV^2] with high accuracy, e.g. for $l = 3$ the accuracy is 0.8%. The values of $M_{\rm str}(l, n_r = 0)$ are given in Table I together with experimental masses with $j = l + s$ and the centroid masses $M_{\rm cog}(l, n_r = 0)$ for $l = 1, 2$.

Table I shows the good agreement between the masses calculated according to Eq. (5), and the experimental masses for $a_4(2040), \rho_5(2350), a_6(2450)$ [1]. Surprisingly, even for a_2 and ρ_3 with $l = 1, 2$, the centroid masses have reasonable values, although the low-lying states have to be studied in the potential-like regime and for them all kinds of the interactions: the confining, the GE, the centrifugal term, are important. In the potential-like regime the RSH can be rewritten in a more convenient form, $H = H_0 + \Delta(\text{str})$, where the unperturbed part H_0 has the form of the Hamiltonian occurring in the spinless Salpeter equation (SSE) ($m_q = 0$) [22,23]:

$$H_0 = 2\sqrt{\boldsymbol{p}^2 + m_q^2} + V_0(r), \tag{6}$$

and the operator p_r^2 is replaced by $\boldsymbol{p}^2$, while the remaining part of the centrifugal term, the so-called string correction,

TABLE I. The masses $M_{\rm str}(l, n_r = 0)$ (in MeV) in the string regime Eq. (4).

l	1	2	3	4	5
$M_{\rm str}(l)$	1265	1664	1979	2249	2489
$M(\exp, j = l + s)$	1318(1)	1689(2)	1982(14)	2330(35)	2450(130)

$$\Delta(\text{str}) = -\frac{l(l+1)\sigma\langle r^{-1}\rangle_{nl}}{8\omega^2(nl)}, \tag{7}$$

is considered as a perturbation. This correction is not very large, ~50–100 MeV for $l = 1, 2$, still it cannot be neglected. In Eq. (7) the variable $\omega(nl)$ is the kinetic energy of a light quark, defined by the solutions of the SSE:

$$\left(2\sqrt{\boldsymbol{p}^2} + V_0(r)\right)\psi_{nl}(r) = M_0(nl)\psi_{nl}(r). \tag{8}$$

To define the solutions of the unperturbed Hamiltonian H_0 with $l = 0$, 1, 2, it is important to use the universal quark-antiquark potential, which has no fitting parameters and therefore allows us to separate physical effects from the artifacts introduced by fitting parameters. This potential has the form of linear plus GE terms (observed on the lattice [33] and derived in the field correlator method [34]), and successfully describes heavy quarkonia spectra [40],

$$V_0(r) = \sigma_0 r + V_{\text{GE}}(r), \tag{9}$$

with $\sigma_0 = 0.18(2)\ \text{GeV}^2$, fixed by the slope of the leading RT. In the GE potential the vector coupling in coordinate space,

$$V_{\text{GE}}(r) = -\frac{4}{3}\frac{\alpha_V(r)}{r}, \tag{10}$$

is taken in two-loop approximation, where it does not depend on the renormalization scheme, and defined via the vector coupling in momentum space:

$$\alpha_V(r) = \frac{2}{\pi}\int_0^\infty \mathrm{d}q \frac{\sin(qr)}{q}\alpha_V(q^2). \tag{11}$$

Here

$$\alpha_V(q^2) = \frac{4\pi}{\beta_0 t}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln t}{t}\right), \tag{12}$$

where for $n_f = 3$, $\beta_0 = 9$, $\beta_1 = 64$ and in the logarithm $t(q^2) = \ln(\frac{q^2+M_B^2}{\Lambda_V^2})$, the vector constant $\Lambda_V(n_f = 3) = (480 \pm 20)$ MeV corresponds to $\Lambda_{\overline{\text{MS}}}(n_f = 3) = (327 \pm 15)$ MeV from pQCD, while the IR regulator $M_B = \sqrt{2\pi\sigma} = 1.13(11)\ \text{GeV}^2$ was defined in Ref. [38] (see the discussion in the Introduction). At $q^2 = 0$ the logarithm

$$t_0 = t(q^2 = 0) = \ln\frac{M_B^2}{\Lambda_V^2} \tag{13}$$

defines the freezing constant, $\alpha_{\text{crit}}(q^2 = 0) = \alpha_V(r \to \infty) = 0.60 \pm 0.04$, which is rather large (for the admissible values, $\Lambda_V = 480 \pm 20$ MeV and $M_B = 1.1$–1.15 GeV). In bottomonium, this strong GE interaction remains important up to high excitations and gives a good description of the charmonium and bottomonium spectra [40].

TABLE II. The masses of the n^3S_1 light mesons (in MeV) for the universal potential with $\sigma = 0.18\ \text{GeV}^2$ and $\alpha_{\text{crit}} = 0.6086$. Experimental data are taken from Refs. [1,7].

$n = n_r + 1$	1	2	3	4
$M(n^3S_1)$	693	1478	2046	2510
experimental [1]	775	1465 (25)	1909(42)	2150(90)
data [7]	775	1493(15)	1861 (17)	2254(22)

However, for the light mesons this universal potential appears to be too strong, giving smaller masses for the $1S$, $1P$, and $1D$ ground states (see Table II). This result does not change if the parameters of the vector coupling vary within the admissible range. It also shows that the dynamics in light mesons, which all lie above open hadronic thresholds, is more complicated due to their large spatial extensions and the appearance of virtual $q\bar{q}$ loops in the Wison loop of large size, and hence modifying the gluon-exchange propagator. As we shall discuss later in Sec. V, the gluon effectively acquires the screening mass due to these loops as obstacles and the color-magnetic confinement [38,41]. For that reason we consider also a screened potential.

The creation of the virtual quark loops (scalars in the $^3P_0$ mechanism) decreases the string tension, making it dependent on the separation r. Due to this flattening effect the masses of excited states decrease, e.g. the mass $M(4S)$ becomes by ~300–350 MeV smaller than for a purely linear potential $\sigma_0 r$. However, it is not so for the ground states with $l = 0$, 1, 2, which have relatively small sizes ($\langle r\rangle \le 1.2$ fm) and are not affected by the flattening effect. Below, we shall find out the direct connection between the parameter responsible for the flattening of the potential, and the slope of the radial RT.

There is one more difference between light-meson masses and those of heavy quarkonia, where the centroid masses just coincide with the eigenvalue (EV) of the SSE. For a light meson its centroid mass $M_{\text{cog}}(nl)$ also includes a negative self-energy contribution $\Delta(\text{SE})$ [22,23,42] and negative string correction $\Delta(\text{str})(l = 1, 2)$, which do not introduce extra parameters. The self-energy term is very important for the mass value, since it gives a contribution to the intercept of the RT. In the case $l = 0$,

$$M_{\text{cog}}(nS) = M_0(nS) + \Delta(\text{SE}),$$
$$\Delta(\text{SE}) = -\frac{3\sigma}{\pi\omega(nl)}. \tag{14}$$

In heavy quarkonia $\Delta(\text{SE}) \sim (1$–$5)$ MeV is very small and can be neglected, while for light-light, K, and ϕ mesons, $\Delta(SE)$ is rather large due to the small value of the kinetic energy $\omega(nl) \sim (400$–$500)$ MeV in the denominator. Because of this term, the squared mass $M^2(nl)$ does not contain a term linear in $M(nl)$ and provides the linear behavior of the RT [10,23]. Notice, that in the RPM a negative subtractive constant (a fitting parameter), usually added to the potential (or the mass) [9], violates the linearity of the orbital and radial RT (see also the discussion in Ref. [15]).

In the mass of the n^3S_1 states, $M(n^3S_1) = M_0(nS) + \Delta(\mathrm{SE}) + \frac{1}{4}\Delta(\mathrm{HF})$, the hyperfine correction is defined as in Ref. [43],

$$\Delta(\mathrm{HF}) = \frac{32\pi\alpha_s(\mu_{\mathrm{hf}})|\psi_{nS}(0)|^2}{9\omega^2(nS)}, \qquad (15)$$

where the kinetic energy $\omega(nS)$ enters in the denominator. Here it is important to underline that in Eq. (15) the coupling $\alpha_s(\mu_{\mathrm{hf}})$ is not an arbitrary parameter. As shown in Ref. [43], this coupling is defined at the universal scale (for all mesons, light and heavy) $\mu_{\mathrm{hf}} \simeq T_g^{-1}$, where $T_g \simeq 0.12$ fm is the vacuum correlation length. Since the scale μ_{hf} is close to the mass of the τ-lepton, the value of $\alpha_s(\mu_{\mathrm{hf}})$ must be close to $\alpha_s(M_\tau) = 0.33(2)$ [1]. We take here $\alpha_s(\mu_{\mathrm{hf}}) = 0.31$, as it was used in Ref. [44] in the analysis of the hyperfine splitting of the B mesons and bottomonium.

In Table II the calculated $\rho(nS)$ masses are given in the typical case, when the universal potential has no screening in the GE term and the freezing constant $\alpha_{\mathrm{crit}} = 0.608$, $\sigma_0 = 0.18\ \mathrm{GeV}^2$.

From Table II one can see (i) that the strong (universal) GE potential gives the $\rho(775)$ mass, as well as the masses of the $M(a_2(1318)) = 1.240$ GeV, $M(\rho_3) = 1.59$ GeV, smaller by ~80 MeV than their experimental values. (ii) On the contrary, for high excitations, where the influence of the GE potential is small, the masses $M(3^3S_1)$ and $M(4^3S_1)$ are by ~150 MeV and ~300 MeV larger than in experiment, irrespective of the strength of the GE potential, if the linear $\sigma_0 r$ potential is used. Therefore one needs to look for another effect (reason), responsible for the strong decrease of the nS masses observed in experiments.

III. THE FLATTENING EFFECT

In Sec. V we present the physical picture explaining the flattening phenomena, observed on the lattice [45] and studied in Ref. [23], while here we give concrete results of our calculations with the confining potential, where the string tension depends on the quark-antiquark separation r,

$$\sigma(r) = \sigma_0 f(r), \qquad \lim_{r\to\infty}\sigma(r) = \sigma_0(1-\gamma). \qquad (16)$$

Here $\sigma_0 = 0.18(2)\ \mathrm{GeV}^2$ and the function $f(r) = 1 - \gamma\frac{\exp(\sigma_0(r-R_0))}{B+\exp(\sigma_0(r-R_0))}$ contains three parameters; two of them, $B \cong 15$–20 and $R_0 \cong (1.2$–$1.4)$ fm, are chosen in such way that the flattening slowly starts at rather large distances, ~1.2 fm, while the lowest-lying states with $l = 0, 1, 2$ are not affected by the flattening effect. At large distances, the string tension goes to the limiting value, $\sigma_{\mathrm{lim}} = \sigma_0(1-\gamma)$. Direct calculations show that the decrease of the $\rho(nS)(n \geq 2)$ masses occur mostly due to the flattening effect and the mass shifts are very sensitive to the value of the parameter γ in $\sigma(r)$. To reach agreement with experiment, the fitting parameter γ is to be taken in the narrow range, $\gamma = (0.43 \pm 0.03)$, however, the values of $\gamma = 0.40$ and 0.45 give rise to different slopes of the ρ trajectory. If $\gamma = 0.40$ is taken, then a contribution from the screened GE potential is more important than for $\gamma = 0.45$ (see Table III), but both variants have common features:

(1) The linear behavior of the ρ trajectory starts with $n_r = 1$, although these RTs have slightly different slopes: for $\gamma = 0.40(0.45)$ the slope $\mu^2(\rho) = 1.40(1.35)\ \mathrm{GeV}^2$.

(2) At the same time the mass difference $\mu_1^2 = M^2(\rho(2S)) - M^2(\rho(1S)) = 1.52(4)\ \mathrm{GeV}^2$ remains relatively large (in both cases) and agrees with $\mu_1^2(\exp) = M^2(\rho(1465)) - M^2(\rho(775)) = (1.55 \pm 0.07)\ \mathrm{GeV}^2$, if the central values of the experimental mass are taken. The reason why μ_1^2 is large, is discussed below, in Sec. IV.

(3) The choice of γ directly determines the slope of the radial RT and therefore it could be extracted from the experimental masses $M(\rho(3S))$ and $M(\rho(4S))$, if they would be measured with better accuracy.

In Table III we give the masses in three cases (in all cases $\sigma_0 = 0.182\ \mathrm{GeV}^2$, $m_q = 0$): in case A there is no screening of the GE potential, i.e., $\delta = 0$; in the cases B and C the exponential form of the screening, $V_{\mathrm{scr}} = V_{\mathrm{GE}}\exp(-\delta r)$ with the screening parameter $\delta = 0.20$ GeV, is taken. In the cases A and B the other parameters coincide,

$$\Lambda_V(n_f = 3) = 465\ \mathrm{MeV}, \qquad M_B = 1.15\ \mathrm{GeV},$$
$$\alpha_{\mathrm{crit}} = 0.5712, \qquad \sigma_0 = 0.182\ \mathrm{GeV}^2,$$
$$\gamma = 0.40, \qquad B = 20, \qquad R_0 = 6.0\ \mathrm{GeV}. \qquad (17)$$

In case C the stronger GE potential, with $\alpha_{\mathrm{crit}} = 0.635$, is taken, while $\gamma = 0.45$ in the flattening potential is larger than in Eq. (17). The other parameters in case C are as follows:

$$\Lambda_V = 500\ \mathrm{MeV}, \qquad M_B = 1.15\ \mathrm{GeV},$$
$$\sigma_0 = 0.182\ \mathrm{GeV}^2, \qquad B = 15, \qquad R_0 = 6.0\ \mathrm{GeV}^2. \qquad (18)$$

For all nS states the hyperfine correction to the masses is calculated with $\alpha_s(\mu_{\mathrm{hf}}) = 0.31$.

As seen from Table III, without screening ($\delta = 0$) the ground state masses of $\rho(775)$, $a_2(1318)$, and $\rho_3(1690)$ appear to be 50–100 MeV smaller than in experiment and

TABLE III. The masses of the lowest-lying states ($l = 0, 1, 2$) and excited n^3S_1 states (in MeV) for the flattening potential: case A with $\gamma = 0.40$, $\delta = 0$ GeV; case B with $\gamma = 0.40$, $\delta = 0.20$ GeV; case C with $\gamma = 0.45$, $\delta = 0.20$ GeV, $\Lambda = 500$ MeV.

	$\delta = 0$	$\delta = 0.20$	$\delta = 0.20$	
State	Case A	Case B	Case C	Exp.
1^3S_1	698[a]	790	774	775 [1]
2^3S_1	1430	1474	1468	1465 [1]
				1493(15) [7]
3^3S_1	1876	1920	1880	1909(42) [1]
				1861(17) [7]
4^3S_1	2172	2239	2170	2150(90) [1]
				2254(22) [7]
1^3P_2	1240	1312	1309	1318(1)
1^3D_3	1590	1696	1690	1689(2)

[a]Here the hyperfine contribution ∼65 MeV is taken into account.

variations of the parameters within reasonable ranges do not change this result. On the contrary, in the case B for the screened GE potential ($\delta = 0.20$ GeV, the other parameters remaining the same, as in case A) a reasonable agreement with experiment is reached. The choice of $\delta = 0.30$ GeV, i.e., stronger suppression of the GE potential, gives rise to large masses of $\rho(775)$ and $\rho(1465)$ and was neglected.

The best agreement with experimental data takes place in the case C Eq. (18), when the screening parameter $\delta = 0.20$ GeV is the same, but $\Lambda_V = 500$ MeV and $\gamma = 0.45$ are larger than in the cases A and B.

Our conclusion is that screening of the universal GE potential is necessary to obtain correct values of the masses of the lowest-lying states with $l = 0$, 1, 2, otherwise they are ∼80 MeV smaller than in experiment. For the higher nS excitations the contribution from the GE potential cannot be neglected and agreement with experiment is reached both in the cases B and C. The masses of the nP and nD ($n_r \geq 1$) states weakly depend on the screened GE potential and will be discussed in the next section.

We give here also the rms radius $R_s(nS) = \langle\sqrt{r^2}\rangle_{nS}$ of $\rho(1S)$ and $\rho(2S)$, which weakly change in all three cases: $R_s(1S) = 0.71(0.72)$ fm in the cases B (C) and a bit smaller, 0.68 fm in the case A, where there is no screening effect. The rms of $\rho(2S)$ is significantly larger, $R_s(2S) = 1.0(1)$ fm in all cases.

IV. RADIAL REGGE TRAJECTORIES

We have shown that the GE potential gives a small contribution to the masses of the high radial excitations ($n_r \geq 1$) and therefore, in first approximation, the GE potential with screening can be neglected. This allows us to reveal more explicitly the role of the flattening effect for formation of the radial RT. The most important contribution to the light-meson masses comes from the EV of the SSE, Eq. (8), the unperturbed part of the RSH, which in the case of the linear $\sigma_0 r$ potential (σ_0 is a constant) is well known. Namely, the EV of the SSE ($m_q = 0$) can be approximated with great accuracy (for $n_r \geq 1$) by the expression [16,23],

$$M_0^2(nl) = \sigma_0(8l + 4\pi n_r + 3\pi). \tag{19}$$

This formula explicitly shows that the EVs $M_0^2(n_r = 0, l)$ for the ground states for given l, lie on the orbital RT with the slope $\beta_0 = 8\sigma_0 = 1.44$ GeV^2, which is ≈27% larger than $\beta(\text{exp}) = 2\pi\sigma_0 = 1.13(1)$ GeV^2, observed in experiment. For the radial excitations, the difference between the slope in Eq. (19) and the one found in experiment is very large: $\mu_0^2 = 4\pi\sigma_0 = 2.26$ GeV^2 is 1.6–2.0 times larger than $\mu^2(\text{exp}) = (1.25 \pm 0.15)$ GeV^2 [8,11]. The question is why such a large difference occurs.

First of all, we look at the contribution to the centroid mass from the self-energy correction, Eq. (14), for which we use the relation, $M_0(nS) = 4\omega_0(nS)$ (valid for $\sigma = \text{const}$) and rewrite $\Delta(\text{SE}) = -\frac{12\sigma_0}{\pi M_0}$; then

$$M_{\text{cog}}(nS) = M_0(nS) - \frac{3.82\sigma_0}{M_0(nS)}. \tag{20}$$

In the squared mass we neglect the small squared self-energy term (although it is not small for the $1S$ state) and obtain

$$M_{\text{cog}}^2(nS) = M_0^2(nS) - 7.6\sigma_0 = \sigma_0(4\pi n_r - 7.6 + 3\pi) \\ = (0.33 + 2.26 n_r)\ \text{GeV}^2. \tag{21}$$

From here one can see that owing to $\Delta(SE)$ the value of M_{cog}^2 is smaller than M_0^2 given in Eq. (19), while the slope $\mu_0^2 = 4\pi\sigma_0 = 2.26(2)$ GeV^2 does not change. Thus we have confirmed the well-known result that the purely linear potential with $\sigma = \text{const}$ produces always a large slope of the radial RT.

The situation strongly changes, if the flattening potential $V_C(r) = \sigma(r)r$ is considered, for which the representation Eq. (19) is not valid anymore [in this case the EV of the SSE will be denoted as $\tilde{M}_0(nS)$]. Our calculations show that

(1) The linear behavior of the ρ RT starts with $n_r = 1$, because for the flattening potential (with $\gamma = 0.40$ or 0.45) the mass difference $\mu_1^2 = \tilde{M}^2(2S) - \tilde{M}^2(1S)$ remains large, $\mu_1^2 \sim 1.87(5)$ GeV^2, being still 20% smaller than $\mu_1^2 = 4\pi\sigma_0$ in Eq. (19).
(2) For the nP and nD states the linear behavior starts with $n_r = 0$.
(3) The slope $\mu^2(l)$ strongly depends on the parameter γ in $\sigma(r)$, Eq. (16), which characterizes the weakening of the confining potential.

The squared EV, $\tilde{M}_0^2(nS)$ (in GeV^2) ($n_r \geq 1$), with $\gamma = 0.40$, 0.45, 0.50 can be approximated as

$$\tilde{M}_0^2 = (2.42 + 1.40 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.40,$$
$$\tilde{M}_0^2 = (2.31 + 1.27 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.45,$$
$$\tilde{M}_0^2 = (2.25 + 1.15 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.50. \qquad (22)$$

(The accuracy of these expressions is $\sim 1\%$.)

From Eq. (22) the important result follows that for the flattening potential the squared EVs of the SSE have a much smaller slope (two times smaller for $\gamma = 0.50$), than in the case of the purely linear potential Eq. (19), which decreases for larger values of γ, i.e., a stronger flattening effect. For the centroid mass the intercept is changed, while the value of the slope is the same, so that the ρ trajectory ($n_r \geq 1$) is

$$M_{\mathrm{cog}}^2(\rho) = (0.77 + 1.40 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.40,$$
$$M_{\mathrm{cog}}^2 = (0.80 + 1.27 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.45,$$
$$M_{\mathrm{cog}}^2 = (0.90 + 1.15 n_r)\ \mathrm{GeV}^2, \quad \text{for } \gamma = 0.50. \qquad (23)$$

In all cases $M_{\mathrm{cog}}(\rho(1450)) = (1.44\text{–}1.47)$ GeV. However, if the exponential screening of V_{GE} and the hyperfine interaction are taken into account, then the slope increases while the intercept does practically not change. In the cases A and C [see the parameters of the GE potential in Eqs. (17), (18)] and for $n_r \geq 1$ we have

$$M^2(n^3S_1) = (0.78 + 1.40(2) n_r)\ \mathrm{GeV}^2 \quad (\gamma = 0.40),$$
$$M^2(n^3S_1) = (0.81 + 1.34(1) n_r)\ \mathrm{GeV}^2 \quad (\gamma = 0.45). \qquad (24)$$

Thus for $\gamma = 0.45$ the calculated ρ trajectory has $\mu^2 = 1.34(1)\ \mathrm{GeV}^2$, in agreement with the results in Refs. [11,26], where $\mu^2 = 1.365(108)\ \mathrm{GeV}^2$ was obtained from the analysis of the Crystal Barrel data [2]. On the contrary, the larger $\mu^2 = 1.40(2)\ \mathrm{GeV}^2$ for $\gamma = 0.40$ agrees with the slope, $\mu^2 = 1.43(13)\ \mathrm{GeV}^2$, predicted in Ref. [27]. Notice, that for $\gamma = 0.45$ a better agreement is obtained for the $\rho(1450)$ mass (see Table III).

In the same way, the radial RTs for the nP and nD states were considered; it appears that for $l = 1$, 2 the linear behavior of the radial RT starts with $n_r = 0$ and the squared EV of the SSE $M_0^2(nP)$ can be approximated as

$$\tilde{M}_0^2(nP) = (3.11 + 1.25 n_r)\ \mathrm{GeV}^2 \quad (\gamma = 0.45). \qquad (25)$$

Then, taking into account the self-energy and string corrections we obtain for the centroid masses,

$$M_{\mathrm{cog}}^2(nP) = (1.64(2) + 1.25 n_r)\ \mathrm{GeV}^2. \qquad (26)$$

From this expression one can obtain the a_{j} radial RT, taking into account the fine-structure splitting, which does practically not change the slope, but introduces a fitting parameter. For that reason we restrict ourselves to the RTs for the centroid masses. Notice that $\mu^2(nP) = 1.25\ \mathrm{GeV}^2$ practically coincides with the slope for the centroid masses of the nS states, if $\gamma = 0.45$.

For the nD trajectory the EVs of the SSE have a smaller slope ($n_r \geq 0$),

$$\tilde{M}_0^2(nD) = (4.36 + 1.11(5) n_r)\ \mathrm{GeV}^2 \quad (\gamma = 0.45) \qquad (27)$$

and

$$M_{\mathrm{cog}}^2(nD) = (2.8(1) + 1.11(5) n_r)\ \mathrm{GeV}^2 \quad (\gamma = 0.45). \qquad (28)$$

Notice, that the slope $\mu^2(l)$ decreases for increasing angular-momentum l. At this point it is important to stress that for physical nD states the GE contribution is much smaller than that for the nS states, and therefore for the ρ_3, ρ_2, and $\rho(n^3D_1)$ trajectories the slopes have to be close to the one given in Eq. (28), where $\mu^2(D) = 1.11(5)\ \mathrm{GeV}^2$, if the fine-structure effects are neglected. Our result is in agreement with $\mu^2(a_2) = 1.00(6)\ \mathrm{GeV}^2$ and $\mu^2(a_1) = (1.084 \pm 0.63)\ \mathrm{GeV}^2$, predicted for the a_1 and a_2 RTs from the analysis of experimental data in Ref. [26].

Now we briefly discuss the reasons why the mass difference $\mu_1^2 = M^2(2S) - M^2(1S)$ is large. The first reason is that this factor is very large for the flattening potential (without GE interaction), where $\mu_1^2 = 1.87(5)\ \mathrm{GeV}^2$ for the squared EV of SSE. This result does not change, if a reasonable choice of the parameters in $\sigma(r)$ is made. Secondly, if the GE interaction is taken into account, then for the $1S$ state, localized at rather small distances, the self-energy and hyperfine corrections decrease the mass difference μ_1^2 but its value remains rather large, $\mu_1^2 = 1.56(6)\ \mathrm{GeV}^2$. This number appears to be very close to what is observed in experiment, $\mu_1^2(\exp) = 1.55(7)\ \mathrm{GeV}^2$ [1], if the central values of the $\rho(775)$ and $\rho(1465)$ masses are used. Just for that reason, the linear behavior of the ρ RT begins with $n_r = 1$, while other radial RTs start with $n_r = 0$.

V. FLATTENING PHENOMENON: THE PHYSICAL PICTURE

The dynamics of light mesons is more complicated than that in heavy quarkonia, since light mesons, as rather extended objects, are sensitive to detailed properties of the confinement mechanism, which also affects the gluon exchanges. Our approach is based on the background perturbation theory (BPT) [46], which takes into account the nonperturbative background with confinement and does not contain unphysical singularities (the Landau ghost poles and IR renormalons), present in standard perturbation theory. Below, we illustrate how the BPT predicts three effects, which are observed in experiment and especially important for light mesons:

(1) Stabilization of the coupling $\alpha_s(q^2)$ at $q^2 \to 0$, $\alpha(0) \equiv \alpha_{\rm crit}$;
(2) Screening of α_s at large distances;
(3) Flattening of the string tension at large distances, $\sigma \to \sigma(r)$.

Item 1. The basic feature of BPT is the gauge-invariant treatment of confinement and gluon exchanges, when both phenomena occur owing to the Wilson loop, where confinement creates the minimal-area surface (the so-called confining film) and the gluon-exchange trajectories are necessarily present inside this surface. As a result, the gluon loops, appearing on these trajectories and responsible for asymptotic freedom, create open loops in the confining film, and this effect strengthens with increasing α_s, leading finally to the saturation of $\alpha_s(q^2)$ which in two-loop approximation is given by $\alpha_{\rm crit} = \frac{4\pi}{\beta_0 t_0}(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln t_0}{t_0})$, where $t_0 = \ln(\frac{M_B^2}{\Lambda^2})$ and with 10% accuracy $M_B^2 = 2\pi\sigma$ [41].

Item 2. For the same reason, the scalar $q\bar{q}$ loops, appearing in the film (of large size), lead to the screening of the GE interaction, since any gluon trajectory, propagating inside the confining film, is interrupted by the scalar loops and those create an effective mass of the gluon. In addition, there is a difference between the free propagation (free Green's function of the gluon) and the gluon propagation inside the surface with confinement. This complicated phenomenon was studied in Ref. [47] for zero temperature and in Ref. [48] for the deconfined phase, where it occurs due to the color-magnetic confinement. This effect at zero temperature, when both color-electric and color-magnetic confinement collaborate, is not yet finally settled and therefore in our paper we exploit the effective screening parameter δ for the screening mass. Our analysis has shown that for the exponential form of screening $\delta = 0.20$ GeV is the preferable value, while suppression of the GE potential is too strong for $\delta = 0.30(0.10)$ GeV.

Item 3. For excited light mesons, confinement occurs in a highly excited string, when the Wilson loop has a free boundary and several typical features, partly discussed above. Namely, there exist

(i) a finite density of the $q\bar{q}$ loops in the confining film, which leads to the dependence of the string tension on r, $\sigma \to \sigma(r)$;
(ii) a possibility to decay, virtually or really, into a pair (or several) mesons, so that if the distance r in the confining potential σr exceeds the separation, $R_{\rm f} \sim 2r_\pi \simeq 1.2$ fm, then the flattening of the potential is expected.

The first feature can also be seen in the T dependence of $\sigma(r)$: when the density of the $q\bar{q}$ loops grows with increasing temperature T, then the potential $V(r,T)$ becomes more and more flat, as it was observed on the lattice [49]. Another manifestation of the flattening phenomenon was recently studied in Ref. [50]: while applying a magnetic field parallel to the confining film, it was observed that the density of the $q\bar{q}$ loops increases and the string tension $\sigma(r)$ flattens, in agreement with the lattice data [51].

Both features, flattening due to a finite $q\bar{q}$ density and the existence of the critical length $R_{\rm f} \sim 2r_\pi$ are embodied in the form of the string tension, Eq. (16), which is used in our paper.

In conclusion we give the rms radius of the $\rho(nS)$ mesons, $r_s = \langle\sqrt{r^2}\rangle_{nS}$, calculated for the sets of the parameters Eqs. (17), (18): for $\rho(775)$, $r_s(1S) = (0.71\text{–}0.73)$ fm and for $\rho(1450)$, $r_s = (0.9\text{–}1.0)$ fm.

VI. THE LEPTONIC WIDTHS OF $\rho(n\,^3S_1)$ AND $\rho(1\,^3D_1)$

The decay constants f_V and leptonic widths of the $\rho(n^3S_1)$ mesons are calculated here, considering them as pure $q\bar{q}$ states, i.e., taking $C_{q\bar{q}} = 1.0$ in the WF given in Eq. (3). For the decay constant in the vector channel f_V we use the expression from Ref. [52], where the correlator of the currents (in different channels) is derived using the functional integral representation and on the final stage expanding this correlator in the complete set of eigenfunctions of the RSH H_0, Eq. (6). This gives

$$f_V^2 = 12\bar{e}_q^2\frac{|\psi_n(0)|^2\xi_V}{M_V(nS)} = \frac{3\bar{e}_q^2|R_n(0)|^2\xi_V}{\pi M_V(nS)}, \tag{29}$$

and

$$\Gamma_{\rm ee}(n^3S_1) = \frac{4\pi\alpha^2 f_V^2\beta_{\rm QCD}}{3M_V}. \tag{30}$$

Here, for a light meson with $m_q = 0$ the relativistic factor $\xi_V(nS)$ is

$$\xi_V = \frac{\omega_n^2 + \frac{1}{3}\boldsymbol{p}^2}{2\omega_n^2}, \tag{31}$$

which for the ground and excited states are almost equal, $\xi(1S) = 0.70(1)$ and $\xi(nS) = 0.72(1)$, $(n = 2,3,4)$, if the static potential with the parameters Eqs. (17), (18) is used (for the ρ-mesons the average $\bar{e}_q^2 = 1/2$). The factor $\beta_{\rm QCD} = 1 - \frac{16}{3\pi}\alpha_s = 0.40$ takes into account the radiative corrections [53] and here we use for all (n^3S_1) states the same coupling $\alpha_s(\mu_s) = 0.353$ (at the scale $\mu_s \sim 1.0$ GeV). If the confining potential flattens at large distances, then the WFs at the origin $R_{\rm nS}(0) \sim (0.36\text{–}0.33)$ GeV$^{3/2}$$(n = 2\text{–}4)$ have close values, while for the ground state the WF $R_{\rm 1S}(0) = (0.376 \pm 0.008)$ GeV$^{3/2}$ is larger, and for the $\rho(775)$ the decay constant and leptonic width are

$$f_V = (245 \pm 6)\ \text{MeV}, \tag{32}$$

where the uncertainty comes from that in the WF at the origin, and

$$\Gamma_{ee}(\rho(775)) = (7.0 \pm 0.3)\ \text{keV}. \tag{33}$$

To calculate the leptonic widths of the higher $\rho(nS)$, it is convenient to use the ratio of the leptonic widths, $\Gamma_{ee}(n^3S_1)/\Gamma_{ee}(\rho(775))$, where the factors $\xi(nS)$ and β_{QCD} drop out. This gives

$$\begin{aligned} \Gamma_{ee}(2^3S_1) &= 0.24\Gamma_{ee}(\rho(775)) = 1.7(1)\ \text{keV}, \\ \Gamma_{ee}(3^3S_1) &= 0.14\Gamma_{ee}(\rho(775)) = 1.0(1)\ \text{keV}, \\ \Gamma_{ee}(4^3S_1) &= 0.096\Gamma_{ee}(\rho(775)) = 0.7(1)\ \text{keV}, \end{aligned} \tag{34}$$

where the uncertainties come from the experimental errors in the $\rho(nS)$ masses and the WFs at the origin. Notice that in a realistic situation the leptonic widths of the excited $\rho(nS)$ mesons may be smaller, if the $q\bar{q}$ component $C_{q\bar{q}}$ in their WFs is less than 1.0.

The leptonic widths of the (n^3D_1) states are calculated defining their WFs at the origin via the second derivative, according to the prescription from Ref. [54]: $R_{nD}(0) = \frac{5R''(0)}{2\sqrt{2}\omega_{nD}^2}$, where $R''_{1D}(0) = 0.026(1)\ \text{GeV}^{7/2}$ and $\omega(1D) = 0.536$ GeV. It gives $R_{1D}(0) = 0.163(3)\ \text{GeV}^{3/2}$, which is not very small due to the small value of the kinetic energy $\omega_{nD} \sim 0.5$ GeV. The other parameters are $\xi(1D) = 0.69$, $\beta_{QCD} = 0.40$, $M(1^3D_1) = 1.72(2)$ GeV, so that the leptonic width

$$\Gamma_{ee}(1^3D_1) = 0.26(5)\ \text{keV} \tag{35}$$

is rather small. However, its value may increase owing to the $2S - 1D$ mixing, and for a mixing angle $\theta = 21°$ the leptonic widths of $\rho(1450)$ and $\rho(1700)$ become almost equal:

$$\begin{aligned} \Gamma_{ee}(\rho(1450)) &= 1.0(1)\ \text{keV}, \\ \Gamma_{ee}(\rho(1700)) &= 0.75(6)\ \text{keV} \quad (\theta = 21°). \end{aligned} \tag{36}$$

Here it was assumed that in the WFs of $\rho(1450)$ and $\rho(1700)$ the $q\bar{q}$ components are equal.

VII. CONCLUSIONS

We have studied the light-meson properties with the use of the RSH, which allows us to investigate the light-meson dynamics without introducing fitting parameters. It appears that the universal static potential, successfully applied to heavy quarkonia, gives rise to small masses of the lowest states with $l = 0, 1, 2$ and at the same time large masses of the excited states. To explain the physical spectrum, two effects, the screening of the GE interaction and the flattening of the confining potential, which appear owing to quark-loop creation, are to be taken into account. We have demonstrated the following properties.

(1) The screening of the GE potential, taken as an exponential function with the screening parameter $\delta = 0.20$ GeV, gives the masses of the lowest-lying states for each l in agreement with experiment.

(2) The slope of the radial RT is very sensitive to the value of the parameter γ, which determines the flattening of the string tension $\sigma(r)$: at large distances $\sigma(r) \to \sigma_0(1-\gamma)$. The parameter γ could be extracted from the experimental masses of $\rho(1900)$ and $\rho(2150)$, if these were measured with better accuracy, while now it is taken from the range $\gamma = 0.43 \pm 0.03$.

(3) From our calculations two values for the slope of the ρ trajectory are obtained, $\mu^2(\rho) = 1.40(2)\ \text{GeV}^2$ for $\gamma = 0.40$ and $\mu^2(\rho) = 1.34(1)\ \text{GeV}^2$ for $\gamma = 0.45$, neither result contradicts the existing experimental data.

(4) The linear behavior of the radial RT starts with $n_r = 0$ for the nP and nD trajectories, while the linear behavior of the ρ trajectory begins with the first excitation, $n_r = 1$, since the large value of the mass difference, $M^2(\rho(1450)) - M^2(\rho(775)) = 1.56(6)\ \text{GeV}^2$ [or the relatively small value of the $\rho(775)$ mass] is a dynamical property of the $1S$ ground state.

(5) The leptonic widths $\Gamma_{ee}(\rho(775)) = 7.0(3)$ keV, $\Gamma_{ee}(\rho(1450)) = 1.7(1)$ keV, $\Gamma_{ee}(\rho(1900)) = 1.0(1)$ keV, $\Gamma_{ee}(2150)) = 0.7(1)$ keV, and $\Gamma_{ee}(\rho(1700)) = 0.26(5)$ keV are calculated considering them as pure $q\bar{q}$ states. If $2S - 1D$ mixing is possible, then for the mixing angle $\theta = 21°$ comparable values of the leptonic widths $\Gamma_{ee}(\rho(1450)) = 1.0(1)$ keV and $\Gamma(\rho(1700)) = 0.75(6)$ keV are obtained.

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