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Dibaryons with two strange quarks and total spin zero in a constituent quark model

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We investigate the symmetry property and construct the wave function of the dibaryon states containing two strange quarks with $S = 0$ in both the flavor SU(3) symmetric and breaking cases. We discuss how the color $\otimes$ isospin $\otimes$ spin states of dibaryon in the symmetry broken case of flavor SU(3) can be extracted from the fully antisymmetric states in flavor SU(3). The stability of the dibaryon against the strong decay into two baryons is then discussed, by using the variational method within a constituent quark model with confining and color-spin interactions. To compare our results with those from lattice QCD in the flavor SU(3) limit, we search for the stable H-dibaryon in a wide range of π meson masses. We find that with the given potential, there is no compact six-quark dibaryon state in the SU(3) flavor symmetry broken case with realistic quark masses as well as in the flavor SU(3) symmetric case in a wide range of quark masses.

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I. INTRODUCTION

Since Jaffe [1–3] suggested the possible existence of tetraquarks and dibrayons in QCD, based on the one-gluon exchange color-spin interaction, multiquark systems have been explored in various models and searched for experimentally over several decades. Attempts to find a stable multiquark system have been made by many researchers using the chromomagnetic model based on the color-spin interaction. For example, for the X(3872), which by now is widely believed to be a $J^{PC} = 1^{++}$ state, Høgassen [4,5] suggested that it could be a tetraquark state with a strong mixing of the color octet-octet component of the two quark-antiquark pair within the color-spin interaction. Silvestre-Brac and Leandri systematically classified dibaryons consisting of light quarks within the flavor SU(3) symmetry [6] and those containing two different types of heavy quarks [7]: These papers have discussed the stability of the multiquark system and its relation to the hyperfine splitting.

The dynamical problem of studying the stability of the multiquark system has been studied mainly using the variational method with a nonrelativistic Hamiltonian, including the confinement and hyperfine potential. The ground states of $qq\bar{q}\bar{q}$ systems with $L = 0$ have been extensively calculated with the harmonic oscillator bases in Refs. [8,9]. Calculations based on the simple Gaussian spatial function have been made in Refs. [10–11] to study the stability of the $qq\bar{q}\bar{q}$ and $qq\bar{Q}\bar{Q}$ ($Q = c$ or b) system, respectively. Two of us [12] have also introduced the correlation between quarks in the Gaussian spatial function to investigate its effect on the stability of the $qq\bar{Q}\bar{Q}$ ($Q = c$ or b) system. In addition to these variational methods, a powerful tool, using the hyperspherical harmonic basis functions, has been developed in Refs. [13,14] to solve the four-body problem in the tetraquark.

The stability study in the dibaryon sector has also been pursued with several other models [15–18]. For dibaryons containing light quarks only, the H-dibaryon is expected to be the most stable state as the color-spin hyperfine splitting is most attractive, even compared to the two Λ baryons. While the model study based on the Goldstone exchange interaction casts some doubt on the existence of the H-dibaryon [19], recent results from lattice QCD have suggested that the H-dibaryon would be bound for massive π mass [20,21]. Also, the H-dibaryon was found to be bound in a chiral constituent quark model calculation [22].

The purpose of this paper is to find the color $\otimes$ isospin $\otimes$ spin states in the SU(3) flavor symmetry broken case appropriate for the dibaryon containing two strange quarks with $S = 0$ that is compatible with a symmetric spatial wave function, and then calculate the mass of the dibaryon by using the variational method, with the color-spin hyperfine potential introduced in Ref. [23]. In particular, by going through the systematic construction of the color $\otimes$ isospin $\otimes$ spin states in general, we find the corresponding state for the symmetry broken case of flavor SU(3). Lastly, we search for the stability of the H-dibaryon in the flavor symmetric limit of SU(3) as a function of the pion mass. Through this work, we are able to verify if a compact H-dibaryon exists in the symmetry breaking case or the symmetric limit of SU(3) within the given Hamiltonian. If the recent lattice result of stable H-dibaryon in the

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massive pion case turns out to be valid, our result strongly suggests that the state should be a hadronic bound state and/or loosely bound molecular state.

This paper is organized as follows. We first present the Hamiltonian, and calculate the mass of both baryon octet and decuplet to determine the fitting parameters of the model, and construct the spatial wave function of the dibaryon in Sec. II. In Sec. III, we construct the color ⊗ isospin ⊗ spin states in both the flavor SU(3) symmetric and broken cases, establishing the relation between the two cases. We show the numerical results obtained from the variational method in Sec. IV. Finally, we summarize the results in Sec. V. The appendixes include some details of the calculations.

II. HAMILTONIAN

To investigate the stability of the dibaryon, we adopt the following nonrelativistic Hamiltonian that contains the confinement and hyperfine potential for the color and spin interaction:

$$H = \sum_{i=1}^{6}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{16}\sum_{i<j}^{6}\lambda_i^c\lambda_j^c(V_{ij}^C + V_{ij}^{SS}), \quad (1)$$

where m_is are the quark masses, $\lambda_i^c/2$ is the color operator of the ith quark for the color SU(3), and V_{ij}^C and V_{ij}^{SS} are the confinement and hyperfine potential, respectively. For the confinement potential, we take the half-linearizing potential and Coulomb potential as follows:

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{(r_{ij})^{1/2}}{a_0} - D. \quad (2)$$

Here, the first term comes from the perturbative one-gluon exchange, and the second from the confining potential. Although confinement is usually modeled by the linearly rising potential, we choose a form that is less repulsive at larger distance and is thus much closer to the string breaking phenomena. We have checked that taking a linearly rising potential does not change the main conclusion of this work as has been also shown to be the case in a more explicit calculation in a similar six-quark configuration reported in Ref. [24].

For the hyperfine potential, we take the potential to be dependent upon spin interaction as follows:

$$V_{ij}^{SS} = \frac{1}{m_i m_j c^4}\frac{\hbar^2 c^2 \kappa'}{(r_{0ij})}\frac{e^{-(r_{ij})^2/(r_{0ij})^2}}{r_{ij}}\sigma_i\cdot\sigma_j. \quad (3)$$

Here, r_{ij} is the distance between interquarks, $|\mathbf{r}_i - \mathbf{r}_j|$, and both (r_{0ij}) and κ' are chosen to depend on the masses of interquarks, given by

$$r_{0ij} = 1\bigg/\left(\alpha + \beta\frac{m_i m_j}{m_i + m_j}\right),$$
$$\kappa' = \kappa_0\left(1 + \gamma\frac{m_i m_j}{m_i + m_j}\right). \quad (4)$$

The hyperfine potential splits the baryon octet as well as baryon decuplet masses. Moreover, in the heavy quark mass limit $m_i \to \infty$, the functional form of the hyperfine potential in Eq. (3) approaches $1/(m_i m_j)\ \delta(r)$. By introducing the additional parameters in Eq. (4), one is able to reproduce the experimental observation that the mass differences between psudoscalar and vector meson for $q\bar{q}$ states decrease slower as a function of the masses than that given by the inverse mass relation given in Eq. (3).

In the Hamiltonian, the fitting parameters have been chosen to reproduce the experimental values of both the baryon octet and decuplet masses using the variational method and typically used constituent quark masses. The fitting parameters are given in Table I.

As in our previous work [24], we choose the color ⊗ isospin ⊗ spin state and the spatial wave function with a simple Gaussian form for the baryons and calculate the mass of both the baryon octet and decuplet with the new fitting parameters using the variational method. The masses are given in Table II.

TABLE I. Parameters fitted to the experimental baryon octet and decuplet masses.

γ	κ	a_0	D	κ_0	α	β	m_u	m_s
0.309 $(\text{GeV})^{-1}$	0.123	1.049 $(\text{GeV})^{-3/2}$	0.994 GeV	0.35 GeV	2.105 GeV	9.164	0.347 GeV	0.596 GeV

TABLE II. The mass of the baryon octet and decuplet obtained from the variational method. The third row shows the experimental data [25] (units are GeV).

(I,S)	$(\frac{1}{2},\frac{1}{2})$ N, P	$(\frac{1}{2},\frac{1}{2})$ Ξ	$(0,\frac{1}{2})$ Λ	$(1,\frac{1}{2})$ Σ	$(\frac{1}{2},\frac{3}{2})$ Ξ^*	$(1,\frac{3}{2})$ Σ^*	$(\frac{3}{2},\frac{3}{2})$ Δ	$(0,\frac{3}{2})$ Ω
Mass	0.974	1.344	1.115	1.217	1.554	1.398	1.233	1.7
Exp	0.938–0.939	1.314–1.321	1.115	1.189–1.197	1.53–1.531	1.382–1.387	1.23–1.234	1.672

In calculating the mass of the dibaryon containing two strange quarks with spin $= 0$, we need the spatial function appropriate for the six-quark system with a certain symmetry property. Since we construct the color $\otimes$ isospin $\otimes$ spin state of the dibaryon to be antisymmetric among particles 1–4, and at the same time antisymmetric between particles 5 and 6, which are denoted by $\{1234\}\{56\}$, the symmetry property of spatial function should be symmetric among particles 1–4, and at the same time symmetric between particles 5 and 6, due to the Pauli principle; we denote the symmetry property of the spatial function by [1234][56].

In order to describe the six-quark system, we consider the center of mass frame, so that the number of suitable Jacobian coordinates of the system is reduced to 5. The five Jacobian coordinates are given by

$$\boldsymbol{x_1} = \frac{1}{\sqrt{2}}(\mathbf{r}_5 - \mathbf{r}_6), \qquad \boldsymbol{x_2} = \frac{1}{2}(\mathbf{r}_1 - \mathbf{r}_2 + \mathbf{r}_3 - \mathbf{r}_4),$$
$$\boldsymbol{x_3} = \frac{1}{2}(\mathbf{r}_1 - \mathbf{r}_2 - \mathbf{r}_3 + \mathbf{r}_4), \quad \boldsymbol{x_4} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4),$$
$$\boldsymbol{x_5} = \frac{1}{\sqrt{12}}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4 - 2\mathbf{r}_5 - 2\mathbf{r}_6). \tag{5}$$

Then, we can construct the spatial wave function of the dibaryon in a Gaussian form, which is used to carry out the variational method, given by

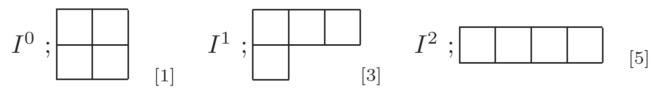
$$R = \exp[-(a(\boldsymbol{x_1})^2 + b(\boldsymbol{x_2})^2 + b(\boldsymbol{x_3})^2 + b(\boldsymbol{x_4})^2 + c(\boldsymbol{x_5})^2)], \tag{6}$$

where a, b, and c are variational parameters. It is easily found that the symmetry of the spatial wave function has the [1234][56] property, required by the color $\otimes$ isospin $\otimes$ spin state of the dibaryon.

III. CLASSIFICATION OF THE DIBARYON CONTAINING TWO STRANGE QUARKS WITH SPIN = 0

A. The state of the dibaryon with respect to isospin states

In this section, we investigate the state of the dibaryon containing two strange quarks with $S = 0$, with flavor symmetry represented by SU(2). The symmetry breaking of SU(3) in flavor part is caused by taking the strange quark mass to be heavier compared to m_u $(=m_d)$. When we choose the symmetry of spatial function to be symmetric under the exchange of any two particles among 1, 2, 3, and 4, and at the same time symmetric under the exchange of two particles between 5 and 6, the fixing of the position of two strange quarks onto the fifth and sixth is convenient to classify the dibaryon state. The four light quarks except for two strange quarks would be characterized by introducing the Young tableau corresponding to isospin states, as follows:

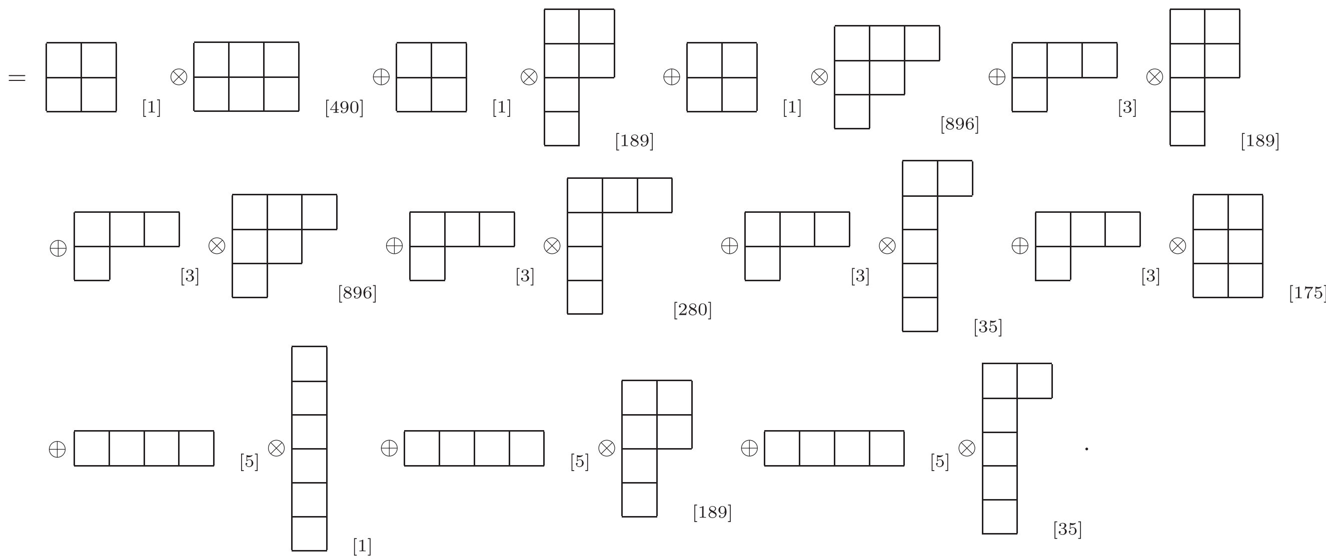


(7)

Here, the dimension of each isospin state is shown below the Young tableau.

In our case where we choose the symmetry of the spatial function of the dibaryon to be [1234][56], the color $\otimes$ isospin $\otimes$ spin state of the dibaryon is chosen to be $\{1234\}\{56\}$ in order to satisfy the Pauli principle. When we fix the positions of the two strange quarks onto the fifth and sixth, the partly antisymmetric state of the color $\otimes$ isospin $\otimes$ spin state can be easily obtained from classifying the multiplets of the direct four product of $[12]_{CIS}$ multiplied by the direct two product of $[6]_{CS}$, which represent the fundamental representation of $SU(12)_{CIS}$ and $SU(6)_{CS}$, respectively. Since the state of $\{1234\}$ by the direct four product of $[12]_{CIS}$ gives the multiplet with dimension 495 corresponding to the Young tableau $[1^4]$, and the state of $\{56\}$ by the direct two product of $[6]_{CS}$ the multiplet with dimension 15 corresponding to the Young tableau $[1^2]$, the dimension of $\{1234\}\{56\}$ is $495 \times 15 = 7425$. This $\{1234\}\{56\}$ state can be decomposed into the direct sum of representation $([2]_I, [6]_{CS})$. The $\{1234\}\{56\}$ state with dimension 7425 decomposed into the direct sum of representation $([2]_I, [6]_{CS})$ is given in Eq. (8). By using the Young tableau, we can easily find that the multiplets in Eq. (8) are $\{1234\}\{56\}$. Because the color state of the dibaryon is supposed to be a physically observable color singlet that corresponds to the Young tableau [2,2,2], and the spin state of the dibaryon in our works is confined to $S = 0$, which corresponds to the Young tableau [3,3], we need the possible color $\otimes$ spin states by combining the color singlet with $S = 0$ state, which we call the CS coupling scheme. Then, only the color $\otimes$ spin states obtained from the color spin (CS) coupling scheme are allowed among the multiplets in Eq. (8). Since the Young tableau corresponding to the color $\otimes$ spin states is [3,3], [2,2,1,1], and $[1^6]$, the $\{1234\}\{56\}$ states of the dibaryon with respect to $S = 0$ are given as $([1]_I, [490]_{CS})$, $([1]_I, [189]_{CS})$, $([3]_I, [189]_{CS})$, $([5]_I, [189]_{CS})$, and $([5]_I, [1]_{CS})$. We note that another Young tableau [4,1,1] obtained from the CS coupling scheme is excluded because the Young tableau [4,1,1] does not belong to the multiplets of the color $\otimes$ spin state in Eq. (8).

$\{1234\}\{56\}_{[7425]}$



(8)

B. Isospin ⊗ color ⊗ spin state of the dibaryon

Before constructing the color ⊗ isospin ⊗ spin state, we emphasize that the $\{1234\}\{56\}$ state of the dibaryon containing two strange quarks with $S = 0$ can be obtained from the CS coupling scheme with respect to isospin states, but can also be derived from a fully antisymmetric color ⊗ flavor ⊗ spin state, in which flavor is SU(3). For this reason, we specifically indicate the flavor state in terms of SU(3) symmetry; as we show later, we find that the symmetry breaking of SU(3) from a fully antisymmetric color ⊗ flavor ⊗ spin state of the dibaryon lead to the $\{1234\}\{56\}$ state.

As was shown in a previous paper by two of us [24], the color singlet and $S = 0$ state of the dibaryon are given by the corresponding Young-Yamanouchi basis, respectively.

(i) Color singlet: five basis functions with the Young tableau [2,2,2],

$$|C_1\rangle=\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5&6\\\hline\end{array}\quad |C_2\rangle=\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline 5&6\\\hline\end{array}\quad |C_3\rangle=\begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline 4&6\\\hline\end{array}\quad |C_4\rangle=\begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline 4&6\\\hline\end{array}$$
$$|C_5\rangle=\begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}. \tag{9}$$

(ii) $S = 0$: five basis functions with the Young tableau [3,3],

$$|S^0_1\rangle=\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\ |S^0_2\rangle=\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\ |S^0_3\rangle=\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}$$
$$|S^0_4\rangle=\begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\ |S^0_5\rangle=\begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}. \tag{10}$$

In order to construct the $\{1234\}\{56\}$ state of the dibaryon, we need to know the color ⊗ spin state, which can be derived from the CS coupling scheme. The CS coupling scheme is technically completed by using the Clebsch-Gordan (CG) coefficient, given by the following formula [26],

$$S([f']p'q'y'[f'']p''q''y''|[f]pqy) = K([f']p'[f'']p''|[f]p)S([f'_{p'}]q'y'[f''_{p''}]q''y''|[f_p]qy), \tag{11}$$

where S in the left-hand (right-hand) side is a CG coefficients of S_n (S_{n-1}) permutation group, and K is an isoscalar factor, which is called the K matrix that factorizes the CG coefficients of S_n into CG coefficients of S_{n-1} multiplied by the isoscalar factor. In this notation, $[f_p]$, which represents the Young tableau associated to S_{n-1}, can be obtained from $[f]$, the Young tableau of S_n, where the pqy represents the row positions of the last three particles in the Young tableau $[f]$, by removing the nth particle. In order to obtain the CG coefficients of S_n, we repeat the process of factorizing the CG coefficients of S_6 further, until Eq. (11) is extended into the following formula [27],

$$
\begin{aligned}
&S([f']p'q'y'r'[f'']p''q''y''r''|[f]pqyr)\\
&\quad = K([f']p'[f'']p''|[f]p)K([f'_{p'}]q'[f''_{p''}]q''|[f_p]q)\\
&\qquad \times K([f'_{p'q'}]y'[f''_{p''q''}]y''|[f_{pq}]y)S\\
&\qquad \times ([f'_{p'q'y'}]r'[f''_{p''q''y''}]r''|[f_{pqy}]r),
\end{aligned} \tag{12}
$$

where S in the third row is the CG coefficient of S_3. When we find out the CG coefficients using Eq. (12), we use the results obtained by Stancu and Pepin [28] about the relevant isoscalar factors for S_4, S_5, and S_6 appearing in Eq. (12), as shown in our previous paper [24].

Then, we can calculate the CG coefficients between the Young tableau [2,2,2] of the color singlet state and [3,3] of $S = 0$ state in making the representation of [3,3], [2,2,1,1], and $[1^6]$ of the CS coupling scheme. The color $\otimes$ spin state is denoted by $|C, S^0\rangle$, all of which are given in Eqs. (13)–(15).

We note that the basis function of the Young tableau is expressed by the Young-Yamanouchi representation, whose symmetry property is symmetric with respect to any neighboring particles that lie in the same row, and is antisymmetric with respect to any neighboring particles that lie in the same column. The color $\otimes$ spin state, which consists of the linear sum of combining the color singlet state with the $S = 0$ state, is presented in Appendix B.

The CS coupling scheme, which represents the color $\otimes$ spin states for Young tableaux [3,3], [2,2,1,1], and $[1^6]$, is given as follows.

(i) CS coupling with Young tableau [2,2,1,1]: nine bases functions,

$$
\begin{aligned}
&|[C,S^0]_1\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline \multicolumn{1}{|c|}{5}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}\quad
|[C,S^0]_2\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline \multicolumn{1}{|c|}{5}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}\quad
|[C,S^0]_3\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}\\
&|[C,S^0]_4\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}\quad
|[C,S^0]_5\rangle = \begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}\quad
|[C,S^0]_6\rangle = \begin{array}{|c|c|}\hline 1&2\\\hline 3&6\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\multicolumn{1}{|c|}{5}\\\cline{1-1}\end{array}\\
&|[C,S^0]_7\rangle = \begin{array}{|c|c|}\hline 1&3\\\hline 2&6\\\hline \multicolumn{1}{|c|}{4}\\\cline{1-1}\multicolumn{1}{|c|}{5}\\\cline{1-1}\end{array}\quad
|[C,S^0]_8\rangle = \begin{array}{|c|c|}\hline 1&4\\\hline 2&6\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\multicolumn{1}{|c|}{5}\\\cline{1-1}\end{array}\quad
|[C,S^0]_9\rangle = \begin{array}{|c|c|}\hline 1&5\\\hline 2&6\\\hline \multicolumn{1}{|c|}{3}\\\cline{1-1}\multicolumn{1}{|c|}{4}\\\cline{1-1}\end{array}.
\end{aligned} \tag{13}
$$

(ii) CS coupling with the Young tableau [3,3]: five bases functions,

$$
\begin{aligned}
&|[C,S^0]_1\rangle = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}\,
|[C,S^0]_2\rangle = \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}\,
|[C,S^0]_3\rangle = \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}\\
&|[C,S^0]_4\rangle = \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}\,
|[C,S^0]_5\rangle = \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}.
\end{aligned} \tag{14}
$$

(iii) CS coupling with the Young tableau $[1^6]$: one basis function,

$$
\begin{aligned}
|[C,S^0]\rangle = \frac{1}{\sqrt{5}}
\Bigg[& -\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline 5&6\\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2&4&6\\\hline\end{array}_{\,S^0}
+\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline 5&6\\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3&4&6\\\hline\end{array}_{\,S^0}
+\begin{array}{|c|c|}\hline 1&2\\\hline 3&5\\\hline 4&6\\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2&5&6\\\hline\end{array}_{\,S^0}\\
& -\begin{array}{|c|c|}\hline 1&3\\\hline 2&5\\\hline 4&6\\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3&5&6\\\hline\end{array}_{\,S^0}
+\begin{array}{|c|c|}\hline 1&4\\\hline 2&5\\\hline 3&6\\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5&6\\\hline\end{array}_{\,S^0}\Bigg].
\end{aligned} \tag{15}
$$

For the isospin part with $I = 0$, the constituent quarks except for two strange quarks of the dibaryon comprise the Young-Yamanouchi basis of the Young tableau [2,2] with dimension 2, given by

$$
\begin{aligned}
\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array} &= \frac{1}{\sqrt{12}} I_1^0\\
&= \frac{1}{\sqrt{12}}(2uudd + 2dduu - udud - uddu - duud - dudu),\\
\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array} &= \frac{1}{2} I_2^0 = \frac{1}{2}(udud - uddu - duud + dudu).
\end{aligned} \tag{16}
$$

Then, we can construct the $\{1234\}\{56\}$ state of the dibaryon with $I = 0$ and $S = 0$ by combining the isospin state with the color $\otimes$ spin state of the Young-Yamanouchi basis [2,2,1,1],

$$
\begin{aligned}
&\{1234\}\{56\}_{[F^{27}; I^0, C, S^0]}\\
&\quad = \frac{1}{\sqrt{2}} s(5)s(6)\\
&\quad \otimes \left[\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array}_{\,I} \otimes \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline \multicolumn{1}{|c|}{5}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}_{\,CS} - \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}_{\,I} \otimes \begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline \multicolumn{1}{|c|}{5}\\\cline{1-1}\multicolumn{1}{|c|}{6}\\\cline{1-1}\end{array}_{\,CS} \right].
\end{aligned} \tag{17}
$$

By F^{27} we mean that this state originally comes from a fully antisymmetric state with a flavor 27 multiplet, as the symmetry breaking of flavor SU(3) is imposed on the constituent quarks. The symmetry property is easily perceived in a sense that the linear sum of the Young-Yamanouchi [2,2] basis ⊗ Young-Yamanouchi [2,2] basis gives $[1^4]$ basis for particles 1–4, and particles 5–6 in the color ⊗ spin state are antisymmetric due to the positions in the same column.

In addition to the state, $\{1234\}\{56\}_{[F^{27};I^0,C,S^0]}$, there is another state, originally coming from a fully antisymmetric state with flavor singlet, as the symmetry breaking of flavor SU(3) is imposed on the constituent quarks,

$$\begin{aligned}\{1234\}\{56\}_{[F^1;I^0,C,S^0]} &= \frac{1}{\sqrt{2}} s(5)s(6)\otimes\Bigg[\young(12,34)_I \otimes \young(135,246)_{CS} \\ &\quad - \young(13,24)_I \otimes \young(125,346)_{CS}\Bigg].\end{aligned} \tag{18}$$

For the isospin part with $I=1$, the constituent quarks, apart from the two strange quarks of the dibaryon, comprise the Young-Yamanouchi basis of the Young tableau [3,1] with dimension 3, given by

$$\begin{aligned}\young(123,4) &= \frac{1}{\sqrt{12}} I_1^1 = \frac{1}{\sqrt{12}}(3uuud - uduu - uudu - duuu),\\ \young(124,3) &= \frac{1}{\sqrt{6}} I_2^1 = \frac{1}{\sqrt{6}}(2uudu - uduu - duuu),\\ \young(134,2) &= \frac{1}{\sqrt{2}} I_3^1 = \frac{1}{\sqrt{2}}(uduu - duuu).\end{aligned} \tag{19}$$

Then, we can construct the $\{1234\}\{56\}$ state of the dibaryon with $I=1$ and $S=0$ by combining the isospin state with the color ⊗ spin state of the Young-Yamanouchi basis [2,2,1,1],

$$\begin{aligned}\{1234\}\{56\}_{[F^{27};I^1,C,S^0]} = \frac{1}{\sqrt{3}} s(5)s(6)\otimes\Bigg[&\young(123,4)_I \otimes \left(-\frac{1}{\sqrt{3}}\young(14,26,3,5)_{CS} + \frac{\sqrt{2}}{\sqrt{3}}\young(14,25,3,6)_{CS}\right)\\ &- \young(124,3)_I \otimes \left(-\frac{1}{\sqrt{3}}\young(13,26,4,5)_{CS} + \frac{\sqrt{2}}{\sqrt{3}}\young(13,25,4,6)_{CS}\right)\\ &- \young(134,2)_I \otimes \left(-\frac{1}{\sqrt{3}}\young(12,36,4,5)_{CS} + \frac{\sqrt{2}}{\sqrt{3}}\young(12,35,4,6)_{CS}\right)\Bigg].\end{aligned} \tag{20}$$

In the same way, the $\{1234\}\{56\}_{[F^{27};I^1,C,S^0]}$ originates from a fully antisymmetric state with a flavor 27 multiplet. In this case, the symmetry property for particles 1–4 is easily perceived due to the fact that the linear sum of the Young-Yamanouchi [3,1] basis ⊗ Young-Yamanouchi [2,1,1] basis gives the $[1^4]$ basis. However, the symmetry property of $\{56\}$ between particles 5 and 6 is not easily obtained from the color ⊗ spin state of the Young-Yamanouchi [2,2,1,1] basis, since particles 5–6 are not positioned in the same column. The symmetry property of the Young-Yamanouchi basis could make it possible to construct $\{56\}$ between particles 5 and 6; for example, if we consider the following formula,

$$\young(14,25,3,6)_{CS} = \frac{3}{\sqrt{8}}\left[(56)\young(14,26,3,5)_{CS} - \frac{1}{3}\young(14,26,3,5)_{CS}\right], \tag{21}$$

where (56) is a permutation operator between particles 5 and 6, then we see the symmetry property between 5 and 6 by the following formula:

$$-\frac{1}{\sqrt{3}}\begin{array}{|c|c|}\hline 1&4\\ \hline 2&6\\ \hline \multicolumn{1}{|c|}{3}\\ \cline{1-1} \multicolumn{1}{|c|}{5}\\ \cline{1-1}\end{array}_{CS}+\frac{\sqrt{2}}{\sqrt{3}}\begin{array}{|c|c|}\hline 1&4\\ \hline 2&5\\ \hline \multicolumn{1}{|c|}{3}\\ \cline{1-1} \multicolumn{1}{|c|}{6}\\ \cline{1-1}\end{array}_{CS}=\frac{\sqrt{3}}{2}\left[(56)\begin{array}{|c|c|}\hline 1&4\\ \hline 2&6\\ \hline \multicolumn{1}{|c|}{3}\\ \cline{1-1} \multicolumn{1}{|c|}{5}\\ \cline{1-1}\end{array}_{CS}-\begin{array}{|c|c|}\hline 1&4\\ \hline 2&6\\ \hline \multicolumn{1}{|c|}{3}\\ \cline{1-1} \multicolumn{1}{|c|}{5}\\ \cline{1-1}\end{array}_{CS}\right]. \tag{22}$$

Therefore, from the right-hand side in Eq. (22), we can show that the left side in Eq. (22) is antisymmetric with respect to the exchange between particles 5 and 6.

For the isospin part with $I = 2$, the constituent quarks, except for the two strange quarks of the dibaryon, comprise the Young-Yamanouchi bases of the Young tableau [4] with dimension 1, given by

$$\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\ \hline\end{array} = I^2 = uuuu. \tag{23}$$

Then, we can construct the $\{1234\}\{56\}$ state of the dibaryon with $I = 2$ and $S = 0$ by combining the isospin state with the color ⊗ spin state of the Young-Yamanouchi basis $[1^6]$;

$$\begin{aligned}&\{1234\}\{56\}_{[F^{28};I^2,C,S^0]}\\ &=\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\ \hline\end{array}_{I}s(5)s(6)\otimes\frac{1}{\sqrt{5}}\left[-\begin{array}{|c|c|}\hline 1&2\\ \hline 3&4\\ \hline 5&6\\ \hline\end{array}_{C}\otimes\begin{array}{|c|c|c|}\hline 1&3&5\\ \hline 2&4&6\\ \hline\end{array}_{S^0}\right.\\ &\quad+\begin{array}{|c|c|}\hline 1&3\\ \hline 2&4\\ \hline 5&6\\ \hline\end{array}_{C}\otimes\begin{array}{|c|c|c|}\hline 1&2&5\\ \hline 3&4&6\\ \hline\end{array}_{S^0}+\begin{array}{|c|c|}\hline 1&2\\ \hline 3&5\\ \hline 4&6\\ \hline\end{array}_{C}\otimes\begin{array}{|c|c|c|}\hline 1&3&4\\ \hline 2&5&6\\ \hline\end{array}_{S^0}\\ &\quad\left.-\begin{array}{|c|c|}\hline 1&3\\ \hline 2&5\\ \hline 4&6\\ \hline\end{array}_{C}\otimes\begin{array}{|c|c|c|}\hline 1&2&4\\ \hline 3&5&6\\ \hline\end{array}_{S^0}+\begin{array}{|c|c|}\hline 1&4\\ \hline 2&5\\ \hline 3&6\\ \hline\end{array}_{C}\otimes\begin{array}{|c|c|c|}\hline 1&2&3\\ \hline 4&5&6\\ \hline\end{array}_{S^0}\right].\end{aligned} \tag{24}$$

In the same way, the $\{1234\}\{56\}_{[F^{28};I^2,C,S^0]}$ originates from a fully antisymmetric state with a flavor 28 multiplet. In this case, the symmetry property is easily understood due to the fact that the color ⊗ spin state of the Young-Yamanouchi basis $[1^6]$ is fully antisymmetric among particles 1–6.

In addition to the state, $\{1234\}\{56\}_{[F^{28};I^2,C,S^0]}$, there is another state, originally coming from a fully antisymmetric state with a flavor 27 multiplet;

$$\{1234\}\{56\}_{[F^{27};I^2,C,S^0]}=\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\ \hline\end{array}_{I}s(5)s(6)\otimes\begin{array}{|c|c|}\hline 1&5\\ \hline 2&6\\ \hline \multicolumn{1}{|c|}{3}\\ \cline{1-1} \multicolumn{1}{|c|}{4}\\ \cline{1-1}\end{array}_{CS} \tag{25}$$

In this case, the symmetry property is easily understood due to the fact that the color ⊗ spin state of the Young-Yamanouchi basis [2,2,1,1] is antisymmetric among particles 1–4, and antisymmetric in the exchange of 5 and 6.

C. Completely antisymmetric flavor ⊗ color ⊗ spin state of the dibaryon with $S = 0$

Until now, we investigated the classification and the flavor ⊗ color ⊗ spin state of the dibaryon, which contains two strange quarks with $S = 0$. From now on, as mentioned in Sec. III B, we show that the isospin ⊗ color ⊗ spin state of the dibaryon with $S = 0$ found in Sec. III B can be explicitly extracted from the completely antisymmetric flavor ⊗ color ⊗ spin state, in the symmetry breaking of SU(3) condition. As described in Ref. [6], from a point of view of SU(3), the Young tableau of flavor state containing two strange quarks for $I = 0$, $I = 1$, and $I = 2$ is represented as follows:

(i) $F = 1$: five basis functions with the Young tableau [2,2,2],

$$|F^1_1\rangle=\begin{array}{|c|c|}\hline 1&2\\ \hline 3&4\\ \hline 5&6\\ \hline\end{array}\quad |F^1_2\rangle=\begin{array}{|c|c|}\hline 1&3\\ \hline 2&4\\ \hline 5&6\\ \hline\end{array}\quad |F^1_3\rangle=\begin{array}{|c|c|}\hline 1&2\\ \hline 3&5\\ \hline 4&6\\ \hline\end{array}\quad |F^1_4\rangle=\begin{array}{|c|c|}\hline 1&3\\ \hline 2&5\\ \hline 4&6\\ \hline\end{array}$$

$$|F^1_5\rangle=\begin{array}{|c|c|}\hline 1&4\\ \hline 2&5\\ \hline 3&6\\ \hline\end{array}.$$

(ii) $F = 27$: nine basis functions with the Young tableau [4,2],

$$|F^{27}_1\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\ \hline 5&6\\ \cline{1-2}\end{array}\quad |F^{27}_2\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&3&5\\ \hline 4&6\\ \cline{1-2}\end{array}\quad |F^{27}_3\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&4&5\\ \hline 3&6\\ \cline{1-2}\end{array}$$

$$|F^{27}_4\rangle=\begin{array}{|c|c|c|c|}\hline 1&3&4&5\\ \hline 2&6\\ \cline{1-2}\end{array}\quad |F^{27}_5\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&3&6\\ \hline 4&5\\ \cline{1-2}\end{array}\quad |F^{27}_6\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&4&6\\ \hline 3&5\\ \cline{1-2}\end{array}$$

$$|F^{27}_7\rangle=\begin{array}{|c|c|c|c|}\hline 1&3&4&6\\ \hline 2&5\\ \cline{1-2}\end{array}\quad |F^{27}_8\rangle=\begin{array}{|c|c|c|c|}\hline 1&2&5&6\\ \hline 3&4\\ \cline{1-2}\end{array}\quad |F^{27}_9\rangle=\begin{array}{|c|c|c|c|}\hline 1&3&5&6\\ \hline 2&4\\ \cline{1-2}\end{array}.$$

(iii) $F = 28$: one basis function with the Young tableau [6],

$$|F^{28}\rangle=\begin{array}{|c|c|c|c|c|c|}\hline 1&2&3&4&5&6\\ \hline\end{array}.$$

In Appendix A, each of the flavor states, which are used to understand our procedure, is presented in detail.

In the same way as described in the previous paper [24], we can construct the fully antisymmetric flavor ⊗ color ⊗ spin state of the dibaryon with $S = 0$ by combining the flavor state with the color ⊗ spin state, which is obtained from CS couping scheme in Sec. III B. For each flavor state, the fully antisymmetric flavor ⊗ color ⊗ spin state is given as follows:

$$
\begin{aligned}
|[F^{27},C,S^0]\rangle = \frac{1}{\sqrt{9}}\Big[& \young(1234,56)_F \otimes \young(15,26,3,4)_{CS} - \young(1235,46)_F \otimes \young(14,26,3,5)_{CS} + \young(1245,36)_F \otimes \young(13,26,4,5)_{CS} - \young(1345,26)_F \otimes \young(12,36,4,5)_{CS} \\
& + \young(1236,45)_F \otimes \young(14,25,3,6)_{CS} - \young(1246,35)_F \otimes \young(13,25,4,6)_{CS} + \young(1346,25)_F \otimes \young(12,35,4,6)_{CS} + \young(1256,34)_F \otimes \young(13,24,5,6)_{CS} \\
& - \young(1356,24)_F \otimes \young(12,34,5,6)_{CS}\Big].
\end{aligned} \tag{26}
$$

Equation (26) is a fully antisymmetric state for $I = 0$, $I = 1$, and $I = 2$, in which the flavor 27 multiplet lies.

$$
\begin{aligned}
|[F^{1},C,S^0]\rangle = \frac{1}{\sqrt{5}}\Big[& \young(12,34,56)_F \otimes \young(135,246)_{CS} - \young(13,24,56)_F \otimes \young(125,346)_{CS} \\
& - \young(12,35,46)_F \otimes \young(134,256)_{CS} - \young(13,25,46)_F \otimes \young(124,356)_{CS} - \young(14,25,36)_F \otimes \young(123,456)_{CS}\Big].
\end{aligned} \tag{27}
$$

Equation (27) is a fully antisymmetric state for $I = 0$, in which the flavor singlet state lies.

$$
\begin{aligned}
|[F^{28},C,S^0]\rangle = \young(123456)_F \otimes \frac{1}{\sqrt{5}}\Big[& - \young(12,34,56)_C \otimes \young(135,246)_{S^0} + \young(13,24,56)_C \otimes \young(125,346)_{S^0} \\
& + \young(12,35,46)_C \otimes \young(134,256)_{S^0} - \young(13,25,46)_C \otimes \young(124,356)_{S^0} + \young(14,25,36)_C \otimes \young(123,456)_{S^0}\Big].
\end{aligned} \tag{28}
$$

Equation (28) is a fully antisymmetric state for $I = 2$, in which the flavor 28 multiplet state lies.

It should be noted that the so-called $\bar{K}$ [26] method is convenient when calculating the expectation value of $-\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ $(i < j = 1 \sim 6)$ for a fully antisymmetric state, as all terms can be identified to $-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6$, on account of the symmetry property. Since only particles 5–6 are relevant in the calculation of the expectation value of $-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6$, we should expand the fully antisymmetric states in terms of the states whose symmetry properties are restricted to those between particles 5 and 6. To achieve our goal, we use the $\bar{K}$ matrix defined in the following equation:

$$
\begin{aligned}
|[f]\widehat{pq}y\rangle = \sum_{y',y''} & \bar{K}([f]\widehat{pq}[f']\widehat{p'q'}|[f'']\widehat{p''q''}) \\
& \times S([f'_{p'q'}]y'[f''_{p''q''}]y''|[f_{pq}]y) \\
& \times |[f']p'q'y'\rangle \otimes |[f'']p''q''y''\rangle,
\end{aligned} \tag{29}
$$

where pq represents the row positions of the last two particles in the Young tableau $[f]$, y, those of the remaining particles, and $\widehat{pq}$ means either symmetry or antisymmetry between the last two particles. By using Eq. (29), $[1^6]_{CS}$ for color and spin part of Eq. (28) is given in a compact form,

$$[1^6]_{CS} = \frac{\sqrt{2}}{\sqrt{5}} \begin{array}{|c|c|}\hline \ & \ \\\hline \ & \ \\\hline \mathbf{5} & \mathbf{6} \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline \ & \ & \mathbf{5} \\\hline \ & \ & \mathbf{6} \\\hline\end{array}_{\,S^0} + \frac{\sqrt{3}}{\sqrt{5}} \begin{array}{|c|c|}\hline \ & \ \\\hline \ & \mathbf{5} \\\hline \ & \mathbf{6} \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline \ & \ & \ \\\hline \ & \mathbf{5} & \mathbf{6} \\\hline\end{array}_{\,S^0}. \tag{30}$$

In fact, the first and second terms on the right-hand side of Eq. (30) can be expanded in terms of the sum of color ⊗ spin states of Eq. (28);

$$\begin{array}{|c|c|}\hline \ & \ \\\hline \ & \ \\\hline \mathbf{5} & \mathbf{6} \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline \ & \ & \mathbf{5} \\\hline \ & \ & \mathbf{6} \\\hline\end{array}_{\,S^0} = -\frac{1}{\sqrt{2}} \begin{array}{|c|c|}\hline 1 & 2 \\\hline 3 & 4 \\\hline 5 & 6 \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\\hline 2 & 4 & 6 \\\hline\end{array}_{\,S^0} + \frac{1}{\sqrt{2}} \begin{array}{|c|c|}\hline 1 & 3 \\\hline 2 & 4 \\\hline 5 & 6 \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\\hline 3 & 4 & 6 \\\hline\end{array}_{\,S^0}, \tag{31}$$

$$\begin{array}{|c|c|}\hline \ & \ \\\hline \ & \mathbf{5} \\\hline \ & \mathbf{6} \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline \ & \ & \ \\\hline \ & \mathbf{5} & \mathbf{6} \\\hline\end{array}_{\,S^0} = \frac{1}{\sqrt{3}} \begin{array}{|c|c|}\hline 1 & 2 \\\hline 3 & 5 \\\hline 4 & 6 \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\\hline 2 & 5 & 6 \\\hline\end{array}_{\,S^0} - \frac{1}{\sqrt{3}} \begin{array}{|c|c|}\hline 1 & 3 \\\hline 2 & 5 \\\hline 4 & 6 \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\\hline 3 & 5 & 6 \\\hline\end{array}_{\,S^0} + \frac{1}{\sqrt{3}} \begin{array}{|c|c|}\hline 1 & 4 \\\hline 2 & 5 \\\hline 3 & 6 \\\hline\end{array}_{\,C} \otimes \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\\hline 4 & 5 & 6 \\\hline\end{array}_{\,S^0}. \tag{32}$$

In order to consider the symmetry breaking of SU(3) in these fully antisymmetric states with respect to isospin I, we need to restrict the position of the two strange quarks to any two among the possible 15 (${}_6C_2$) places. In our work, the fixing of the positions of the two strange quarks onto the fifth and sixth is performed by taking the cases where the strange quark resides in other positions to zero; $s(1)$, $s(2)$, $s(3)$, and $s(4) \to 0$. Then, Eq. (26) exactly becomes equal to $1/\sqrt{15}\,\{1234\}\{56\}_{[F^{27};I^0,C,S^0]}$ for $I = 0$ in Eq. (17) under the condition, because the other terms of the flavor 27 multiplet except for $|F_8^{27}\rangle$ and $|F_9^{27}\rangle$ vanish as shown in Appendix A. The factor $1/\sqrt{15}$, which will become clear later, is related to the symmetry property of the fully antisymmetric state. As another example for $I = 1$, Eq. (26) exactly becomes equal to the $1/\sqrt{15}\,\{1234\}\{56\}_{[F^{27};I^1,C,S^0]}$ in Eq. (20), because $|F_1^{27}\rangle$, $|F_8^{27}\rangle$, and $|F_9^{27}\rangle$ terms of the flavor 27 multiplet vanish as shown in Appendix A. With this approach, we find that the completely antisymmetric flavor ⊗ color ⊗ spin state in Eqs. (26)–(28) becomes the $\{1234\}\{56\}$ states obtained in Sec. III B with respect to I.

We need to investigate more to make the completely antisymmetric flavor ⊗ color ⊗ spin state in terms of $\{1234\}\{56\}$ state prior to finishing this subsection. According to the way of fixing two strange quarks, there are 14 more partially antisymmetric states in addition to the $\{1234\}\{56\}$ state, where the two strange quarks are located on the fifth and sixth position in the $\{1234\}\{56\}$ states. These states are given as follows: $\{3456\}\{12\}$, $\{2456\}\{13\}$, $\{2356\}\{14\}$, $\{2346\}\{15\}$, $\{2345\}\{16\}$, $\{1456\}\{23\}$, $\{1356\}\{24\}$, $\{1346\}\{25\}$, $\{1345\}\{26\}$, $\{1256\}\{34\}$, $\{1246\}\{35\}$, $\{1245\}\{36\}$, $\{1236\}\{45\}$, $\{1235\}\{46\}$, in addition to $\{1234\}\{56\}$. These states are orthonormal to each other, because one has at least a strange quark in a different position, in contrast to the others. Moreover, the states for I are also obtained from the completely antisymmetric flavor ⊗ color ⊗ spin state, by performing the same procedure with two strange quarks in different positions presented above. For any $[1^6]$ with respect to I, the completely antisymmetric state can be explicitly expressed by

$$\begin{aligned}[1^6] = \frac{1}{\sqrt{15}}[&\{3456\}\{12\} + \{2456\}\{13\} + \{2356\}\{14\} \\ &+ \{2346\}\{15\} + \{2345\}\{16\} + \{1456\}\{23\} \\ &+ \{1356\}\{24\} + \{1346\}\{25\} + \{1345\}\{26\} \\ &+ \{1256\}\{34\} + \{1246\}\{35\} + \{1245\}\{36\} \\ &+ \{1236\}\{45\} + \{1235\}\{46\} + \{1234\}\{56\}].\end{aligned} \tag{33}$$

As interesting points, we find that the other states in Eq. (33) except for the $\{1234\}\{56\}$ state can be directly obtained from the $\{1234\}\{56\}$ state, by using Eq. (37), which are identities obtained from Eq. (33) itself. In Eq. (37), (ij) is the permutation operator that exchanges between i and j. From Eq. (37), we can introduce a formula so as to simplify our arguments: we define the following formula from the fact that the $\{1234\}\{56\}$ state can be extracted from the completely antisymmetric $[1^6]$ state by taking $s(1)$, $s(2)$, $s(3)$, and $s(4) \to 0$ so that

$$\begin{aligned}&\lim_{s(1),s(2),s(3),s(4)\to 0}[1^6] \equiv \frac{1}{\sqrt{15}}\{1234\}\{56\}, \\ &\lim_{s(1),s(2),s(3),s(5)\to 0}[1^6] \equiv \frac{1}{\sqrt{15}}\{1235\}\{46\}, \\ &\lim_{s(1),s(2),s(3),s(6)\to 0}[1^6] \equiv \frac{1}{\sqrt{15}}\{1236\}\{45\}, \\ &\ldots\ldots, \\ &\lim_{s(3),s(4),s(5),s(6)\to 0}[1^6] \equiv \frac{1}{\sqrt{15}}\{3456\}\{12\}.\end{aligned} \tag{34}$$

The acting of the permutation operator (ij) of the above equation on both sides can be defined so as to satisfy Eq. (37), given by

$$\begin{aligned}(ij)\lim_{s(1),s(2),s(3),s(4)\to 0}[1^6] &\equiv \lim_{s((ij)(1)),s((ij)(2)),s((ij)(3)),s((ij)(4))\to 0}(ij)[1^6] \\ &\equiv \lim_{s((ij)(1)),s((ij)(2)),s((ij)(3)),s((ij)(4))\to 0}(-)[1^6] \\ &= \frac{1}{\sqrt{15}}(ij)\{1234\}\{56\}, \end{aligned} \tag{35}$$

where the minus sign appearing in the third line of Eq. (35) is due to the fully antisymmetric property of $[1^6]$ when acted upon by any permutation operator. When we apply (15) and (26) on both sides of the first line in Eq. (34), we have the same equation to satisfy Eq. (37),

$$\begin{aligned}(15)(26)\lim_{s(1),s(2),s(3),s(4)\to 0}[1^6] &= \lim_{s(5),s(6),s(3),s(4)\to 0}(15)(26)[1^6] \\ &= \lim_{s(5),s(6),s(3),s(4)\to 0}[1^6] = \frac{1}{\sqrt{15}}\{3456\}\{12\}. \end{aligned} \tag{36}$$

Therefore, we find that $\{3456\}\{12\} = (15)(26)\ \{1234\}\{56\}$.

$$\begin{aligned} &\{3456\}\{12\} = (15)(26)\{1234\}\{56\}, \quad \{2456\}\{13\} = (15)(36)\{1234\}\{56\}, \quad \{2356\}\{14\} = (15)(46)\{1234\}\{56\}, \\ &\{2346\}\{15\} = -(16)\{1234\}\{56\}, \quad \{2345\}\{16\} = -(15)\{1234\}\{56\}, \quad \{1456\}\{23\} = (25)(36)\{1234\}\{56\}, \\ &\{1356\}\{24\} = (25)(46)\{1234\}\{56\}, \quad \{1346\}\{25\} = -(26)\{1234\}\{56\}, \quad \{1345\}\{26\} = -(25)\{1234\}\{56\}, \\ &\{1256\}\{34\} = (35)(46)\{1234\}\{56\}, \quad \{1246\}\{35\} = -(36)\{1234\}\{56\}, \quad \{1245\}\{36\} = -(35)\{1234\}\{56\}, \\ &\{1236\}\{45\} = -(46)\{1234\}\{56\}, \quad \{1235\}\{46\} = -(45)\{1234\}\{56\}. \end{aligned} \tag{37}$$

Using Eq. (35), it is easily found that the right-hand side of Eq. (33) is fully antisymmetric. Also, it should be noted that the expectation value of $-\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ in terms of the $[1^6]$ state with respect to I is simply reduced to that in terms of the $\{1234\}\{56\}$ state. For example, we have the following equation for $I=0$, coming from the flavor singlet:

$$\begin{aligned}\left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{|[F^1,C,S^0]\rangle} &= \frac{1}{15}\Bigg[\left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{3456\}\{12\}_{[F^1;I^0,C,S^0]}} + \left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{2456\}\{13\}_{[F^1;I^0,C,S^0]}} \\ &\quad + \ldots\ldots, + \\ \left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}}\Bigg] &= \left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}}. \end{aligned} \tag{38}$$

IV. CALCULATION OF THE EXPECTATION VALUE OF THE HYPERFINE POTENTIAL

In this section, we calculate the expectation value of the hyperfine potential of the dibaryon in terms of the isospin ⊗ color ⊗ spin state found in Sec. III. Even though the expectation value of the hyperfine potential of the dibaryon can be directly obtained from the full flavor ⊗ color ⊗ spin wave function of the state, the symmetry property of the state enables us to approach the expectation value of the hyperfine potential of the dibaryon in terms of the fully antisymmetric state mentioned in Sec. III C, namely, in terms of the isospin ⊗ color ⊗ spin state. In calculating the expectation value of the hyperfine potential of the dibaryon, it is more convenient to use the well-established formula [29], which is applicable for the fully antisymmetric state with SU(2) flavor, given by

$$\begin{aligned}&-\sum_{i<j}^{N}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j \\ &\quad = \left[\frac{4}{3}N(N-6) + 4I(I+1) + \frac{4}{3}S(S+1) + 2C_c\right], \end{aligned} \tag{39}$$

where N is the total number of quarks in a system, and $C_c = \frac{1}{4}\lambda^c\lambda^c$, that is, the quadratic Casimir operator of color SU (3) in the system of N quarks. To apply this formula to the $\{1234\}\{56\}$ state found in Sec. III C, we need to know the color state among particles 1–4, and between 5 and 6, because both of the color states are not in color singlet states. We can reexpress the $\{1234\}\{56\}$ state for I by considering another way of constructing the $\{1234\}\{56\}$ state;

$$
\begin{aligned}
|[C,I^0,S^0]_1\rangle = \frac{1}{\sqrt{3}}\Bigg[&\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C \otimes\left(-\frac{1}{2}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{2}}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}-\frac{1}{2}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}\right)\\
&-\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C \otimes\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}+\frac{1}{2}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}-\frac{1}{2}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}\right)\\
&+\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}_C \otimes\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{2}}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}\right)\Bigg]\otimes s(5)s(6),
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
|[C,I^0,S^0]_2\rangle = \frac{1}{\sqrt{2}}\Bigg[&\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}_C \otimes\left(-\frac{1}{\sqrt{2}}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{2}}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&5\\3&4&6\end{bmatrix}_{S^0}\right)\\
&-\begin{bmatrix}1&3\\2&4\\5&6\end{bmatrix}_C \otimes\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1&2\\3&4\end{bmatrix}_I\otimes\begin{bmatrix}1&2&5\\3&4&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{2}}\begin{bmatrix}1&3\\2&4\end{bmatrix}_I\otimes\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}_{S^0}\right)\Bigg]\otimes s(5)s(6),
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
|[C,I^1,S^0]\rangle = \frac{1}{\sqrt{3}}\otimes s(5)s(6)\Bigg[&\begin{bmatrix}1&2&3\\4\end{bmatrix}_I \otimes\left(\frac{2}{\sqrt{6}}\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{6}}\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{6}}\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}\right)\\
&-\begin{bmatrix}1&2&4\\3\end{bmatrix}_I \otimes\left(\frac{1}{\sqrt{6}}\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{6}}\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{3}}\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{3}}\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}\right)\\
&-\begin{bmatrix}1&3&4\\2\end{bmatrix}_I \otimes\left(-\frac{1}{\sqrt{6}}\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{3}}\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}+\frac{1}{\sqrt{3}}\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}-\frac{1}{\sqrt{6}}\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}\right)\Bigg],
\end{aligned}
\tag{42}
$$

$$
|[C,I^2,S^0]_1\rangle = \frac{1}{\sqrt{3}}\Bigg[\begin{bmatrix}1&2&3&4\end{bmatrix}_I\otimes\left(\begin{bmatrix}1&2\\3&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&4\\2&5&6\end{bmatrix}_{S^0}-\begin{bmatrix}1&3\\2&5\\4&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&4\\3&5&6\end{bmatrix}_{S^0}+\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&3\\4&5&6\end{bmatrix}_{S^0}\right)\Bigg]\otimes s(5)s(6), \tag{43}
$$

$$
|[C,I^2,S^0]_2\rangle = \frac{1}{\sqrt{2}}\Bigg[\begin{bmatrix}1&2&3&4\end{bmatrix}_I\otimes\left(\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}_C\otimes\begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}_{S^0}-\begin{bmatrix}1&3\\2&4\\5&6\end{bmatrix}_C\otimes\begin{bmatrix}1&2&5\\3&4&6\end{bmatrix}_{S^0}\right)\Bigg]\otimes s(5)s(6). \tag{44}
$$

In the case for $I=0$, Eq. (17) and Eq. (18) can be rewritten in terms of Eqs. (40)–(41) as

$$\{1234\}\{56\}_{[F^1;I^0,C,S^0]} = -\frac{\sqrt{3}}{2}|[C,I^0,S^0]_1\rangle - \frac{1}{2}|[C,I^0,S^0]_2\rangle,$$
$$\{1234\}\{56\}_{[F^{27};I^0,C,S^0]} = \frac{1}{2}|[C,I^0,S^0]_1\rangle - \frac{\sqrt{3}}{2}|[C,I^0,S^0]_2\rangle. \tag{45}$$

In the case for $I=2$, both Eq. (24) and Eq. (25) can be rewritten in terms of Eqs. (43)–(44) as

$$\{1234\}\{56\}_{[F^{27};I^2,C,S^0]} = \frac{\sqrt{2}}{\sqrt{5}}|[C,I^2,S^0]_1\rangle + \frac{\sqrt{3}}{\sqrt{5}}|[C,I^2,S^0]_2\rangle,$$
$$\{1234\}\{56\}_{[F^{28};I^2,C,S^0]} = \frac{\sqrt{3}}{\sqrt{5}}|[C,I^2,S^0]_1\rangle - \frac{\sqrt{2}}{\sqrt{5}}|[C,I^2,S^0]_2\rangle. \tag{46}$$

Since the states in Eqs. (40)–(44) have definite symmetry properties, which are antisymmetric among particles 1–4, and at same time antisymmetric between 5 and 6, the states become eigenstates for both $-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ and $-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6$, respectively. Therefore, the states are orthonormal to each other. If we focus on the Young-Yamanouchi basis, which is partly represented with a solid line in Eqs. (40)–(44), the isospin, color, and spin states among particles 1–4 can be easily calculated. For example, in Eq. (40), among particles 1–4, the isospin state is in a $I=0$ due to the Young-Yamanouchi basis [2,2], and the spin state is in a $S=1$ due to the Young-Yamanouchi basis [3,1]; the color state is in a color singlet for three quarks and in a color triplet for one quark, while, for two quarks 5 and 6 without a solid line, the isospin state is in a $I=1$ due to $s(5)s(6)$ represented by the symmetric Young-Yamanouchi basis [2]; the spin state is in a $S=1$ due to the same reason as the isospin state; and the color state is in an antitriplet. In this way, the eigenstates are characterized by the following equations:

$$-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[C,I^0,S^0]_1\rangle = -\frac{16}{3}|[C,I^0,S^0]_1\rangle$$
$$-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6|[C,I^0,S^0]_1\rangle = \frac{8}{3}|[C,I^0,S^0]_1\rangle, \tag{47}$$

$$-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[C,I^0,S^0]_2\rangle = -4|[C,I^0,S^0]_2\rangle$$
$$-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6|[C,I^0,S^0]_2\rangle = 4|[C,I^0,S^0]_2\rangle, \tag{48}$$

$$-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[C,I^1,S^0]\rangle = \frac{8}{3}|[C,I^1,S^0]\rangle$$
$$-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6|[C,I^1,S^0]\rangle = \frac{8}{3}|[C,I^1,S^0]\rangle, \tag{49}$$

$$-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[C,I^2,S^0]_1\rangle = \frac{56}{3}|[C,I^2,S^0]_1\rangle$$
$$-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6|[C,I^2,S^0]_1\rangle = \frac{8}{3}|[C,I^2,S^0]_1\rangle, \tag{50}$$

$$-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[C,I^2,S^0]_2\rangle = 20|[C,I^2,S^0]_2\rangle$$
$$-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6|[C,I^2,S^0]_2\rangle = 4|[C,I^2,S^0]_2\rangle. \tag{51}$$

Using the above equations, Eqs. (45)–(46) and Eq. (42), the expectation value of both $-\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ and $-\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6$ in terms of the $\{1234\}\{56\}$ state for I found in Sec. III B can be calculated, instead of directly taking the expectation by means of the $\{1234\}\{56\}$ state. For $-\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ between i and j ($i=1,2,3,4$, $j=5,6$), where the number of choices for i and j is 8, we note that each of the expectation values is the same as that of $-\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5$, because the states have the $\{1234\}\{56\}$ symmetry. In order to calculate the expectation value of $-\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5$, we must take advantage of Eq. (38).

For the case of $I=0$ where the flavor state is in a singlet before the symmetry breaking of SU(3), the expectation value of the hyperfine potential in terms of a fully antisymmetric state, $|[F^1,C,S^0]\rangle$, is rewritten, by using Eq. (38);

$$\left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{|[F^1,C,S^0]\rangle} = \left\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}} = \left\langle -\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\right\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}}$$
$$+ 8\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}} + \langle -\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_5\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}}. \tag{52}$$

Then, we have enough equations to calculate the $\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle_{\{1234\}\{56\}_{[F^1;I^0,C,S^0]}}$. Using Eq. (45),

$$\langle -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle_{|[F^1,C,S^0]\rangle} = \frac{3}{4}\langle -\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle_{|[C,I^0,S^0]_1\rangle} + \frac{3}{4}\langle -\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6\rangle_{|[C,I^0,S^0]_1\rangle} + \frac{1}{4}\langle -\sum_{i<j}^{N=4}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle_{|[C,I^0,S^0]_2\rangle} + \frac{1}{4}\langle -\lambda_5^c\lambda_6^c\sigma_5\cdot\sigma_6\rangle_{|[C,I^0,S^0]_2\rangle} + 8\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle_{[F^1;I^0,C,S^0]}. \tag{53}$$

In the first line of Eq. (53), the expectation value is −24, by making use of another formula [29], which is very well known for flavor SU(3) symmetry, given by

$$-\sum_{i<j}^{N}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j = \left[N(N-10)+\frac{4}{3}S(S+1)+2C_c+4C_F\right], \tag{54}$$

where $C_F=\frac{1}{4}\lambda^F\lambda^F$. Then, we can calculate the $\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle_{[F^1;I^0,C,S^0]}$, since the other terms in Eq. (53) are given by Eqs. (47)–(48). In this way, we can find the expectation value of $-\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j$ about the other $\{1234\}\{56\}$ state for I, without taking the trouble of direct calculation.

The process for calculating the expectation value for the cross terms between different states follows a similar path. For the cross term between $\{1234\}\{56\}_{[F^1;I^0,C,S^0]}$ and $\{1234\}\{56\}_{[F^{27};I^0,C,S^0]}$, the terms other than the expectation value of the $\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle$ are obtained by using Eq. (45) and Eqs. (47)–(48), while the expectation value of the $\langle -\lambda_1^c\lambda_5^c\sigma_1\cdot\sigma_5\rangle$ is obtained by the following equation:

$$\langle [F^{27},C,S^0]| -\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j|[F^1,C,S^0]\rangle = 0. \tag{55}$$

The final results for the matrix elements are summarized in Table III. The corresponding matrix elements for the baryon in isospin symmetric states are given in Table IV.

It is interesting to discuss the hyperfine factors for the difference between a dibaryon state and its lowest two baryon threshold. In the SU(3) symmetric limit, all quarks have the same mass. In this case, the hyperfine factors appearing in the three columns in Tables III–IV contribute with equal strength to the potential. Therefore, the factor for the $I=0, F^1$ state is $6\times(-5/6)+8\times(-11/4)+3=-24$, while that for the two Λ state is −16, resulting in an attractive contribution with a factor −8. However, consider the limit where the strange quark mass becomes infinitely heavy. Then, the contribution from the second and third column in Tables III–IV does not contribute. This does not have any effect on the two Λ threshold but the dibaryon hyperfine factor now becomes just −5. Hence, it is natural that the dibaryon could become stable when the strange quark mass decreases. However, when the SU(3) symmetric mass increases, the attraction will become smaller. As will be borne out in the next section, although one still cannot find a bound state, the general tendency seems to be true.

For the expectation value of $\lambda_i^c\lambda_j^c$ appearing in the confinement potential, the matrix elements are calculated in the same procedure as in the hyperpotential case, and using the following:

$$\sum_{i<j}^{N=6}\lambda_i^c\lambda_j^c = -\frac{8}{3}N+2C_c. \tag{56}$$

We summarize all matrix elements for both hyperfine and confinement potential of the dibaryon in Tables III and V. To compare matrix elements for hyperfine potential of

TABLE III. The matrix element of $-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ for hyperfine potential of the dibaryon with respect to isospin and flavor.

Isospin Flavor	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i<j=1$–4	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i=1$–4, $j=5, 6$	$-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ $i=5, j=6$
$I=0, F^1$	−5/6	−11/4	3
$I=0, F^{27}$	−13/18	13/12	11/3
Cross terms	$1/(6\sqrt{3})$	$-1/(4\sqrt{3})$	$1/\sqrt{3}$
$I=1, F^{27}$	4/9	1/3	8/3
$I=2, F^{28}$	16/5	16/5	16/5
$I=2, F^{27}$	146/45	−28/15	52/15
Cross terms	$-2\sqrt{2}/(15\sqrt{3})$	$\sqrt{2}/(5\sqrt{3})$	$-4\sqrt{2}/(5\sqrt{3})$

TABLE IV. The matrix element of $-\langle\lambda_i^c\lambda_j^c\sigma_i\cdot\sigma_j\rangle$ for hyperfine potential of the baryon spectrum.

Baryon	$-\langle\lambda_1^c\lambda_2^c\sigma_1\cdot\sigma_2\rangle$	$-\langle\lambda_1^c\lambda_3^c\sigma_1\cdot\sigma_3\rangle$	$-\langle\lambda_2^c\lambda_3^c\sigma_2\cdot\sigma_3\rangle$
N	−8/3	−8/3	−8/3
Λ	−8	0	0
Ξ	8/3	−16/3	−16/3
Σ	8/3	−16/3	−16/3

the dibaryon, we show that of the baryon in Table IV. For matrix elements for confinement potential of the baryon, it is very easy to show from Eq. (56) that $-\langle\lambda_i^c\lambda_j^c\rangle$ ($i<j=1$, 2, 3) is 8/3. We note that direct calculations of the expectation value of both hyperfine and confinement potential in terms of the $\{1234\}\{56\}$ state, which is obtained in Sec. III B, is equal to those mentioned above.

V. NUMERICAL RESULTS

In this section, we analyze the numerical results obtained from the variational method, by using a total wave function that consists of the spatial function in Eq. (6) as the trial function and the color ⊗ isospin ⊗ spin state constructed in the previous section. Since there are two color ⊗ isospin ⊗ spin states for $I=0$, given by $\{1234\}\{56\}_{[F^1;I^0,C,S^0]}$ and $\{1234\}\{56\}_{[F^{27};I^0,C,S^0]}$, the expectation value of the Hamiltonian is a 2 by 2 matrix in terms of the two color ⊗ isospin ⊗ spin states. Therefore, the ground state for $I=0$ must be represented as the mixing form of $\{1234\}\{56\}_{[F^1;I^0,C,S^0]}$ and $\{1234\}\{56\}_{[F^{27};I^0,C,S^0]}$. For the same reason, the ground state for $I=2$ must be represented as a mixed state of $\{1234\}\{56\}_{[F^{27};I^2,C,S^0]}$ and $\{1234\}\times\{56\}_{[F^{28};I^2,C,S^0]}$.

Table VI shows the result of the analysis for the mass of the dibaryons containing two strange quarks with $S=0$, with respect to I. Table VII shows the matrix element of the expectation value of the Hamiltonian for $I=0$ and $I=2$. As we see in Table VI, there are no bound dibaryons against the strong decay into two baryons. It should be emphasized that our approach only probes compact six-quark states based on color confining and color-spin potential. To probe molecular configuration, we have to include possible long range forces induced by meson exchange potentials. Moreover, the trial spatial wave function should have sufficient parameters to allow for largely separated two baryon states. Therefore, the nonexistence proves that there are no compact six-quark dibaryon states possible. In $I=0$ and $I=2$, the $\{1234\}\{56\}_{[F^1;I^0,C,S^0]}$ state and $\{1234\}\{56\}_{[F^{27};I^2,C,S^0]}$ state are overwhelmingly dominant terms in each ground state, respectively, so that the mixing effect is nearly negligible.

So far, we investigated the stability of the dibaryon in the realistic case of broken SU(3) flavor, where we took m_s to be heavier than m_u ($=m_d$). We now consider the possibility of a stable H-dibaryon in the flavor SU(3) symmetric limit. Such a possibility was recently proposed by lattice

TABLE V. The matrix element of $-\langle\lambda_i^c\lambda_j^c\rangle$ for confinement potential with respect to isospin and flavor.

Isospin Flavor	$-\langle\lambda_i^c\lambda_j^c\rangle$ $i<j=1$–4	$-\langle\lambda_i^c\lambda_j^c\rangle$ $i=1$–4, $j=5$, 6	$-\langle\lambda_i^c\lambda_j^c\rangle$ $i=5$, $j=6$
$I=0$, F^1	7/6	11/12	5/3
$I=0$, F^{27}	5/6	17/12	−1/3
Cross terms	$1/(2\sqrt{3})$	$-\sqrt{3}/4$	$\sqrt{3}$
$I=1$, F^{27}	4/3	2/3	8/3
$I=2$, F^{28}	16/15	16/15	16/15
$I=2$, F^{27}	14/15	19/15	4/15
Cross terms	$-2\sqrt{2}/(5\sqrt{3})$	$\sqrt{6}/5$	$-4\sqrt{6}/5$

TABLE VI. The mass of the dibaryon with respect to the (I, $S=0$) state. The binding energy E_B is taken to be the difference between the mass of the dibaryon and the lowest two baryon threshold. The unit of the mass and the variational parameters are GeV, and fm^{-2}, respectively.

(I,S)	(0,0)	(1,0)	(2,0)
Type	*uuddss*	*uuudss*	*uuuuss*
Mass	2.549	2.88	2.918
Variational parameters	$a=2.3$, $b=1.7$, $c=2.1$	$a=1.9$, $b=1.1$, $c=1.1$	$a=1.5$ $b=0.9$, $c=1.7$
Decay mode	ΛΛ	ΞN	ΣΣ
E_B	0.319	0.562	0.484

TABLE VII. The matrix for the expectation values of the Hamiltonian. The upper and lower matrices are for $I = 0$ and $I = 2$, respectively.

Basis functions	$uuddss$
$[F^{27}; I^0, C, S^0]$, $[F^1; I^0, C, S^0]$	$\begin{pmatrix} 2.967 & -0.0018 \\ -0.0018 & 2.549 \end{pmatrix}$
Basis functions	$uuuuss$
$[F^{28}; I^2, C, S^0]$, $[F^{27}; I^2, C, S^0]$	$\begin{pmatrix} 3.219 & -0.0168 \\ -0.0168 & 2.919 \end{pmatrix}$

calculation [20,21]. In Ref. [21], using the baryon-baryon potential extracted from lattice QCD in the flavor SU(3) limit, the H-dibaryon was found to be stable for pseudoscalar meson mass of 673–1015 MeV. Under such a circumstance, since the strange quark mass m_s is identified with the m_u, the flavor $\otimes$ color $\otimes$ spin state should be fully antisymmetric, only if we choose the spatial function to be fully symmetric. So, for the flavor $\otimes$ color $\otimes$ spin state with full antisymmetry, we use the $|[F^1, C, S^0]\rangle$ state in Eq. (27), and for the spatial function with full symmetry, we use that introduced in our previous paper [24]. Moreover, to compare our work with the result of the previous paper, we try to search for the existence of the stable H-dibaryon in a wide range of π meson mass.

For this purpose, we keep most of the parameters in Eq. (1) the same, but refit D and κ_0 to better reproduce the meson spectrum. The fitting parameters, including m_c and m_b, are given in Table VIII. The meson masses obtained from the variational method with the fitting parameters are given in Table IX.

We now increase π mass smoothly by varying the constituent quark mass from 347 to 917 MeV. The resulting mass difference between the H-dibaryon and two Λ baryons is plotted in Fig. 1 as a function of the pion mass. First, it should be noted that when m_s is reduced to m_u, the mass difference decreases to below 170 MeV, which is smaller than that of the realistic case given in Table VI. This result is consistent with the hyperfine interaction becoming larger as discussed in the previous section. On the other hand, as one increases the pion mass, as can be seen in Fig. 1, it is found that the mass difference between the H-dibaryon and two Λ baryons increases monotonously. This seems to be a consequence of the overall weakening of the hyperfine interaction due to the prefactors proportional to the inverse quark masses. The wave function is also becoming more compact as can be seen from Fig. 2, which makes the contribution from the additional kinetic term larger. Hence, one can conclude that even in the flavor SU(3) symmetric limit, there is no stable compact H-dibaryon for a wide range of pion masses. But to probe a possible molecular bound state, further input in the Hamiltonian and trial wave function would be needed.

TABLE VIII. Parameters fitted to the experimental meson masses.

Parameter	D	κ_0	m_u	m_c	m_b
Value	0.954	0.252	0.347	1.793	5.23
Unit	GeV	GeV	GeV	GeV	GeV

TABLE IX. Meson masses obtained from the variational method. The third row indicates the experimental data [25]. (Units are GeV.)

Meson	π	ρ	D	D^*	η_c	J/ψ	Υ	η_b
Mass	0.146	0.775	1.892	2.024	2.989	3.096	9.398	9.471
Exp	0.139	0.775	1.869	2.006	2.983	3.096	9.398	9.46

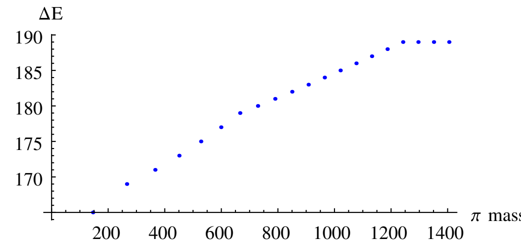


FIG. 1. The mass difference (ΔE) between the H-dibaryon and two Λ baryons as a function of the pion mass in the SU(3) limit. (Units are MeV.)

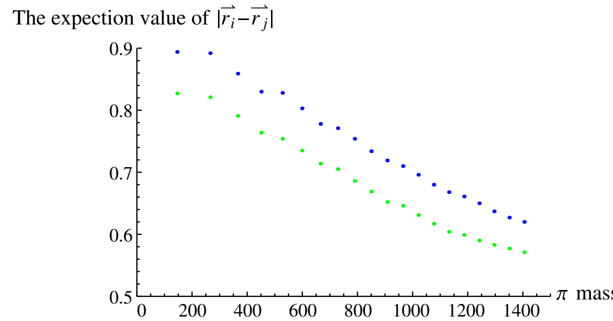


FIG. 2. The expectation value of the distance between inter-quarks ($\langle|\mathbf{r}_i - \mathbf{r}_j|\rangle$) for the H-dibaryon (blue dotted curve) and Λ baryon (green dotted curve) as a function of the pion mass in the SU(3) limit. The expectation value of $|\mathbf{r}_i - \mathbf{r}_j|$ is the same for any i and j ($i < j$) as the spatial wave function for both the H-dibaryon and Λ baryon are fully symmetric. (Units are x axis MeV and y axis fm.)

VI. SUMMARY

For the dibaryon containing two strange quarks with $S = 0$, such as $uuddss$, $uuudss$, and $uuuuss$, we have constructed the flavor ⊗ color ⊗ spin state both in the flavor SU(3) symmetric and breaking cases appropriate for the symmetric spatial wave function, satisfying the Pauli principle. We showed that the possible dibaryon states could be classified in terms of the symmetry property of the Young tableau. It is found that the symmetry breaking of flavor SU(3) reduces the fully antisymmetric states in flavor SU(3) to the color ⊗ isospin ⊗ spin states in flavor SU(2).

In order to investigate the stability of the dibaryon, we adopted the nonrelativistic Hamiltonian, including confinement and hyperfine potential, and calculated the mass of the dibaryon, by using the variational method. We conclude that there are no compact bound dibaryons states that are stable against the strong decay into two baryons in both the symmetric and symmetry breaking limit of flavor SU(3). It should be noted that the spatial wave function that we used as shown in Sec. II is limited to symmetric configurations. However, it is important to improve the spatial part to include possible correlations between quarks as was taking into account in Ref. [30] through the orbital mixed symmetry [4,2]. We leave such improvements as an important future work. Further improvements in the potential as well as a more sophisticated spatial trial wave function are needed to probe the largely separated two baryon molecular bound state.

ACKNOWLEDGMENTS

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APPENDIX A: FLAVOR STATE IN SU(3)

In Appendix A, we systematically investigate how to find out the flavor state, in which lies SU(3), by introducing a generator operator. As mentioned in Sec. III C, we presented the flavor states, which correspond to the Young-Yamanouchi basis. In dealing with each flavor multiplet, in particular, it should be noted that for a given flavor multiplet, the representation of the flavor states is different with respect to isospin. First, we consider the flavor 27 multiplet with nine dimensions, concerning $I=0$, $I=1$, and $I=2$. For a given I, once we find one state among the flavor multiplets, the others can be easily obtained by applying a permutation operator.

1. Flavor 27 multiplet with $I=0$

The $|F_1^{27}\rangle$ state with $I=0$ is obtained by introducing the following generator operator,

$$G^{27}_{1,I=0} \equiv \frac{1}{\sqrt{12}}\frac{1}{2!}\frac{1}{2!}\sum_{\sigma\in S_4}\sigma A^{27}_{1,I=0}, \tag{A1}$$

where $A^{27}_{1,I=0} = (A_{15}A_{26} + A_{25}A_{16})$, $A_{ij} = 1—(ij)$, and S_4 is a permutation group for particles 1–4.

The action of the $A^{27}_{1,I=0}$ on $u(1)u(2)s(3)s(4)d(5)d(6)$ gives $2uussdd+2ddssuu—udssud—udssdu—dussud—dussdu$, which is denoted by $I_1^0 s(3)s(4)$, because the four quarks except for two strange quarks $s(3)$ and $s(4)$ are in the I_1^0 represented in Eq. (16). Since $\pi\sum_{\sigma\in S_4}\sigma = \sum_{\sigma\in S_4}\sigma$ for $\pi\in S_4$ and $(56)A^{27}_{1,I=0} = A^{27}_{1,I=0}(56)$, the $G^{27}_{1,I=0}$ $u(1)u(2)s(3)s(4)d(5)d(6)$ represent the $|F_1^{27}\rangle$ state with $I=0$, whose symmetry property has [1234][56].

2. Flavor 27 multiplet with $I=1$

The $|F_1^{27}\rangle$ state with $I=1$ is obtained by introducing the following generator operator,

$$G^{27}_{1,I=1} \equiv \frac{1}{8\sqrt{3}}\frac{1}{3!}\sum_{\sigma\in S_4}\sigma S_{56}A^{27}_{1,I=1}, \tag{A2}$$

where $A^{27}_{1,I=1} = (A_{15}A_{26} + A_{25}A_{36} + A_{35}A_{16})$, and $S_{56} = 1 + (56)$. The action of the $G^{27}_{1,I=1}$ on $u(1)u(2)u(3)\times s(4)d(5)s(6)$ gives the $|F_1^{27}\rangle$ state with $I=1$, whose symmetry property has [1234][56], for the same reason.

3. Flavor 27 multiplet with $I=2$

The $|F_1^{27}\rangle$ state with $I=2$ is obtained by introducing the following generator operator,

$$G^{27}_{1,I=2} \equiv \frac{1}{4\sqrt{15}}\frac{1}{2!}\frac{1}{2!}\sum_{\sigma\in S_4}\sigma A^{27}_{1,I=2}, \tag{A3}$$

where $A^{27}_{1,I=2} = (A_{15}A_{26} + A_{25}A_{16})$.

The action of the $A^{27}_{1,I=2}$ on $u(1)u(2)u(3)u(4)s(5)s(6)$ gives $2uuuuss+2ssuuuu—suuuus—suuusu—usuusu—usuuus$, which is denoted by $2I^2s(5)s(6)+2I^2s(1)s(2)—I^2s(1)s(6)—I^2s(1)s(5)—I^2s(2)s(5)—I^2s(2)s(6)$, because the four quarks except for two strange quarks are in the I^2 represented in Eq. (23). Then, $G^{27}_{1,I=2}$ $u(1)u(2)u(3)u(4)\times s(5)s(6)$ represent the $|F_1^{27}\rangle$ state with $I=2$, whose symmetry property has [1234][56], for the same reason.

4. Flavor 28 multiplet with $I=2$

For the $|F^{28}\rangle$ state with one dimension, the state with $I=2$ is obtained by introducing the following generator operator,

$$G^{28}_{I=2} \equiv \frac{1}{\sqrt{15}}\frac{1}{4!}\frac{1}{2!}\sum_{\sigma\in S_6}\sigma, \tag{A4}$$

where S_6 is a permutation group for particles 1–6.

The action of the $G^{28}_{I=2}$ on $u(1)u(2)u(3)u(4)s(5)s(6)$ gives the $|F^{28}\rangle$ state with $I=2$, whose symmetry property has [123456], because $\pi\sum_{\sigma\in S_6}\sigma = \sum_{\sigma\in S_6}\sigma$ for $\pi\in S_6$.

5. Flavor singlet with $I=0$

For the flavor singlet, it is convenient to deal with the $|F^1_5\rangle$ state, because the flavor singlet state comes from the flavor singlet for particles 1–3, and the flavor singlet for particles 4–6. The $|F^1_5\rangle$ state with $I=0$ is obtained by introducing the following generator operator,

$$G^1_{5,I=0} \equiv \frac{1}{6}\sum_{\sigma\in S_3}(-1)^\sigma\sigma\sum_{\pi\in S_3}(-1)^\pi\pi, \tag{A5}$$

where $(-1)^\sigma$ is 1 if σ is an even permutation, and -1 if σ is an odd permutation, and the first permutation group S_3 is applied on particles 1–3, and the second on particles 4–6. The action of the $G^1_{5,I=0}$ on $u(1)d(2)s(3)u(4)d(5)s(6)$ gives the $|F^1_5\rangle$ state with $I=0$, whose symmetry property has $\{123\}\{456\}$, because $\pi\sum_{\sigma\in S_3}(-1)^\sigma\sigma = (-1)^\pi\sum_{\sigma\in S_3}(-1)^\sigma\sigma$ for $\pi\in S_3$.

F^{27} multiplet for $I=0$:

$$|F^{27}_1\rangle = \frac{1}{6\sqrt{2}}[I^0_1s(1)s(2)+I^0_1s(1)s(3)+I^0_1s(1)s(4)+I^0_1s(2)s(3)+I^0_1s(2)s(4)+I^0_1s(3)s(4)],$$

$$|F^{27}_2\rangle = \frac{\sqrt{2}}{3\sqrt{15}}\left[-\frac{1}{4}I^0_1s(1)s(2)+\frac{3}{2}I^0_2s(1)s(2)-\frac{1}{4}I^0_1s(1)s(3)+\frac{3}{2}I^0_2s(1)s(3)+I^0_1s(1)s(5)+\frac{1}{4}I^0_1s(1)s(4)-\frac{1}{4}I^0_1s(2)s(3)\right.$$
$$\left.+\frac{3}{2}I^0_2s(2)s(3)+I^0_1s(2)s(5)+\frac{1}{4}I^0_1s(2)s(4)+I^0_1s(3)s(5)+\frac{1}{4}I^0_1s(3)s(4)\right],$$

$$|F^{27}_3\rangle = \frac{1}{2\sqrt{15}}\left[-\frac{1}{3}I^0_1s(1)s(2)-I^0_2s(1)s(2)-\frac{1}{6}I^0_1s(1)s(4)+\frac{1}{6}I^0_1s(1)s(3)+\frac{3}{2}I^0_2s(1)s(4)+\frac{1}{2}I^0_2s(1)s(3)-\frac{1}{6}I^0_1s(1)s(5)\right.$$
$$+\frac{3}{2}I^0_2s(1)s(5)-\frac{1}{6}I^0_1s(2)s(4)+\frac{1}{6}I^0_1s(2)s(3)+\frac{3}{2}I^0_2s(2)s(4)+\frac{1}{2}I^0_2s(2)s(3)-\frac{1}{6}I^0_1s(2)s(5)+\frac{3}{2}I^0_2s(2)s(5)$$
$$\left.+I^0_1s(4)s(5)+\frac{1}{3}I^0_1s(3)s(5)+\frac{1}{3}I^0_1s(3)s(4)\right],$$

$$|F^{27}_4\rangle = \frac{1}{\sqrt{45}}\left[-\frac{1}{4}I^0_1s(1)s(3)-\frac{3}{4}I^0_2s(1)s(3)-\frac{1}{4}I^0_1s(1)s(4)-\frac{3}{4}I^0_2s(1)s(4)-\frac{1}{4}I^0_1s(1)s(5)-\frac{3}{4}I^0_2s(1)s(5)+\frac{1}{4}I^0_1s(2)s(4)\right.$$
$$+\frac{1}{4}I^0_1s(2)s(3)+\frac{3}{2}I^0_2s(3)s(4)+\frac{3}{4}I^0_2s(2)s(4)+\frac{3}{4}I^0_2s(2)s(3)+\frac{1}{4}I^0_1s(2)s(5)+\frac{3}{2}I^0_2s(3)s(5)+\frac{3}{4}I^0_2s(2)s(5)$$
$$\left.+\frac{3}{2}I^0_2s(4)s(5)\right],$$

$$|F^{27}_5\rangle = \frac{1}{2\sqrt{15}}\left[-\frac{1}{3}I^0_1s(1)s(2)-I^0_2s(1)s(2)-\frac{1}{3}I^0_1s(1)s(3)-I^0_2s(1)s(3)+I^0_1s(1)s(6)+\frac{1}{3}I^0_1s(1)s(5)+\frac{1}{3}I^0_1s(1)s(4)\right.$$
$$-\frac{1}{3}I^0_1s(2)s(3)-I^0_2s(2)s(3)+I^0_1s(2)s(6)+\frac{1}{3}I^0_1s(2)s(5)+\frac{1}{3}I^0_1s(2)s(4)+I^0_1s(3)s(6)+\frac{1}{3}I^0_1s(3)s(5)$$
$$\left.+\frac{1}{3}I^0_1s(3)s(4)\right],$$

$$|F^{27}_6\rangle = \frac{\sqrt{3}}{4\sqrt{10}}\left[-\tfrac{4}{9}I^0_1s(1)s(2)+\tfrac{2}{3}I^0_2s(1)s(2)-\tfrac{2}{9}I^0_1s(1)s(4)+\tfrac{2}{9}I^0_1s(1)s(3)-I^0_2s(1)s(4)-\tfrac{1}{3}I^0_2s(1)s(3)-\tfrac{1}{6}I^0_1s(1)s(6)\right.$$
$$+\frac{3}{2}I^0_2s(1)s(6)-\frac{1}{18}I^0_1s(1)s(5)+\frac{1}{2}I^0_2s(1)s(5)-\frac{2}{9}I^0_1s(2)s(4)+\frac{2}{9}I^0_1s(2)s(3)-I^0_2s(2)s(4)-\frac{1}{3}I^0_2s(2)s(3)$$
$$-\frac{1}{6}I^0_1s(2)s(6)+\frac{3}{2}I^0_2s(2)s(6)-\frac{1}{18}I^0_1s(2)s(5)+\frac{1}{2}I^0_2s(2)s(5)+I^0_1s(4)s(6)+\frac{1}{3}I^0_1s(3)s(6)+\frac{1}{3}I^0_1s(4)s(5)$$
$$\left.+\frac{1}{9}I^0_1s(3)s(5)+\frac{4}{9}I^0_1s(3)s(4)\right],$$

$$|F_7^{27}\rangle = \frac{1}{2\sqrt{10}}\left[-\frac{1}{3}I_1^0 s(1)s(3) + \frac{1}{2}I_2^0 s(1)s(3) - \frac{1}{3}I_1^0 s(1)s(4) + \frac{1}{2}I_2^0 s(1)s(4) - \frac{1}{4}I_1^0 s(1)s(6) - \frac{3}{4}I_2^0 s(1)s(6)\right.$$
$$- \frac{1}{12}I_1^0 s(1)s(5) - \frac{1}{4}I_2^0 s(1)s(5) + \frac{1}{3}I_1^0 s(2)s(4) + \frac{1}{3}I_1^0 s(2)s(3) - I_2^0 s(3)s(4) - \frac{1}{2}I_2^0 s(2)s(4)$$
$$- \frac{1}{2}I_2^0 s(2)s(3) + \frac{1}{4}I_1^0 s(2)s(6) + \frac{3}{2}I_2^0 s(3)s(6) + \frac{3}{4}I_2^0 s(2)s(6) + \frac{1}{12}I_1^0 s(2)s(5) + \frac{1}{2}I_2^0 s(3)s(5)$$
$$\left.+ \frac{1}{4}I_2^0 s(2)s(5) + \frac{3}{2}I_2^0 s(4)s(6) + \frac{1}{2}I_2^0 s(4)s(5)\right],$$
$$|F_8^{27}\rangle = \frac{1}{4\sqrt{10}}\left[-\frac{2}{3}I_1^0 s(1)s(2) - \frac{1}{3}I_1^0 s(1)s(3) - \frac{1}{3}I_1^0 s(1)s(4) - \frac{1}{2}I_1^0 s(1)s(5) - \frac{3}{2}I_2^0 s(1)s(5) - \frac{1}{2}I_1^0 s(1)s(6) - \frac{3}{2}I_2^0 s(1)s(6)\right.$$
$$- \frac{1}{3}I_1^0 s(2)s(3) - \frac{1}{3}I_1^0 s(2)s(4) - \frac{1}{2}I_1^0 s(2)s(5) - \frac{3}{2}I_2^0 s(2)s(5) - \frac{1}{2}I_1^0 s(2)s(6) - \frac{3}{2}I_2^0 s(2)s(6) + \frac{2}{3}I_1^0 s(3)s(4)$$
$$\left.+ I_1^0 s(3)s(5) + I_1^0 s(3)s(6) + I_1^0 s(4)s(5) + I_1^0 s(4)s(6) + 2I_1^0 s(5)s(6)\right],$$
$$|F_9^{27}\rangle = \frac{\sqrt{3}}{4\sqrt{10}}\left[\frac{1}{3}I_1^0 s(1)s(3) - \frac{1}{3}I_1^0 s(1)s(4) - \frac{1}{2}I_1^0 s(1)s(5) + \frac{1}{2}I_2^0 s(1)s(5) - \frac{1}{2}I_1^0 s(1)s(6) + \frac{1}{2}I_2^0 s(1)s(6) - \frac{1}{3}I_1^0 s(2)s(3)\right.$$
$$+ \frac{1}{3}I_1^0 s(2)s(4) + \frac{1}{2}I_1^0 s(2)s(5) - \frac{1}{2}I_2^0 s(2)s(5) + \frac{1}{2}I_1^0 s(2)s(6) - \frac{1}{2}I_2^0 s(2)s(6) - I_2^0 s(3)s(5) - I_2^0 s(3)s(6)$$
$$\left.+ I_2^0 s(4)s(5) + I_2^0 s(4)s(6) + 2I_2^0 s(5)s(6)\right]. \tag{A6}$$

F^{27} multiplet for $I = 1$:

$$|F_1^{27}\rangle = -\frac{1}{12\sqrt{3}}I_1^1 s(1)s(2) - \frac{1}{12\sqrt{3}}I_1^1 s(1)s(3) - \frac{1}{12\sqrt{3}}I_1^1 s(1)s(4) + \frac{1}{8\sqrt{3}}I_1^1 s(1)s(5) + \frac{1}{8\sqrt{3}}I_1^1 s(1)s(6)$$
$$- \frac{1}{12\sqrt{3}}I_1^1 s(2)s(3) - \frac{1}{12\sqrt{3}}I_1^1 s(2)s(4) + \frac{1}{8\sqrt{3}}I_1^1 s(2)s(5) + \frac{1}{8\sqrt{3}}I_1^1 s(2)s(6) - \frac{1}{12\sqrt{3}}I_1^1 s(3)s(4)$$
$$+ \frac{1}{8\sqrt{3}}I_1^1 s(3)s(5) + \frac{1}{8\sqrt{3}}I_1^1 s(3)s(6) + \frac{1}{8\sqrt{3}}I_1^1 s(4)s(5) + \frac{1}{8\sqrt{3}}I_1^1 s(4)s(6) - \frac{1}{6\sqrt{3}}I_2^1 s(1)s(2)$$
$$- \frac{1}{6\sqrt{3}}I_2^1 s(1)s(3) - \frac{1}{6\sqrt{3}}I_2^1 s(1)s(4) - \frac{1}{6\sqrt{3}}I_2^1 s(2)s(3) - \frac{1}{6\sqrt{3}}I_2^1 s(2)s(4) - \frac{1}{6\sqrt{3}}I_2^1 s(3)s(4),$$
$$|F_2^{27}\rangle = -\frac{\sqrt{5}}{36}I_1^1 s(1)s(2) - \frac{\sqrt{5}}{36}I_1^1 s(1)s(3) + \frac{\sqrt{5}}{36}I_1^1 s(1)s(4) - \frac{\sqrt{5}}{72}I_1^1 s(1)s(5) - \frac{1}{72\sqrt{5}}I_1^1 s(1)s(6) - \frac{\sqrt{5}}{36}I_1^1 s(2)s(3)$$
$$+ \frac{\sqrt{5}}{36}I_1^1 s(2)s(4) - \frac{\sqrt{5}}{72}I_1^1 s(2)s(5) - \frac{1}{72\sqrt{5}}I_1^1 s(2)s(6) + \frac{\sqrt{5}}{36}I_1^1 s(3)s(4) - \frac{\sqrt{5}}{72}I_1^1 s(3)s(5) - \frac{1}{72\sqrt{5}}I_1^1 s(3)s(6)$$
$$+ \frac{\sqrt{5}}{24}I_1^1 s(4)s(5) + \frac{1}{24\sqrt{5}}I_1^1 s(4)s(6) + \frac{1}{6\sqrt{5}}I_1^1 s(5)s(6) + \frac{1}{18\sqrt{5}}I_2^1 s(1)s(2) + \frac{1}{18\sqrt{5}}I_2^1 s(1)s(3)$$
$$- \frac{1}{18\sqrt{5}}I_2^1 s(1)s(4) - \frac{2}{9\sqrt{5}}I_2^1 s(1)s(5) + \frac{2}{9\sqrt{5}}I_2^1 s(1)s(6) + \frac{1}{18\sqrt{5}}I_2^1 s(2)s(3) - \frac{1}{18\sqrt{5}}I_2^1 s(2)s(4)$$
$$- \frac{2}{9\sqrt{5}}I_2^1 s(2)s(5) + \frac{2}{9\sqrt{5}}I_2^1 s(2)s(6) - \frac{1}{18\sqrt{5}}I_2^1 s(3)s(4) - \frac{2}{9\sqrt{5}}I_2^1 s(3)s(5) + \frac{2}{9\sqrt{5}}I_2^1 s(3)s(6)$$
$$- \frac{1}{3\sqrt{5}}I_3^1 s(1)s(2) - \frac{1}{3\sqrt{5}}I_3^1 s(1)s(3) - \frac{1}{3\sqrt{5}}I_3^1 s(2)s(3),$$

$$\begin{aligned}
|F_3^{27}\rangle = & -\frac{\sqrt{10}}{36}I_1^1s(1)s(2)+\frac{\sqrt{10}}{72}I_1^1s(1)s(3)-\frac{\sqrt{10}}{72}I_1^1s(1)s(4)-\frac{\sqrt{10}}{72}I_1^1s(1)s(5)-\frac{\sqrt{10}}{360}I_1^1s(1)s(6)+\frac{\sqrt{10}}{72}I_1^1s(2)s(3)\\
& -\frac{\sqrt{10}}{72}I_1^1s(2)s(4)-\frac{\sqrt{10}}{72}I_1^1s(2)s(5)-\frac{\sqrt{10}}{360}I_1^1s(2)s(6)+\frac{\sqrt{10}}{36}I_1^1s(3)s(4)+\frac{\sqrt{10}}{36}I_1^1s(3)s(5)+\frac{\sqrt{10}}{180}I_1^1s(3)s(6)\\
& +\frac{\sqrt{10}}{90}I_2^1s(1)s(2)-\frac{\sqrt{10}}{180}I_2^1s(1)s(3)+\frac{\sqrt{10}}{180}I_2^1s(1)s(4)+\frac{\sqrt{10}}{180}I_2^1s(1)s(5)-\frac{\sqrt{10}}{180}I_2^1s(1)s(6)-\frac{\sqrt{10}}{180}I_2^1s(2)s(3)\\
& +\frac{\sqrt{10}}{180}I_2^1s(2)s(4)+\frac{\sqrt{10}}{180}I_2^1s(2)s(5)-\frac{\sqrt{10}}{180}I_2^1s(2)s(6)-\frac{\sqrt{10}}{90}I_2^1s(3)s(4)-\frac{\sqrt{10}}{90}I_2^1s(3)s(5)+\frac{\sqrt{10}}{90}I_2^1s(3)s(6)\\
& -\frac{\sqrt{10}}{30}I_2^1s(4)s(5)+\frac{\sqrt{10}}{30}I_2^1s(4)s(6)+\frac{\sqrt{10}}{30}I_2^1s(5)s(6)+\frac{\sqrt{10}}{30}I_3^1s(1)s(2)-\frac{\sqrt{10}}{60}I_3^1s(1)s(3)-\frac{\sqrt{10}}{20}I_3^1s(1)s(4)\\
& -\frac{\sqrt{10}}{20}I_3^1s(1)s(5)+\frac{\sqrt{10}}{20}I_3^1s(1)s(6)-\frac{\sqrt{10}}{60}I_3^1s(2)s(3)-\frac{\sqrt{10}}{20}I_3^1s(2)s(4)-\frac{\sqrt{10}}{20}I_3^1s(2)s(5)+\frac{\sqrt{10}}{20}I_3^1s(2)s(6),
\end{aligned}$$

$$\begin{aligned}
|F_4^{27}\rangle = & -\frac{\sqrt{30}}{72}I_1^1s(1)s(3)-\frac{\sqrt{30}}{72}I_1^1s(1)s(4)-\frac{\sqrt{30}}{72}I_1^1s(1)s(5)-\frac{\sqrt{30}}{360}I_1^1s(1)s(6)+\frac{\sqrt{30}}{72}I_1^1s(2)s(3)+\frac{\sqrt{30}}{72}I_1^1s(2)s(4)\\
& +\frac{\sqrt{30}}{72}I_1^1s(2)s(5)+\frac{\sqrt{30}}{360}I_1^1s(2)s(6)+\frac{\sqrt{30}}{180}I_2^1s(1)s(3)+\frac{\sqrt{30}}{180}I_2^1s(1)s(4)+\frac{\sqrt{30}}{180}I_2^1s(1)s(5)-\frac{\sqrt{30}}{180}I_2^1s(1)s(6)\\
& -\frac{\sqrt{30}}{180}I_2^1s(2)s(3)-\frac{\sqrt{30}}{180}I_2^1s(2)s(4)-\frac{\sqrt{30}}{180}I_2^1s(2)s(5)+\frac{\sqrt{30}}{180}I_2^1s(2)s(6)+\frac{\sqrt{30}}{60}I_3^1s(1)s(3)+\frac{\sqrt{30}}{60}I_3^1s(1)s(4)\\
& +\frac{\sqrt{30}}{60}I_3^1s(1)s(5)-\frac{\sqrt{30}}{60}I_3^1s(1)s(6)-\frac{\sqrt{30}}{60}I_3^1s(2)s(3)-\frac{\sqrt{30}}{60}I_3^1s(2)s(4)-\frac{\sqrt{30}}{60}I_3^1s(2)s(5)+\frac{\sqrt{30}}{60}I_3^1s(2)s(6)\\
& -\frac{1}{\sqrt{30}}I_3^1s(3)s(4)-\frac{1}{\sqrt{30}}I_3^1s(3)s(5)+\frac{1}{\sqrt{30}}I_3^1s(3)s(6)-\frac{1}{\sqrt{30}}I_3^1s(4)s(5)+\frac{1}{\sqrt{30}}I_3^1s(4)s(6)+\frac{1}{\sqrt{30}}I_3^1s(5)s(6),
\end{aligned}$$

$$\begin{aligned}
|F_5^{27}\rangle = & -\frac{\sqrt{10}}{180}I_1^1s(1)s(2)-\frac{\sqrt{10}}{180}I_1^1s(1)s(3)+\frac{\sqrt{10}}{180}I_1^1s(1)s(4)+\frac{\sqrt{10}}{180}I_1^1s(1)s(5)+\frac{\sqrt{10}}{90}I_1^1s(1)s(6)-\frac{\sqrt{10}}{180}I_1^1s(2)s(3)\\
& +\frac{\sqrt{10}}{180}I_1^1s(2)s(4)+\frac{\sqrt{10}}{180}I_1^1s(2)s(5)+\frac{\sqrt{10}}{90}I_1^1s(2)s(6)+\frac{\sqrt{10}}{180}I_1^1s(3)s(4)+\frac{\sqrt{10}}{180}I_1^1s(3)s(5)+\frac{\sqrt{10}}{90}I_1^1s(3)s(6)\\
& -\frac{\sqrt{10}}{60}I_1^1s(4)s(5)-\frac{\sqrt{10}}{30}I_1^1s(4)s(6)-\frac{\sqrt{10}}{30}I_1^1s(5)s(6)+\frac{\sqrt{10}}{45}I_2^1s(1)s(2)+\frac{\sqrt{10}}{45}I_2^1s(1)s(3)-\frac{\sqrt{10}}{45}I_2^1s(1)s(4)\\
& -\frac{\sqrt{10}}{45}I_2^1s(1)s(5)+\frac{\sqrt{10}}{45}I_2^1s(1)s(6)+\frac{\sqrt{10}}{45}I_2^1s(2)s(3)-\frac{\sqrt{10}}{45}I_2^1s(2)s(4)-\frac{\sqrt{10}}{45}I_2^1s(2)s(5)+\frac{\sqrt{10}}{45}I_2^1s(2)s(6)\\
& -\frac{\sqrt{10}}{45}I_2^1s(3)s(4)-\frac{\sqrt{10}}{45}I_2^1s(3)s(5)+\frac{\sqrt{10}}{45}I_2^1s(3)s(6)+\frac{\sqrt{10}}{15}I_3^1s(1)s(2)+\frac{\sqrt{10}}{15}I_3^1s(1)s(3)+\frac{\sqrt{10}}{15}I_3^1s(2)s(3),
\end{aligned}$$

$$\begin{aligned}
|F_6^{27}\rangle = & \frac{\sqrt{5}}{90}I_1^1s(1)s(2)-\frac{\sqrt{5}}{180}I_1^1s(1)s(3)+\frac{\sqrt{5}}{180}I_1^1s(1)s(4)-\frac{\sqrt{5}}{90}I_1^1s(1)s(5)-\frac{\sqrt{5}}{45}I_1^1s(1)s(6)-\frac{\sqrt{5}}{180}I_1^1s(2)s(3)\\
& +\frac{\sqrt{5}}{180}I_1^1s(2)s(4)-\frac{\sqrt{5}}{90}I_1^1s(2)s(5)-\frac{\sqrt{5}}{45}I_1^1s(2)s(6)-\frac{\sqrt{5}}{90}I_1^1s(3)s(4)+\frac{\sqrt{5}}{45}I_1^1s(3)s(5)+\frac{2\sqrt{5}}{45}I_1^1s(3)s(6)\\
& -\frac{2\sqrt{5}}{45}I_2^1s(1)s(2)+\frac{\sqrt{5}}{45}I_2^1s(1)s(3)-\frac{\sqrt{5}}{45}I_2^1s(1)s(4)-\frac{\sqrt{5}}{180}I_2^1s(1)s(5)+\frac{\sqrt{5}}{180}I_2^1s(1)s(6)+\frac{\sqrt{5}}{45}I_2^1s(2)s(3)\\
& -\frac{\sqrt{5}}{45}I_2^1s(2)s(4)-\frac{\sqrt{5}}{180}I_2^1s(2)s(5)+\frac{\sqrt{5}}{180}I_2^1s(2)s(6)+\frac{2\sqrt{5}}{45}I_2^1s(3)s(4)+\frac{\sqrt{5}}{90}I_2^1s(3)s(5)-\frac{\sqrt{5}}{90}I_2^1s(3)s(6)\\
& +\frac{\sqrt{5}}{30}I_2^1s(4)s(5)-\frac{\sqrt{5}}{30}I_2^1s(4)s(6)+\frac{\sqrt{5}}{15}I_2^1s(5)s(6)+\frac{\sqrt{5}}{15}I_3^1s(1)s(2)-\frac{\sqrt{5}}{30}I_3^1s(1)s(3)-\frac{\sqrt{5}}{10}I_3^1s(1)s(4)\\
& +\frac{\sqrt{5}}{20}I_3^1s(1)s(5)-\frac{\sqrt{5}}{20}I_3^1s(1)s(6)-\frac{\sqrt{5}}{30}I_3^1s(2)s(3)-\frac{\sqrt{5}}{10}I_3^1s(2)s(4)+\frac{\sqrt{5}}{20}I_3^1s(2)s(5)-\frac{\sqrt{5}}{20}I_3^1s(2)s(6),
\end{aligned}$$

$$|F_7^{27}\rangle = \frac{\sqrt{15}}{180} I_1^1 s(1)s(3) + \frac{\sqrt{15}}{180} I_1^1 s(1)s(4) - \frac{\sqrt{15}}{90} I_1^1 s(1)s(5) - \frac{\sqrt{15}}{45} I_1^1 s(1)s(6) - \frac{\sqrt{15}}{180} I_1^1 s(2)s(3) - \frac{\sqrt{15}}{180} I_1^1 s(2)s(4)$$
$$+ \frac{\sqrt{15}}{90} I_1^1 s(2)s(5) + \frac{\sqrt{15}}{45} I_1^1 s(2)s(6) - \frac{\sqrt{15}}{45} I_2^1 s(1)s(3) - \frac{\sqrt{15}}{45} I_2^1 s(1)s(4) - \frac{\sqrt{15}}{180} I_2^1 s(1)s(5) + \frac{\sqrt{15}}{180} I_2^1 s(1)s(6)$$
$$+ \frac{\sqrt{15}}{45} I_2^1 s(2)s(3) + \frac{\sqrt{15}}{45} I_2^1 s(2)s(4) + \frac{\sqrt{15}}{180} I_2^1 s(2)s(5) - \frac{\sqrt{15}}{180} I_2^1 s(2)s(6) + \frac{\sqrt{15}}{30} I_3^1 s(1)s(3) + \frac{\sqrt{15}}{30} I_3^1 s(1)s(4)$$
$$- \frac{\sqrt{15}}{60} I_3^1 s(1)s(5) + \frac{\sqrt{15}}{60} I_3^1 s(1)s(6) - \frac{\sqrt{15}}{30} I_3^1 s(2)s(3) - \frac{\sqrt{15}}{30} I_3^1 s(2)s(4) + \frac{\sqrt{15}}{60} I_3^1 s(2)s(5) - \frac{\sqrt{15}}{60} I_3^1 s(2)s(6)$$
$$- \frac{1}{\sqrt{15}} I_3^1 s(3)s(4) + \frac{1}{2\sqrt{15}} I_3^1 s(3)s(5) - \frac{1}{2\sqrt{15}} I_3^1 s(3)s(6) + \frac{1}{2\sqrt{15}} I_3^1 s(4)s(5)$$
$$- \frac{1}{2\sqrt{15}} I_3^1 s(4)s(6) + \frac{1}{\sqrt{15}} I_3^1 s(5)s(6),$$

$$|F_8^{27}\rangle = -\frac{\sqrt{15}}{90} I_1^1 s(1)s(2) + \frac{\sqrt{15}}{180} I_1^1 s(1)s(3) + \frac{\sqrt{15}}{180} I_1^1 s(1)s(4) + \frac{\sqrt{15}}{180} I_1^1 s(2)s(3) + \frac{\sqrt{15}}{180} I_1^1 s(2)s(4) - \frac{\sqrt{15}}{90} I_1^1 s(3)s(4)$$
$$- \frac{\sqrt{15}}{45} I_2^1 s(1)s(2) + \frac{\sqrt{15}}{90} I_2^1 s(1)s(3) + \frac{\sqrt{15}}{60} I_2^1 s(1)s(4) + \frac{\sqrt{15}}{60} I_2^1 s(1)s(5) + \frac{\sqrt{15}}{60} I_2^1 s(1)s(6) + \frac{\sqrt{15}}{90} I_2^1 s(2)s(3)$$
$$+ \frac{\sqrt{15}}{90} I_2^1 s(2)s(4) + \frac{\sqrt{15}}{60} I_2^1 s(2)s(5) + \frac{\sqrt{15}}{60} I_2^1 s(2)s(6) - \frac{\sqrt{15}}{45} I_2^1 s(3)s(4) - \frac{\sqrt{15}}{30} I_2^1 s(3)s(5) - \frac{\sqrt{15}}{30} I_2^1 s(3)s(6)$$
$$- \frac{\sqrt{15}}{30} I_2^1 s(4)s(5) - \frac{\sqrt{15}}{30} I_2^1 s(4)s(6) + \frac{\sqrt{15}}{20} I_3^1 s(1)s(5) + \frac{\sqrt{15}}{20} I_3^1 s(1)s(6) + \frac{\sqrt{15}}{20} I_3^1 s(2)s(5) + \frac{\sqrt{15}}{20} I_3^1 s(2)s(6),$$

$$|F_9^{27}\rangle = \frac{\sqrt{5}}{60} I_1^1 s(1)s(3) - \frac{\sqrt{5}}{60} I_1^1 s(1)s(4) - \frac{\sqrt{5}}{60} I_1^1 s(2)s(3) + \frac{\sqrt{5}}{60} I_1^1 s(2)s(4) + \frac{\sqrt{5}}{30} I_2^1 s(1)s(3) - \frac{\sqrt{5}}{30} I_2^1 s(1)s(4)$$
$$- \frac{\sqrt{5}}{20} I_2^1 s(1)s(5) - \frac{\sqrt{5}}{20} I_2^1 s(1)s(6) - \frac{\sqrt{5}}{30} I_2^1 s(2)s(3) + \frac{\sqrt{5}}{30} I_2^1 s(2)s(4) + \frac{\sqrt{5}}{20} I_2^1 s(2)s(5) + \frac{\sqrt{5}}{20} I_2^1 s(2)s(6)$$
$$+ \frac{\sqrt{5}}{20} I_3^1 s(1)s(5) + \frac{\sqrt{5}}{20} I_3^1 s(1)s(6) - \frac{\sqrt{5}}{20} I_3^1 s(2)s(5) - \frac{\sqrt{5}}{20} I_3^1 s(2)s(6) - \frac{\sqrt{5}}{10} I_3^1 s(3)s(5) - \frac{\sqrt{5}}{10} I_3^1 s(3)s(6)$$
$$+ \frac{\sqrt{5}}{10} I_3^1 s(4)s(5) + \frac{\sqrt{5}}{10} I_3^1 s(4)s(6). \tag{A7}$$

F^{27} multiplet for $I = 2$:

$$|F_1^{27}\rangle = \frac{1}{4\sqrt{15}} [12 I^2 s(5)s(6) - 3I^2 s(4)s(6) - 3I^2 s(3)s(6) - 3I^2 s(2)s(6) - 3I^2 s(1)s(6) - 3I^2 s(4)s(5) - 3I^2 s(3)s(5)$$
$$- 3I^2 s(2)s(5) - 3I^2 s(1)s(5) + 2I^2 s(3)s(4) + 2I^2 s(2)s(4) + 2I^2 s(1)s(4) + 2I^2 s(2)s(3) + 2I^2 s(1)s(3)$$
$$+ 2I^2 s(1)s(2)],$$

$$|F_2^{27}\rangle = \frac{1}{12} [2I^2 s(1)s(2) + 2I^2 s(1)s(3) - 2I^2 s(1)s(4) + I^2 s(1)s(5) - 3I^2 s(1)s(6) + 2I^2 s(2)s(3) - 2I^2 s(2)s(4)$$
$$+ I^2 s(2)s(5) - 3I^2 s(2)s(6) - 2I^2 s(3)s(4) + I^2 s(3)s(5) - 3I^2 s(3)s(6) - 3I^2 s(4)s(5) + 9I^2 s(4)s(6)],$$

$$|F_3^{27}\rangle = \frac{1}{6\sqrt{2}} [2I^2 s(1)s(2) - I^2 s(1)s(3) + I^2 s(1)s(4) + I^2 s(1)s(5) - 3I^2 s(1)s(6) - I^2 s(2)s(3) + I^2 s(2)s(4)$$
$$+ I^2 s(2)s(5) - 3I^2 s(2)s(6) - 2I^2 s(3)s(4) - 2I^2 s(3)s(5) + 6I^2 s(3)s(6)],$$

$$|F_4^{27}\rangle = \frac{1}{2\sqrt{6}} [I^2 s(1)s(3) + I^2 s(1)s(4) + I^2 s(1)s(5) - 3I^2 s(1)s(6) - I^2 s(2)s(3) - I^2 s(2)s(4) - I^2 s(2)s(5)$$
$$+ 3I^2 s(2)s(6)],$$

$$|F_5^{27}\rangle = \frac{1}{3\sqrt{2}}[I^2s(1)s(2) + I^2s(1)s(3) - I^2s(1)s(4) - I^2s(1)s(5) + I^2s(2)s(3) - I^2s(2)s(4) - I^2s(2)s(5) - I^2s(3)s(4) - I^2s(3)s(5) + 3I^2s(4)s(5)],$$

$$|F_6^{27}\rangle = \frac{1}{6\sqrt{2}}[2I^2s(1)s(2) - I^2s(1)s(3) + I^2s(1)s(4) - 2I^2s(1)s(5) - I^2s(2)s(3) + I^2s(2)s(4) - 2I^2s(2)s(5) - 2I^2s(3)s(4) + 4I^2s(3)s(5)],$$

$$|F_7^{27}\rangle = \frac{1}{2\sqrt{3}}[I^2s(1)s(3) + I^2s(1)s(4) - 2I^2s(1)s(5) - I^2s(2)s(3) - I^2s(2)s(4) + 2I^2s(2)s(5)],$$

$$|F_8^{27}\rangle = \frac{1}{2\sqrt{3}}[2I^2s(1)s(2) - I^2s(1)s(3) - I^2s(1)s(4) - I^2s(2)s(3) - I^2s(2)s(4) + 2I^2s(3)s(4)],$$

$$|F_9^{27}\rangle = \frac{1}{2}[I^2s(1)s(3) - I^2s(1)s(4) - I^2s(2)s(3) + I^2s(2)s(4)]. \tag{A8}$$

F^{28} multiplet for $I = 2$:

$$|F^{28}\rangle = \frac{1}{\sqrt{15}}[I^2s(1)s(2) + I^2s(1)s(3) + I^2s(1)s(4) + I^2s(1)s(5) + I^2s(1)s(6) + I^2s(2)s(3) + I^2s(2)s(4) + I^2s(2)s(5) + I^2s(2)s(6) + I^2s(3)s(4) + I^2s(3)s(5) + I^2s(3)s(6) + I^2s(4)s(5) + I^2s(4)s(6) + I^2s(5)s(6)]. \tag{A9}$$

F^1 multiplet for $I = 0$:

$$|F_1^1\rangle = \frac{1}{8\sqrt{18}}[4I_1^0s(5)s(6) - 2I_1^0s(4)s(6) - 2I_1^0s(4)s(5) + 3I_2^0s(2)s(5) - 2I_1^0s(2)s(3) + 4I_1^0s(1)s(2) - 2I_1^0s(1)s(3) + 3I_2^0s(2)s(6) + 3I_2^0s(1)s(5) - 2I_1^0s(3)s(6) + I_1^0s(2)s(6) + 4I_1^0s(3)s(4) - 2I_1^0s(3)s(5) - 2I_1^0s(2)s(4) + I_1^0s(2)s(5) + I_1^0s(1)s(6) + 3I_2^0s(1)s(6) + I_1^0s(1)s(5) - 2I_1^0s(1)s(4)],$$

$$|F_2^1\rangle = \frac{\sqrt{2}}{16\sqrt{3}}[4I_2^0s(5)s(6) - 2I_2^0s(4)s(6) - 2I_2^0s(4)s(5) + 2I_2^0s(3)s(5) + I_2^0s(2)s(5) - 2I_1^0s(1)s(3) + 2I_1^0s(1)s(3) + 2I_2^0s(3)s(6) + I_2^0s(2)s(6) - I_2^0s(1)s(5) - I_1^0s(2)s(6) + 2I_1^0s(2)s(4) - I_1^0s(2)s(5) + I_1^0s(1)s(6) - I_2^0s(1)s(6) + I_1^0s(1)s(5) - 2I_1^0s(1)s(4)],$$

$$|F_3^1\rangle = \frac{1}{8\sqrt{6}}[2I_1^0s(4)s(6) - 2I_1^0s(4)s(5) + 2I_2^0s(2)s(4) - 2I_2^0s(2)s(3) + 4I_2^0s(1)s(2) - 2I_2^0s(1)s(3) - I_2^0s(2)s(6) + I_2^0s(2)s(5) + 2I_2^0s(1)s(4) - 2I_1^0s(3)s(6) + I_1^0s(2)s(6) + 2I_1^0s(3)s(5) - I_1^0s(2)s(5) + I_1^0s(1)s(6) - I_2^0s(1)s(6) - I_1^0s(1)s(5) + I_2^0s(1)s(5)],$$

$$|F_4^1\rangle = \frac{1}{24\sqrt{2}}[6I_2^0s(4)s(6) - 6I_2^0s(4)s(5) + 4I_2^0s(3)s(4) - 6I_2^0s(2)s(3) + 6I_2^0s(1)s(3) - 2I_2^0s(3)s(6) + 2I_2^0s(3)s(5) + 2I_2^0s(2)s(4) - 2I_2^0s(1)s(4) - 3I_1^0s(2)s(6) - I_2^0s(2)s(6) + 3I_1^0s(2)s(5) + I_2^0s(2)s(5) + 3I_1^0s(1)s(6) + I_2^0s(1)s(6) - 3I_1^0s(1)s(5) - I_2^0s(1)s(5)],$$

$$|F_5^1\rangle = \frac{1}{6}[I_2^0s(3)s(6) - I_2^0s(3)s(5) + I_2^0s(3)s(4) - I_2^0s(2)s(6) + I_2^0s(2)s(5) - I_2^0s(2)s(4) + I_2^0s(1)s(6) - I_2^0s(1)s(5) + I_2^0s(1)s(4)]. \tag{A10}$$

We note that the flavor multiplet bases are orthonormal to each other, that is, $\langle F_k^i|F_l^j\rangle = \delta_{ij}\delta_{kl}$.

APPENDIX B: CS COUPLING

In Appendix B, we present the color ⊗ spin basis, which is obtained from the CS coupling scheme. As mentioned in Sec. III B, the CG coefficient of combining the color singlet basis with the $S=0$ basis is calculated by using Eq. (11). The color ⊗ spin basis represented by the Young-Yamanouci basis [2,2,1,1] and [3,3] is given as follows. Young-Yamanouci basis [2,2,1,1]:

$$\begin{aligned}
|[C,S^0]_1\rangle &= -\frac{\sqrt{6}}{4}|C_1\rangle\otimes|S^0_4\rangle+\frac{\sqrt{6}}{4}|C_2\rangle\otimes|S^0_5\rangle+\frac{\sqrt{6}}{12}|C_3\rangle\otimes|S^0_2\rangle-\frac{\sqrt{3}}{6}|C_3\rangle\otimes|S^0_1\rangle-\frac{\sqrt{6}}{12}|C_4\rangle\otimes|S^0_3\rangle\\
&\quad-\frac{\sqrt{3}}{6}|C_5\rangle\otimes|S^0_3\rangle,\\
|[C,S^0]_2\rangle &= -\frac{\sqrt{3}}{6}|C_4\rangle\otimes|S^0_1\rangle-\frac{\sqrt{6}}{12}|C_4\rangle\otimes|S^0_2\rangle+\frac{\sqrt{3}}{6}|C_5\rangle\otimes|S^0_2\rangle-\frac{\sqrt{6}}{12}|C_3\rangle\otimes|S^0_3\rangle+\frac{\sqrt{6}}{4}|C_2\rangle\otimes|S^0_4\rangle\\
&\quad+\frac{\sqrt{6}}{4}|C_1\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_3\rangle &= -\frac{\sqrt{3}}{6}|C_1\rangle\otimes|S^0_1\rangle-\frac{1}{3}|C_3\rangle\otimes|S^0_1\rangle+\frac{\sqrt{6}}{12}|C_1\rangle\otimes|S^0_2\rangle-\frac{\sqrt{2}}{3}|C_3\rangle\otimes|S^0_2\rangle-\frac{\sqrt{6}}{12}|C_2\rangle\otimes|S^0_3\rangle\\
&\quad+\frac{\sqrt{2}}{3}|C_4\rangle\otimes|S^0_3\rangle-\frac{1}{3}|C_5\rangle\otimes|S^0_3\rangle-\frac{\sqrt{6}}{12}|C_3\rangle\otimes|S^0_4\rangle+\frac{\sqrt{6}}{12}|C_4\rangle\otimes|S^0_5\rangle+\frac{\sqrt{3}}{6}|C_5\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_4\rangle &= -\frac{\sqrt{3}}{6}|C_2\rangle\otimes|S^0_1\rangle-\frac{1}{3}|C_4\rangle\otimes|S^0_1\rangle-\frac{\sqrt{6}}{12}|C_2\rangle\otimes|S^0_2\rangle+\frac{\sqrt{2}}{3}|C_4\rangle\otimes|S^0_2\rangle+\frac{1}{3}|C_5\rangle\otimes|S^0_2\rangle\\
&\quad-\frac{\sqrt{6}}{12}|C_1\rangle\otimes|S^0_3\rangle+\frac{\sqrt{2}}{3}|C_3\rangle\otimes|S^0_3\rangle+\frac{\sqrt{6}}{12}|C_4\rangle\otimes|S^0_4\rangle-\frac{\sqrt{3}}{6}|C_5\rangle\otimes|S^0_4\rangle+\frac{\sqrt{6}}{12}|C_3\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_5\rangle &= \frac{2}{3}|C_5\rangle\otimes|S^0_1\rangle+\frac{\sqrt{3}}{6}|C_2\rangle\otimes|S^0_2\rangle+\frac{1}{3}|C_4\rangle\otimes|S^0_2\rangle-\frac{\sqrt{3}}{6}|C_1\rangle\otimes|S^0_3\rangle-\frac{1}{3}|C_3\rangle\otimes|S^0_3\rangle\\
&\quad-\frac{\sqrt{3}}{6}|C_4\rangle\otimes|S^0_4\rangle+\frac{\sqrt{3}}{6}|C_3\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_6\rangle &= -\frac{\sqrt{6}}{6}|C_1\rangle\otimes|S^0_1\rangle+\frac{\sqrt{2}}{6}|C_3\rangle\otimes|S^0_1\rangle+\frac{\sqrt{3}}{6}|C_1\rangle\otimes|S^0_2\rangle+\frac{1}{3}|C_3\rangle\otimes|S^0_2\rangle-\frac{\sqrt{3}}{6}|C_2\rangle\otimes|S^0_3\rangle\\
&\quad-\frac{1}{3}|C_4\rangle\otimes|S^0_3\rangle+\frac{\sqrt{2}}{6}|C_5\rangle\otimes|S^0_3\rangle-\frac{\sqrt{3}}{6}|C_3\rangle\otimes|S^0_4\rangle+\frac{\sqrt{3}}{6}|C_4\rangle\otimes|S^0_5\rangle+\frac{\sqrt{6}}{6}|C_5\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_7\rangle &= -\frac{\sqrt{6}}{6}|C_2\rangle\otimes|S^0_1\rangle+\frac{\sqrt{2}}{6}|C_4\rangle\otimes|S^0_1\rangle-\frac{\sqrt{3}}{6}|C_2\rangle\otimes|S^0_2\rangle-\frac{1}{3}|C_4\rangle\otimes|S^0_2\rangle-\frac{\sqrt{2}}{6}|C_5\rangle\otimes|S^0_2\rangle\\
&\quad-\frac{\sqrt{3}}{6}|C_1\rangle\otimes|S^0_3\rangle-\frac{1}{3}|C_3\rangle\otimes|S^0_3\rangle+\frac{\sqrt{3}}{6}|C_4\rangle\otimes|S^0_4\rangle-\frac{\sqrt{6}}{6}|C_5\rangle\otimes|S^0_4\rangle+\frac{\sqrt{3}}{6}|C_3\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_8\rangle &= -\frac{\sqrt{2}}{3}|C_5\rangle\otimes|S^0_1\rangle+\frac{\sqrt{6}}{6}|C_2\rangle\otimes|S^0_2\rangle-\frac{\sqrt{2}}{6}|C_4\rangle\otimes|S^0_2\rangle-\frac{\sqrt{6}}{6}|C_1\rangle\otimes|S^0_3\rangle+\frac{\sqrt{2}}{6}|C_3\rangle\otimes|S^0_3\rangle\\
&\quad-\frac{\sqrt{6}}{6}|C_4\rangle\otimes|S^0_4\rangle+\frac{\sqrt{6}}{6}|C_3\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_9\rangle &= \frac{\sqrt{2}}{\sqrt{15}}|C_5\rangle\otimes|S^0_1\rangle-\frac{\sqrt{2}}{\sqrt{15}}|C_4\rangle\otimes|S^0_2\rangle+\frac{\sqrt{2}}{\sqrt{15}}|C_3\rangle\otimes|S^0_3\rangle-\frac{\sqrt{3}}{\sqrt{10}}|C_2\rangle\otimes|S^0_4\rangle+\frac{\sqrt{3}}{\sqrt{10}}|C_1\rangle\otimes|S^0_5\rangle. \qquad \text{(B1)}
\end{aligned}$$

Young-Yamanouci basis [3,3]:

$$
\begin{aligned}
|[C,S^0]_1\rangle &= \frac{1}{2}[|C_1\rangle\otimes|S^0_2\rangle + |C_2\rangle\otimes|S^0_3\rangle + |C_3\rangle\otimes|S^0_4\rangle + |C_4\rangle\otimes|S^0_5\rangle],\\
|[C,S^0]_2\rangle &= \frac{1}{2}|C_1\rangle\otimes|S^0_1\rangle + \frac{\sqrt{2}}{4}|C_1\rangle\otimes|S^0_2\rangle - \frac{\sqrt{2}}{4}|C_2\rangle\otimes|S^0_3\rangle - \frac{\sqrt{2}}{4}|C_3\rangle\otimes|S^0_4\rangle + \frac{\sqrt{2}}{4}|C_4\rangle\otimes|S^0_5\rangle\\
&\quad - \frac{1}{2}|C_5\rangle\otimes|S^0_5\rangle,\\
|[C,S^0]_3\rangle &= -\frac{\sqrt{2}}{4}|C_1\rangle\otimes|S^0_3\rangle + \frac{1}{2}|C_2\rangle\otimes|S^0_1\rangle - \frac{\sqrt{2}}{4}|C_2\rangle\otimes|S^0_2\rangle + \frac{\sqrt{2}}{4}|C_3\rangle\otimes|S^0_5\rangle + \frac{\sqrt{2}}{4}|C_4\rangle\otimes|S^0_4\rangle\\
&\quad + \frac{1}{2}|C_5\rangle\otimes|S^0_4\rangle,\\
|[C,S^0]_4\rangle &= -\frac{\sqrt{2}}{4}|C_1\rangle\otimes|S^0_4\rangle + \frac{\sqrt{2}}{4}|C_2\rangle\otimes|S^0_5\rangle + \frac{1}{2}|C_3\rangle\otimes|S^0_1\rangle - \frac{\sqrt{2}}{4}|C_3\rangle\otimes|S^0_2\rangle + \frac{\sqrt{2}}{4}|C_4\rangle\otimes|S^0_3\rangle\\
&\quad + \frac{1}{2}|C_5\rangle\otimes|S^0_3\rangle,\\
|[C,S^0]_5\rangle &= \frac{\sqrt{2}}{4}|C_1\rangle\otimes|S^0_5\rangle + \frac{\sqrt{2}}{4}|C_2\rangle\otimes|S^0_4\rangle + \frac{\sqrt{2}}{4}|C_3\rangle\otimes|S^0_3\rangle + \frac{1}{2}|C_4\rangle\otimes|S^0_1\rangle + \frac{\sqrt{2}}{4}|C_4\rangle\otimes|S^0_2\rangle\\
&\quad - \frac{1}{2}|C_5\rangle\otimes|S^0_2\rangle.
\end{aligned}
\tag{B2}
$$

[1] R. L. Jaffe, Multiquark hadrons. 2. Methods, Phys. Rev. D **15**, 281 (1977).

[2] R. L. Jaffe, Multiquark hadrons. 1. The phenomenology of (2 quark 2 antiquark) mesons, Phys. Rev. D **15**, 267 (1977).

[3] R. L. Jaffe, Perhaps a Stable Dihyperon, Phys. Rev. Lett. **38**, 195 (1977).

[4] H. Hgaasen, E. Kou, J. M. Richard, and P. Sorba, Isovector and hidden-beauty partners of the X(3872), Phys. Lett. B **732**, 97 (2014).

[5] H. Hogaasen, J. M. Richard, and P. Sorba, A chromomagnetic mechanism for the X(3872) resonance, Phys. Rev. D **73**, 054013 (2006).

[6] B. Silvestre-Brac and J. Leandri, Systematics of q-6 systems in a simple chromomagnetic model, Phys. Rev. D **45**, 4221 (1992).

[7] J. Leandri and B. Silvestre-Brac, Dibaryon states containing two different types of heavy quarks, Phys. Rev. D **51**, 3628 (1995).

[8] B. Silvestre-Brac and C. Semay, Systematics of $L = 0\ q-2$ anti-$q-2$ systems, Z. Phys. C **57**, 273 (1993).

[9] B. Silvestre-Brac and C. Semay, Spectrum and decay properties of diquonia, Z. Phys. C **59**, 457 (1993).

[10] D. M. Brink and F. Stancu, Role of hidden color states in 2 q 2 anti-q systems, Phys. Rev. D **49**, 4665 (1994).

[11] D. M. Brink and F. Stancu, Tetraquarks with heavy flavors, Phys. Rev. D **57**, 6778 (1998).

[12] W. Park and S. H. Lee, Color-spin wave functions of heavy tetraquark states, Nucl. Phys. **A925**, 161 (2014).

[13] J. Vijande, E. Weissman, A. Valcarce, and N. Barnea, Are there compact heavy four-quark bound states?, Phys. Rev. D **76**, 094027 (2007).

[14] J. Vijande, E. Weissman, N. Barnea, and A. Valcarce, Do $c\bar{c}n\bar{n}$ bound states exist?, Phys. Rev. D **76**, 094022 (2007).

[15] A. Gal, Meson assisted dibaryons, Acta Phys. Pol. B **47**, 471 (2016).

[16] T. Sakai, K. Shimizu, and K. Yazaki, *H* Dibaryon, Prog. Theor. Phys. Suppl. **137**, 121 (2000).

[17] N. Kodama, M. Oka, and T. Hatsuda, H-dibaryon in the QCD sum rule, Nucl. Phys. **A580**, 445 (1994).

[18] M. Oka and S. Takeuchi, Instanton induced interaction and the strange dibaryons, Nucl. Phys. **A524**, 649 (1991).

[19] F. Stancu, S. Pepin, and L. Y. Glozman, How the H particle, unravels the quark dynamics, Phys. Rev. D **57**, 4393 (1998).

[20] S. R. Beane *et al.* (NPLQCD Collaboration), Evidence for a Bound H-Dibaryon from Lattice QCD, Phys. Rev. Lett. **106**, 162001 (2011).

[21] T. Inoue, N. Ishii, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, K. Murano, H. Nemura, and K. Sasaki, Bound H-Dibaryon in Flavor SU(3) Limit of Lattice QCD, Phys. Rev. Lett. **106**, 162002 (2011).

[22] T. F. Carames and A. Valcarce, Examination of the H-dibaryon within a chiral constituent quark model, Phys. Rev. C **85**, 045202 (2012).

[23] R. K. Bhaduri, L. E. Cohler, and Y. Nogami, A unified potential for mesons and baryons, Nuovo Cimento A **65**, 376 (1981).

[24] W. Park, A. Park, and S. H. Lee, Dibaryons in a constituent quark model, Phys. Rev. D **92**, 014037 (2015).
[25] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).
[26] F. Stancu, Group theory in subnuclear physics, Oxford Stud. Nucl. Phys. **19**, 1 (1996).
[27] S. Pepin and F. Stancu, On a three-body confinement force in hadron spectroscopy, Phys. Rev. D **65**, 054032 (2002).
[28] F. Stancu and S. Pepin, Isoscalar factors of the permutation group, Few-Body Syst. Suppl. X **26**, 113 (1999).
[29] A. T. M. Aerts, P. J. G. Mulders, and J. J. de Swart, Multibaryon states in the bag model, Phys. Rev. D **17**, 260 (1978).
[30] F. Stancu and L. Wilets, The important configurations in six-quark *NN* states, Phys. Rev. C **38**, 1145 (1988).