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Aharonov-Bohm phase in high density quark matter

Chandrasekhar Chatterjee* and Muneto Nitta†

Department of Physics, and Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan

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Stable non-Abelian vortices, which are color magnetic flux tubes as well as superfluid vortices, are present in the color-flavor locked phase of dense quark matter with diquark condensations. We calculate the Aharanov-Bohm phases of charged particles, that is, electrons, muons, and color-flavor locked mesons made of tetraquarks around a non-Abelian vortex.

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I. INTRODUCTION

The concept of the superfluidity of nuclear matter inside neutron stars was suggested a long time ago by Migdal [1]. The mechanism of the Cooper pair formation inside a superconductor due to the electron-phonon interaction can be extended to the nuclear matter inside a neutron star at sufficiently high density and low temperature, leading to superfluidity and/or superconductivity [2–4]. Several astrophysical observations indicate that this is likely the case. Pulsar glitches [5], that is, the sudden speedup of the rotation frequency of the star, were proposed to be explained by a sudden unpinning of a large number of superfluid vortices in the inner crust of the star [6]; the observed long-time relaxation after pulsar glitches can be explained by two components of normal and superfluid neutrons [4]; and the cooling process of a neutron star was proposed to be explained by the formation of superconducting or superfluid gaps [7,8].

At much higher density, quarks are expected to form Cooper pairs to show color superconductivity [9,10]; see Refs. [11,12] as a review. The two-flavor superconducting (2SC) phase in which up and down quarks participate in condensations are expected to be realized at intermediate density, while the color-flavor locked (CFL) phase in which up, down, and strange quarks participate in condensations is expected to be realized at asymptotically high density. The Ginzburg-Landau free energy in the CFL phase was derived in Refs. [13–15]. While magnetic flux tubes are created in type-II metallic superconductor in the presence of magnetic field, color magnetic flux tubes are present stably in the CFL phase [16–19]. These flux tubes are superfluid vortices created by a rapid rotation of a superconductor; see Ref. [20] as a review. This color magnetic flux is a non-Abelian vortex carrying collective coordinates parametrizing Nambu-Goldstone modes $\mathbb{C}P^2 \simeq \mathrm{SU}(3)_{\mathrm{C+F}}/[\mathrm{SU}(2)\times \mathrm{U}(1)]$ localized around the vortex core that are gapless excitations propagating along the vortex [21,22]. Such vortices will form a vortex lattice in rotating color superconductors, showing color (anti)ferromagnetism [23].

The Aharanov-Bohm (AB) effect [24] is a quantum mechanical effect that occurs when a charged particle scatters from a solenoid with nonzero magnetic flux inside. Outside the solenoid, the field strength is zero everywhere, and the wave function of the particle vanishes at the center of the solenoid. Nevertheless, when a particle goes around the solenoid, it picks up the phase known as the AB phase, leading to a nontrivial differential scattering cross section. The AB effect was experimentally confirmed [25] and has been studied in various nanomaterials in condensed matter physics. Now, the investigation is not only limited to materials but is also explored in various areas of fundamental physics such as cosmology, particle physics, and field theory. Vortices or cosmic strings exhibiting the AB effect, namely, “AB cosmic strings,” were studied extensively [26]. In particular, AB strings feel friction as a consequence of the AB effect [27,28]. AB cosmic strings may give a possible observational signature of string theory [29,30]. The AB effect around non-Abelian vortices in supersymmetric gauge theory was found in Ref. [31], and it has been extended [32,33] to the non-Abelian AB phase [34]. In the context of dense quark matter, the AB effect caused by a color magnetic flux tube was discussed before in the 2SC phase [35], in which the authors discussed scatterings of electrons, muons, and ungapped quarks via the AB effect. The friction of vortices and effects on the transport of particles were also discussed. However, color magnetic flux tubes in the 2SC phase are unstable to decay.

In this paper, we investigate the AB effect of a color magnetic flux tube (non-Abelian vortex) stably existing in the CFL phase of dense quark matter. In the presence of the electromagnetic interaction, a $\mathrm{U}(1)_{\mathrm{em}}$ subgroup of the flavor symmetry $\mathrm{SU}(3)_{\mathrm{F}}$ is gauged. Consequently, an effective potential term on the $\mathbb{C}P^2$ space is induced, resulting in stable and metastable vortices with color magnetic fluxes correspond to generators commuting with $\mathrm{U}(1)_{\mathrm{em}}$ [36]. The minimum energy configuration is the one found by Balachandran, Digal and Matsuura (BDM) [16] and the metastable vortices corresponding to the $\mathbb{C}P^1$

*chandra.chttrj@gmail.com, chandra@phys-h.keio.ac.jp
†nitta@phys-h.keio.ac.jp

subspace of which the isometry SU(2) commutes with $\mathrm{U(1)_{em}}$ (hereafter, we call them $\mathbb{C}P^1$ vortices). We calculate the AB phases of charged particles, that is, electrons, muons, and CFL mesons made of four quarks, around a stable BDM vortex or a metastable $\mathbb{C}P^1$ vortex in the CFL phase. Since the nontrivial AB phase generates frictional force on vortices as denoted above, it can affect the conductivity of the vortex particle system, and it may create anisotropy in the density of particles in the bulk, which remains as a future problem.

The rest of this paper is organized as follows. In Sec. II, we first give a brief introduction of the Ginzburg-Landau (GL) free energy in the CFL phase and non-Abelian vortices in the absence of the electromagnetic interaction. Then, in Sec. III, we introduce the electromagnetic interaction by gauging a $\mathrm{U(1)_{em}}$ subgroup of the $\mathrm{SU(3)_F}$ flavor symmetry and calculate the AB phases for gapless excitation of the CFL phase that are scattered by non-Abelian vortices. We make a comment on the effect of the strange quark mass, in the presence of which the AB phase remains nontrivial. Section IV is devoted to a summary and discussion.

II. GINZBURG-LANDAU FREE ENERGY AND NON-ABELIAN VORTICES IN THE CFL PHASE

In this section, we first introduce the GL description of the CFL phase and study non-Abelian vortices based on the GL description.

A. Ginzburg-Landau free energy

The GL description for the order parameter is appropriate at temperatures close to the critical temperature T_c for the CFL phase transition. Here, the GL order parameters are the diquark condensates $\Phi_{L/R}$ defined by

$$\begin{aligned}\Phi_{\mathrm{L}}{}_a^A &\sim \epsilon_{abc}\epsilon^{ABC} q_{\mathrm{L}}{}_b^B \mathcal{C} q_{\mathrm{L}}{}_c^C,\\ \Phi_{\mathrm{R}}{}_a^A &\sim \epsilon_{abc}\epsilon^{ABC} q_{\mathrm{R}}{}_b^B \mathcal{C} q_{\mathrm{R}}{}_c^C,\end{aligned} \tag{1}$$

where $q_{L/R}$ stand for left- and right-handed quarks with a, b, c as fundamental color $[\mathrm{SU(3)_C}]$ and A, B, C as fundamental flavor $[\mathrm{SU(3)_{L/R}}]$ indices. The order parameters $\Phi_{L/R}$ transform as a bifundamental representation of color and flavor groups. It was found that positive parity states are favored compared to the one with negative parity as a ground state. A convenient choice of order parameters for symmetry breaking would be taken as $\Phi_L = -\Phi_R \equiv \Phi$. Then, the order parameter Φ can be regarded as a bifundamental representation of the symmetry group $\mathrm{U(1)_B} \times \mathrm{SU(3)_C} \times \mathrm{SU(3)_F}$. Here, $\mathrm{U(1)_B}$ is the global Abelian transformation of baryon number conservation, and the flavor group $\mathrm{SU(3)_F}$ is the diagonal subgroup $\mathrm{SU(3)_{L+R}}$ of the total flavor group $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$. The GL free energy can be written in terms of the order parameter Φ as [13–15]

$$\begin{aligned}\Omega = \mathrm{Tr}\left[\frac{1}{4\lambda_3}F_{ij}^2 + \frac{\varepsilon_3}{2}F_{0i}^2 + K_3\mathcal{D}_i\Phi^\dagger\mathcal{D}_i\Phi\right] + \alpha\mathrm{Tr}(\Phi^\dagger\Phi)\\ + \beta_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \beta_2\mathrm{Tr}[(\Phi^\dagger\Phi)^2] + \frac{3\alpha^2}{4(\beta_1+3\beta_2)},\end{aligned} \tag{2}$$

where $i, j = 1, 2, 3$ are indices for space coordinates, λ_3 is a magnetic permeability, and ε_3 is a dielectric constant for gluons.

The GL parameters $\alpha = 4N(\mu)\log\frac{T}{T_c}$, $\beta_1 = \beta_2 = \frac{7\zeta(3)}{8(\pi T_c)^2}N(\mu) \equiv \beta$ and $K_3 = \frac{7\zeta(3)}{12(\pi T_c)^2}N(\mu)$ are obtained from the weak-coupling calculations, which are valid at a sufficiently high density [13,14]. Here, μ stands for the quark chemical potential, and we also have taken $\lambda_0 = \epsilon_0 = \lambda_3 = \epsilon_3 = 1$. We have introduced the density of state $N(\mu)$ at the Fermi surface $N(\mu) = \frac{\mu^2}{2\pi^2}$.

B. Non-Abelian vortices

Let us first briefly review a few salient features of the non-Abelian vortices in the CFL phase in the absence of the electromagnetic interaction.

The covariant derivative and the field strength of gluons are defined by $\mathcal{D}_\mu\Phi = \partial_\mu\Phi - ig_s A_\mu^a T^a\Phi$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s[A_\mu, A_\nu]$. Here, μ and ν are indices for spacetime coordinates, and g_s stands for the $\mathrm{SU(3)_C}$ coupling constant. The transformation properties of the field Φ can be written as

$$\begin{aligned}\Phi' = e^{i\theta_B}\mathrm{U_C}\Phi\mathrm{U_F^{-1}}, \qquad e^{i\theta_B} \in \mathrm{U(1)_B},\\ \mathrm{U_C} \in \mathrm{SU(3)_C}, \qquad \mathrm{U_F} \in \mathrm{SU(3)_F}.\end{aligned} \tag{3}$$

There is a redundancy in the action of the discrete symmetries, and the actual symmetry group is given by

$$\mathrm{G} = \frac{\mathrm{SU(3)_C} \times \mathrm{SU(3)_F} \times \mathrm{U(1)_B}}{\mathbb{Z}_3 \times \mathbb{Z}_3}. \tag{4}$$

In the ground state $\langle\Phi\rangle = \Delta_{\mathrm{CFL}}\mathbf{1_3}$ with $\Delta_{\mathrm{CFL}} \equiv \sqrt{-\frac{\alpha}{8\beta}}$, the full symmetry group G is spontaneously broken down to

$$\mathrm{H} \simeq \frac{\mathrm{SU(3)_{C+F}}}{\mathbb{Z}_3}. \tag{5}$$

The order parameter space is $\mathrm{G/H} \simeq \frac{\mathrm{SU(3)\times U(1)}}{\mathbb{Z}_3} = \mathrm{U(3)}$. It can be easily noticed that $\pi_1(\mathrm{G/H}) = \mathbb{Z}$. This nonzero fundamental group implies the existing vortices. Since the broken $\mathrm{U(1)_B}$ is a global symmetry, the vortices are global vortices or superfluid vortices [16]. The structure of these vortices can be understood by the orientation and winding of the configuration of the condensed scalar field Φ far away from the vortex core perpendicular to the vortex direction. We place a vortex along the z direction.

At the large distance R from the vortex core, the condensation can have a configuration like

$$\Phi(R,\theta) = \Delta_{\rm CFL}\begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \Delta_{\rm CFL}\exp i\left[\frac{\theta}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + i\frac{\theta}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\right]. \quad (6)$$

This can be rewritten as

$$\Phi(R,\theta) = e^{ig_s\int A\cdot dl} e^{i\frac{\theta}{3}}\Phi(R,0), \quad (7)$$

with A proportional to $\mathrm{diag}(2,-1,-1)$. From Eq. (7), the minimum energy condition yields

$$D_i\Phi = -i\frac{\epsilon_{ij}x_j}{3r^2}\Phi, \qquad r\to R \quad (8)$$

at a large distance. From this boundary construction, one can write down the ansatz as

$$\Phi(r,\theta) = \Delta_{\rm CFL}\begin{pmatrix} e^{i\theta}f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix},$$
$$A_i(r) = -\frac{1}{3g_s}\frac{\epsilon_{ij}x_j}{r^2}[1-h(r)]\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (9)$$

The form of the profiles $f(r)$ and $h(r)$ can be calculated numerically with the boundary condition

$$f(0)=0, \qquad \partial_r g(r)|_0 = 0,$$
$$h(0)=1, \qquad f(\infty)=g(\infty)=\Delta_{\rm CFL},$$
$$h(\infty)=0. \quad (10)$$

The vortex configuration in Eq. (9) breaks the unbroken symmetry $SU(3)_{C+F}$ in the ground state into a subgroup $SU(2)\times U(1)$ inside the vortex core. This breaking results in Nambu-Goldstone modes parametrizing a coset space,

$$\frac{SU(3)}{SU(2)\times U(1)} \simeq \mathbb{C}P^2. \quad (11)$$

The low-energy excitation and interaction of these zero modes can be calculated by the effective $\mathbb{C}P^2$ sigma model action [21]. Generic solutions on the $\mathbb{C}P^2$ space can be found by just applying a global transformation by a reducing matrix,

$$\mathrm{U} = \frac{1}{\sqrt{X}}\begin{pmatrix} 1 & -B^\dagger \\ B & X^{\frac{1}{2}}Y^{-\frac{1}{2}} \end{pmatrix},$$
$$X = 1 + B^\dagger B,$$
$$Y = \mathbf{1}_3 + BB^\dagger, \quad (12)$$

where $B = \{B_1, B_2\}$ are inhomogeneous coordinates of the $\mathbb{C}P^2$. The vortex solution with a generic orientation and in the regular gauge takes the form

$$\Phi(r,\theta) = \Delta_{\rm CFL}\mathrm{U}\begin{pmatrix} e^{i\theta}f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}\mathrm{U}^\dagger,$$
$$A_i(r) = -\frac{\epsilon_{ij}x_j}{3g_s r^2}[1-h(r)]\mathrm{U}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\mathrm{U}^\dagger. \quad (13)$$

III. AHARONOV-BOHM PHASES AROUND A NON-ABELIAN VORTEX

As mentioned in Introduction, the AB effect [24] is a quantum mechanical effect that occurs when a charged particle scatters from a solenoid with nonzero magnetic flux inside. It leads to the differential scattering cross section

$$\frac{d\sigma}{d\vartheta} = \frac{\sin^2(\pi\varphi)}{2\pi k \sin^2(\frac{\vartheta}{2})}, \qquad \varphi = \frac{q}{2\pi}\times \mathrm{Flux}. \quad (14)$$

Here, q is the electric charge of a scattering particle, k is the momentum perpendicular to the string, and ϑ is the scattering angle. The scattering cross section depends on the flux of the solenoid in a nontrivial way. In the case of a vortex carrying a nonquantized flux, the same thing occurs [26]. Although particles can get inside a vortex core, we have the same formula as far as when we consider paths far from the vortex core.

Non-Abelian vortices similar to those in the CFL phase in dense QCD were found in the CFL phase in supersymmetric gauge theories [37–39]; see Refs. [40–43] as a review. When one gauges a U(1) subgroup of the flavor group, non-Abelian vortices become AB strings [31]. This was extended to non-Abelian gauging [32,33]. As for a non-Abelian vortex in the CFL phase, the AB effect appears once we introduce the electromagnetic interaction $[U(1)_{\rm em}]$ as a subgroup of the flavor symmetry group, as in the case of supersymmetric theories. So, it would be interesting to determine the value of φ for the scattering of particles that are the relevant low-energy excitation in the CFL phase. In the CFL phase, electrons, muons, and Nambu-Goldstone bosons, e.g., the CFL mesons, can be considered as fundamental excitations in the bulk.[1] Here, we calculate the AB phases of electrons, muons, and the CFL mesons present in the bulk.

A. Electromagnetic interactions of non-Abelian vortices

Here, we introduce $\mathrm{U}(1)_{\mathrm{em}}$ generator as a part of the flavour symmetry $\mathrm{SU}(3)_{\mathrm{F}}$:

$$Q = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{15}$$

Massless symmetry is realized by a linear combination of color and the $\mathrm{U}(1)_{\mathrm{em}}$ subgroup. To see exactly which gauge field remains unbroken, let us look at the covariant derivative on the order parameter:

$$\mathcal{D}_\mu \Phi = \partial_\mu \Phi - i g_s A_\mu \Phi - ie A^{\mathrm{em}}_\mu \Phi Q. \tag{16}$$

When the order parameter is in a diagonal form Φ_{diag}, the covariant derivative can be written as

$$\mathcal{D}_\mu \Phi_{\mathrm{diag}} = \partial_\mu \Phi_{\mathrm{diag}} - i(g_s A^a_\mu \mathrm{H}^a - e A^{\mathrm{em}}_\mu Q)\Phi_{\mathrm{diag}}. \tag{17}$$

Here, we have taken only the color diagonal gauge fields, and $\mathrm{H}^a = \{\mathrm{T}^8, \mathrm{T}^3\}$ are generators of the Cartan subalgebra of the SU(3) Lie algebra. The massive and massless diagonal gauge fields in the bulk can be expressed as (see, e.g., Ref. [11])

$$A^{\mathrm{M}}_\mu = \frac{g_s}{g_M} A^8_\mu - \frac{\eta e}{g_{\mathrm{M}}} A^{\mathrm{em}}_\mu, \qquad A^q_\mu = \frac{\eta e}{g_M} A^8_\mu + \frac{g_s}{g_M} A^{\mathrm{em}}_\mu, \tag{18}$$

respectively, where $\eta = \frac{2}{\sqrt{3}}$ and $g^2_{\mathrm{M}} = g^2_s + \eta^2 e^2$. All fields living in the bulk interact with A^q effectively as an effective electromagnetic interaction $\tilde{\mathrm{U}}(1)^{\mathrm{em}}$ generated by A^q.

The original electromagnetic gauge potential can be written as

$$A^{\mathrm{em}}_\mu = \frac{g_s}{g_M} A^q_\mu - \frac{\eta e}{g_M} A^{\mathrm{M}}_\mu. \tag{19}$$

So, the effective electromagnetic coupling for a particle with charge q becomes

$$\frac{q g_s}{\sqrt{g_s^2 + \eta^2 e^2}}. \tag{20}$$

Construction of vortices with electromagnetic interaction can be understood from the winding of scalar field and the covariant derivative defined above.

[1]The AB effect can be realized if there exist charged asymptotic states in the bulk of the condensate. Color charged quasiparticle quarks cannot exist in the bulk freely because of condensation. The quark condensate screens color charges in the bulk.

The existence of $\mathrm{U}(1)_{\mathrm{em}}$ breaks the global $\mathrm{SU}(3)_{\mathrm{C+F}}$ invariance to $\mathrm{SU}(2) \times \mathrm{U}(1)$, and consequently the $\mathbb{C}P^2$ Nambu-Goldstone zero modes become massive, leaving the BDM vortices and CP_1 vortices as (meta)stable configurations.

1. BDM vortices

In this case, the scalar field configuration at large distance R can be described as

$$\Phi(R, \theta) = \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{21}$$

We can rewrite this in terms of a global $\mathrm{U}(1)_{\mathrm{B}}$ rotation added with rotation in color and electromagnetic action as

$$\Phi(R, \theta) = e^{i g_s \int A \cdot dl} e^{i\frac{\theta}{3}} \Phi(R, 0) e^{-ie \int A^{\mathrm{em}} Q \cdot dl}. \tag{22}$$

From this boundary condition, one can write down the ansatz as

$$\Phi(r, \theta) = \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\theta} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix},$$
$$A^{\mathrm{M}}_i(r)\mathrm{T}^8 = -\frac{\epsilon_{ij} x_j}{3 g_{\mathrm{M}} r^2}[1 - h(r)] \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{23}$$

The form of the profiles $f(r)$ and $h(r)$ can be calculated numerically from the equations of motion with boundary condition in Eq. (10) [16].

2. $\mathbb{C}P^1$ vortices

A $\mathbb{C}P^1$ sector at $|B| \to \infty$ solutions of Eq. (13) remains gapless [36] even in the presence of the electromagnetic interaction. The vortex configurations can be written as

$$\Phi(r, \theta) = \Delta_{\mathrm{CFL}} \begin{pmatrix} g(r) & 0 & 0 \\ 0 & e^{i\theta} f(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix},$$
$$A^{\mathrm{M}}_i(r)\mathrm{T}^8 = \frac{1}{6 g_{\mathrm{M}}} \frac{\epsilon_{ijx_j}}{r^2}[1 - h(r)] \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$
$$A^3_i(r)\mathrm{T}^3 = -\frac{1}{2 g_s} \frac{\epsilon_{ijx_j}}{r^2}[1 - h(r)] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{24}$$

It is clear from Eq. (24) that the existence of this vortex spontaneously breaks the global U(2) invariance acting on the lower-right 2 by 2 block, and this breaking generates $\mathbb{C}P^1$ Nambu-Goldstone modes. It is important to note that the configuration of A^{M}_{μ} in Eq. (24) has a factor $-\frac{1}{2}$ compared with that in Eq. (23).

B. Aharonov-Bohm phases of electrons and muons

The AB scattering of electrons or muons in the CFL phase can be understood by writing the Dirac equation in the vortex background,

$$(\partial\!\!\!/ - ieA\!\!\!/^{\mathrm{em}} + iM_{e/\mu_e})\psi_{e/\mu_e} = 0, \tag{25}$$

where ψ_{e/μ_e} are the Dirac fields for electrons and muons with masses M_{e/μ_e}. Using Eq. (19), we can write

$$\left(\partial\!\!\!/ - i\frac{eg_s}{g_{\mathrm{M}}}A^q_{\mu} + i\frac{\eta e^2}{g_{\mathrm{M}}}A^{\mathrm{M}}_{\mu} + iM_{e/\mu_e}\right)\psi_{e/\mu_e} = 0. \tag{26}$$

The second term is just the coulomb term with effective charge $\frac{eg_s}{g_{\mathrm{M}}}$, but for the AB scattering, the last term would be important. This can be understood in another way. The AB phase for electrons or muons can be defined as

$$\varphi_{e/\mu_e} = -\frac{e}{2\pi}\oint A^{\mathrm{em}}\cdot dl. \tag{27}$$

So, according to Eq. (19), we may calculate the above integral as

$$\varphi_{e/\mu_e} = \frac{\eta e^2}{2\pi g_{\mathrm{M}}}\oint A^{\mathrm{M}}\cdot dl. \tag{28}$$

Here, we have used the fact that

$$\oint A^q \cdot dl = 0. \tag{29}$$

A^{m}_i can be determined for the BDM case and for the $\mathbb{C}P^1$ case from Eqs. (23) and (24). So, the AB phase around a BDM vortex can be calculated as

$$\varphi^{\mathrm{BDM}}_{e/\mu_e} = \frac{\eta e^2}{2\pi g_{\mathrm{M}}}\oint A^{\mathrm{M}}\cdot dl = \frac{\eta e^2}{2\pi g_{\mathrm{M}}}\times\frac{\eta 2\pi}{g_{\mathrm{M}}} = \frac{2e^2}{3g_s^2+2e^2}, \tag{30}$$

while the AB phase $\varphi^{\mathrm{CP}^1}_{e/\mu_e}$ around a $\mathbb{C}P^1$ vortex can be determined as

$$\varphi^{\mathbb{C}P^1}_{e/\mu_e} = \frac{\eta e^2}{2\pi g_{\mathrm{M}}}\oint A^{\mathrm{M}}\cdot dl = -\frac{\eta e^2}{2\pi g_{\mathrm{M}}}\times\frac{\eta\pi}{2g_{\mathrm{M}}} = -\frac{e^2}{3g_s^2+2e^2}. \tag{31}$$

C. Aharonov-Bohm phases of CFL mesons

At high density, the chiral symmetry breaking generates Nambu-Goldstone bosons, known as the CFL mesons. The CFL mesons can be expressed using a composite operator of the diquark field as

$$\Sigma^{AB}_{\mathrm{CFL}} = \Phi^{\dagger\mathrm{L}}_{Aa}\Phi^{\mathrm{R}}_{aB}. \tag{32}$$

Here, a and A and B are the color and flavor indices, respectively. In terms of quarks, the CFL mesons can be expressed as [44]

$$\Sigma^{AB}_{\mathrm{CFL}} \sim \epsilon^{ACD}\epsilon^{BEF}\bar{q}_{\mathrm{L}(a}{}^{C}\bar{q}_{\mathrm{L}b)}{}^{D}q_{\mathrm{R}(a}{}^{E}q_{\mathrm{R}b)}{}^{F}, \tag{33}$$

where $(\ldots)$ denotes the antisymmetrization of indices. The electromagnetic $\mathrm{U}(1)_{\mathrm{em}}$ group acts on this operator as

$$\Sigma'_{\mathrm{CFL}} = e^{ieQ\alpha}\Sigma_{\mathrm{CFL}}e^{-ieQ\alpha}, \tag{34}$$

where Q is defined by Eq. (15). So, the charge can be measured by computing the simple commutator $[Q,\Sigma_{\mathrm{CFL}}]$. As we know, Q is basically the T^8 generator of SU(3), and Σ_{CFL} could also be expanded in SU(3) generators. There are only four components of Σ_{CFL} that do not commute with Q, which can be written as

$$\begin{pmatrix} 0 & \Sigma^{1+}_{\mathrm{CFL}} & \Sigma^{2+}_{\mathrm{CFL}} \\ \Sigma^{1-}_{\mathrm{CFL}} & 0 & 0 \\ \Sigma^{2-}_{\mathrm{CFL}} & 0 & 0 \end{pmatrix}. \tag{35}$$

So, the charges of Σ_{CFL} mesons can be determined as

$$q = \{0, \pm e\}. \tag{36}$$

In terms of quarks, the charged CFL mesons are

$$\begin{aligned} \Sigma^{1+}_{\mathrm{CFL}} &= \Sigma^{12}_{\mathrm{CFL}} \sim \bar{d}_{\mathrm{L}}\bar{s}_{\mathrm{L}}s_{\mathrm{R}}u_{\mathrm{R}}, \quad & \Sigma^{1-}_{\mathrm{CFL}} &= \Sigma^{21}_{\mathrm{CFL}} \sim \bar{u}_{\mathrm{L}}\bar{s}_{\mathrm{L}}s_{\mathrm{R}}d_{\mathrm{R}} \\ \Sigma^{2+}_{\mathrm{CFL}} &= \Sigma^{13}_{\mathrm{CFL}} \sim \bar{s}_{\mathrm{L}}\bar{d}_{\mathrm{L}}d_{\mathrm{R}}u_{\mathrm{R}}, \quad & \Sigma^{2-}_{\mathrm{CFL}} &= \Sigma^{31}_{\mathrm{CFL}} \sim \bar{u}_{\mathrm{L}}\bar{d}_{\mathrm{L}}d_{\mathrm{R}}s_{\mathrm{R}}. \end{aligned} \tag{37}$$

The AB phases φ_{CFL} for charged CFL mesons $\Sigma^{\pm i}$ can be expressed by using Eqs. (30) and (31). The AB phases for $\mathbb{C}P^1$ vortices ($\varphi^{\mathbb{C}P^1}_{\mathrm{CFL}}$) and BDM vortices ($\varphi^{\mathrm{BDM}}_{\mathrm{CFL}}$) can be calculated as

$$\varphi^{\mathrm{BDM}}_{\mathrm{CFL}} = \pm\frac{2e^2}{3g_s^2+2e^2}, \qquad \varphi^{\mathbb{C}P^1}_{\mathrm{CFL}} = \mp\frac{e^2}{3g_s^2+2e^2}. \tag{38}$$

D. Strange quark mass

The importance of $\mathbb{C}P^1$ vortices can be understood if we study the vortices at an intermediate density regime, which is more relevant in the core of neutron star. In this case, the mass of the strange quark (m_s) becomes admissible and cannot be neglected. The potential in Eq. (2) has to be changed by terms like $\mathrm{Tr}[\Phi^{\dagger}\{(\alpha+\frac{2\epsilon}{3})\mathbf{1}+\epsilon T^3\}\Phi]$, where $\epsilon \propto m_s^2$. This potential would generate instabilities in the effective theory of non-Abelian vortices. The general $\mathbb{C}P^2$ vortices would decay radically with lifetime of order 10^{-21} sec, as estimated in

Ref. [22] for the case in which $\mu \sim 500$ MeV, $\Delta \sim 10$ MeV, and $m_s \sim 150$ MeV. Only one type of $\mathbb{C}P^1$ vortex corresponding to a single point (0,1,0) in full $\mathbb{C}P^2$ moduli space would survive. So, only one of $\mathbb{C}P^1$ vortices becomes a stable vortex in the presence of the strange quark mass [22] as mentioned above. Therefore, in such a situation, all vortices have the AB phase $\varphi_{e/\mu_e}^{\mathbb{C}P^1}$.

IV. SUMMARY AND DISCUSSION

We have calculated the phases of the AB scattering of the gapless fundamental excitations in the CFL phase of dense quark matter and have found nontrivial AB phases due to the scattering of electrons, muons, and CFL mesons with vortices. The nontrivial AB phases arise because the flux due to the $\mathrm{U}(1)_{\mathrm{em}}$ gauge field shares a fraction of the total magnetic flux present inside vortices and the existence of particles with electric charges present in the bulk of the dense QCD medium as gapless excitations. In the absence of the electromagnetism, non-Abelian vortices are degenerate and can be rotated in the $\mathbb{C}P^2$ moduli space, resulting in the effective action written as the $\mathbb{C}P^2$ sigma model. The presence of $\mathrm{U}(1)_{\mathrm{em}}$ as a subgroup of flavor breaks the SU(3) global invariance and generates a potential in the $\mathbb{C}P^2$ model. In this case, only stable vortices are those for which the color gauge field direction and $\mathrm{U}(1)_{\mathrm{em}}$ directions are parallel. We have found a mismatch in the AB phases between scattering with BDM vortices (corresponding to the $B = 0$ point in the $\mathbb{C}P^2$ moduli space) and $\mathbb{C}P^1$ vortices (corresponding to the $B = \infty$ submanifold on the $\mathbb{C}P^2$ moduli space). This mismatch arises because of the fact that the orientation of color flux to the $\mathbb{C}P^1$ direction changes the fraction of the flux shared by color magnetic field. So, the fraction of electromagnetic flux changes automatically.

The AB scattering off non-Abelian vortices present in the CFL phase is important property of the particles present in the bulk of the CFL phase medium, as was discussed for unstable vortices in the 2SC phase [35]. We will discuss transportation properties of particles, the friction of vortices in the CFL phase, and possible implications on physics of neutron stars.

In this paper, we have discussed the AB scattering of a single vortex. In the CFL phase under rotation, a vortex lattice will be formed. The interaction of the electromagnetic field with a vortex lattice was discussed in Ref. [45], showing that the lattice behaves as a polarizer. The AB scattering of charged particles inside a vortex lattice should be an interesting future direction.

We have discussed the AB scattering of charged particles due to the electromagnetic field in the presence of a non-Abelian vortex. Non-Abelian vortices are color magnetic fluxes having non-Abelian fluxes, too. Since gluons are massive, the AB phase is usually thought to be absent, but they may give a global analog of the AB phase. Colored particles in the nontrivial representation of the color $\mathrm{SU}(3)_{\mathrm{C}}$ group may have such a phase. The interaction of quasiquarks with a non-Abelian vortex [46–48] and the interaction of gluons with a vortex [49] were studied before. The presence or absence of the (global) AB phases of these colored particles should be clarified.

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