Dissipation and decoherence effects on a moving particle in front of a dielectric plate

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In this work, we consider a particle moving in front of a dielectric plate and study two of the most relevant effects of the vacuum field fluctuations: the dissipation and the decoherence of the particle's internal degrees of freedom. We consider the particle to follow a classical, macroscopically fixed trajectory. To study the dissipative effects, we calculate the in-out effective action by functionally integrating over the vacuum field and the microscopic degrees of freedom of both the plate and the particle. This in-out effective action develops an imaginary part and, hence, a nonvanishing probability for the decay (because of friction) of the initial vacuum state. We analyze how the dissipation is affected by the relative velocity between the particle and the plate and the properties of the microscopic degrees of freedom. In order to study the effects of decoherence over the internal degrees of freedom of the particle, we calculate the closed time path or Schwinger-Keldysh influence action, by functionally integrating over the vacuum field and the microscopic degrees of freedom of the plate. We estimate the decoherence time as the time needed by two different quantum configurations (of the internal degree of freedom of the particle) to be possible to differentiate from one another. We analyze the way in which the presence of the mirror affects the decoherence and the possible ways to maximize or reduce its effects.

I. INTRODUCTION

Over the past few years, increasing attention has been paid to the interaction between a particle and a (perfect or imperfect) mirror or any dielectric surface [1–7]. One of the main interests has been to calculate the frictional force exerted over the particle by the plate, mediated by the vacuum field fluctuations. As in the case of the quantum friction between two plates [8–10], there is still no general agreement about the nature of this frictional force. These frictional effects are interesting, macroscopically observable consequences of the quantum nature of microscopic systems. However, frictional and normal (Casimir) forces are not the only effects of the vacuum quantum fluctuations. Any quantum system that interacts with an environment will suffer the process of decoherence, which is one of the main ingredients necessary to understand the quantum-classical transition. The vacuum field is, clearly, an environment that cannot be switched off, since any particle (charged or with a nonvanishing dipole moment) will unavoidably interact with the electromagnetic field fluctuations. The effects of the electromagnetic field over the coherence of the quantum state of a particle, and the way in which this effect is modified by the presence of a conducting plate, has already been studied for interference experiments [11,12]. However, in the many studies of the quantum friction over a moving particle in front of a dielectric plate, the effects of decoherence have not yet been taken into consideration. In this work, we will study the decoherence of the internal degree of freedom of the particle. The loss of coherence of the particle's dipolar moment becomes relevant in any Ramsey interferometry experiment, where the depolarization of the atom could be macroscopically observed by means of the Ramsey fringes. In the case of a Rydberg atom, this phenomenon could be also observed as a decay of the Raby oscillations [13,14].

In this work, we consider a neutral particle moving in front of an imperfect mirror. The trajectory of the particle will be, in this paper, kept as an externally fixed variable. This accounts for many of the cases of interest, for example, when the particle is the tip of an atomic force microscope. When we specify the system, we will have the particle moving at a constant velocity *v*, as in the most popular scenario in the literature [1,2]. We are interested in the dynamics of the internal degree of freedom of the particle, that we will model as a quantum harmonic oscillator, being a simple model for the particle's electric dipole, and that will be coupled in position to the vacuum field. We will also use a simple model for the microscopic degrees of freedom of the mirror, as we have done in a previous work [15]: a set of uncoupled harmonic oscillators, each of them also interacting locally with the vacuum field. Even though this is a simple model, it allows us to calculate some relevant quantities without much further assumption.

We will study the effects both of friction and of decoherence over the moving particle, from the perspective

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of the quantum field theory. In order to do so, we will use two different but similar approaches. To study the friction, we will follow the procedure presented in our previous work [15], where we studied the friction generated by the relative motion of two dielectric plates. We will calculate the imaginary part of the in-out effective action, that will account for the dissipative effects. To study the decoherence, we will switch to the closed time path (CTP) formalism, calculate the action of influence of the environment (vacuum field + plate) over the particle, and use it to obtain an estimation of the decoherence time, following an approach similar to the one in a previous work by some of us [16].

The structure of this paper is as follows: In Sec. II, we define the system we study and present the formalism in Minkowski space. We then calculate the in-out effective action for a specific microscopic model and obtain an expression for the imaginary part of the effective action (and hence the dissipative effects over the system) as a function of the relative velocity. In Sec. III, we review the Schwinger-Keldysh formalism, obtain a general expression for the CTP influence action for the internal degree of freedom of the particle, and use it to obtain the stochastic equations of motion for the particle's internal degree of freedom. In Sec. IV, we present a way to estimate the decoherence time and then analyze the way in which it is modified by the presence of the plate. Finally, in Sec. V, we show our conclusions.

II. IN-OUT EFFECTIVE ACTION AND FRICTION

A. The system

Let us consider a specific system, even though the formalism that is going to be developed among the next sections is general and could be used to study different problems. In the current work, we are interested in the dissipative effects over a particle that moves parallel to a flat, dielectric surface. The vacuum field shall be a non-massive scalar field $\phi(x)$ obeying the Klein-Gordon equation, interacting with both the particle and the internal degrees of freedom of the plate $\psi(x)$. The particle moves in a macroscopic, externally fixed, unidimensional trajectory, in a plane parallel to the plate. The distance a between the particle and the plate is also kept constant by an external source. We shall call x_1 the direction of movement of the particle and x_3 the direction perpendicular to the plate. The particle also has an internal degree of freedom that we shall call q. A scheme of the system under consideration is shown in Fig. 1. We may write the classical action for the system as

$$S[\phi,\psi,q] = S_0^{\text{vac}}[\phi] + S_0^{\text{pl}}[\psi] + S_0^{\text{part}}[q] + S_{\text{int}}^{\text{pl}}[\phi,\psi] \\ + S_{\text{int}}^{\text{part}}[\phi,q], \quad (1)$$

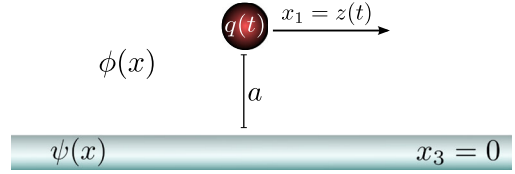where the Klein-Gordon action for the vacuum field, neglecting boundary terms, is given by

FIG. 1. A simple scheme of the system under consideration, where $\phi(x)$ is the vacuum field, $\psi(x)$ are the internal degrees of freedom of the plate, and $q(t)$ are the internal degrees of freedom of the particle, which follows a macroscopic trajectory $z(t)$ in the x_1 direction.

$$S_0^{\text{vac}}[\phi] = -\frac{1}{2}\int dx\phi(x)[\partial_\mu\partial^\mu - i\epsilon]\phi(x). \quad (2)$$

The generating functional for the system will be given by

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}\psi\mathcal{D}q e^{iS[\phi,\psi,q]}. \quad (3)$$

The internal degrees of freedom of the plates can be integrated out, resulting in an effective interaction potential $V(x,x')$ for the vacuum field. This procedure has already been performed in Ref. [17], and the resulting classical action is

$$S[\phi,q] = S_{\text{eff}}[\phi] + S_0^{\text{part}}[q] + S_{\text{int}}^{\text{part}}[\phi,q], \quad (4)$$

with

$$S_{\text{eff}}[\phi] = S_0[\phi] + \int dxdx'\phi(x)V(x,x')\phi(x'). \quad (5)$$

The internal degree of freedom of the particle interacts with the vacuum field through a current that we shall call $j(x)$. This current contains the information about the position and trajectory of the particle and the strength of the coupling. The interaction term is, then,

$$S_{\text{int}}^{\text{part}}[\phi,q] = i\int dx\phi(x)j(x). \quad (6)$$

It is worth noticing that, at this point, the current $j(x)$ could make the vacuum field interact with any system of any given geometry: We have not yet specified that we are studying a moving, punctual particle.

B. The in-out effective action

We would now like to obtain the effective action for the particle. That is, we are going to functionally integrate over the vacuum field, to obtain the generating functional

$$\mathcal{Z}[q] = \int \mathcal{D}\phi e^{iS[\phi,q]} = e^{iS_0[q]}\int \mathcal{D}\phi e^{iS_{\text{eff}}[\phi]+iS_{\text{int}}[\phi,q]}. \quad (7)$$

This functional integral can be written as

$$\mathcal{Z}[q] = \int \mathcal{D}\phi \exp\left[-\frac{1}{2}\int dx dx' \phi(x)A(x,x')\phi(x')\right.$$
$$\left. -\int dx \phi(x)j(x)\right]$$
$$= (\det A)^{-\frac{1}{2}}\exp\left[\frac{1}{2}\int dx dx' j(x)A^{-1}(x,x')j(x')\right], \quad (8)$$

where

$$A(x,x') = i[\partial_\mu\partial^\mu - i\epsilon]\delta(x-x') - iV(x-x'). \quad (9)$$

We need to obtain an operator A^{-1} such that

$$A^{-1}(x,x')A(x,x') = A(x,x') = A^{-1}(x,x') = \delta(x-x'). \quad (10)$$

In order to do so, we may write $A(x,x') = iA_0(x,x') + A_1(x,x')$, where

$$A_0(x,x') = \delta(x-x')(\partial_\mu\partial^\mu - i\epsilon),$$
$$A_1(x,x') = -iV(x,x').$$

Now, the effective potential $V(x,x')$ is proportional to the coupling constant λ between the vacuum field and the internal degrees of freedom of the plates. If this coupling is weak, then we can assume $A_1 \ll A_0$ and obtain A^{-1} as an expansion in powers of λ. Keeping up to first order in λ, we write

$$A^{-1}(x,x) \approx -i(\mathbb{1} + iA_0^{-1}A_1)A_0^{-1}. \quad (11)$$

And it is easy to prove that the operator from Eq. (11) satisfies

$$A^{-1}A = AA^{-1} = \mathbb{1} + \mathcal{O}(\lambda^2). \quad (12)$$

Now, recalling the definition of A_0, it is clear that A_0^{-1} is a Green function of the Klein-Gordon equation. We take it to be the Feynman propagator $A_0^{-1}(x,x') = G_F(x,x')$:

$$G_F(x,x') = \int \frac{dp}{(2\pi)^4}e^{-ip_\mu(x^\mu - x'^\mu)}\frac{1}{p_\mu p^\mu + i\epsilon}. \quad (13)$$

The desired operator can be written as

$$A^{-1}(x,x') = -i\left(G_F(x,x')\right.$$
$$\left. + \int dy dy' G_F(x,y)V(y,y')G_F(y',x')\right). \quad (14)$$

The only part of Eq. (8) remaining to be calculated is the normalization factor $(\det A)^{1/2}$. However, this factor does not contribute to the connected diagrams: It involves only the interaction between the plate and the vacuum and has no effect on the physics of the moving particle [or any other system we might be interested in that could be introduced through the current $j(x)$]. That being said, we might write the generating functional for the particle

$$\mathcal{Z}[q] = \exp\left\{iS_0[q] - \frac{i}{2}\int dx dx' j(x)G_F(x,x')j(x')\right.$$
$$-\frac{i}{2}\int dx dy dy' dx' j(x)G_F(x,y)V(y,y')$$
$$\left. \times G_F(y',x')j(x')\right\}. \quad (15)$$

The effective action for the particle can be written as

$$\Gamma[q] = S_0[q] + S_1[q] + S_2[q], \quad (16)$$

where S_1 is the action that contains the influence of the vacuum over the particle, as if there were no dielectric plate, and is defined by

$$S_1[q] = -\frac{1}{2}\int dx dx' j(x)G_F(x,x')j(x'). \quad (17)$$

The term S_2 accounts for the influence of the plate over the particle, mediated by the vacuum field, and is defined by

$$S_2[q] = -\frac{1}{2}\int dx dy dy' dx' j(x)G_F(x,y)V(y,y')$$
$$\times G_F(y',x')j(x'). \quad (18)$$

We might think of the effect of the plate on the particle as if mediated by a new effective propagator $\tilde{G}(x,x')$:

$$\tilde{G}(x,x') \equiv \int dy dy' G_F(x,y)V(y,y')G_F(y',x'), \quad (19)$$

and then S_2 can be written as

$$S_2[q] = -\frac{1}{2}\int dx dx' j(x)\tilde{G}(x,x')j(x'). \quad (20)$$

But we are also interested in calculating the imaginary part of the in-out effective action for the whole system, since this quantity accounts for the dissipative effects over the system. The following calculations will be completely analogous to the ones we have performed in Ref. [15] for the case of two plates in relative parallel motion.

To obtain the in-out effective action for the whole system, we need to integrate out the internal degree of freedom of the particle q; that is, we need to functionally integrate Eq. (15) over q:

$$\mathcal{Z}_{\text{sys}} = \int \mathcal{D}q \mathcal{Z}[q]. \qquad (21)$$

But, instead of performing the functional integrations in this order (first over ψ, then over ϕ, and last over q), we go back a few steps and integrate over q before we integrate over the vacuum field ϕ. The reason for doing this is purely of mathematical simplicity, and it does not affect the final expression for the effective action. We then obtain an expression with two effective potentials: one accounting for the interaction of the plate with the vacuum field and another accounting for the interaction of the particle with the vacuum field. If we take into consideration only the terms involving both the plate and the particle, then the effective action for small values of the coupling constants is [15]

$$\Gamma_I \approx \frac{-i}{2} \int \frac{dp}{(2\pi)^4} \frac{dq}{(2\pi)^4} G_F(p) V_{\text{plate}}(p,q) G_F(q) V_{\text{particle}}(q,p). \qquad (22)$$

C. Specific model and results

Now we would like to specify a concrete system of study. As we have already said in Sec. II A, even though, for the sake of clarity, through this work we have talked about a particle and a plate, this situation has not yet been specified. We have, so far, a complex, massless scalar field ϕ that interacts with another field ψ, that we call the internal degrees of freedom of a plate but could be associated to any other system that we might be interested in. When we integrate those degrees of freedom out, we obtain a non-local effective potential $V(x, x')$ that contains the information about the characteristics of the other system. It could be a thin plate, a half-space, or any other geometry, with any kind of internal degrees of freedom. But if we do consider a thin, infinitesimal plate, occupying the $x_3 = 0$ plate, with internal degrees of freedom that interact locally in space with the vacuum field, then the effective potential will have the form [17]

$$V_{\text{plate}}(q, p) = (2\pi)^3 \lambda(p_0) \delta(p_0 - q_0) \delta^{(2)}(\mathbf{p}_\parallel - \mathbf{q}_\parallel). \qquad (23)$$

And if the particle is considered to be punctual, moving along the x_1 axis with a constant velocity v, at a fixed distance $x_3 = a$ above the plate, and interacting locally in position with the vacuum field, then the effective potential results in

$$V_{\text{particle}}(q, p)$$
$$= 2\pi g(p_0 - vp_1)\delta(p_0 - q_0 - v(p_1 - q_1))e^{-ia(q_3 - p_3)}. \qquad (24)$$

These potentials take into account the geometry of the plate and the particle, but the information about their internal degrees of freedom and the nature of their interaction with the vacuum field are yet to be specified by the λ and g functions. The difference between a moving particle and a moving plate becomes clearer in position space, where the potential is localized in position by means of three Dirac δ functions: one indicating that the particle is always on $x_3 = a$ (this δ function was also present for the moving plate), another that the particle is always at $x_2 = 0$, and the third one indicating that it is always at $x_1 = vt$.

By using the explicit shape of the potentials and integrating over q_0, q_1 and q_2, we find

$$\Gamma_I \approx \frac{iT}{2(2\pi)^4} \int d^4p \, d_3 G(p_0, p_1, p_2, p_3)\lambda(p_0)$$
$$\times G(p_0, p_1, p_2, q_3)g(p_0 - vp_1)e^{-i(q_3 - p_3)a}, \qquad (25)$$

where T is the total time of flight of the particle. We consider the internal degrees of freedom of the plate to be uncoupled harmonic oscillators of frequency Ω, each of them interacting locally in position with the vacuum field with a coupling constant λ; and the internal degree of freedom of the particle will also be a harmonic oscillator of frequency ω_0, also interacting linearly and locally in position with the vacuum field, with a coupling constant g. This results [15] in

$$\lambda(\omega) = \frac{-\lambda^2}{\omega^2 - \Omega^2 + i\epsilon},$$
$$g(\omega) = \frac{-g^2}{\omega^2 - \omega_0^2 + i\epsilon}. \qquad (26)$$

The remaining integrations are performed in the exact same way as the integrations in Ref. [15]. The analytic result in $2 + 1$ dimensions is

$$\text{Im}\Gamma_I \approx \frac{Tv\pi\lambda^2 g^2}{32\tilde{\Omega}\tilde{\omega}_0} \frac{e^{-\frac{2}{v}\sqrt{(\tilde{\omega}_0 + \tilde{\Omega})^2 - v^2\tilde{\Omega}^2}}}{(\tilde{\omega}_0 + \tilde{\Omega})^2 - v^2\tilde{\Omega}^2}, \qquad (27)$$

where $\tilde{\Omega} = \Omega a$ and $\tilde{\omega}_0 = \omega_0 a$ are the dimensionless frequencies (a is the distance between the particle and the plate). We show in Fig. 2 the imaginary part of the effective action as a function of the relative velocity v.

As we mentioned before, the imaginary part of the effective action implies the excitation of internal degrees of freedom on the mirror that influence the particle through the vacuum field. This signals a noncontact frictional effect. We can see that the dissipative effects are strongly suppressed as $v \to 0$. This exponential vanishing of the dissipation effects has already been found, using different approaches, in previous works [1,2]. It is worth noticing that our coupling constant g is the analog to the electric dipole moment d appearing in the models used by other authors, since it accounts for the interaction between the particle's polarizability and the electromagnetic (vacuum) field. This means that the results presented here correspond
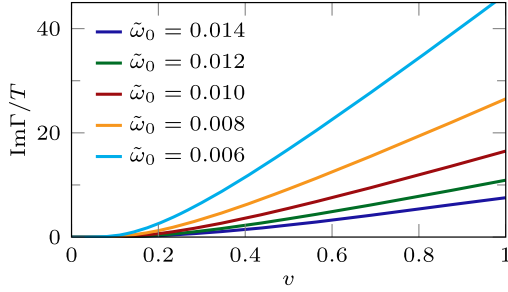
FIG. 2. The imaginary part of the in-out effective action as a function of v, for $\tilde{\Omega} = \Omega a = 0.01$ and $\lambda = 0.01$, in units of g^2.

to the d^2 contribution to the friction, and we will calculate the d^4 contribution within this approach in a future work. Last, let us recall that the λ^2 factor accounts for the interaction between the internal degrees of freedom of the plate, and this information is usually contained in the dielectric permittivity of the material, so there is no analog to our λ factor appearing in the literature.

It is possible to see, from the general expression of Eq. (27), that the case of resonance, for $\Omega = \omega_0$, coincides exactly with the expression of the imaginary part of the effective action per unit of area, for the case of two plates of frequency Ω, studied in Ref. [15]. This is not that surprising, since in our model the plate and the particle or other plate only interact locally and the harmonic oscillators of the plate(s) are uncoupled. In that work, we had set the coupling constant between the two plates to be equal, and that is the reason why our result was of the order of λ^4. It is important to notice that it does not mean that our result is to be compared with the g^4 results appearing in the literature, since a factor λ^2 is to be considered part of the dielectric permittivity of the material.

III. CTP ACTION OF INFLUENCE AND DECOHERENCE

A. The Schwinger-Keldysh or closed time path formalism

Up to this section, we have considered the effective action for the system (the particle) and the influence action that mainly describes the dynamics of the particle after integration of the quantum fields. This in-out effective action cannot be applied in a straightforward way to the derivation of the equations of motion, since they would become neither real nor causal. As is well known, in order to get the correct effective equations of motion, one should compute the in-in, Schwinger-Keldysh, or closed time path effective action (CTPEA) [18], which also has information on the stochastic dynamics. The CTPEA is defined as

$$e^{-i\Gamma_{\text{CTP}}[q^+, q^-]} = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{iS[q^+, \phi^+] - iS[q^-, \phi^-]}$$

$$\equiv \int \mathcal{D}\phi e^{iS^C[q, \phi]}, \qquad (28)$$

where in the last step we have introduced the CTP complex temporal path $\mathcal{C}$, going from minus to plus infinity $\mathcal{C}_+$ and backwards $\mathcal{C}_-$, with a decreasing (infinitesimal) imaginary part. Time integration over the contour $\mathcal{C}$ is defined by $\int_{\mathcal{C}} dt = \int_{\mathcal{C}_+} dt \int_{\mathcal{C}_-} dt$. The field in the last step is defined by $\phi(x, t) = \phi_{\pm}(x, t)$ if $t \in \mathcal{C}_{\pm}$. The equation above is useful, because it has the structure of the usual in-out or the Euclidean effective action, and we are able to use the results of the last section but properly changing the contour of integration. This would lead to the appearance of the CTP propagators $G_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}', t')$, where α (β) = $\pm$ indicates that t (t') is on $\mathcal{C}_{\pm}$. For the free scalar field, the CTP propagators are

$$G_{++}(x, x') = \Delta_F(x, x') = -i\langle T\phi(x)\phi(x')\rangle,$$

$$G_{--}(x, x') = \Delta_D(x, x') = i\langle \hat{T}\phi(x)\phi(x')\rangle,$$

$$G_{+-}(x, x') = \Delta^-(x, x') = -i\langle \phi(x')\phi(x)\rangle,$$

$$G_{-+}(x, x') = \Delta^+(x, x') = +i\langle \phi(x)\phi(x')\rangle.$$

These CTP propagators satisfy the following properties that will prove to be useful below: $(G_{++})^* = G_{--}$, $(G_{+-})^* = G_{-+}$, and $G_{++} - G_{--} = G_{+-} - G_{-+}$.

We are now in a position to write the actions describing the influence of the environment on the particle within this formalism. We will do this in a general way, and later we will come back to our specific model. By performing the change in the contour of integration, S_1 results [see Eq. (17)]:

$$S_1^{\text{IF}}[q^+, q^-] = -\frac{1}{2} \int dx\, dy [j^+(x) G_{++}(x, y) j^+(y)$$

$$- j^-(x) G_{--}(x, y) j^-(y)$$

$$- j^+(x) G_{+-}(x, y) j^-(y)$$

$$+ j^-(x) G_{-+}(x, y) j^+(y)], \qquad (29)$$

where $j_{\pm}(x)$ is evaluated over $\mathcal{C}_{\pm}$. It is useful to define

$$\Delta j(x) = \frac{j^+(x) - j^-(x)}{2},$$

$$\Sigma j(x) = \frac{j^+(x) + j^-(x)}{2}.$$

By writing Eq. (29) in terms of Δj and Σj, one obtains four different terms. However, by recalling the definition and properties of the different CTP propagators, it is easy to see that one of the terms vanishes, and two of the remaining terms are identical, with the result

$$S_1^{\text{IF}}[q^+, q^-] = -\frac{1}{2} \int dx dy \{ \Delta j(x) [G_{++}(x, y) - G_{--}(x, y)$$
$$- G_{-+}(x, y) + G_{+-}(x, y)] \Delta j(y)$$
$$+ 2\Delta j(x) [G_{++}(x, y) + G_{--}(x, y)$$
$$+ G_{-+}(x, y) + G_{+-}(x, y)] \Sigma j(y) \}.$$

With some further considerations concerning the properties of the propagators mentioned above, it is possible to define the noise (diffusion) kernel

$$N_1(x, y) \equiv -i(G_{++}(x, y) - G_{--}(x, y)) = 2\mathrm{Im}G_{++}(x, y),$$
$$(30)$$

associated with fluctuations, the source of decoherence effects; and the dissipation kernel

$$D_1(x, y) \equiv \frac{1}{2} [G_{++}(x, y) + G_{--}(x, y) + G_{+-}(x, y)$$
$$+ G_{-+}(x, y)]$$
$$= 2\Theta(x_0 - y_0) \mathrm{Re}G_{++}(x - y). \quad (31)$$

Both kernels are real, and the dissipation kernel is explicitly causal. The influence action results, then, in

$$S_1^{\text{IF}}[q^+, q^-] = -\int dx dy [i\Delta j(x) N_1(x, y) \Delta j(y)$$
$$+ 2\Delta j(x) D_1(x, y) \Sigma j(y)]. \quad (32)$$

This action accounts for the interaction of the particle with the vacuum, and all the propagators involved in the calculations correspond to free, scalar fields. Furthermore, in order to take the dielectric plate into consideration, we have to look at the S_2 term from Eq. (20), which contains the effect of the plate over the particle up to first order in the coupling constant λ. By changing the contour of integration, we obtain an expression analogous to Eq. (29) but changing the free CTP propagator $G_{\alpha\beta}$ for an effective CTP propagator $\tilde{G}_{\alpha\beta}$, defined by

$$\tilde{G}_{\alpha,\beta}(x, y) \equiv \int dx' dy' [G_{\alpha+}(x, x') V_{++}(x', y') G_{+\beta}(y', y)$$
$$- G_{\alpha-}(x, x') V_{--}(x', y') G_{-\beta}(y', y)$$
$$- G_{\alpha+}(x, x') V_{+-}(x', y') G_{-\beta}(y', y)$$
$$+ G_{\alpha-}(x, x') V_{-+}(x', y') G_{+\beta}(y', y)], \quad (33)$$

where $G_{\alpha\beta}$ is the free CTP propagator and $V_{\alpha\beta}$ is the CTP expression for the effective potential. Both G and V, being CTP Green functions, fulfill the relation $G_{++} - G_{--} = G_{+-} - G_{-+}$ and $V_{++} - V_{--} = V_{+-} - V_{-+}$, which allows us to prove that $\tilde{G}_{++} - \tilde{G}_{--} = \tilde{G}_{+-} - \tilde{G}_{-+}$, and thus

$$S_2^{\text{IF}}[q^+, q^-] = \int dx dy [i\Delta j(x) N_2(x, y) \Delta j(y)$$
$$+ 2\Delta j(x) D_2(x, y) \Sigma j(y)], \quad (34)$$

with

$$N_2(x, y) \equiv \tilde{G}_{++}(x, y) + \tilde{G}_{--}(x, y) = 2\mathrm{Re}\tilde{G}_{++}(x, y),$$
$$(35)$$

where we also used that $\tilde{G}_{++} = (\tilde{G}_{--})^*$. The dissipation kernel

$$D_2(x, y) \equiv \frac{-i}{2} [\tilde{G}_{++}(x, y) - \tilde{G}_{--}(x, y)$$
$$- \tilde{G}_{+-}(x, y) + \tilde{G}_{-+}(x, y)]. \quad (36)$$

On general grounds, the real and imaginary parts of the influence action can be associated with the dissipation and noise, respectively, and can be related by some integral equation known as the fluctuation-dissipation relation. As we shall see, the dissipation will be present in the generalized Langevin-like equation of motion for the internal degrees of freedom of the particle, and the noise kernel will be relevant in defining the correlation function for the noise source.

B. Our model

We will now consider, as we have done in Sec. II C, an infinitesimally thin plate occupying the plane $x_3 = 0$, formed by a set of uncoupled harmonic oscillators of frequency Ω, each of them interacting locally in position with the vacuum field with a coupling constant λ. We have already found the CTP expression for the effective potential in a previous work [15]:

$$V^{\text{CTP}}(x - y) = \int \frac{dp}{(2\pi)^4} e^{-ip_\mu(x^\mu - y^\mu)} \lambda^2$$
$$\times \begin{pmatrix} \frac{1}{p_0^2 - \Omega^2 + i\epsilon} & \frac{i\pi\delta(p_0 + \Omega)}{\Omega} \\ \frac{i\pi\delta(p_0 - \Omega)}{\Omega} & \frac{1}{p_0^2 - \Omega^2 - i\epsilon} \end{pmatrix}. \quad (37)$$

The same kind of reasoning applies to the current $j(x)$. We now want it to describe a particle moving in the direction x_1, describing a trajectory $z(t)$, at a fixed distance a from the plate. That is, its macroscopic coordinates are given by $x^\mu(t) = (z(t), 0, a, t)$ and are kept fixed by an external source. On the other hand, we describe its internal degree of freedom as a unidimensional harmonic oscillator $q(t)$ of frequency ω_0. This is the variable whose dynamics we now wish to study. So we have

$$j(x) = gq(t)\delta(x_1 - z(t))\delta(x_2)\delta(x_3 - a), \quad (38)$$

where g is the coupling constant between the vacuum and the internal degree of freedom of the particle. Now, as the classic trajectory $z(t)$ is macroscopic and externally fixed, we might assume that it remains the same in the different branches of the CTP integral. That is, we will from now on assume that $z^+(t) = z^-(t)$, where $z^\pm(t)$ is the classical function $z(t)$ integrated over the contour $\mathcal{C}_\pm$. This means that

$$\Delta j(x) = g\Delta q(t)\delta(x_1 - z(t))\delta(x_2)\delta(x_3),$$

$$\Sigma j(x) = g\Sigma q(t)\delta(x_1 - z(t))\delta(x_2)\delta(x_3).$$

With these specific considerations, we can define, for $N \equiv N_1 + N_2$,

$$N(t - t') \equiv g^2 \int d\mathbf{x}d\mathbf{x}'\delta(x_1 - z(t))\delta(x_2)\delta(x_3 - a)N$$
$$\times (x - x')\delta(x_1' - z(t'))\delta(x_2')\delta(x_3'), \quad (39)$$

and an analogous expression is obtained for $D \equiv D_1 + D_2$.

Within this model, we can write explicit expressions for D_1, D_2, N_1 and N_2 as integrals in momentum space. If the trajectory is $z(t) = vt$ for constant v, then

$$D_1(t - t') = 2g^2\Theta(t - t')\text{Re}$$
$$\times \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-i(p_0 - vp_1)(t-t')}}{p_0^2 - p_1^2 - p_2^2 - p_3^2 + i\epsilon}, \quad (40)$$

$$N_1(t - t') = 2g^2\text{Im} \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-i(p_0 - vp_1)(t-t')}}{p_0^2 - p_1^2 - p_2^2 - p_3^2 + i\epsilon}, \quad (41)$$

$$D_2(t - t') = \frac{\lambda^2 g^2}{2}\Theta(t - t')\text{Re} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p_0^2 - \Omega^2 + i\epsilon}$$
$$\times \frac{e^{-i(p_0 - vp_1)(t-t')+iia\sqrt{p_0^2-p_1^2-p_2^2+i\epsilon}}}{p_0^2 - p_1^2 - p_2^2 + i\epsilon}, \quad (42)$$

$$N_2(t - t') = \frac{\lambda^2 g^2}{2}\text{Im} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p_0^2 - \Omega^2 + i\epsilon}$$
$$\times \frac{e^{i(p_0 - vp_1)(t-t')+2ia\sqrt{p_0^2-p_1^2-p_2^2+i\epsilon}}}{p_0^2 - p_1^2 - p_2^2 + i\epsilon}. \quad (43)$$

Therefore, the influence action for the moving particle in front of a dielectric plate is given, formally, by

$$S^{\text{IF}}[q^+, q^-] = \int dtdt'[i\Delta q(t)N(t, t')\Delta q(t')$$
$$+ 2\Delta q(t)D(t, t')\Sigma q(t')]. \quad (44)$$

C. Real and causal equations of motion

From Secs. III and III B, it is easy to see that the (CTP) influence action has a real part, generated by the dissipation kernels D_1 and D_2, and an imaginary part, generated by the noise kernels N_1 and N_2. In order to obtain the real and

causal equations of motion for $q(t)$, we must functionally derive $S[q^+, q^-] = S_0[q^+] - S_0[q^-] + S^{\text{IF}}[q^+, q^-]$ with respect to q^+ and set $q^+ = q^- = q$. However, this procedure would lead us to the equations for the mean value of $q(t)$, and the stochastic effect of the noise would not appear.

In order to see the influence of the noise on the equations of motion, it is necessary to consider a realization of the noise over the system, considering an stochastic source of noise $\xi(t)$. Following the well-known procedure for open quantum systems, we consider this stochastic force to have a Gaussian probability distribution given by

$$P[\xi(t)] = N_\xi \exp\left\{-\frac{1}{2}\int dtdt'\xi(t)N^{-1}(t, t')\xi(t')\right\}. \quad (45)$$

It is easy to see that the inclusion of this stochastic source can be performed by adding to the generating functional the factor

$$\int \mathcal{D}\xi P[\xi]e^{-i\int dt\Delta q(t)\xi(t)} = e^{i\int dtdt'\Delta q(t)N(t,t')\Delta q(t')}, \quad (46)$$

which, as is shown in the right-hand side of the last equation, leaves the generating functional unaltered. The modified influence action is, then,

$$\hat{S}^{\text{IF}}[q^+, q^-, \xi] = 2\int dtdt'\Delta q(t)D(t, t')\Sigma q(t')$$
$$- \int dt\Delta q(t)\xi(t). \quad (47)$$

And the equation of motion will be given by

$$\left.\frac{\delta(S_0[q^+] - S_0[q^-] + \hat{S}^{\text{IF}}[q^+, q^-, \xi])}{\delta q^+}\right|_{q^+=q^-=q} = 0. \quad (48)$$

The result is

$$\ddot{q}(t) + \omega_0^2 q(t) + \int dt'D(t, t')q(t') = \xi(t), \quad (49)$$

where it is easy to see that the dissipation on the system comes from the kernel $D(t, t')$, and the fluctuations are generated by the stochastic force $\xi(t)$, that must fulfill, according to Eq. (45),

$$\langle\xi(t)\rangle = 0,$$

$$\langle\xi(t)\xi(t')\rangle = N(t, t').$$

This is a generalized Langevin equation, with classical noise ξ and dissipation, that satisfies the fluctuation-dissipation theorem. Each part of the environment that we include leads to a further *dissipative* term on the left-hand side of Eq. (49) (i.e., D_1 and D_2 as we named before) with a countervailing noise term on the right-hand side. Of

course it is very difficult to solve this stochastic equation analytically. It is difficult to imagine an *ab initio* derivation of the dissipative and noise terms from the full quantum theory. In this sense, a reasonable alternative is to analyze phenomenological stochastic equations numerically and check the robustness of the predictions against different choices of the dissipative kernels and of the type of noise.

IV. DECOHERENCE OF THE PARTICLE'S INTERNAL DEGREES OF FREEDOM

The absence of quantum interference between the stationary phase solutions to the classical stochastic equations, $q(t)$, is manifested through the increasing decay of the nondiagonal terms of the reduced density matrix $\rho[q^+, q^-, t]$. This leads to the crucial notion of a decoherence time t_D, after which ρ (or, more exactly, its real part) is effectively diagonal. Equation (49) is the classical stochastic equation that we are looking for but, as it stands, is guaranteed only to describe classical degrees of freedom $q_{\text{class}}(t)$ after decoherence (see in another context Refs. [19,20]).

The notion of consistent histories provides an alternative approach to classicality, opposed to trying to solve the master equation, and it is normally used in open quantum systems. Quantum evolution can be considered as a coherent superposition of fine-grained histories. Since we need to be able to distinguish different classical system configurations evolving in time, we work in the basis of amplitudes $q(t)$. If one defines the c number $q(t)$ as specifying a fine-grained history, the quantum amplitude for that history is $\Psi \sim e^{S[q]}$ (we work in units in which $\hbar = 1$) [21]. In the quantum open system approach that we have adopted here, we are concerned with coarse-grained histories

$$\Psi[\alpha] = \int \mathcal{D}q \, e^{S[q]} \alpha[q], \qquad (50)$$

where $\alpha[q]$ is the filter function that defines the coarse-graining. In the first instance this filtering corresponds to tracing over all the degrees of freedom of the composite environment. From this we define the decoherence functional for two coarse-grained histories as

$$\mathcal{D}[\alpha^+, \alpha^-] = \int \mathcal{D}q^+ \mathcal{D}q^- e^{i(S[q^+] - S[q^-])} \alpha^+[q^+] \alpha^-[q^-]. \quad (51)$$

$\mathcal{D}[\alpha^+, \alpha^?]$ does not factorize because the histories $q^\pm$ are not independent. Decoherence means physically that the different coarse-graining histories making up the full quantum evolution acquire individual reality and may therefore be assigned definite probabilities in the classical sense. A necessary and sufficient condition for the validity of the sum rules of probability theory (i.e., no quantum interference terms) is [22]

$$\text{Re}\mathcal{D}[\alpha^+, \alpha^-] \approx 0, \qquad (52)$$

when $\alpha^+ \neq \alpha^-$ (although in most cases the stronger condition $\mathcal{D}[\alpha^+, \alpha^-] \approx 0$ holds) [23]. Such histories are consistent [24].

For our particular application, we wish to consider as a single coarse-grained history all those fine-grained ones where the solution $q(t)$ remains close to a prescribed classical configuration q_{cl}. The filter function takes the form $\alpha_{\text{cl}}[q] = \int \mathcal{D}J e^{i \int J(q - q_{\text{cl}})} \alpha_{\text{cl}}[J]$. Using $Jq = \int dx J(x) q[x]$, we may write the decoherence functional between two classical histories in terms of the closed path time generating functional. In principle, we can examine adjacent general classical solutions for their consistency but, in practice, it is simpler to restrict ourselves to particular solutions $q^\pm$, according to the nature of the decoherence that we are studying. All in all, the decoherence functional results in

$$\mathcal{D}[q_{\text{cl}}^+, q_{\text{cl}}^-] \approx F[q_{\text{cl}}^+, q_{\text{cl}}^-], \qquad (53)$$

where $F[q_{\text{cl}}^+, q_{\text{cl}}^-]$ is the Feynman-Vernon influence functional (IF) [25]. The influence functional is written in terms of the influence action $S^{\text{IF}}[q^+, q^-]$ as $F = \exp[-iS^{\text{IF}}]$. As a result,

$$|\mathcal{D}[q_{\text{cl}}^+, q_{\text{cl}}^-]| \sim \exp\{i\text{Im}S^{\text{IF}}[q_{\text{cl}}^+, q_{\text{cl}}^-]\}, \qquad (54)$$

where S^{IF} is the contribution to the action due to the, in our case, composite environment. From this viewpoint, once we have chosen the classical solutions of interest, adjacent histories become consistent at the time t_D, for which $1 \approx \text{Im}S^{\text{IF}}|_{t=t_D}$ [21]. This decoherence time will become relevant in any Ramsey interferometry experiment [13]. For times larger than t_D, the Ramsey fringes will no longer be distinguishable, expressing the depolarization of the atom.

A. Imaginary part of S_1^{IF}

Let us recall the expression for $N_1(t, t')$ given by Eq. (30) and consider two classical trajectories $q_{\text{cl}}(t)$ with different amplitudes but the same frequency:

$$\Delta q_{\text{cl}}(t) = \Delta q_0 \cos(\omega_0 t). \qquad (55)$$

So we can write an expression for the imaginary part of the influence action for the particle in the presence of the vacuum field (ignoring the plate) as

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{-g^2\Delta q_0^2}{4}\mathrm{Im}\int \frac{dp_0 d\mathbf{p}}{(2\pi)^4}\frac{1}{p_0^2 - \mathbf{p}^2 + i\epsilon}$$

$$\times \left\{\int dt dt' e^{i(p_0 - vp_1 + \omega_0)(t-t')}\right.$$

$$+ \int dt dt' e^{i(p_0 - vp_1 - \omega_0)(t-t')}$$

$$+ \int dt dt' e^{i(p_0 - vp_1 + \omega_0)(t+t')}$$

$$\left.+ \int dt dt' e^{i(p_0 - vp_1 - \omega_0)(t+t')}\right\}.$$

All the temporal integrations result in Dirac delta functions. The last two terms vanish because the resulting deltas cannot be fulfilled simultaneously. From each nonvanishing term, an infinite $\delta(0)$ is obtained, accounting for the total time of integration T (time of flight of the particle). Thus the integrals over p_0 can trivially be performed, and the remaining terms are

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{-g^2\Delta q_0^2 T}{4}$$

$$\times \mathrm{Im}\int \frac{dp_0 d\mathbf{p}}{(2\pi)^2}\frac{\delta(p_0 + vp_1 - \omega_9)\delta(p_0 + vp_1 + \omega_0)}{p_0^2 - \mathbf{p}^2 + i\epsilon}.$$
(56)

Now, if we take the limit $\epsilon \to 0$, then

$$\frac{1}{p_0^2 - p_{\parallel}^2 + i\epsilon} \to \mathrm{pv}\left(\frac{1}{p_0^2 - \mathbf{p}^2}\right) - i\pi\delta(p_0^2 - \mathbf{p}^2), \quad (57)$$

where pv denotes the Cauchy principal value. So we have

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{-g^2\Delta q_0^2 T}{4}\int \frac{dp_0 d\mathbf{p}}{(2\pi)^2}[\delta(p_0 + vp_1 - \omega_0)$$

$$+ \delta(p_0 + vp_1 + \omega_0)]\delta(p_0^2 - \mathbf{p}^2).$$
(58)

Let us now consider the case in $2 + 1$ dimensions, setting $p_2 \equiv 0$ (x_2 is the direction with translational symmetry); then, integrating over p_0, we have

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{\pi g^2\Delta q_0^2 T}{4}\int \frac{dp_1 dp_3}{2\pi}[\delta((-vp_1 + \omega_0)^2$$

$$- (p_1^2 + p_3^2)) + \delta((vp_1 + \omega_0)^2 - (p_1^2 + p_3^2))],$$
(59)

and, assuming the δ functions as functions of p_3, we have four terms:

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{\pi g^2\Delta q_0^2 T}{4}\int \frac{dp_1 dp_3}{2\pi}$$

$$\times \left[\frac{\delta(p_3 - \sqrt{(v^2-1)p_1^2 \pm 2\omega_0 vp_1 + \omega_0^2})}{2\sqrt{(v^2-1)p_1^2 \pm 2\omega_0 vp_1 + \omega_0^2}}\right.$$

$$\left.+ \frac{\delta(p_3 + \sqrt{(v^2-1)p_1^2 \pm 2\omega_0 vp_1 + \omega_0^2})}{2\sqrt{(v^2-1)p_1^2 \pm 2\omega_0 vp_1 + \omega_0^2}}\right]. \quad (60)$$

Since the integration is over the real values of p_3, the integral of these δ functions can be nonvanishing only if the squared root is a real number. It is easy to see, by looking carefully at the argument of the squared root, that the "+" terms can be nonzero only if

$$\frac{-\omega_0}{1+v} < p_1 < \frac{\omega_0}{1-v} \quad (61)$$

and the "−" terms are nonvanishing only if

$$\frac{-\omega_0}{1-v} < p_1 < \frac{\omega_0}{1+v}. \quad (62)$$

So we can write the remaining integral over a bounded set of p_1:

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{g^2\Delta q_0^2 T}{32\pi}\left[\int_{-\frac{\omega_0}{1+v}}^{\frac{\omega_0}{1-v}} dp_1 \frac{1}{\sqrt{(v^2-1)p_1^2 + 2\omega_0 vp_1 + \omega_0^2}}\right.$$

$$\left.+ \int_{-\frac{\omega_0}{1-v}}^{\frac{\omega_0}{1+v}} dp_1 \frac{1}{\sqrt{(v^2-1)p_1^2 - 2\omega_0 vp_1 + \omega_0^2}}\right].$$
(63)

Now the integrands are of the form $(-a(p_1 \pm b)^2)^{-1/2} + c$, with $a = (1 - v^2)$, $b = \frac{\omega_0 v}{1-v^2}$ and $c = \frac{\omega_0^2}{1-v^2}$. Thus, we have to take carefully the limit when p_1 tends to the integration boundaries, and we find that the result of each integral is $\frac{\pi}{\sqrt{1-v^2}}$, which means that

$$\mathrm{Im}S_1^{\mathrm{IF}} = \frac{g^2\Delta q_0^2 T}{16\sqrt{1-v^2}}. \quad (64)$$

B. Imaginary part of S_2^{IF}

Let us recall the expression for $N_2(t, t')$ given by Eq. (35) and consider two classical trajectories $q(t)$ with different amplitudes but the same frequency:

$$\Delta q_{\mathrm{cl}}(t) = \Delta q_0 \cos(\omega_0 t). \quad (65)$$

By inserting these results in Eq. (16), we are able to write an expression for the imaginary part of the action of influence, given by

$$\text{Im}S_2^{\text{IF}} = \frac{g^2\lambda^2\pi^2\Delta q_0^2}{2}\text{Im}\int\frac{dp_0 dp_\parallel}{(2\pi)^3}\frac{e^{2ia\sqrt{p_0^2-p_\parallel^2+i\epsilon}}}{(p_0^2-p_\parallel^2+i\epsilon)(p_0^2-\Omega^2+i\epsilon)}\left\{\int dt dt' e^{i(p_0-vp_1+\omega_0)(t-t')} + \int dt dt' e^{i(p_0-vp_1-\omega_0)(t-t')}\right.$$

$$\left. + \int dt dt' e^{i(p_0-vp_1+\omega_0)(t+t')} + \int dt dt' e^{i(p_0-vp_1-\omega_0)(t+t')}\right\}.$$

Following the same procedure as before, we find

$$\text{Im}S_2^{\text{IF}} = \frac{g^2\pi^2\Delta q_0^2\lambda^2 T}{2}\text{Im}\int\frac{dp_\parallel}{(2\pi)^2}\frac{e^{2ia\sqrt{(vp_1-\omega_0)^2-p_\parallel^2+i\epsilon}}}{((vp_1-\omega_0)^2-p_\parallel^2+i\epsilon)((vp_1-\omega_0)^2-\Omega^2+i\epsilon)} + \omega_0 \leftrightarrow -\omega_0. \tag{66}$$

We will now consider, again, the case of $2+1$ dimensions, setting $p_2 = 0$. By changing variables $k \equiv vp_1$, we have

$$\text{Im}S_2^{\text{IF}} = \frac{g^2\pi^2\Delta q_0^2\lambda^2 T}{2}\text{Im}\int\frac{dk}{(2\pi)}\frac{e^{\frac{2ia}{v}\sqrt{P(k)+i\epsilon}}}{(P(k)+i\epsilon)(A(k)+i\epsilon)} + \omega_0 \leftrightarrow -\omega_0. \tag{67}$$

Here $P(k) = (k-\omega_0)^2 v^2 - k^2$ and $A(k) = (k-\omega_0)^2 - \Omega^2$. We will begin by considering the first term. If $P(k)$ had a definite sign, we could get rid of the $i\epsilon$ accompanying it. Since $P(k)$ has two real roots, located at $k_+ = \frac{v\omega_0}{1+v}$ and $k_- = -\frac{v\omega_0}{1-v}$, we will consider three different regions:

$$\text{(I)} \rightarrow \frac{v\omega_0}{1+v} < k \quad P(k) < 0,$$

$$\text{(II)} \rightarrow -\frac{v\omega_0}{1-v}k < \frac{v\omega_0}{1+v} \quad P(k) > 0,$$

$$\text{(III)} \rightarrow k < -\frac{v\omega_0}{1-v} \quad P(k) < 0.$$

Now, within each region, $P(k)$ has a definite sign, and the limit $\epsilon \rightarrow 0$ can be taken on the terms involving $P(k)$.

On the other hand, the integrand will have two poles associated with the zeros of $A(k)$, which are located at $k_1 = \omega_0 + \Omega$ and $k_2 = \omega_0 - \Omega$. It is easy to see that k_1 is always located in region III and that k_2 could be in any region, depending on the values of the external parameters of the problem: v, ω_0 and Ω. With a little algebra, we find that k_2 is in region II if and only if

$$\frac{\omega_0}{1+v} < \Omega < \frac{\omega_0}{1-v}. \tag{68}$$

This condition is important in order to explicitly calculate the integrals.

Let us start computing the integral over region II. In order to do so, we will have in mind that, when $\epsilon \rightarrow 0$,

$$\frac{1}{A(k)+i\epsilon} \rightarrow \text{pv}\left(\frac{1}{A(k)}\right) - i\pi\delta(A(k)), \tag{69}$$

and, since $P(k) > 0$ in this region, we may write

$$\exp\left(\frac{2ia}{v}\sqrt{P(k)}\right)$$

$$= \cos\left(\frac{2a}{v}\sqrt{P(k)}\right) + i\sin\left(\frac{2a}{v}\sqrt{P(k)}\right).$$

If the condition (68) is fulfilled, then $A(k)$ will have a zero in the integration region, and $\delta(A(k))$ will be nonzero. We will have two terms contributing to the imaginary part of the effective action. The first one will be

$$\int_{k_-}^{k_+}\frac{dk}{2\pi}(-v\pi)\frac{\cos\left[\frac{2a}{v}\sqrt{P(k)}\right]}{P(k)}\delta(A(k))$$

$$= -\frac{v}{4\Omega}\frac{\cos\left[\frac{2a}{v}\sqrt{\Omega^2 v^2 - (\omega_0-\Omega)^2}\right]}{\Omega^2 v^2 - (\omega_0-\Omega)^2}, \tag{70}$$

where we have inserted the explicit expressions for $P(k_2)$ and $|A'(k_2)|$. The other term is

$$\frac{v}{2\pi}\int_{k_-}^{k_+}dk\frac{\sin\left(\frac{2ia}{v}\sqrt{P(k)}\right)}{P(k)}\text{pv}\left(\frac{1}{A(k)}\right). \tag{71}$$

Now, if the condition (68) is not satisfied, then there the integrand has no poles on the integration region, and we can take $\epsilon \rightarrow 0$ everywhere. This results in one term contributing to the imaginary part of the effective action:

$$\frac{v}{2\pi}\int_{k_-}^{k_+}dk\frac{\sin\left(\frac{2ia}{v}\sqrt{P(k)}\right)}{P(k)A(k)}. \tag{72}$$

Let us now compute the integral over regions I and III. Since in this region $P(k) < 0$, we may write

$$\text{Im}\left\{\left[\int_{-\infty}^{k_-}dk+\int_{k_+}^{+\infty}\right]\underbrace{\frac{e^{-\frac{2a}{v}\sqrt{-P(k)}}}{P(k)}}_{\in\mathbb{R}}\right\}v\frac{1}{A(k)+i\epsilon}, \quad (73)$$

and, when we take $\epsilon \to 0$, the only contributions to the imaginary part will come from the $-i\pi\delta$ term. The only remaining question is how many zeros $A(k)$ has in the integration region. As we already said, k_1 is always in region III, so we will always have a contribution coming from k_1, that is,

$$\frac{v}{4\Omega}\frac{e^{-\frac{2a}{v}\sqrt{(\omega_0+\Omega)^2-\Omega^2v^2}}}{(\omega_0+\Omega)^2-\Omega^2v^2}. \quad (74)$$

And, when the condition (68) is not fulfilled, then k_2 is also in region I or in region III, so we will have an extra contribution of the form

$$\frac{v}{4\Omega}\frac{e^{-\frac{2a}{v}\sqrt{(\omega_0-\Omega)^2-\Omega^2v^2}}}{(\omega_0-\Omega)^2-\Omega^2v^2}. \quad (75)$$

So far, we have computed the result of the first term on Eq. (66). The procedure for computing the second term is completely analogous, so we are now in a position to write the result for the imaginary part of the action of influence. We define

$$\mathcal{S}_1=\frac{1}{2\pi}\int_{-\frac{v\omega_0}{1-v}}^{\frac{v\omega_0}{1+v}}dk\frac{\sin[\frac{2a}{v}\sqrt{(k-\omega_0)^2v^2-k^2}]}{((k-\omega_0)^2v^2-k^2)((k-\omega_0)^2-\Omega^2)}$$
$$+\frac{1}{2\pi}\int_{-\frac{v\omega_0}{1+v}}^{\frac{v\omega_0}{1-v}}dk\frac{\sin[\frac{2a}{v}\sqrt{(k+\omega_0)^2v^2-k^2}]}{((k+\omega_0)^2v^2-k^2)((k+\omega_0)^2-\Omega^2)}$$
$$+\frac{1}{2\Omega}\frac{e^{\frac{2a}{v}\sqrt{(\omega_0+\Omega)^2-v^2\Omega^2}}}{(\omega_0+\Omega)^2-v^2\Omega^2}+\frac{1}{2\Omega}\frac{e^{\frac{2a}{v}\sqrt{(\omega_0-\Omega)^2-v^2\Omega^2}}}{(\omega_0-\Omega)^2-v^2\Omega^2}$$
$$(76)$$

and

$$\mathcal{S}_2=\frac{1}{2\pi}\int_{-\frac{v\omega_0}{1-v}}^{\frac{v\omega_0}{1+v}}dk\frac{\sin[\frac{2a}{v}\sqrt{(k-\omega_0)^2v^2-k^2}]}{(k-\omega_0)^2v^2-k^2}\text{pv}$$
$$\times\left(\frac{1}{(k-\omega_0)^2-\Omega^2}\right)-\frac{1}{2\Omega}\frac{\cos[\frac{2a}{v}\sqrt{v^2\Omega^2-(\omega_0-\Omega)^2}]}{v^2\Omega^2-(\omega_0-\Omega)^2}$$
$$+\frac{1}{2\pi}\int_{-\frac{v\omega_0}{1+v}}^{\frac{v\omega_0}{1-v}}dk\frac{\sin[\frac{2a}{v}\sqrt{(k+\omega_0)^2v^2-k^2}]}{(k+\omega_0)^2v^2-k^2}\text{pv}$$
$$\times\left(\frac{1}{(k-\omega_0)^2+\Omega^2}\right)+\frac{1}{2\Omega}\frac{e^{\frac{2a}{v}\sqrt{(\omega_0+\Omega)^2-v^2\Omega^2}}}{(\omega_0+\Omega)^2-v^2\Omega^2},$$
$$(77)$$

where the remaining integrals over k can be performed numerically. The imaginary part of the action of influence is then given by

$$\text{Im}S_2^{\text{IF}}=\frac{g^2\pi^2\Delta q_0^2\lambda^2Tv}{2}\begin{cases}\mathcal{S}_1 & \text{if }\Omega<-\frac{\omega_0}{1+v}\text{ or }\frac{\omega_0}{1+v}<\Omega,\\ \mathcal{S}_2 & \text{if }-\frac{\omega_0}{1+v}<\Omega<\frac{\omega_0}{1+v}.\end{cases}$$
$$(78)$$

C. Estimation of the decoherence time

We are now in a position to estimate the decoherence time for the particle. The total imaginary part of the action of influence (up to second order in λ) is

$$\text{Im}S^{\text{IF}}=\frac{g^2\Delta q_0^2T}{2}\left(\frac{1}{8\sqrt{1-v^2}}\right.$$
$$\left.+\lambda^2v\begin{cases}\mathcal{S}_1 & \text{if }\Omega<-\frac{\omega_0}{1+v}\text{ or }\frac{\omega_0}{1+v}<\Omega\\ \mathcal{S}_2 & \text{if }-\frac{\omega_0}{1+v}<\Omega<\frac{\omega_0}{1+v}\end{cases}\right). \quad (79)$$

Now, as we have detailed in the beginning of this section, the decoherence time can be estimated by the time of flight of the particle when $\text{Im}S^{\text{IF}}\sim 1$, so that

$$t_D\sim\frac{2}{g^2\Delta q_0^2}\frac{1}{\left(\frac{1}{8\sqrt{1-v^2}}+\lambda^2v\begin{cases}\mathcal{S}_1 & \text{if }\Omega<-\frac{\omega_0}{1+v}\text{ or }\frac{\omega_0}{1+v}<\Omega\\ \mathcal{S}_2 & \text{if }-\frac{\omega_0}{1+v}<\Omega<\frac{\omega_0}{1+v}\end{cases}\right)},$$
$$(80)$$

where $\mathcal{S}_1$ and $\mathcal{S}_2$ are defined in Eqs. (76) and (77), respectively. After numerically performing these integrals, we are in a position to plot, in Fig. 3(a), the estimation of the decoherence time as a function of the velocity of the particle, for different values of the coupling constant between the plate and the vacuum, λ. There is a global factor $A=\frac{2}{g^2\Delta q_0^2}$ that shows that the decoherence time is reduced for larger values of the coupling constant between the particle and the vacuum field and for larger difference in the amplitudes of the classical trajectories under consideration.

Now, as can be seen in Fig. 3(a), the presence of the plate reduces the decoherence time but only for nonvanishing relative velocity. For a particle that is at rest or moving at very small velocities with respect to the plate, the decoherence time is not reduced, even for greater values of the coupling constant between the plate and the vacuum field (and hence stronger interaction between the particle and the plate, mediated by the vacuum field). Now, for nonvanishing velocities, the decoherence of the particle is greater the stronger the coupling between the plate and the vacuum field.

In Fig. 3(b), t_D is shown as a function of the plate's characteristic dimensionless frequency $\tilde{\Omega}$, for different values of its macroscopic velocity v. In the graphic, a clear minimum appears for every value of v, and it is located in $\tilde{\Omega}=0.03=\tilde{\omega}_0$. This means that the decoherence is maximal in the resonant case, hence making the decoherence time
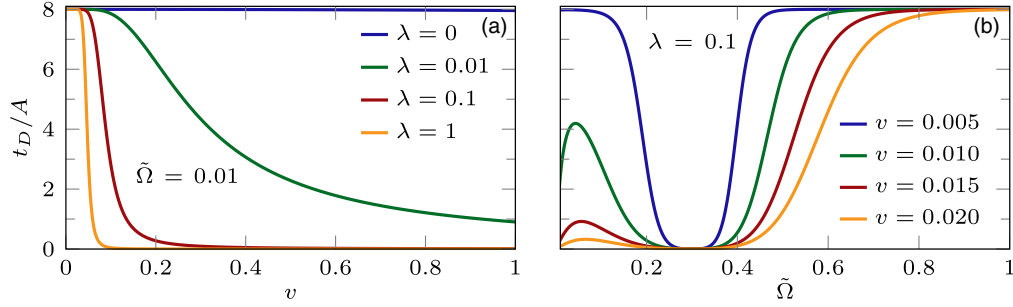
FIG. 3.   Estimation of the decoherence time, in units of a global factor $A = \frac{2}{g^2 \Delta q_0^2}$, as a function of (a) the relative velocity v and (b) the dimensionless material frequency $\tilde{\Omega}$, for $\tilde{\omega}_0 = 0.03$.

vanish. Far from the resonance, that is, for values of $\Omega \gg \omega_0$ (or the opposite limit when possible), the decoherence time tends to the limiting value that corresponds to the case $\lambda = 0$ (the case with no plate).

From the results shown above, we can see that, in this simple model, the presence of the plate enhances the decoherence over the particle. The decoherence effects can be maximized by an appropriate choice of the atom's fine-grained history (Δq_0 the classical amplitude difference in our example) and the plate's material [for a relation between the $\lambda(\omega)$ function of the material and its dielectric permittivity $\epsilon(\omega)$, see Ref. [26]]. As has already been discussed [11], the decoherence effects given solely by the vacuum field are neglectable. The coupling between any neutral atom and the vacuum field is very weak, resulting in an almost infinite decoherence time. However, from our results it can be seen that, by carefully choosing the plate's material [i.e., close to the resonance in Fig. 3(b)], the decoherence time could be drastically reduced. We also show that the relative velocity contributes to the enhancing of the decoherence effects.

V. CONCLUSIONS

In this paper, we have used a functional approach to study the effects of quantum vacuum fluctuations on a particle moving parallel to an imperfect mirror. We have presented a simple model in which the vacuum field is a massless, real, scalar field coupled to the microscopic degrees of freedom of the mirror and the internal degree of freedom of the particle. In our simple model, the plate is formed by uncoupled unidimensional harmonic oscillators, each of them interacting locally in position with the vacuum field. The macroscopic trajectory of the particle was externally fixed, and its internal degree of freedom was also a unidimensional harmonic oscillator, also coupled in position with the scalar field, resulting in a dipolelike interaction.

We first studied the dissipative effects over the system. In order to do that, we followed the same approach we had used in our previous work [15]: We calculate the in-out effective action for the whole system, which presented an imaginary part that accounted for the nonvanishing probability of decay of the system's initial state due to friction.

We found that these dissipative effects depended on the relative velocity between the particle and the mirror and on the characteristic of the materials. We also found that, within our simple model, the imaginary part of the effective action for a particle moving in front of a mirror is the same as the imaginary part of the effective action per unit of area for two mirrors in parallel relative motion [15].

Then, we switched to the Schwinger-Keldysh or CTP formalism in order to study the decoherence suffered by the particle due to its interaction with the vacuum field fluctuations and its effective (mediated by the vacuum field) interaction with the mirror. We calculated the CTP effective action describing the influence of the vacuum field and the microscopic degrees of freedom of the mirror over the internal degree of freedom of the particle. This action of influence allowed us to estimate the decoherence time, in which two different quantum states for the particle could no longer be distinguished from one another. We found the decoherence time to decrease with the presence of the mirror and to be minimized when the characteristic frequencies of the mirror and the particle are the same (resonant case). We also found that the relative velocity between the mirror and the plate increases the decoherence, as does the increasing of any of the coupling constants. We expect that, in a Ramsey-like interference experiment, the parameters of our model could be chosen in a way that would maximize the decoherence effects. As we have modeled the atom as the lowest energy levels of a harmonic oscillator, it is possible to choose its characteristic frequency close to the resonance with the plate. In addition, with a nonvanishing relative velocity, we expect decoherence effects to be observed by means of the attenuation of the contrast in the Ramsey fringes.

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