

Influence of relativistic effects on satellite-based clock synchronizationJieci Wang,^{1,2,*} Zehua Tian,¹ Jiliang Jing,¹ and Heng Fan^{2,†}¹*Department of Physics, Collaborative Innovation Center for Quantum Effects, and Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, People's Republic of China*²*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China*

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Clock synchronization between the ground and satellites is a fundamental issue in future quantum telecommunication, navigation, and global positioning systems. Here, we propose a scheme of near-Earth orbit satellite-based quantum clock synchronization with atmospheric dispersion cancellation by taking into account the spacetime background of the Earth. Two frequency entangled pulses are employed to synchronize two clocks, one at a ground station and the other at a satellite. The time discrepancy of the two clocks is introduced into the pulses by moving mirrors and is extracted by measuring the coincidence rate of the pulses in the interferometer. We find that the pulses are distorted due to effects of gravity when they propagate between the Earth and the satellite, resulting in remarkably affected coincidence rates. We also find that the precision of the clock synchronization is sensitive to the source parameters and the altitude of the satellite. The scheme provides a solution for satellite-based quantum clock synchronization with high precision, which can be realized, in principle, with current technology.

DOI: [10.1103/PhysRevD.93.065008](https://doi.org/10.1103/PhysRevD.93.065008)**I. INTRODUCTION**

High precision synchronization of clocks plays an important role in modern society and scientific research [1,2]; examples include navigation, global positioning, tests of general relativity theory, and long baseline interferometry in radio astronomy, as well as gravitational wave observation. Two standard classical protocols for clock synchronization are Einstein's synchronization scheme [3] and Eddington's slow clock transfer [4]. The former requires operational exchange of light pulses between the distant clocks, and the latter is based on sending a locally synchronized clock from one part to other parts. Recently, quantum strategies have been exploited to improve the accuracy of clock synchronization. A few quantum clock synchronization (QCS) proposals [5–16] and experiments [17] are reported. It is shown that the schemes based on quantum mechanics can gain significant improvements in precision over their classical counterparts.

On one hand several satellite-based quantum optics experiences [18] are feasible with current technology, such as satellite quantum communication [19–27] and quantum tagging [28], as well as gravity probes using beam interferometers [29,30] and atomic clocks [31,32] to test the principle of equivalence. Among these experiments the synchronization of clocks between a satellite and a ground

station [33] is an essential step. Besides, a satellite-based quantum network of clocks is promising to act as a single world clock with unprecedented stability and accuracy approaching the limit set by quantum mechanics, and there is also a security advantage [34]. On the other hand time dilation [32] is a concern because of relativistic effects of the Earth on QCS. The influence of relativistic effects on quantum systems [35–43] is a focus of study in recent years because such studies provide insights into some key questions in quantum mechanics and relativity, such as nonlocality, causality, and the information paradox of black holes. Relativistic effects are particularly significant for the quantum versions of the Eddington scheme [6–8] because one must assume that the transfer is performed “adiabatically slowly” [44] and the spacetime is flat such that relativistic effects are negligible. However, time dilation induced by the Earth's spacetime curvature is experimentally observed for a change in height of 0.33 m [32], and thus cannot be neglected.

In this paper we propose a practical scheme for satellite-based QCS. We let two observers, Alice and Bob, exchange two frequency entangled pulses between a ground station and a satellite. The influence of gravitational redshift on the frequency of a pulse can be eliminated by an opposite gravitational blueshift. By assuming the clocks have the same precision [5,7,8], clock synchronization can be realized by identifying the time discrepancies. In our scheme the time discrepancies are introduced through adding or subtracting optical path differences into the pulses, and the coincidence rate of the pulses in the

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interferometer as a function of the time discrepancy between the clocks is measured. In actual satellite-based quantum information processing tasks [19,21–23,26,27,45], and similarly in protocols of QCS, the main errors are induced by photon loss and the dispersion effects of the atmosphere through which the pulses travel. Therefore, we employ frequency entangled light, instead of the entangled NOON state, which is vulnerable for photon loss [46], as well as the dispersion cancellation technology [8,47,48] to eliminate the influence of atmospheric scattering. We find that the coincidence rate of interferometry is remarkably affected by the spacetime curvature of the Earth. We also find that the precision of the clock synchronization is sensitive to the light source parameters.

The outline of the paper is as follows. In Sec. II we briefly introduce the sketch of the experimental setup. In Sec. III we discuss how the Earth’s spacetime affects the propagation of photons. In Sec. IV we study the feasibility of satellite-based QCS and how the effect of the Earth’s gravity will disturb it. In the last section we discuss the experimental feasibility of our scheme and give a brief summary.

II. THE SCHEME

The sketch of our proposal for satellite-based QCS is described in Fig. 1. The quantum optical technology of our proposal is based on the Hong, Ou, and Mandel (HOM) interferometer [49]. We assume that Alice works on the surface of the Earth ($r = r_A$) with her own clock, while Bob works on a satellite at constant radius $r = r_B > r_A$. The clocks have the same accuracy; thus, the clock synchronization problem is reduced to the problem of identifying the time discrepancy between the clocks. Alice sends two frequency entangled beams produced by a parametric down-converter crystal (PDC) source to Bob, and Bob bounces them back to Alice again. Those two pulses are

named signal (S) beam and idler (I) beam. By exchanging entangled beams between Alice and Bob, a “conveyor belt” [8] for time information is established. After propagating through different optical paths, the signal and idler beams interfere at the 50/50 beam splitter and are measured by the detectors. To introduce timing information into the beams, Alice and Bob use moving mirrors with constant speed v to add or subtract optical path differences (OPD) from the beams [8]. Alice and Bob come to an agreement on the starting time τ_0 of their mirrors in advance. Since they do not have a synchronized clock to start with, they can only start the mirrors at time τ_0^i ($i = a, b$) relative to the time readings of their own clocks, which are different due to different locations. As described in Fig. 1, at the ground station, Alice starts a moving mirror to add an OPD $\delta l_a^I(t)$ to the idler beam [8] and to subtract the same amount of OPD $\delta l_a^S(t)$ from the signal beam, which was bounced back from Bob. At the satellite Bob subtracts an OPD $\delta l_b^I(t)$ from the idler beam and adds an identical OPD $\delta l_b^S(t)$ to the signal beam. The linear time dependent OPDs are given by

$$\begin{aligned} \delta l_a^I(t) &= v(t - \tau_0^a), & \delta l_b^I(t) &= -v(t - \tau_0^b), \\ \delta l_a^S(t) &= -v(t - \tau_0^a), & \delta l_b^S(t) &= v(t - \tau_0^b), \end{aligned} \quad (1)$$

where τ_0^a and τ_0^b are the starting times (proper time) as measured by Alice’s and Bob’s clocks, respectively. In Eq. (1) t is the time point (coordinate time) of the coincidence detection of the signal and idler photons, i.e., the time when the photons’ quantum state is measured. By assuming the quantum state is instantaneously collapsed by the measurements made on the surface of the Earth, we can agree that the collapsing time is identical, $t = t^a = t^b$, even if Alice’s and Bob’s reading times are different, $\tau^a \neq \tau^b$, at this moment. From Eq. (1) we can see that the delays are proportional to the time interval between the starting time τ_0^i for the moving of the mirrors and the time t

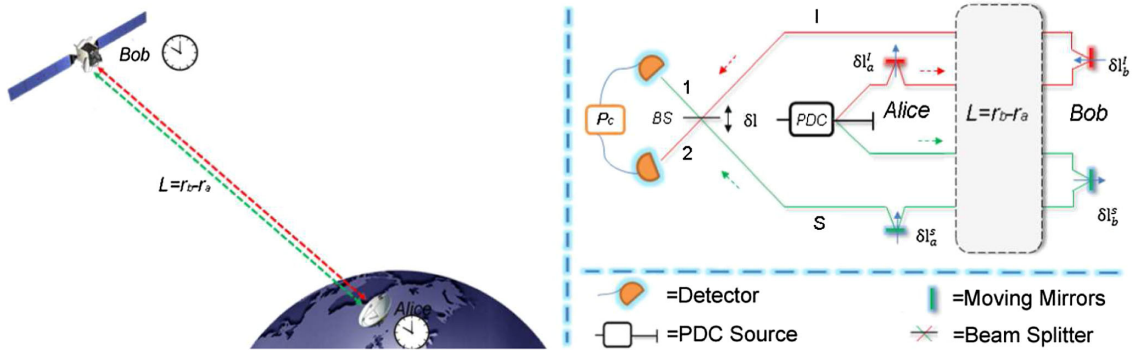


FIG. 1. Sketch of the experimental setup for satellite-based QCS. Alice sends two frequency entangled beams produced by a parametric down-converter crystal (PDC) source to Bob who bounces them back to Alice again. Alice adds an OPD $\delta l_a^I(t)$ to the idler beam [8] and subtracts the same amount of OPD $\delta l_a^S(t)$ from the signal beam, which was bounced back from Bob. At the satellite Bob subtracts an OPD $\delta l_b^I(t)$ from the idler beam and adds an identical OPD $\delta l_b^S(t)$ to the signal beam. Finally, we let the pulses interfere at the 50/50 beam splitter (BS) and be detected at the measurement setups. The coincidence rate P_c is measured as a function of the time discrepancy $\tau_0^b - \tau_0^a$ between the clocks.

when the photons are detected. If the proportionality constant v in Eq. (1) and the starting time reading τ_0^i on Alice's and Bob's clocks are identical, the quantity of OPD at Alice's point will be *zero* after an exchange period. Therefore, the final OPD is totally produced from the starting time discrepancy $\Delta\tau = \tau_0^b - \tau_0^a$ between Alice's clock and Bob's clock. Then the signal and idler pulses interfere at the beam splitter (BS) and click the detectors. We show in Sec. IV that the final difference of optical path lengths is affected by a factor depending on the starting time discrepancy $\Delta\tau$. By measuring the photon coincidence rate P_c at the output ports 1 and 2 of the beam splitter, one may acquire very precise information on OPD in the two arms. Thus, it is sufficient to measure the photon coincidence rate to recover the exact time discrepancy between Alice's clock and Bob's clock. Then Alice tells Bob by classical communication to adjust his clock time according to the time discrepancy. By using this scheme Alice and Bob may check how much the accuracy is disturbed by the gravity-induced spacetime curvature of the Earth, and inversely, they can precisely measure the curvature via an atomic clock, which is the most accurate setup currently available in the world.

III. EARTH'S SPACETIME ON THE PROPAGATION OF PHOTONS

Now we describe the propagation of photons from the Earth to a satellite by taking Earth's gravity into consideration. The Earth's spacetime curvature will influence the light pulses during their propagation between the ground station and the satellite. We know that the Earth rotates slowly with an angular velocity at the equator of $\omega_E = 7.2921151247 \times 10^{-5}$ rad/s or at a linear speed of $v_E = 465$ ms $^{-1}$, which is much slower than the speed of light. Therefore, the Schwarzschild metric [50–52] is a sufficient approximation for the Earth's spacetime, as has been discussed in [27,45,50]. The Schwarzschild metric is given by [51,52]

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2)$$

where $r_s := \frac{2GM}{c^2}$ is the Earth's Schwarzschild radius, M is the mass of the Earth, c is the speed of light in vacuo, and G is the gravitational constant.

A photon can be properly modeled by a wave packet of electromagnetic fields [53] with a distribution $F_{\omega_{K,0}}^{(K)}$ of modes peaked around the frequencies $\omega_{K,0}$ [54,55], where $K = A, B$ means Alice or Bob, respectively. The annihilation operator for a photon from the point of view of Alice or Bob takes the form

$$a_{\omega_{K,0}}(t_K) = \int_0^{+\infty} d\omega \Omega_K e^{-i\omega t_K} F_{\omega_{K,0}}^{(K)}(\omega_K) a_{\omega_K}, \quad (3)$$

where ω_K is the physical frequencies as measured in their labs. At time τ_A , Alice prepares a wave packet $F_{\omega_{A,0}}^{(A)}$ and sends it to Bob, who will receive it at a different proper time $\tau_B = \Delta\tau + \sqrt{f(r_B)/f(r_A)}\tau_A$, where $f(r_A) = 1 - \frac{r_s}{r_A}$, $\Delta\tau$ is propagation time of the wave packet, and the factor $\sqrt{f(r_B)/f(r_A)}\tau_A$ indicates the relativistic time dilation. The wave packet received is modified due to the spacetime curvature of the Earth and takes the form $F_{\omega_{B,0}}^{(B)}$. By employing the definition of proper time, it is easy to show that the time evolution for the modes has the form $i\partial_{\tau_K}\phi_{\omega_K} = \omega_K\phi_{\omega_K}$, where $\phi_{\omega_K}^{(u)}$ are the quantum states of the modes corresponding to the operators a_{ω_K} [27]. This equation indicates that the physical frequency ω_K measured by an observer at the position r_K is $\omega_K = f(r_K)^{-1/2}\omega$. Then we find that Bob will receive a mode with frequency $\omega_B = \sqrt{\frac{f(r_A)}{f(r_B)}}\omega_A$ if a sharp frequency mode with ω_A was sent by Alice. This is the well-known gravitational redshift effect, which was predicted by Einstein in 1911 and experimentally verified in 1960 [56]. In our scheme such a classical gravitational effect is designed to be eliminated by an opposite gravitational blueshift factor $\sqrt{\frac{f(r_B)}{f(r_A)}}$ because the signal will be sent downward from a satellite to the Earth.

However, such a nonlinear gravitational effect is found to influence the fidelity of the quantum channel between Alice and Bob [27,45] and will inevitably affect the accuracy of the satellite-based QCS as well. The mode \bar{a}' received by Bob can be decomposed in terms of the mode a prepared by Alice and an orthogonal mode a_{\perp} [27,45,57],

$$\bar{a}' = \Theta a + \sqrt{1 - \Theta^2} a_{\perp}, \quad (4)$$

where Θ is the wave packet overlap between the distributions $F_{\omega_{B,0}}^{(B)}(\omega_B)$ and $F_{\omega_{A,0}}^{(A)}(\omega_B)$,

$$\Theta = \int_0^{+\infty} d\omega_A F_{\omega_{A,0}}^{(A)*}(\omega_B) F_{\omega_{B,0}}^{(B)}(\omega_B), \quad (5)$$

which describes the fidelity of the channel between Alice and Bob. For a perfect channel, one has $\Theta = 1$.

IV. SPACETIME CURVATURE ON SATELLITE-BASED QCS

The emitted signal and idler beams from the PDC initially share an entangled state [8],

$$|\psi\rangle = \int d\omega_1 d\omega_2 F(\omega_1, \omega_2) a^\dagger(\omega_1) a^\dagger(\omega_2) |0\rangle, \quad (6)$$

where $a^\dagger(\omega_1)$ and $a^\dagger(\omega_2)$ are creation operators of the first and the second photons, respectively. For a monochromatic pump this state can be rewritten as

$$|\psi\rangle = \int d\omega F_{\omega_0}(\omega) |\omega_0 + \omega\rangle_I |\omega_0 - \omega\rangle_S, \quad (7)$$

where $|\omega\rangle_S$ and $|\omega\rangle_I$ are the states for the signal and idler pulses, respectively, and $F_{\omega_0}(\omega)$ is the spectral distribution function of the down-converted photons [47,48]. Now let us briefly discuss how quantum entanglement is useful to quantum clock synchronization and the advantage of the quantum clock synchronization scheme. The main advantage of a quantum strategy is that we can employ quantum uncertainty and coherence time of the frequency entangled photons. From Eq. (7) we can see that although the sum frequency $2\omega_0$ is certain, the down-shifted frequencies $|\omega_0 - \omega\rangle_S$ and $|\omega_0 + \omega\rangle_I$ are highly uncertain. The frequencies are largely determined by the pass bands of the interference filters that are inserted in the down-shifted beams [49]. These pass bands have been found on the order of 5×10^{12} Hz, which corresponds to a coherence time for each photon on the order of 100 fs [49]. Therefore, they are able to measure a time interval of or better than the coherence time of the photons (50 fs), with an accuracy of 1 fs (10^{-15} s). Also, the most important task for the quantum clock synchronization is to determine the time interval by measuring the rate at which photons are detected in coincidence, which relates to the coherence length of the photon wave packet and entanglement of the photons.

As stated before Alice adds an OPD $\delta l'_a(t)$ to the idler beam [8] and subtracts the same amount of OPD $\delta l'_a(t)$ from the signal beam that was bounced back from Bob. At the satellite Bob subtracts an OPD $\delta l'_b(t)$ from the idler beam and adds an identical OPD $\delta l'_b(t)$ to the signal beam. Then we let the pulses interfere at the 50/50 BS and be measured in the HOM interferometer. The detected coincidence rate P_c at the detectors is given by the Mandel formula [58],

$$P_c \propto \int_t dt_1 dt_2 \langle \psi | \mathcal{E}_1^{(-)} \mathcal{E}_2^{(-)} \mathcal{E}_2^{(+)} \mathcal{E}_1^{(+)} | \psi \rangle, \quad (8)$$

where t is the interaction time of the detectors, and the electromagnetic fields at time t_j at the output of the beam splitter can be defined by

$$\begin{aligned} \mathcal{E}_j^{(+)} &= i \int d\omega \sqrt{\frac{\hbar\omega}{4\pi\zeta c}} a_j(\omega) e^{-i\omega(t_j - x_j/c)} \\ &= (\mathcal{E}_j^{(-)})^\dagger, \quad \text{for } j = 1, 2, \end{aligned} \quad (9)$$

where ζ is the beam cross section, and x_j is the position of the moving mirrors' detector. The relationship between the output and input fields of the beam of the moving mirrors can be obtained by performing Lorentz transformations on the input fields [8]. For example, the factor $a_j(\omega) e^{-i\omega(\tau_0^j - x_0^j/c)}$ after the beams outflow from the moving mirrors evolves to $\sqrt{\chi} a_j(\chi\omega) e^{-i\omega \frac{2\beta}{1-\beta}(\tau_0^j - x_0^j/c)}$, where $\beta = \frac{v}{c}$ is the Lorentz transformation factor, $\chi = \frac{1+\beta}{1-\beta}$ denotes the Doppler shift introduced by the moving mirrors, and x_0^j is the distance of the beam's delay. For the time delays defined in Eq. (1), we have $x_0^j = x_0$ for all j . Then the beams are sent from the Earth to the satellite and are influenced by the dissipation of the atmosphere and the spacetime curvature. The former produces a phase discrepancy $i\kappa_j^i(\omega)$, and the latter can be described by Eq. (5). Taking the signal beam as an example, the full procedure can be expressed by the transformations between the annihilation operators when the photons are sent from the Earth to the satellite,

$$\begin{aligned} a'_S(\omega) &= \Theta_1 \sqrt{\chi} a_S \left(\chi \sqrt{\frac{f(r_A)}{f(r_B)}} \omega \right) e^{-i\sqrt{\frac{f(r_A)}{f(r_B)}} \omega \Upsilon_1 + i\kappa_1^S(\omega)} \\ &\quad + \sqrt{1 - \Theta_1^2} a_S^\perp \left(\chi \sqrt{\frac{f(r_A)}{f(r_B)}} \omega \right), \end{aligned} \quad (10)$$

where $\Upsilon_1 = \frac{2\beta}{1-\beta}(\tau_0^b - x_0/c) - L/c$ and Θ_1 is the wave packet overlap between the distributions when the pulse is sent from the Earth to the satellite. We can see that the annihilation operator $a_S(\omega)$ at Alice's laboratory evolves into $a'_S(\omega)$ when observing at Bob's laboratory, where L is the distance between the Earth and the satellite. Again, the annihilation operator for the signal beam evolves into $a''_S(\omega)$ before entering the 50/50 BS,

$$\begin{aligned} a''_S(\omega) &= \Theta_1 \Theta_2 a_1(\omega) e^{i\omega \Upsilon_2 + i\kappa_1^S(\omega) + i\kappa_2^S(\omega/\chi)} \\ &\quad + \Lambda a_S^\perp(\omega), \end{aligned} \quad (11)$$

where $\Upsilon_2 = \frac{2\beta}{1+\beta}(\tau_0^a - \tau_0^b + \frac{x_0}{c}) + (L/\chi + L')/c$, $\Lambda = \Theta_2 \sqrt{1 - \Theta_1^2} + \sqrt{1 - \Theta_2^2}$, L' denotes the distance between Bob and the BS, and Θ_2 is the mode overlap between the distributions when the pulse is received from the satellite. In Eq. (11) the terms κ_1^S and κ_2^S describe the effect of the dispersive atmosphere on the signal beams on their way to and from the satellite, respectively. Notice that the Doppler shift introduced by the first mirror is ω/χ , and the frequency itself is reevaluated at ω again because the second mirror moves in the opposite direction [8]. The analogous procedures can be applied to the idler beam propagation process, yielding the final operator transformations,

$$a_I''(\omega) = \Theta_1 \Theta_2 a_I(\omega) e^{-i\omega \Upsilon_3 + i\kappa_I^s(\omega/\chi) + i\kappa_I^f(\omega)} + \Lambda a_I^{\dagger}(\omega),$$

where $\Upsilon_3 = \frac{2\beta}{1+\beta}(\tau_0^a - \tau_0^b - \frac{x_0}{c}) - (L/\chi + L')/c$. Because the distance between the BS and the PDC source is much smaller than the distance between the satellite and the Earth, we assume that $L = L'$. At the output of the 50/50 BS, the modes are found to be $a_1(\omega_1) = \frac{1}{\sqrt{2}}[ia_I''(\omega_1)e^{-i\omega_1\delta l/c} + a_S''(\omega_1)]$ and $a_2(\omega_2) = \frac{1}{\sqrt{2}}[ia_S''(\omega_2) + a_I''(\omega_2)e^{-i\omega_2\delta l/c}]$, where δl is the delay introduced to relate the coincidence rate P_c with the path length. Then the coincidence rate P_c defined in (8) is obtained as

$$P_c \propto \int d\omega_1 d\omega_2 \langle \psi | a_1^{\dagger}(\omega_1) a_2^{\dagger}(\omega_2) a_1(\omega_1) a_2(\omega_2) | \psi \rangle \\ = \int d\omega_1 d\omega_2 |\langle 0 | a_1(\omega_1) a_2(\omega_2) | \psi \rangle|^2. \quad (12)$$

The matrix element $\langle 0 | a_1(\omega_1) a_2(\omega_2) | \psi \rangle$ in (12) is given by

$$\langle 0 | a_1(\omega_1) a_2(\omega_2) | \psi \rangle \\ = \frac{1}{2} (\Theta_1 \Theta_2)^2 \delta(\omega_1 + \omega_2 - 2\omega_0) \\ \times e^{i\varphi} \phi(\omega_1 - \omega_0) [1 - e^{2i(\omega_1 - \omega_0)\Upsilon_4 - i\Delta\kappa(\omega_1)}], \quad (13)$$

where $\Upsilon_4 = \frac{4\beta}{1+\beta}(\tau_0^b - \tau_0^a) - \delta l/c$, and φ is an overall phase term that will disappear by taking the modulus [8], and the contribution of the dispersion terms is

$$\Delta\kappa(\omega) = \kappa_I^S(\omega) - \kappa_f^I(\omega) + \kappa_f^S(\omega') - \kappa_I^S(\omega') \\ + \kappa_I^I(\omega') - \kappa_f^S\left(\frac{\omega_1}{\chi}\right) + \kappa_f^S\left(\frac{\omega}{\chi}\right) - \kappa_I^I\left(\frac{\omega}{\chi}\right), \quad (14)$$

where $\omega' = 2\omega_0 - \omega$. If the properties of the beams propagating through different optical paths are such that $\kappa_I^S = \kappa_f^I$ and $\kappa_f^S = \kappa_I^I$, the dispersion effect of the atmosphere is erased. Such conditions can be satisfied by allowing the ‘‘from’’ idler beam to propagate at a distance less than the spatial inhomogeneities of the atmosphere from the ‘‘to’’ signal beam and, equivalently, by allowing the ‘‘to’’ idler beam to propagate less than the ‘‘from’’ signal beam [8]. Here, the ‘‘from’’ beam denotes the beam from Alice to Bob and vice versa. Then the dispersion suffered by one of the photons can cancel that suffered by the other photon. These two photons can remain totally coincident after propagating through different optical paths. Substituting Eq. (13) into Eq. (12), we can obtain

$$P_c = (\Theta_1 \Theta_2)^2 \int d\omega |\mathcal{F}|^2 \left(1 - \cos \left[\frac{2\omega}{c} \left(\frac{4v\Delta\tau}{1+\beta} - \delta l \right) \right] \right), \quad (15)$$

where $\mathcal{F} = F_{\omega_0}(\omega)$. From Eq. (15) we can see that the coincidence rate P_c is directly related to the time discrepancy $\Delta\tau = \tau_0^b - \tau_0^a$ of Alice’s and Bob’s clocks. That is to say Alice’s and Bob’s clocks can be synchronized by using the measured coincidence rate of interferometry.

To be explicit we next only consider the case in which $F_{\omega_0}(\omega)$ is a Gaussian wave packet $F_{\omega_0}(\omega) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\omega-\omega_0)^2}{4\sigma^2}}$, where σ is the Gaussian width. The wave packet overlaps Θ_1 and Θ_2 are found to be

$$\Theta_{1(2)} = \sqrt{\frac{2\Delta_{1(2)}}{1 + \Delta_{1(2)}^2}} e^{-\frac{\vartheta^2 \omega_0^2}{4\sigma^2(1 + \Delta_{1(2)}^2)}}, \quad (16)$$

where $\Delta_{1(2)} = 1 \pm \vartheta$, and the signs \pm occur for $r_A < r_B$ or $r_A > r_B$. In Eq. (16), we define $\vartheta = \sqrt{\frac{f(r_A)}{f(r_B)}} - 1$, and $\omega_{A,0} = \omega_{B,0} = \omega_0$ is assumed. The modes will be perfectly overlapped ($\Theta = 1$) when Alice and Bob are in a flat spacetime $f(r_A) = f(r_B) = 1$. For the typical resources used in quantum optics experiments, the relation $\vartheta \ll (\frac{\vartheta\omega_0}{\sigma})^2 \ll 1$ should be satisfied [27,45,59], which yields $\Theta_1 = \Theta_2 \sim 1 - \frac{\vartheta^2 \omega_0^2}{8\sigma^2}$. Then we find that the coincidence rate P_c has the form

$$P_c = \left(1 - \frac{\vartheta^2 \omega_0^2}{8\sigma^2} \right)^4 [1 - e^{-2\sigma^2(\delta l - \frac{4v\Delta\tau}{1+\beta})^2/c^2}]. \quad (17)$$

We can see that the coincidence rate P_c has the factor $(1 - \frac{\vartheta^2 \omega_0^2}{8\sigma^2})^4$ compared to that of the flat spacetime case, where $P_c^f = 1 - e^{-2\sigma^2(\delta l - \frac{4v\Delta\tau}{1+\beta})^2/c^2}$ [8]. We define the effect of spacetime curvature on the accuracy of clock synchronization as the relative disturbance of coincidence rate

$$\Delta_p = \frac{P_c^f - P_c}{P_c^f} = 1 - \left(1 - \frac{\vartheta^2 \omega_0^2}{8\sigma^2} \right)^4. \quad (18)$$

It is now clear that the relative disturbance of coincidence rate Δ_p depends on the spacetime parameter ϑ and the characteristics of the PDC source.

In Fig. 2 we plot the relative disturbance of the Earth’s spacetime curvature on the coincidence rate Δ_p as a function of the distance r_B between the satellite and the Earth’s core for the fixed light source parameters. It is shown that Δ_p increases as the distance increases; i.e., the accuracy of clock synchronization depends on the altitude of the satellite, which also verifies that the spacetime curvature remarkably influences the running of the atomic clocks [32]. This result is very different from that of Ref. [8], in which the coincidence rate is independent of the distance L between Alice and Bob when the spacetime curvature of the Earth is not considered.

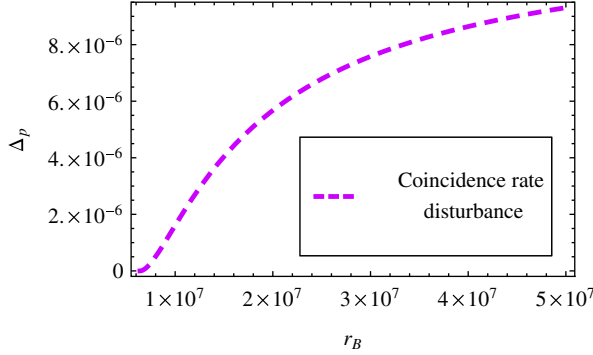


FIG. 2. The relative disturbance of coincidence rate Δ_p as a function of the altitude of the satellite r_B . The parameters of the PDC light source are fixed as $\omega_0 = 700$ THz and $\sigma = 100$ MHz.

In Fig. 3 we plot the relative disturbance Δ_p over the peak frequency ω_0 and bandwidth σ of the PDC. It is shown that the disturbance of accuracy depends sensitively on the bandwidth σ of the source, which is similar to the flat spacetime case [8]. However, here we find that the disturbance on accuracy also depends on the peak frequency of the pulses, which is different from that of Ref. [8] where the accuracy is independent of the peak frequency. In this paper we are particularly interested in two typical cases in which the QCS are performed between the ground station and either a low earth orbit (LEO) or a geostationary earth orbit (GEO) satellite.

The LEO case: The typical distance from the Earth to a LEO satellite is about 400 km, which yields $r_A = 6.371 \times 10^6$ m and $r_B = 6.771 \times 10^6$ m. Considering that the Schwarzschild radius of the Earth is $r_S = 9$ mm, it is found that $\delta \sim -\frac{1}{2}(\frac{r_S}{r_B} - \frac{r_S}{r_A}) = 4.17 \times 10^{-11}$. We employ a typical PDC source with a wavelength of 369.5 nm (corresponding to $\omega_0 = 812$ THz) and $\sigma = 100$ MHz

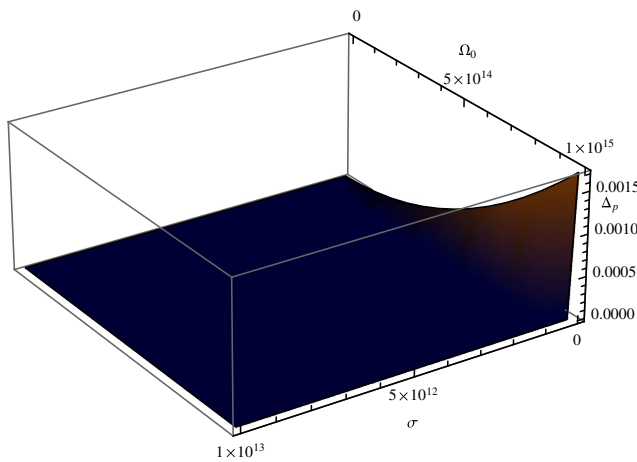


FIG. 3. The relative disturbance of coincidence rate Δ_p as a function of the source parameters ω_0 and σ for the fixed distance $r_B = 6.771 \times 10^6$ m (LEO).

[the relation $\delta \ll (\frac{\delta\omega_0}{\sigma})^2 \ll 1$ is satisfied]. A light source with such peak frequency and bandwidth is available, for example, in trapped ion experiments [59]. The relative disturbance of the spacetime curvature on the coincidence rate is obtained as $\Delta_p^L = 5.73993 \times 10^{-8}$. The achievement of an optical lattice clock with accuracy at the 10^{-18} level has been reported in Refs. [60,61]. If we would like to synchronize two clocks up to a time discrepancy of $\tau = 1$ ns (as in Ref. [8], or a much lower level $\tau = 100$ ns as presented in Ref. [7]), the correction of the Earth's spacetime curvature effect will reach 10^{-17} s during a single synchronization process. Such a correction is comparable to the accuracy of the atom clocks and thus should be considered for the QCS between clocks in future satellite-based applications. Therefore, we can safely arrive at the conclusion that the spacetime curvature is *not negligible* when the synchronization is performed by LEO satellites.

The GEO case: The typical distance L between a GEO satellite and the ground is about 3.6×10^7 m. Therefore, the distance between the Earth and the satellite r_B is about $r_B = 42.371 \times 10^6$ m, which yields $\delta \sim -\frac{1}{2}(\frac{r_S}{r_B} - \frac{r_S}{r_A}) = 6 \times 10^{-10}$. In this case the relative disturbance of the spacetime curvature on the coincidence rate is $\Delta_p^G = 1.18729 \times 10^{-5}$. We find that the disturbance of the spacetime curvature on the coincidence rate for the GEO satellites becomes even *more remarkable* than that of the LEO case. We remark that the current GPS satellites have $r_B \approx 2.7 \times 10^7$ m. In the QCS scheme, the spacetime curvature is also remarkable.

Error analysis: It is worth mentioning that the velocity variations of the moving mirrors may induce some errors on the coincidence rate. However, the order of magnitudes of the movement speed of the mirrors is much smaller than the velocity of light, let alone the velocity variation of the mirrors. To be specific the typical velocity of the moving mirrors is 10^{-1} m/s. Let us suppose that this velocity has one percent of variation, say 10^{-3} m/s, which is much smaller than the velocity of light. Note that the relative disturbance of the spacetime curvature on the coincidence rate is on the order of 10^{-8} for the LEO satellites and on the order of 10^{-5} for the GEO satellites, which are at least 3 orders of magnitude larger than that of the velocity variations of the mirrors. Therefore, the systematic errors induced by the velocity variation of the mirrors can be safely ignored in the scheme.

V. DISCUSSIONS

We have proposed a practical satellite-based QCS scheme with the advantages of dispersion cancellation and robust frequency entangled pulses of light, taking the effects of the spacetime curvature of the Earth into consideration. The spacetime background of the Earth is described by the Schwarzschild metric, and the quantum

optics parts of our proposal are based on the HOM interferometer. By eliminating the gravitational redshift and blueshift of the laser pulses and the atmospheric dispersion cancellation, the accuracy of clock synchronization in our quantum scheme can be very high, showing that Δ_p is close to unity. Our proposal can be implemented, in principle, with current available technologies. To be specific optical sources with the required peak frequency and bandwidth have been achieved by the trapped ion experiments [59]. The feasibility of photon exchanges between a satellite and a ground station has been experimentally demonstrated [21] by the Matera Laser Ranging Observatory (MLRO) in Italy. Most recently they have reported the operation of experimental satellite quantum communications [22] by sending selected satellites laser pulses. Our scheme can also be generalized to the quantum

clock network cases [6,34]. The results should be significant both for determining the accuracy of clock synchronization and for our general understanding of time discrepancy in future satellite-based quantum systems.

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