

Head-on collision of ultrarelativistic particles in ghost-free theories of gravity

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We study linearized equations of a ghost-free gravity in four- and higher-dimensional spacetimes. We consider versions of such a theory where the nonlocal modification of the \square operator has the form $\square \exp[(-\square/\mu^2)^N]$, where $N = 1$ or $N = 2n$. We first obtain the Newtonian gravitational potential for a point mass for such models and demonstrate that it is finite and regular in any number of spatial dimensions $d \geq 3$. The second result of the paper is calculation of the gravitational field of an ultrarelativistic particle in such theories. And finally, we study a head-on collision of two ultrarelativistic particles. We formulated conditions of the apparent horizon formation and showed that there exists a mass gap for mini-black-hole production in the ghost-free theory of gravity. In the case when the center-of-mass energy is sufficient for the formation of the apparent horizon, the latter has two branches, the outer and the inner ones. When the energy increases the outer horizon tends to the Schwarzschild-Tangherlini limit, while the inner horizon becomes closer to $r = 0$.

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I. INTRODUCTION

Singularities are inherent properties of general relativity. It is generally believed that the Einstein-Hilbert action should be modified in spacetime domains where the curvature becomes large. Such a modification is required, for example, when one includes in the theory quantum corrections, connected with particle creation and vacuum polarization effects. At a more fundamental level, the modification of the gravity equation might be required if the gravity is described as an emergent phenomenon. In such a case the Einstein equations are nothing but the low energy limit of the corresponding more fundamental background theory. The string theory is a well-known example. It is convenient to introduce two (generally different) energy scale parameters μ and $\tilde{\mu}$. The corresponding length scales are $\lambda = \mu^{-1}$ and $\tilde{\lambda} = \tilde{\mu}^{-1}$. We assume that when the spacetime curvature \mathcal{R} is much less than λ^{-2} , the corrections to the Einstein equations are small. These corrections become comparable with other terms of the Einstein equations at $\mathcal{R} \sim \lambda^{-2}$, and for higher values of the curvature they play an important role. We assume that one can use the classical metric $g_{\mu\nu}$ for the description of the gravitational field. For example, one can understand it as a quantum average of some metric operator, $g_{\mu\nu} = \langle \hat{g}_{\mu\nu} \rangle$. This means that the quantum gravity effects, and in particular fluctuation of the metric, are small. In other words, one can use the effective action approach to study spacetime properties in this domain. The second parameter, $\tilde{\lambda}$, defines the scale when effective action description breaks

down and the quantum nature of the gravitational field becomes important.

In studies of the singularity problem in modified gravity it is usually assumed that $\tilde{\mu} \gg \mu$. In the present paper we also use this assumption and discuss some aspects of the singularity problem in the framework of the classical modified gravity equations.

There exist a wide class of the modified theories of gravity proposed to solve fundamental problems of black holes and cosmology. We consider a special class of such theories, namely theories with higher derivatives. Important features of such theories can be clarified already in a simple approximation when the gravitational field is weak and can be described as the perturbation on the flat spacetime background. Such an analysis was performed by Stelle [1]. In particular, he demonstrated that the Newtonian gravitational potential of a point mass located at $\vec{r} = 0$ can be made finite at this point, if the higher derivative terms are included in the gravity equations. Detailed analyses of this problem can be found in recent papers [2,3].

However, the higher derivative gravity, as well as any theory with higher derivatives, has a fundamental problem. In a general case the propagator of such a theory contains two or more poles, and, as a result, it almost always contains ghostlike excitations (see, e.g., [1,4,5]). Presence of the excitations with negative energy results in an instability of the theory and the possibility of an empty space decay. This is a special case of a very general phenomenon known as Ostrogradsky instability [6] (see discussion in [5]).

In the higher-derivative theory a standard box operator \square , which enters the field equations, is changed to the operator $P(\square)\square$, where $P(z)$ is a polynomial. The poles of

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$P^{-1}(z)$ correspond to additional degrees of freedom. However, there exists an interesting option of theories where $P^{-1}(z)$ is an entire function of z and hence it does not have poles in the complex plane. Such a modification of the gravitational equations is called ghost-free (GF) gravity (see, e.g., [7–15] and references therein). GF gravity contains an infinite number of derivatives and, hence, it is nonlocal. Theories of this type were considered a long time ago (see, e.g., [16–20]). They appear naturally also in the context of noncommutative geometry deformation of the Einstein gravity [21,22] (see a review [23] and references therein). The initial value problem in nonlocal theories was studied in [5,24]. The application of the ghost-free theory of gravity to the problem of singularities in cosmology and black holes can be found in [25–31]. Static and dynamical solutions of the linearized equations of the ghost-free gravity in four and higher dimensions were studied in [32,33]. Recently the consequences of the ghost-free modifications of higher-dimensional gravity on the entropy of black holes and on cosmological models have been studied [34].

In this paper we continue study of the linearized equations of the GF gravity. In Secs. II–IV we study solutions for a static gravitational field in the Newtonian approximation in different models of the GF gravity. Namely, we consider a class of the GF_N theories of gravity with $P(\square) = \exp[(-\square/\mu^2)^N]$. A static solution of the linearized equations for $N = 1$ in four-dimensional spacetime was found in [8,26] (see also [33]). In this paper we generalize this result to the higher-dimensional case and obtain new solutions for GF_{2n} theories in the spacetime with an arbitrary number d of spatial dimensions. In Sec. V we used these results to obtain a solution of the GF gravity describing a gravitational field of an ultrarelativistic particle. We succeeded to find a generalization of the famous Aichelburg-Sexl solution [35] to the GF gravity in an arbitrary number of dimensions. In Sec. VI we used the obtained solutions to study the apparent horizon formation in the head-on collision of two ultrarelativistic particles. This problem for the general theory of relativity in four dimensions was first solved by Penrose [36]. Later, this result was generalized for a collision with a nonzero impact parameter in four and higher dimensions [37–40]. In the present paper we show that in the GF gravity a similar process has two important new features: (i) the apparent horizon is not formed if the center-of-mass energy of the particles, E , is smaller than some critical value E_{crit} , which depends on the scale parameter μ , the type of the theory, and the number of spacetime dimensions; (ii) if the energy is larger than E_{crit} the apparent horizon besides the usual outer part always has another inner branch. We discuss the obtained results in the last section.

In the present paper we use units in which $\hbar = c = 1$ and sign conventions adopted in the book [41].

II. NEWTONIAN LIMIT OF HIGHER-DIMENSIONAL HIGHER-DERIVATIVE EQUATIONS

Let us consider a static gravitational field perturbation on a flat background and write the corresponding metric in the form

$$\begin{aligned} ds^2 &= -(1 + 2\varphi)dt^2 + (1 - 2\psi + 2\varphi)d\ell^2, \\ d\ell^2 &= \delta_{ik}dx^i dx^k, \quad x^i = (x^1, \dots, x^d). \end{aligned} \quad (2.1)$$

Here and later we denote by $d = D - 1$ a number of spatial dimensions. We also have

$$\begin{aligned} h_{00} &= -2\varphi, \quad h_{ij} = -2(\psi - \varphi)\delta_{ij}, \\ h &= 2[(d + 1)\varphi - d\psi]. \end{aligned}$$

By substituting these expressions into the gravity equations (A5) one gets

$$\begin{aligned} a(\Delta)\Delta\psi &= \kappa_d \left(\tau_{00} + \frac{1}{d-1} \delta^{ij} \tau_{ij} \right), \\ [a(\Delta) - dc(\Delta)]\Delta\varphi + (d-1)c(\Delta)\Delta\psi &= \kappa_d \tau_{00}. \end{aligned}$$

Here $\kappa_d = 8\pi G^{(D)}$ and $D = d + 1$ is the total number of spacetime dimensions. In the Newtonian approximation $\delta^{ij} \tau_{ij} = 0$ and the first of these equations takes the form

$$a(\Delta)\Delta\psi = \kappa_d \tau_{00}. \quad (2.2)$$

For the gravity theory with $c = a$ the equations simplify and one obtains

$$\psi = \frac{d-1}{d-2} \varphi, \quad (2.3)$$

and the metric (2.1) takes the form

$$ds^2 = -(1 + 2\varphi)dt^2 + \left(1 - \frac{2}{d-2} \varphi \right) d\ell^2. \quad (2.4)$$

For a point mass m the energy density has the form $\tau_{00} = m\delta^d(x)$. Then for the Einstein gravity, where $a = c = 1$ one has

$$\varphi = -\frac{\kappa_d m \Gamma(\frac{d}{2})}{2(d-1)\pi^{d/2} r^{d-2}}. \quad (2.5)$$

In four dimensions $D = 4$ ($d = 3$)

$$\varphi = -\frac{\kappa_3 m}{8\pi r}. \quad (2.6)$$

III. STATIC SOLUTIONS OF LINEARIZED EQUATIONS IN GHOST-FREE GRAVITY

A. Ghost-free gravity

The Newtonian potential (2.5) is evidently singular at $r = 0$. One can regularize it and make it finite at $r = 0$ by modifying the gravity equations in the ultraviolet (UV) domain. For example, one may assume that $a(\square)$ and $c(\square)$ are polynomials of the \square operator. If these functions obey the condition $a(0) = c(0) = 1$, the theory correctly reproduces the standard results of general relativity in the infrared regime, that is in the domain where $r \rightarrow \infty$. In a general case such a theory possesses ghosts. These ghosts are new degrees of freedom which are connected with extra poles of the operators a^{-1} and c^{-1} which give contributions to the propagator with a wrong (negative) sign. However, there exists an option to use such functions $a^{-1}(z)$ and $c^{-1}(z)$ that are entire functions of the complex z -variable which do not have poles. It happens, for example, when $a(z)$ and $c(z)$ are of the form $\exp(P(z))$, where $P(z)$ is a polynomial. A modified gravity which contains such regular form factors is called *ghost-free (GF) gravity*. In the present paper we focus on the special class of the theories of GF gravity. Namely, we assume that

$$a(\square) = c(\square) = \exp((-\square/\mu^2)^N). \quad (3.1)$$

We denote such a theory GF_N . We restrict ourselves by considering the cases $N = 1$ and $N = 2n$, which are of the most interest for applications.

The exponent of the operator can be written in the form of a convergent series of the powers of this operator. However, it is not a good idea to “approximate” the exponent by the polynomial which is obtained by keeping a finite number of terms in this series. The inverse operator will have extra poles and the ghost will be present for such truncation. That is why our first goal is to present these nonlocal objects in the form of an integral transform which contains a well-defined kernel.

B. Potential ψ_d and Green functions in GF theories

Consider the equation for the potential ψ_d created by a point massive particle placed at a point x'

$$\hat{F}\psi_d = \kappa_d m \delta^d(x - x'), \quad (3.2)$$

where the operator \hat{F} is defined on the d -dimensional Euclidean space. It is assumed to be a function of the Laplace operator

$$\hat{F} = \tilde{F}(-\Delta), \quad \tilde{F}(\xi) = -\xi a(-\xi). \quad (3.3)$$

The Euclidean Green function $D_d(x, x')$ of this operator is the solution of the problem

$$\hat{F}D_d(x, x') = -\delta^d(x - x') \quad (3.4)$$

with vanishing boundary conditions at infinity. Formally it can be treated as a matrix element

$$D_d(x, x') = \langle x | \hat{D} | x' \rangle \quad (3.5)$$

of the operator

$$\begin{aligned} \hat{D} &= -\hat{F}^{-1}, & \hat{D} &= \tilde{D}(-\Delta), \\ \tilde{D}(\xi) &= -\frac{1}{\tilde{F}(\xi)} = \frac{1}{\xi a(-\xi)}. \end{aligned} \quad (3.6)$$

The momentum space calculations of $D_d(x, x')$ are presented in Appendix B. The result reads (B6)

$$\begin{aligned} D_d(x, x') &= \frac{1}{4\pi} \int_0^\infty d\eta \tilde{D}(\eta) \left(\frac{\sqrt{\eta}}{2\pi|x - x'|} \right)^{\frac{d}{2}-1} \\ &\times J_{\frac{d}{2}-1}(\sqrt{\eta}|x - x'|), \end{aligned} \quad (3.7)$$

In Sec. V and Sec. VI we will use this Green function to study a gravitational field created by ultrarelativistic particles. For this purpose it is useful to have another representation of the Green function, where the Bessel function is replaced by its integral representation

$$\begin{aligned} J_\nu(z) &= \left(\frac{z}{2} \right)^\nu \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dt t^{-\nu-1} \exp\left(t - \frac{z^2}{4t}\right), \\ c &> 0. \end{aligned}$$

Then after the change of the integration variable

$$t = i\eta\tau, \quad \eta > 0, \quad (3.8)$$

the Green function can be written in the form

$$\begin{aligned} D_d(x, x') &= \frac{1}{2\pi} \int_0^\infty d\eta \tilde{D}(\eta) \\ &\times \int_{-\infty-ic}^{\infty-ic} \frac{d\tau}{(4\pi i\tau)^{d/2}} e^{i\tau\eta + i\frac{(x-x')^2}{4\tau}}. \end{aligned} \quad (3.9)$$

Note that the last integral contains the expression which is known as the heat kernel of the Laplace operator in a d -dimensional flat Euclidean space:

$$K_d(x, x'|\tau) = \frac{1}{(4\pi i\tau)^{d/2}} e^{i\frac{(x-x')^2}{4\tau}}. \quad (3.10)$$

The heat kernel obeys the equation

$$i\partial_\tau K_d(x, x'|\tau) + \Delta K_d(x, x'|\tau) = 0 \quad (3.11)$$

and the condition

$$\lim_{\tau \rightarrow 0} K_d(x, x'|\tau) = \delta^d(x - x'). \quad (3.12)$$

It describes the amplitude

$$K_d(x, x'|\tau) = \langle x | e^{i\tau\Delta} | x' \rangle. \quad (3.13)$$

In flat space, because of the symmetries of the system in question, both the Green function D_d and the potential ψ_d are the functions of a distance r between the points only:

$$D_d = D_d(r), \quad \psi_d = \psi_d(r), \quad r = \sqrt{(x - x')^2}. \quad (3.14)$$

The potential at the point x created by the massive particle located at the point x' is

$$\psi_d = -\kappa_d m D_d(r). \quad (3.15)$$

IV. GRAVITATIONAL POTENTIAL IN LINEARIZED GF GRAVITY THEORIES

A. General properties of GF theories

All GF theories of gravity are assumed to reproduce Einstein gravity in the low energy regime, i.e., at large scales. In particular it means that the functions $a(\xi)$ and $c(\xi)$ approach smoothly to 1 at small ξ :

$$a(\xi) = 1 + O(\xi), \quad c(\xi) = 1 + O(\xi). \quad (4.1)$$

Then we have the functions $\tilde{F}(\xi) = -\xi + O(\xi)$ and $\tilde{D}(\xi) = 1/\xi + O(1)$. This property and (3.7), (3.15) guarantee that in the limit of large distances one gets a universal asymptotic for the potential for all these GF theories:

$$\psi_d(r)|_{r \rightarrow \infty} = -\kappa_d m \frac{\Gamma(\frac{d}{2} - 1)}{4\pi^{d/2} r^{d-2}}. \quad (4.2)$$

Obviously, as it should be, it exactly reproduces the gravitational potential (2.5) in the higher-dimensional Einstein gravity theory.

The asymptotic of the potential at small distances is theory dependent. Our particular interest is in GF_N theories, where

$$a(-\xi) = \exp((\xi/\mu^2)^N) \quad (4.3)$$

and $N = 1$ or an even integer number. The parameter μ characterizes the scale where the nonlocality becomes important. One can show that for all GF_N gravities the potential ψ_d is finite at small r . For these theories the

asymptotic at $r\mu \rightarrow 0$ can be computed explicitly. Let us substitute (4.3) to (3.6), (3.7), (3.15) and change the integration variable $\eta = z^2/r^2$. Then we have

$$\psi_d(r) = -\frac{\kappa_d m}{(2\pi)^{d/2} r^{d-2}} \int_0^\infty dz z^{\frac{d}{2}-2} e^{-\frac{z^2}{r^2} \mu^{2N}} J_{\frac{d}{2}-1}(z). \quad (4.4)$$

One can see that in the limit when $r\mu \rightarrow 0$ only small arguments of the Bessel function contribute to the integral (4.4). Therefore, one can substitute there an expansion

$$J_{\frac{d}{2}-1}(z) = \frac{(\frac{z}{2})^{\frac{d}{2}-1}}{\Gamma(d/2)} \left[1 - \frac{z^2}{2d} + \frac{z^4}{8d(d+2)} + O(z^6) \right]. \quad (4.5)$$

Then taking the integrals in (4.4) one obtains

$$\psi_d(r) \sim -\kappa_d m \frac{\mu^{d-2} [\Gamma(\frac{d-2}{2N}) - \frac{r^2 \mu^2}{2d} \Gamma(\frac{d}{2N})]}{(4\pi)^{d/2} N \Gamma(\frac{d}{2})} + O(r^4 \mu^4). \quad (4.6)$$

One can see that the leading term is finite and proportional to $\kappa_d m \mu^{d-2}$. Moreover the next term in the expansion is proportional to r^2 that guarantees regularity of the metric at $r = 0$.

There are other interesting universal properties of the potentials in generic GF gravities. For example, because the distance r in the integral (3.7) does not enter the function \tilde{D} and due to the properties of the derivatives of Bessel functions it is clear that there is a universal relation

$$D_{d+2}(r) = -\frac{1}{2\pi r} \frac{\partial}{\partial r} D_d(r). \quad (4.7)$$

For the potentials, considered as functions of the radial distance r , this property leads to the relation

$$\frac{1}{\kappa_{d+2}} \psi_{d+2}(r) = -\frac{1}{\kappa_d} \frac{1}{2\pi r} \frac{\partial}{\partial r} \psi_d(r), \quad (4.8)$$

provided the mass parameter m is the same in d and $(d+2)$ dimensions.

B. Potential in GF₁ theory

The static potential ψ_d in the GF₁ theory satisfies the equation

$$\exp(-\Delta/\mu^2) \Delta \psi_d = \kappa_d m \delta^d(x - x'), \quad (4.9)$$

so that

$$\tilde{F}(\xi) = -\xi e^{\xi/\mu^2}, \quad \tilde{D}(\xi) = \frac{1}{\xi} e^{-\xi/\mu^2}. \quad (4.10)$$

Substitution of this expression into (3.7) and change of the integration variable $\eta = z^2/r^2$ leads to

$$D_d(r) = \frac{1}{(2\pi)^{d/2} r^{d-2}} \int_0^\infty dz z^{\frac{d}{2}-2} e^{-\frac{z^2}{r^2}} J_{\frac{d}{2}-1}(z) \\ = \frac{\gamma(\frac{d}{2}-1, \frac{r^2\mu^2}{4})}{4\pi^{d/2} r^{d-2}}, \quad (4.11)$$

where $\gamma(n, x)$ is the lower incomplete gamma function [42]. At large distance $r \gg \mu^{-1}$ this expression reproduces the static Green function of the d -dimensional Laplace operator

$$G_d(x, x') = \frac{\Gamma(\frac{d}{2}-1)}{4\pi^{d/2} r^{d-2}}. \quad (4.12)$$

For small distances $r \ll \mu^{-1}$ the Green function $D_d(r)$ is a regular function of r and is of the form

$$D_d(r) = \frac{2\mu^{d-2}}{(d-2)(4\pi)^{d/2}} \left(1 - \frac{d-2}{d} r^2 \mu^2\right) + \dots \quad (4.13)$$

The potential ψ_d is given by

$$\psi_d = -\kappa_d m D_d(x, x') = -\kappa_d m \frac{\gamma(\frac{d}{2}-1, \frac{r^2\mu^2}{4})}{4\pi^{d/2} r^{d-2}}. \quad (4.14)$$

In four-dimensional spacetime ($d = 3$) we reproduce the results of [8,23,26,33,43]

$$\psi_3 = -\kappa_3 m \frac{\text{erf}(r\mu/2)}{4\pi r}. \quad (4.15)$$

In the case of five-dimensional spacetime ($d = 4$) we obtain an even simpler expression:

$$\psi_4 = -\kappa_4 m \frac{1 - \exp(-r^2\mu^2/4)}{4\pi^2 r^2}. \quad (4.16)$$

The potentials ψ_d in an arbitrary number of dimensions qualitatively look alike. They are negative and finite at $r = 0$. At larger distances they become more shallow and at $r \gg \mu^{-1}$ quickly approach the Einstein asymptotic (4.2).

C. Potential in GF₂ theory

When $N = 2$ the operator \hat{F} corresponds to

$$a(\Delta) = \exp(\Delta^2/\mu^4) \quad (4.17)$$

and, hence,

$$\tilde{F}(\xi) = -\xi e^{\xi^2/\mu^4}, \quad \tilde{D}(\xi) = \frac{1}{\xi} e^{-\xi^2/\mu^4}. \quad (4.18)$$

Then the potential takes the form

$$\psi_d(r) = -\frac{\kappa_d m \mu^{d-2}}{d(d-2)2^{\frac{3d}{2}-2}\pi^{\frac{d-1}{2}}} \\ \times \left[\frac{d}{\Gamma(\frac{d}{4})} {}_1F_3\left(\frac{d}{4} - \frac{1}{2}; \frac{1}{2}, \frac{d}{4}, \frac{d}{4} + \frac{1}{2}; y^2\right) \right. \\ \left. - \frac{2(d-2)y}{\Gamma(\frac{d}{4} + \frac{1}{2})} {}_1F_3\left(\frac{d}{4}; \frac{3}{2}, \frac{d}{4} + 1, \frac{d}{4} + \frac{1}{2}; y^2\right) \right], \quad (4.19)$$

where

$$y = \frac{r^2\mu^2}{16}, \quad (4.20)$$

and ${}_pF_q$ is the generalized hypergeometric function (see, e.g., [42]).

Qualitatively the potentials for different parameters N and in different dimensions d look similar. Figures 1–2 show examples of the gravitational potential for $d = 3$ and $d = 4$ in two cases, $N = 1$ and $N = 2$.

D. Potential in GF_N theories

Similar results in terms of the generalized hypergeometric functions can be derived for an arbitrary GF_N theory. For all these theories the asymptotic at large distances is governed by the (4.2) and the asymptotic at small distances is given by (4.6). Let us present here only one more explicit example of the potential in GF₄ gravity:

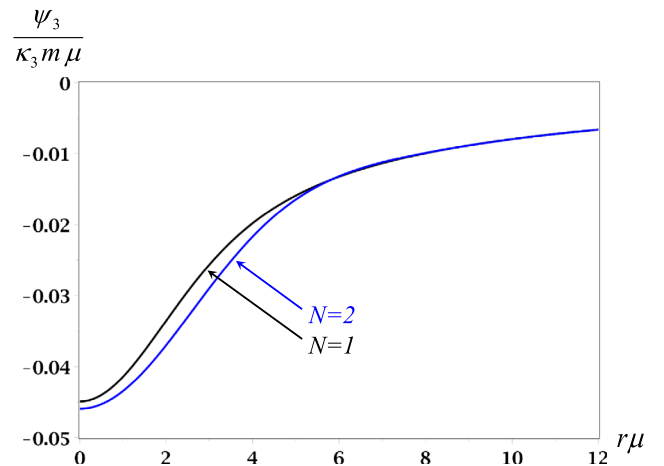


FIG. 1. This plot shows the function $\psi_3(r)$ for $N = 1$ and $N = 2$.

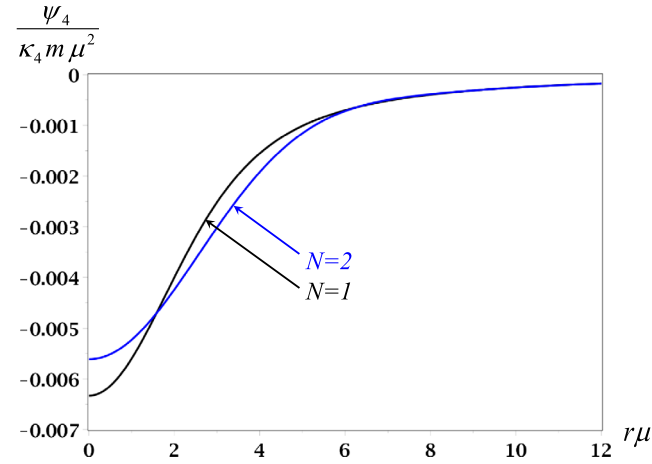


FIG. 2. This plot shows the function $\psi_4(r)$ for $N = 1$ and $N = 2$.

$$\begin{aligned} \psi_d(r) = & -A \left[d(d+2)(d+4)^2 \Gamma\left(\frac{d-2}{8}\right) B_1 \right. \\ & - 8y(d+2)(d+4)^2 \Gamma\left(\frac{d}{8}\right) B_2 \\ & + 32y^2(d+4)^2 \Gamma\left(\frac{d+2}{8}\right) B_3 \\ & \left. - \frac{2048}{3} y^3 \Gamma\left(\frac{d+12}{8}\right) B_4 \right], \end{aligned} \quad (4.21)$$

where

$$\begin{aligned} B_1 = & {}_1F_7\left(\frac{d-2}{8}; \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{d}{8}, \frac{d}{8} + 1, \frac{d}{8} + \frac{3}{4}, \frac{d}{8} + \frac{1}{2}, \frac{d}{8} + \frac{1}{4}; \frac{y^4}{256}\right), \\ B_2 = & {}_1F_7\left(\frac{d}{8}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{d}{8} + 1, \frac{d}{8} + \frac{3}{4}, \frac{d}{8} + \frac{1}{2}, \frac{d}{8} + \frac{1}{4}; \frac{y^4}{256}\right), \\ B_3 = & {}_1F_7\left(\frac{d+2}{8}; \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{d}{8} + 1, \frac{d}{8} + \frac{3}{4}, \frac{d}{8} + \frac{1}{2}, \frac{d}{8} + \frac{1}{4}; \frac{y^4}{256}\right), \\ B_4 = & {}_1F_7\left(\frac{d+4}{8}; \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{d}{8} + 1, \frac{d}{8} + \frac{3}{4}, \frac{d}{8} + \frac{1}{2}, \frac{d}{8} + \frac{1}{4}; \frac{y^4}{256}\right), \end{aligned}$$

and the coefficient

$$\begin{aligned} A = & \frac{\kappa_d m \mu^{d-2}}{2^{2d-\frac{1}{2}} \pi^{\frac{d-3}{2}} d(d-2)(d+2)(d+4)} \\ & \times \frac{1}{\Gamma\left(\frac{d-2}{8}\right) \Gamma\left(\frac{d}{8}\right) \Gamma\left(\frac{d+2}{8}\right) \Gamma\left(\frac{d+12}{8}\right)}. \end{aligned}$$

Expressions for the potentials become more complicated for higher N and we do not present them here.

V. PENROSE LIMIT

Let us demonstrate now that obtained static solutions of the GF gravity can be used to find the gravitational field of an ultrarelativistic object. In the standard 4D Einstein gravity such a limiting metric is known as an Aichelburg-Sexl metric [35]. This metric was generalized to the case of higher dimensions and for the spinning objects (called gyratons) in papers [44–46]. In this section we obtain a metric created by an ultrarelativistic object moving in D -dimensional spacetime (nonspinning gyraton metric) in GF theories of gravity. As we shall see a key role in this derivation is played by the heat kernel representation (3.9) of the Green function $D_d(x, x')$.

Consider the metric in the following form:

$$\begin{aligned} ds^2 = & -(1 + 2\varphi_d) dt^2 + (1 - 2\psi_d + 2\varphi_d)(dy^2 + d\zeta_\perp^2), \\ x = & (y, \zeta_\perp), \quad \zeta_\perp = (\zeta^2, \dots, \zeta^{d+1}). \end{aligned} \quad (5.1)$$

Let us boost this metric in the y -direction

$$\begin{aligned} \bar{t} = & \gamma(y - \beta t), \quad \bar{y} = \gamma(t - \beta y), \quad \gamma = (1 - \beta^2)^{-1/2}, \end{aligned} \quad (5.2)$$

and introduce null coordinates

$$u = \bar{t} - \bar{y}, \quad v = \bar{t} + \bar{y}. \quad (5.3)$$

In the relativistic limit, when the boost velocity is close to the speed of light, i.e., $\beta \rightarrow 1$, the boost factor $\gamma \rightarrow \infty$. In this limit $dt \sim \gamma du$ and $dy \sim -\gamma du$. Then the line element (5.1) becomes

$$ds^2 = -dudv + d\zeta_\perp^2 + \Phi_d du^2, \quad (5.4)$$

where

$$\Phi_d = -2 \lim_{\gamma \rightarrow \infty} (\gamma^2 \psi_d). \quad (5.5)$$

For a point particle of mass m the Penrose limit corresponds to ultrarelativistic limit $\gamma \rightarrow \infty$ with the condition that an energy $E = \gamma m$ of the particle is kept fixed.

The gravitational potential ψ_d [see (3.15), (3.9)] can be presented in the form

$$\begin{aligned} \psi_d = & -\kappa_d m D_d(r) \\ = & -\frac{\kappa_d m}{2\pi} \int_0^\infty d\eta \tilde{D}(\eta) \int_{-\infty}^\infty \frac{d\tau}{(4\pi i \tau)^{d/2}} e^{i\eta\tau} e^{i\frac{\tau^2}{4\tau}}. \end{aligned} \quad (5.6)$$

One can see that the boost affects only the last exponent in this integral representation.

Taking into account that after the boost

$$\begin{aligned} y &\rightarrow -\gamma u, & r^2 &\rightarrow \gamma^2(u - u')^2 + \rho^2, \\ \rho^2 &= (\zeta_\perp - \zeta'_\perp)^2, \end{aligned} \quad (5.7)$$

and using the delta-function representation

$$\lim_{\gamma \rightarrow \infty} \frac{\gamma}{\sqrt{4\pi i\tau}} e^{i\frac{\gamma^2 u^2}{4\tau}} = \delta(u), \quad (5.8)$$

we obtain

$$\Phi_d = \mathcal{F}_d(\rho)\delta(u - u'). \quad (5.9)$$

Here

$$\mathcal{F}_d(\rho) = \frac{\kappa_d E}{\pi} \int_0^\infty d\eta \tilde{D}(\eta) \int_{-\infty}^\infty \frac{d\tau}{(4\pi i\tau)^{(d-1)/2}} e^{i\eta\tau} e^{i\frac{\rho^2}{4\tau}}. \quad (5.10)$$

Comparison of this integral expression with (5.6) leads to the observation that the function $\mathcal{F}_d(\rho)$ is proportional to the gravitational potential defined in space of one dimension less, i.e., in the space orthogonal to the particle motion:

$$\mathcal{F}_d(\rho) = 2\kappa_d E D_{d-1}(\rho) = -2 \frac{\kappa_d E}{\kappa_{d-1} m} \psi_{d-1}(\rho). \quad (5.11)$$

This property is valid for arbitrary GF_N theories of gravity.

Using the property (4.8), which is also valid for a generic GF_N gravity, we derive a relation

$$\partial_\rho \mathcal{F}_d(\rho) = 4\pi \tilde{E} \rho \psi_{d+1}(\rho), \quad \tilde{E} = \frac{\kappa_d E}{\kappa_{d+1} m}. \quad (5.12)$$

This relation will be useful for the study of gravitational effects in collisions of ultrarelativistic particles (gyratons [45–47]) in the next sections.

VI. APPARENT HORIZON FORMATION FOR HEAD-ON COLLISION OF THE ULTRARELATIVISTIC PARTICLES

Our next goal is to use the obtained results to study head-on collision of the ultrarelativistic particles in the GF theories of gravity. We use an approach developed by Penrose [36] and D'Eath and Payne [48–50] and approximate the colliding particles by *gyratons*. A schematic picture of such a process is shown in Fig. 3. It shows two-particle motion in the center-of-mass frame. Each of the particles moves with the velocity of light. Particle 1 moves from the left to the right along the y -direction, while particle 2 moves in the opposite direction. The null lines, representing their trajectories, belong to $u = 0$ and $v = 0$ null planes, correspondingly. The gravitational field of

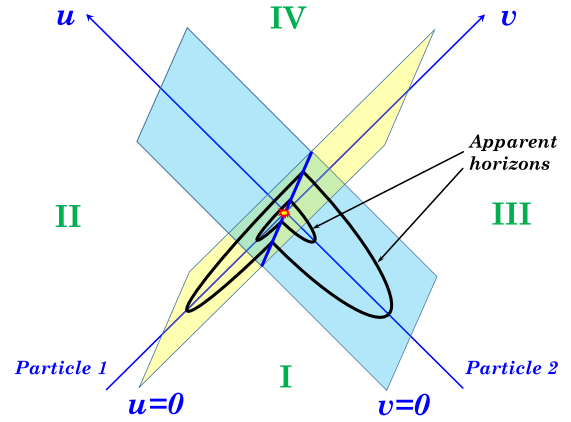


FIG. 3. Head-on collision of two ultrarelativistic particles.

these particles is localized on the plane $u = 0$ (for particle 1) and $v = 0$ (for particle 2). The intersection of two null planes is the $(d - 1)$ -dimensional transverse plane. In the regions *I*, *II*, and *III*, outside the $u = 0$ and $v = 0$ null planes the metric is flat and null rays in these domains are nothing but null straight lines. However, when such a ray passes either through $u = 0$ or $v = 0$ planes, it is scattered by the gravitational field of the corresponding particle.

Our purpose is to study formation of the apparent horizon in such a process. Let us remember that a *trapped surface* is a compact spacelike $(d - 1)$ -dimensional surface which has the property that both of the null congruences orthogonal to it are not expanding. We focus on the outgoing congruence. One calls a trapped surface a *marginally trapped surface* if the outer normals to it have zero convergence [51]. In a spherically symmetric spacetime one may consider spherical slices and define an apparent horizon as a d -dimensional surface which on each of the slices coincides with the marginally trapped surface.

The problem of ultrarelativistic particle collision in general relativity was discussed recently in connection with possible mini-black-hole creation in colliders [37–39,47]. Eardley and Giddings [37] demonstrated that a problem of existence of the apparent horizon can be reduced to a special boundary-value problem for an elliptic (Poisson) equation in a flat spacetime. Generalizations of these results to the collision of shock waves on AdS background were also considered in [52,53]. The problem is greatly simplified for the case of the head-on collision and can be solved analytically in any number of spacetime dimensions. In the present paper we follow their approach. Let us write the metric (5.4) in the form

$$ds^2 = -d\bar{u}d\bar{v} + d\bar{\zeta}_\perp^2 + \Phi_d d\bar{u}^2, \quad (6.1)$$

$$\Phi = \mathcal{F}_d(\bar{\rho})\delta(\bar{u}), \quad \bar{\rho} = \sqrt{\bar{\zeta}_\perp^2}. \quad (6.2)$$

It is possible to show that geodesics and their tangent vectors are not continuous in these coordinates

(see e.g., [37]). One can change the coordinates so that both geodesics and their tangent vectors will be continuous in the new coordinates. The new coordinates in the domain II are defined as follows:

$$\begin{aligned}\bar{u} &= u, & \bar{\zeta}_i &= \zeta_i + \frac{u}{2} \nabla_i \Phi \vartheta(u), \\ \bar{v} &= v + \Phi \vartheta(u) + \frac{1}{4} u \vartheta(u) (\nabla \Phi)^2.\end{aligned}\quad (6.3)$$

A similar transformation (with a change $u \leftrightarrow v$) should be made in the domain III .

The metric (6.1) in the new coordinates takes the form

$$\begin{aligned}ds^2 &= -dudv + [H_{ik}^{(1)} H_{jk}^{(1)} + H_{ik}^{(2)} H_{jk}^{(2)} - \delta_{ij}] d\zeta_i d\zeta_k, \\ H_{ij}^{(1)} &= \delta_{ij} + \frac{1}{2} \nabla_i \nabla_j \Phi u \vartheta(u), \\ H_{ij}^{(2)} &= \delta_{ij} + \frac{1}{2} \nabla_i \nabla_j \Phi v \vartheta(v).\end{aligned}\quad (6.4)$$

We consider a special marginally trapped surface \mathcal{S} which consists of two parts $\mathcal{S}_u \mathcal{S}_v$. In coordinates (u, v, ζ_i) a position of \mathcal{S}_u and \mathcal{S}_v on two incoming null planes is described by equations

$$\{v = -\Psi(\rho), u = 0\} \quad \text{and} \quad \{u = -\Psi(\rho), v = 0\}, \quad (6.5)$$

respectively. These two $(d-1)$ -dimensional surfaces intersect at $(d-2)$ -dimensional boundary \mathcal{C} , located at $u = v = 0$. The function Ψ is positive inside the boundary \mathcal{C} and vanishes at \mathcal{C} . The internal (induced) geometry of \mathcal{S}_u and \mathcal{S}_v are the geometry of a half of a $(d-1)$ -dimensional round sphere, their intersection \mathcal{C} being a round $(d-2)$ -dimensional sphere. For the head-on collision the function $\Psi(\rho)$, which enters both equations in (6.5), is the same. In [37] it was shown that the outer null normals have zero convergence in \mathcal{S}_u and \mathcal{S}_v if

$$\nabla^2(\Psi - \mathcal{F}_d) = 0. \quad (6.6)$$

A condition that both normals (in \mathcal{S}_u and \mathcal{S}_v) coincide at their boundary \mathcal{C} implies

$$(\nabla \Psi)^2 = 4. \quad (6.7)$$

Denote $\chi = \Psi - \mathcal{F}_d$ and by ρ_C the radius ρ at the boundary. Then

$$\nabla^2 \chi = 0, \quad \chi_C = -\mathcal{F}_d(\rho_C). \quad (6.8)$$

Hence one can put $\chi = -\mathcal{F}_d(\rho_C)$ inside \mathcal{C} so that

$$\Psi = \mathcal{F}_d(\rho) - \mathcal{F}_d(\rho_C). \quad (6.9)$$

The condition (6.7) takes the form

$$(\nabla \mathcal{F}_d)^2|_C = 4. \quad (6.10)$$

Using (5.12) one gets

$$2\pi\kappa_d E \rho D_{d+1}(\rho) = 1. \quad (6.11)$$

In terms of a dimensionless coordinate $x = \mu\rho$, dimensionless energy $\tilde{E} = 2\pi\mu^{d-2}\kappa_d E$, and a dimensionless profile function

$$P_d(x) \equiv x D_{d+1}(x/\mu) / \mu^{d-1}, \quad (6.12)$$

this condition reads

$$P_d(x) = \frac{1}{\tilde{E}}. \quad (6.13)$$

All functions $P_d(\rho)$ look similar (see Figs. 4 and 5). They vanish at $x = 0$ and then grow, reach maximum, and then decrease to a universal asymptotic that does not depend on the parameter N , though it depends on d . The plots in Figs. 6 and 7 show solutions of Eq. (6.13). The apparent horizon exists for the energy obeying the condition $\tilde{E} \geq \tilde{E}_{\text{critical}}$. In this energy domain it has at least two branches, inner and outer. At $\tilde{E} = \tilde{E}_{\text{critical}}$ they meet and the apparent horizon disappears [54]. This behavior resembles qualitatively that of the colliding relativistic extended sources [55]. This resemblance is not accidental. One can rearrange Laplace operators in (3.2) and move $a(\Delta)^{-1}$ to the right-hand side of the equation. Then it can be identically rewritten as

$$\Delta \psi_d = j, \quad j = \kappa_d m a(\Delta)^{-1} \delta^d(x - x'). \quad (6.14)$$

When acting on the localized source, the operator $a(\Delta)^{-1}$ delocalizes it and makes j become effectively an extended current for the traditional Laplace equation (6.14). In this sense the analogy of effects in the ghost-free gravities and for the colliding extended sources [55] becomes evident.

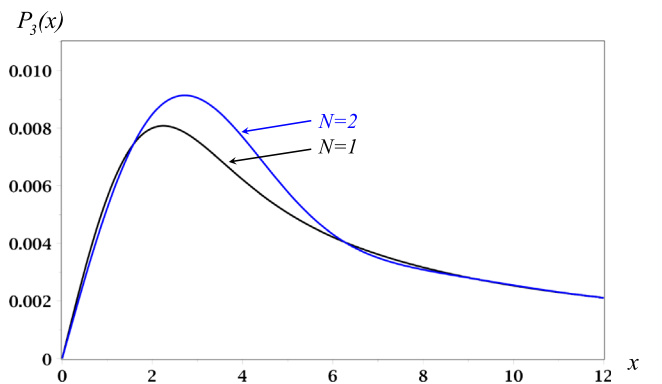


FIG. 4. The plot shows function $P_3(x)$ for $N = 1, 2$.

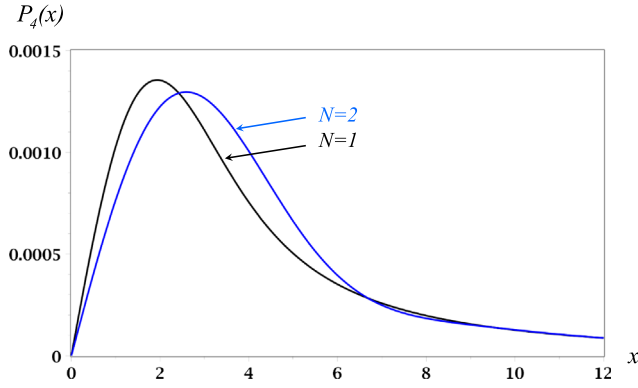


FIG. 5. The plot shows function $P_4(x)$ for $N = 1, 2$.

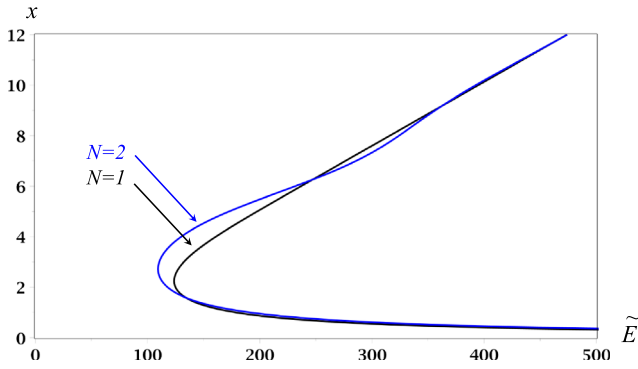


FIG. 6. The plot shows the radius $x = \mu\rho$ of outer and inner apparent horizons as a function of the energy \tilde{E} for $d = 3$ and $N = 1, 2$.

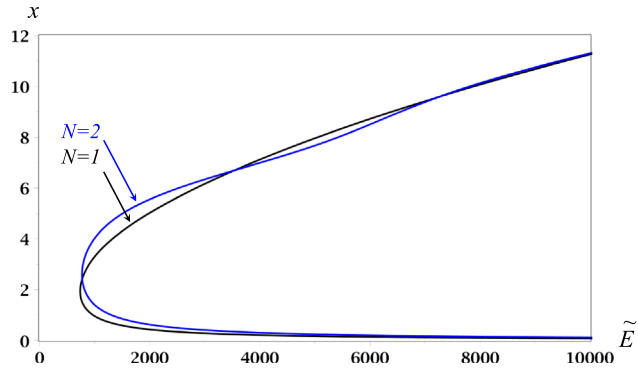


FIG. 7. The plot shows the radius $x = \mu\rho$ of outer and inner apparent horizons as a function of the energy \tilde{E} for $d = 4$ and $N = 1, 2$.

VII. SUMMARY AND DISCUSSION

In this paper we discussed an application of the linearized equations of the ghost-free theory of gravity to three connected problems. First, we calculated the gravitational potential of a point mass in the Newtonian limit and showed that GF modification of gravity works as a regularizer. Namely, this potential is regular at the origin. This property

is valid for GF_1 and GF_{2n} theories in any number of spatial dimensions $d \geq 3$. This is a generalization of the earlier obtained result for GF_1 for $d = 3$ [8,26] and for $d > 3$ [33]. The second main result of the paper is calculation of the gravitational field of an ultrarelativistic particle in the GF theories. The obtained metrics are generalizations of the famous four-dimensional Aichelburg-Sexl metric [35] of general relativity. Again, the obtained metrics are solutions of the equations of the GF_N gravity equations ($N = 1$ and $N = 2n$) in a spacetime with an arbitrary number of dimensions $d + 1$. And finally, we used these results to study an apparent horizon formation in the head-on collision of two ultrarelativistic particles. Our main conclusion is that in such a process there exists a mass gap for the mini-black-hole formation. If μ is the characteristic mass scale of the corresponding ghost-free theory, then in order for a mini black hole to be formed in the collision, the center-of-mass energy E should be of the order of or larger than $(G^{(d)}\mu^{d-2})^{-1}$. Another important feature of the process is that when the apparent horizon is formed, it has two branches: outer and inner marginally trapped surfaces. Both of them have the geometry of the sphere. When the center-of-mass energy increases, the inner part becomes closer to the point until it reaches the scale $\tilde{\lambda}$, where the model we used breaks down.

This result is again valid for any GF_N theory ($N = 1$ and $N = 2n$) in any number of dimensions. It can be considered as some indication that for such theories the inner singularity of a black hole might be absent and there exists a closed apparent horizon. Such a model was proposed in [56] and discussed later in many publications. It should be emphasized that most of the results, related to the study of the models with closed apparent horizons, beyond a linear approximation, were obtained without using concrete dynamical equations. In this sense they are phenomenological. It is a real challenge to obtain solutions for a dynamical collapse in the modifications of the Einstein theory which are UV complete. In particular, in order to arrive at a definite conclusion concerning the structure of a black hole interior in the GF gravity one needs to perform analysis in the complete version of such a theory, which includes nonlinear effects.

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APPENDIX A: LINEARIZED EQUATIONS OF THE HIGHER-DERIVATIVE MODIFICATION OF THE GRAVITATIONAL EQUATIONS IN HIGHER DIMENSIONS

In order to obtain linearized equations of a theory of gravity with higher derivatives in higher dimensions

one can follow a similar derivation in four dimensions presented in the papers [8,12]. In this appendix we collected the corresponding formulas for further reference.

The main steps of this derivation are the following. One considers first a covariant action which besides the Einstein term contains also a part S_q which is quadratic in curvature. The latter may contain an arbitrary number of covariant derivatives acting on each of the curvature tensors. One can always move the derivatives acting on the first Riemann tensor to the position, when it acts on the other one. This can be achieved by using integration by parts. The number of derivatives may even be infinite, so that a theory is nonlocal. Since each of the Riemann curvature tensors has four indices, the maximal total number of derivatives with “free” indices is eight. All other derivatives can be combined in functions of the covariant box operator. In order to achieve this it might be required to commute the derivatives. But this operation produces terms which are of the third order in the curvature so that they should be neglected in the adopted approximation. Using symmetry properties of the curvature tensor, Bianchi identities and commutativity of the covariant derivatives in the adopted approximation one finally obtains the following expression for S_q [8,12]:

$$S = \frac{1}{2\kappa_d} \int dx \sqrt{-g} [R + RF_1(\square)R + R_{\mu\nu}F_2(\square)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}F_3(\square)R^{\mu\nu\lambda\sigma}].$$

Here $\kappa_d = 8\pi G^{(D)}$ and $G^{(D)}$ is the gravitational coupling constant in D -dimensional spacetime. In four dimensions the value of this constant is fixed by the requirement that the Poisson equation for the gravitational potential in the Newtonian limit have a standard form. There is an ambiguity in the normalization of $G^{(D)}$ in higher dimensions. We fix it by requiring the Einstein-Hilbert action to have the same form in all dimensions.

This general form of the quadratic in curvature action can be further simplified using the following observation [2,57]: the “Gauss-Bonnet structures” of the form ($k \geq 1$)

$$\begin{aligned} *R^{\alpha\beta\gamma\sigma}\square^k R_{\alpha\beta\gamma\sigma} &= R^{\alpha\beta\gamma\sigma}\square^k R_{\alpha\beta\gamma\sigma} - 4R^{\alpha\beta}\square^k R_{\alpha\beta} \\ &+ R\square^k R = O(R^3) + \text{div.} \end{aligned} \quad (\text{A1})$$

in arbitrary dimensions are all of the third and higher order in curvature plus total divergence terms. As a

result, the general higher derivative action can be written in the form which contains only two arbitrary functions of the box operator [12].

To obtain the linearized equation we write the action in the form

$$\begin{aligned} S &= \frac{1}{2\kappa_d} (S_0 + S_1 + S_2 + S_3), \\ S_0 &= \int dx \sqrt{-g} R, \\ S_1 &= \int dx \sqrt{-g} R F_1(\square) R, \\ S_2 &= \int dx \sqrt{-g} R_{\mu\nu} F_2(\square) R^{\mu\nu}, \\ S_3 &= \int dx \sqrt{-g} R_{\mu\nu\lambda\sigma} F_3(\square) R^{\mu\nu\lambda\sigma}. \end{aligned}$$

We use the following expressions for the variations of the objects that enter the above action and keep only the terms that are quadratic in perturbations

$$\begin{aligned} S_0 &= - \int dx \left(-\frac{1}{2} h^{\mu\nu} \square h_{\mu\nu} + h^{\mu\nu} \partial_\mu \partial_\alpha h^\alpha{}_\nu \right. \\ &\quad \left. - h^{\mu\nu} \partial_\mu \partial_\nu h + \frac{1}{2} h \square h \right), \\ S_1 &= \int dx (h^{\mu\nu} F_1(\square) \partial_\mu \partial_\nu \partial_\alpha \partial_\beta h^{\alpha\beta} \\ &\quad - 2h^{\mu\nu} \square F_1(\square) \partial_\mu \partial_\nu h + h \square^2 F_1(\square) h), \\ S_2 &= \frac{1}{4} \int dx (2h^{\mu\nu} F_2(\square) \partial_\mu \partial_\nu \partial_\alpha \partial_\beta h^{\alpha\beta} \\ &\quad - 2h^{\mu\nu} \square F_2(\square) \partial_\mu \partial_\alpha h^\alpha{}_\nu - 2h^{\mu\nu} \square F_2(\square) \partial_\mu \partial_\nu h \\ &\quad + h^{\mu\nu} \square^2 F_2(\square) h_{\mu\nu} + h \square^2 F_2(\square) h), \\ S_3 &= \int dx (h^{\mu\nu} F_3(\square) \partial_\mu \partial_\nu \partial_\alpha \partial_\beta h^{\alpha\beta} \\ &\quad + h^{\mu\nu} \square^2 F_3(\square) h^{\mu\nu} - 2h^{\mu\nu} \square F_3(\square) \partial_\mu \partial_\alpha h^\alpha{}_\nu). \end{aligned}$$

Let us write the total linearized action S in the form

$$\begin{aligned} S &= \frac{1}{2\kappa_d} \int dx \left(\frac{1}{2} h^{\mu\nu} a \square h_{\mu\nu} + h^{\mu\nu} b \partial_\mu \partial_\alpha h^\alpha{}_\nu \right. \\ &\quad \left. + h^{\mu\nu} c \partial_\mu \partial_\nu h + \frac{1}{2} h d \square h \right. \\ &\quad \left. + \frac{1}{2} h^{\mu\nu} \frac{f}{\square} \partial_\mu \partial_\nu \partial_\alpha \partial_\beta h^{\alpha\beta} \right). \end{aligned} \quad (\text{A2})$$

Then we have

$$\begin{aligned}
a &= 1 + \frac{1}{2}F_2\Box + 2F_3\Box, \\
b &= -1 - \frac{1}{2}F_2\Box - 2F_3\Box, \\
c &= 1 - 2F_1\Box - \frac{1}{2}F_2\Box, \\
d &= -1 + 2F_1\Box + \frac{1}{2}F_2\Box, \\
f &= 2F_1\Box + F_2\Box + 2F_3\Box.
\end{aligned} \tag{A3}$$

It is easy to see that the form factors a , b , c , d , f satisfy the identities

$$a + b = 0, \quad c + d = 0, \quad b + c + f = 0. \tag{A4}$$

The equations of motion, obtained from (A2), are

$$\begin{aligned}
&a(\Box)\Box h_{\mu\nu} + b(\Box)\partial_\sigma(\partial_\nu h_\mu^\sigma + \partial_\mu h_\nu^\sigma) \\
&+ c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) + \eta_{\mu\nu}d(\Box)\Box h \\
&+ f(\Box)\Box^{-1}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma h^{\rho\sigma} = -2\kappa_d\tau_{\mu\nu}.
\end{aligned} \tag{A5}$$

Here

$$\tau^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{Matter}}}{\delta g_{\mu\nu}}. \tag{A6}$$

Let us remember that $\eta_{\mu\nu}$ is a metric in D -dimensional Minkowski spacetime, and partial derivatives and the \Box operator are written in Cartesian coordinates in this space. Let us emphasize that the number of independent arbitrary functions of the box operator, as well as the form of the equations, is the same as in the four-dimensional case. However, the dimensional gravitational coupling constant $G^{(D)}$ depends on the number of dimensions. We also show in Sec. II that the form of the equations for static gravitational potentials, which contain contractions of the form $\eta_{\mu\nu}h^{\mu\nu}$, would be explicitly dependent on D .

APPENDIX B: GRAVITATIONAL POTENTIAL IN MOMENTUM SPACE

Let us write the gravitational potential ψ_d in terms of the modes in momentum space

$$\psi_d(x) = \int \frac{d^d\mathbf{k}}{(2\pi)^d} e^{i\mathbf{k}x} \bar{\psi}(\mathbf{k}). \tag{B1}$$

Here $\mathbf{k} = k_i$ is the d -dimensional vector of momentum. Similarly we have

$$D_d(x, x') = \int \frac{d^d\mathbf{k}}{(2\pi)^d} e^{i\mathbf{k}(x-x')} \bar{D}(\mathbf{k}). \tag{B2}$$

From (3.15) and (3.6) one can derive

$$\begin{aligned}
\bar{\psi}(\mathbf{k}) &= -\kappa_d m \bar{D}(\mathbf{k}), \\
\bar{D}(\mathbf{k}) &= \frac{1}{k^2 a(-k^2)} = \tilde{D}(k^2), \quad k = |\mathbf{k}|.
\end{aligned}$$

Using the spherical symmetry of the system we get

$$d^d\mathbf{k} = dk d\theta k^{d-1} \sin^{d-2}\theta A_{d-2},$$

$$A_{d-2} = 2 \frac{\pi^{(d-1)/2}}{\Gamma(\frac{d-1}{2})},$$

where A_{d-2} is the area of a unit sphere S^{d-2} . In spherical coordinates

$$k_i(x^i - x'^i) = kr \cos(\theta), \quad r = |x - x'|. \tag{B3}$$

Then the Green function reads

$$\begin{aligned}
D_d(x, x') &= 2 \frac{\pi^{(d-1)/2}}{\Gamma(\frac{d-1}{2})} \int_0^\infty \frac{dk}{(2\pi)^d} \frac{k^{d-3}}{a(-k^2)} \\
&\times \int_0^\pi d\theta \sin^{d-2}\theta e^{ikr \cos(\theta)}.
\end{aligned} \tag{B4}$$

Integration over θ gives the expression for the Green function $D_d(x, x')$ in terms of an integral from the Bessel function

$$D_d(x, x') = \frac{1}{2\pi} \int_0^\infty \frac{dk}{ka(-k^2)} \left(\frac{k}{2\pi r}\right)^{\frac{d-1}{2}} J_{\frac{d-1}{2}}(kr).$$

Both the potential ψ_d and the Green function $D_d(x, x')$ depend only on the distance r between points. Change of the integration variables leads to the following equivalent forms ($z = kr$):

$$D_d(r) = \frac{1}{(2\pi)^{d/2} r^{d-2}} \int_0^\infty dz \frac{z^{\frac{d-4}{2}}}{a(-z^2/r^2)} J_{\frac{d-1}{2}}(z), \tag{B5}$$

and ($\eta = z^2/r^2$)

$$D_d(r) = \frac{1}{4\pi} \int_0^\infty d\eta \tilde{D}(\eta) \left(\frac{\sqrt{\eta}}{2\pi r}\right)^{\frac{d-1}{2}} J_{\frac{d-1}{2}}(\sqrt{\eta}r). \tag{B6}$$

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