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Probing the constancy of the speed of light with future galaxy survey: The case of SKA and Euclid

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In [V. Salzano, M. P. Dąbrowski, and R. Lazkoz, Phys. Rev. Lett. 114, 101304 (2015)] a new method to measure the speed of light through baryon acoustic oscillations was introduced. Here, we describe in much more detail the theoretical basis of that method and its implementation, and we give some newly updated results about its application to forecast data. In particular, we show that SKA will be able to detect a 1% variation (if any) in the speed of light at the 3σ level. Smaller signals will be hardly detectable by already-planned future galaxy surveys, but we give indications of what sensitivity requirements a survey should fulfill in order to be successful.

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I. INTRODUCTION

The speed of light is one of the most fundamental constants of nature playing a significant role in basic physical laws such as the Maxwell equations, special and general relativity equations, atomic and particle physics equations, and many others. In other words, it influences the vast areas of physics which deal with both microscopic and astronomical scales. Because of its crucial meaning, the speed of light was officially announced in 1983 to take a fixed value by the Bureau International des Poids et Mesures [1]. Many measurements of the speed of light have been performed, beginning with the famous, though inaccurate, measurement by Rømer and Huygens in 1675, continuing with Bradley, Fizeau, and Michaelson, and ending in a laser interferometric measurement by [2]. However, in view of the contemporary theories of physics such as multidimensional theories of gravity within the framework of superstring and brane theories (see, e.g., [3]), some physicists argue that the values of physical constants like the gravitational constant G, the fine-structure constant α, the electron-to-proton mass ratio m_e/m_p, and the speed of light c may become dynamical (represented by some scalar fields, for example), and so they can evolve in time and space. (For a review of such a variation, see [4].)

The option which probably has the strongest impact on the whole of physics is the variability of c. Such an idea even dates back to Einstein himself [5], but it has attracted more interest recently due to the fact that it can provide an alternative solution for the classic problems of noninflationary cosmology, such as the horizon and flatness problems. Having such advantages, the theories of a varying speed of light—in short, VSL theories—are often regarded as controversial because they are usually not formulated in a proper framework of dynamical scalar field theory [6], allowing a special choice of the frame in which the speed of light is constant [7,8], though there are some attempts at a more proper formulation [9]. In particular, a comparison of those theories with experimental data seems to still be missing [10]. In this paper, following our brief previous study [11], we will try to put constraints on some varying speed of light theories by using future galaxy surveys such as the Square Kilometer Array (SKA) [12], Euclid [13,14], and the Wide-Field Infrared Survey Telescope WFIRST-2.4 [15], showing how the huge number of galaxies which will be collected can be used as a probe for the constancy of c.

One of the main signals that can be detected by a galaxy survey is related to the baryon acoustic oscillations (BAOs) [16]. Theoretically well established since 1970 [17], they have only relatively recently been recognized as one of the most useful and promising probes for studying dark energy and cosmology [18,19]. At the present stage, even though they are very helpful, they are still far from being of optimal use; this is the reason why they are among the main objectives of many important ongoing and future Earth-based and spatial surveys, such as SKA, Euclid, WFIRST-2.4, the Baryon Oscillation Spectroscopic Survey (BOSS) [20], the Extended BOSS Survey (eBOSS) [21], the Dark Energy Spectroscopic Instrument (DESI) [22], and the Hobby-Eberly Telescope Dark Energy Experiment (HETDEX) [23]. Basically, BAO observational outputs are measurements of the sound horizon at late times, as it is imprinted in the clustering of a large scale structure. It is generally considered to be a standard ruler, i.e., an "object" whose size is constant in time and which can be used as a

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stick to calibrate/measure cosmological distances (for problems and alternatives, see [24]). Its magnitude can be exactly calculated from the theory and it is approximately equal to 150 Mpc in physical units. This is just the value which can be measured, with the best precision possible, from the cosmic microwave background (CMB) observations. The latest data release from Planck [25] gives us a value of $r_s(z_{\rm rec}) = 144.81 \pm 0.24$ Mpc for the baseline model, exhibiting a very weak cosmological model dependence. Given the strong correlation between photons (measured by CMB) and baryons, such a distance should be imprinted in the large scale structure, and so it is. Measuring the distribution of galaxies in space, as well as their redshifts, and analyzing their correlation function, it is possible to observe the typical correlation length which, as expressed in comoving units, corresponds exactly to the sound horizon. Being more precise, it corresponds to the sound horizon not at recombination, but rather at a later epoch defined as "dragging redshift" [18], at $z_{\rm drag} \approx 1060$. Of course, galaxy distribution is three dimensional, and the sound horizon should be measured in three different directions: two are on the projected sky and one is in the radial direction. The former are said to be the tangential modes, the latter the radial. They can be defined as

$$y_t(z) = \frac{D_A(z)}{r_s(z_{\rm rec})} \quad \text{and} \quad y_r(z) = \frac{c}{H(z) r_s(z_{\rm rec})},$$

where c is the speed of light, z is the cosmological redshift, D_A is the angular diameter distance, H is the Hubble function, and $r_s(z_{\rm dec})$ is the sound horizon, evaluated at recombination (or dragging epoch).

At this point, we do not yet have such a good signal in order to have accurate measurements for y_t and y_r separately. We have good measurements of quantities combining D_A and H as, for example, the average distance

$$D_V = \left[(1+z)^2 cz \frac{D_A^2}{H}\right]^{1/3}, \tag{1}$$

or the Alcock-Paczyński distortion parameter

$$F = (1+z) D_A \frac{H}{c}. \tag{2}$$

There are some trials to obtaining independent information for D_A and H [26,27], but they are not yet fully competitive. With future surveys, with a larger number of galaxies available, this will be possible eventually, and it will reveal itself as a necessary requirement for our method to be applied.

The same galaxies used for detecting BAOs—or, at least, a fraction of them—can also be used as *cosmic chronometers*. The seminal idea of cosmic chronometers was first described in [28] and then progressively extended and used for cosmological analysis in [29,30]. It is based on the *differential age method*: the key is to find a "cosmological clock" able to return the variation of Universe age with redshift. If one has this clock, then one simply has to measure the age difference Δt between two redshifts separated by Δz, and calculate from these the derivative $dz/dt \approx \Delta z/\Delta t$. This latter quantity then would be directly related to the Hubble function, defined as

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}.$$

If such a method were possible, we would have a measurement of the Hubble function free from any assumptions about the nature of the metric, which normally affects, for example, the definition of cosmological distances. It was proposed in [28] that the role of such clocks could be played by passively evolving early-type galaxies (ETGs). How to use them has also been shown, and what order of constraints should be expected from them. Since then, a lot of work and improvements have been made: now we have better stellar population models; we have a much larger number of observations (see, e.g., [29], in which $\approx 10^4$ galaxies were analyzed) and very deep in redshift (up to $z \sim 2$); as well as more precise tools to calibrate the clocks (the 4000 Å break in ETG spectra). And this scenario can still be improved using future galaxy surveys in the optical, as Euclid and WFIRST-2.4, which should observe at least ten times more galaxies (and ETG, eventually) than now.

The paper is organized as follows: in Sec. II we will describe the theoretical background underlying the proposed method; in Sec. III we will describe in detail all of the steps involved in the building of our method; and, finally, in Sec. IV we will apply our method to some particular cases and then discuss the results.

II. THEORETICAL BASIS

The possibility of constraining VSL theories from a large scale structure is strictly related to the definition of one of the quantities that can be measured in a galaxy (BAO) survey, i.e., the angular diameter distance, $D_A(z)$. This distance is defined as

$$D_A(z) = \frac{1}{1+z}\int_0^z \frac{c_0}{H(z')} dz', \tag{3}$$

where c_0 is the speed of light. From now on, we will assume the convention for defining $c_0 \equiv 299792.458\ \mathrm{km\,s^{-1}}$ as the value of the speed of light [1]. This is, of course, assumed to be constant in a standard scenario (and in most of physics these days), while, in a VSL theory, it is equal to the speed of light evaluated *here and now*.

A very well-known, and somewhat counterintuitive, property of $D_A(z)$ is that it rises up to a maximum at some redshift, which we will call z_M, and then starts to decline

[31]. Starting from its definition, an equivalent way to set the problem is to say that the angular size of a given escaping object diminishes while it is going farther from us up to some point, where it reaches a minimum, before it starts rising again. Both pictures basically tell us that early times objects (or, at least, older than some redshift z_M) look closer than late times ones. The explanation behind this peculiar behavior is a mix of geometric facts (curvature, non-Euclidean space) and the dynamical history of our Universe [32].

The exact location of the maximum, i.e., the redshift z_M, depends on the cosmological model, which enters the definition of $H(z)$. In order to have a general idea of the range possibly covered by z_M, and compatible with the most updated set of cosmological probes available, we have considered the Chevallier-Polarski-Linder (CPL) [33] $w + w_a$ plikHM_TTTEEE_lowTEB_BAO_post_lensing best fit from the Planck 2015 release [34]. We have considered a total of 10^4 cosmological models, derived from varying the cosmological parameters consistently with the 1σ confidence intervals defined for the previous parametrization. Of course, $w + w_a$ is only one of the many dark energy models available, but it is somewhat used as a "reference" model in the literature. Moreover, the large errors on its parameters —in particular, on the dynamical dark energy equation of state parameter w_a—make us confident of having explored a very large set of cosmological scenarios compatible with observational data, thus making our estimation for the range of z_M highly conservative. We have checked to see that z_M lies in the range [1.4, 1.75] for more than 99% of 10^4 random cosmological models chosen as described above. This is a quite narrow redshift range and, very interestingly, it will be covered by many surveys in the future (SKA, Euclid, and WFIRST-2.4), so we will have good quality data in such a range.

Given the dependence of z_M on the cosmological model, one could think about using it as a further tool to constrain, for example, dark energy properties, in addition to the most used probes in cosmology. Unfortunately, the large degeneracy between the cosmological parameters in such a narrow range, as was shown in the previous case for the CPL parametrization, makes z_M of no real use in such a case [35]. However, we have found a different and very interesting way for which z_M can be usefully used to explore the nature of our Universe.

In fact, a very interesting relation (in the context of testing the validity of VSL) exists between the angular diameter distance and the maximum redshift, which is easy to derive and intrinsic to its definition: the mathematical condition for the maximum of a function is that the derivative with respect to a variable vanishes. In the case of D_A, we find that the condition $\partial D_A(z)/\partial z = 0$, when evaluated at z_M, corresponds to the relation

$$D_A(z_M) = \frac{c_0}{H(z_M)} \Rightarrow D_A(z_M)H(z_M) = c_0; \qquad (4)$$

i.e., the multiplication of the angular diameter distance and the Hubble function, both evaluated at the maximum redshift, will give the value of the speed of light. It is worth underlining here that such a relation is not fully model independent, but it is based on two hypothesis: a Friedmann-Robertson-Walker metric of the background, and no spatial curvature, i.e., $k = 0$. While the former is quite general and is used as an assumption in most of the models on the market (despite the fact that some non-Friedmannian models are theoretically studied anyway [36]), the latter has to be proven not to invalidate our results. At the least, it can be shown that even in the case of $k \neq 0$, Eq. (4) still has some validity. Allowing the curvature to differ from zero, the angular diameter distance is defined as

$$D_A(z) = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}(1+z)}\sinh\left(\sqrt{\Omega_k}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{D_C(z)}{1+z} & \text{for } \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}(1+z)}\sin\left(\sqrt{|\Omega_k|}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0, \end{cases} \qquad (5)$$

where $D_H = c_0/H_0$ is the Hubble distance, $D_C(z) = D_H \int_0^z dz'/E(z')$ is the line-of-sight comoving distance, $E(z) = H(z)/H_0$, and $\Omega_k \equiv kc_0^2/H_0^2$ is the dimensionless curvature density parameter today. One can easily check that the condition for the maximum in D_A is now generalized into [37]

$$\frac{D_A(z_M)H(z_M)}{c_0} = \begin{cases} \cosh\left(\sqrt{\Omega_k}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ 1 & \text{for } \Omega_k = 0 \\ \cos\left(\sqrt{|\Omega_k|}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0. \end{cases} \qquad (6)$$

From the previous expression, we can easily quantify what the "error" is in using our Eq. (4), assuming null curvature. Using the Planck 2015 data release base_omegak_ plikHM_TTTEEE_lowTEB_BAO_H070p6_JLA_post_ lensing model, we have $\Omega_k = 0.0008 \pm 0.002$ at the 68% (and ± 0.004 at the 95%) confidence level. Assuming for z_M the value of 1.59 (the maximum for the considered reference model), we obtain a correction $\lesssim 0.05\%$ or, equivalently, the contribution of curvature, in the redshift range of concern, is 3 to 4 orders of magnitude less than the leading order which is of interest for us. This result is also in agreement with the recent estimations presented in [38]. Finally, the consistency of the curvature with a null value is generally assumed to be an indication of no spatial curvature; this makes us confident about the use of Eq. (4).

Equation (4) itself is very interesting already at this stage: it states that it will be possible to measure the speed of light *cosmologically*. So far, this has been done only in laboratories on Earth [2], and such measurements are officially used to establish the value of c_0 [1]. Here, we

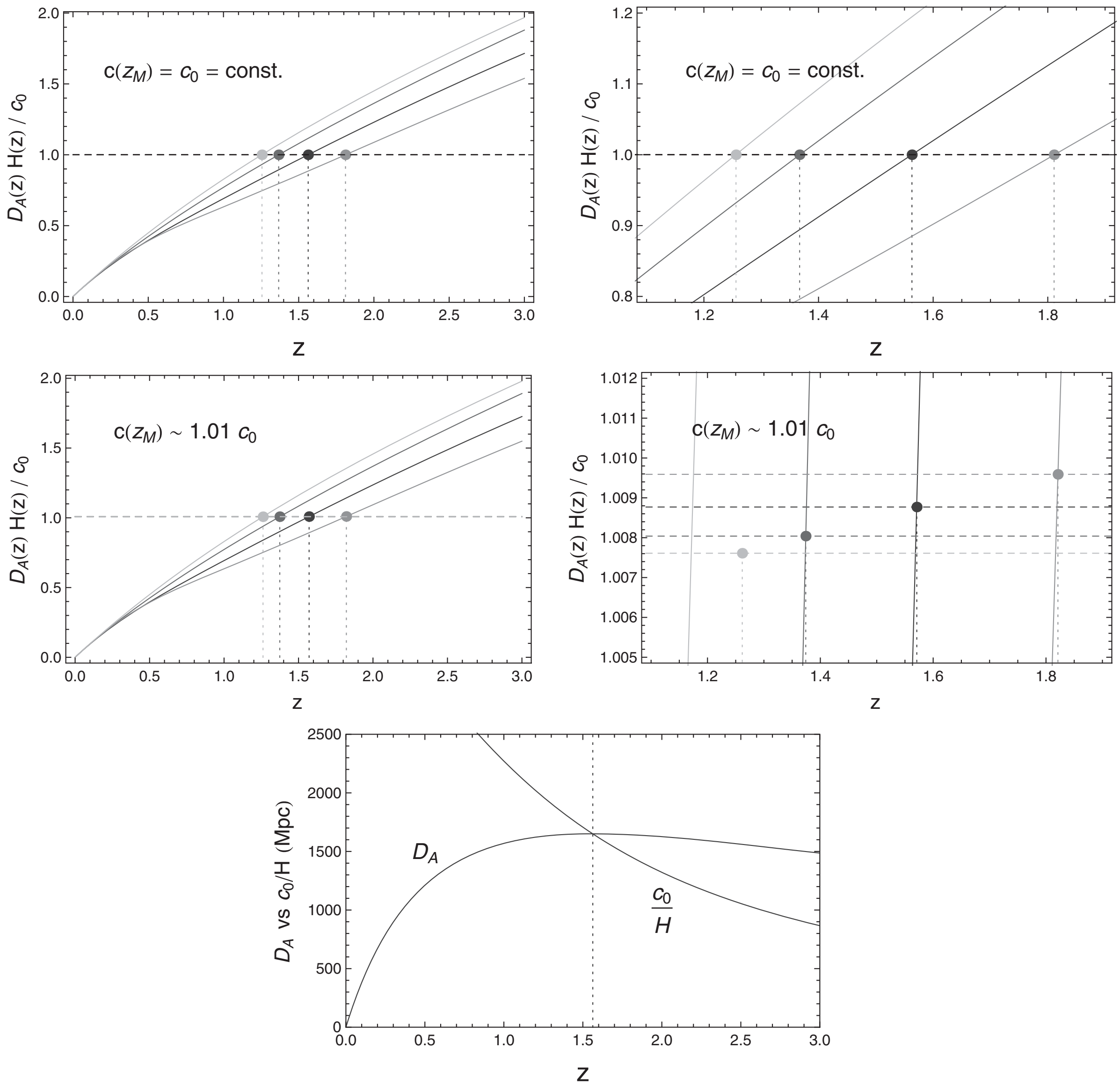

FIG. 1. (Top panel) Speed of light measurement through $D_A \cdot H$ evaluated at the maximum redshift z_M when the speed of light is constant. (Middle panel) Speed of light measurement through $D_A \cdot H$ evaluated at the maximum redshift z_M when the speed of light is a redshift function (see the forthcoming sections for its formulation); each model recovers its own value for the speed of light. (Bottom panel) Determination of the maximum redshift z_M.

have a first ever way to measure the speed of light out of Earth, out of the Solar System, out even of our Galaxy. This measurement is *direct*, imprinted in the clustering of galaxies, as direct as any other measurement that can be done in any terrestrial laboratory.

We also point out another important property of Eq. (4): it is valid independent of the cosmological model, or, in other words, the measurement of D_A and H at the maximum redshift z_M is unequivocally equal to the value of the speed of light at that time. In Fig. 1, in the top panels, we plot the quantity $D_A \cdot H$ for different models, chosen from the 10^4 described above, having different cosmological backgrounds, but assuming a constant speed of light. It is clearly shown that, independent of the value of z_M, which is actually dependent on the cosmological background, the quantity $D_A \cdot H$ can change its profile but, when evaluated at z_M, is *always* equal to c_0. We anticipate here some discussion from the next section, in order to show, in the same Fig. 1, but in the middle panels, what happens when the speed of light is varying. In that case, again, we change the cosmological background as above *and* assume a varying speed of light (more details on how this can be done are in the forthcoming sections). From left to right, each model has its own maximum, as well as $D_A \cdot H$ asymptotics, and also, given that the speed of light is a function of redshift, its own value for $c(z_M)$. Now, of course, $c(z_M)$ will be different for each model because it is a function of redshift, and z_M changes from one value to another. However, the quantity $D_A \cdot H$, evaluated at z_M, still has exactly the value, equal to $c(z_M)$, which we expect from the theoretical model we used as input.

As a consequence, one could argue that a VSL theory might be constrained directly from the total observations, with no need of any alternative method. Even if this is, in theory, true, in reality there would be many caveats and conceptual flaws, similar (if not worse) to those one finds when exploring dark energy properties, and ones which would make it impossible to constrain with good accuracy any VSL. Whether theoretically based or phenomenologically given, a VSL, faced in this way, would be just an uncertainty (or an "ignorance") adding up to other well-known uncertainties ("ignorances") from the cosmological side like, for example, dark energy equation of state or density. Actually, we ignore, in the same way, the "right" dark energy behavior and the right VSL theory, and they are also degenerate. In fact, VSL's were originally introduced as an alternative to inflation (or any accelerated expansion), given that a higher speed of light in the past would solve the horizon event. But a VSL might also mimic dark energy: instead of an energy-mass fluid, dark energy could be explained, totally or in part, as a "virtual" effect coming from VSL. Again, we stress that our method is different: we will measure the speed of light directly—no indirect inference will be applied.

Thus, our algorithm has to pass through two main steps: (1) the detection of the maximum redshift z_M in D_A and (2) the measurement of the speed of light at z_M using $D_A \cdot H$. In the following subsections, we will describe in detail the basis for both steps, highlighting problems and solutions.

A. Maximum detection by BAOs

Equation (4), stated in a different form, can be written as

$$y_t(z_M) = y_r(z_M), \tag{7}$$

where y_t and y_r are the tangential and radial modes (apart from a multiplicative term equal to the sound horizon which appears on both the right-hand and left-hand sides, and having no influence on our results) which will be directly measured by a BAO survey in the next future.

Equation (7) is very important for our purposes because it helps to state the determination of the maximum in an easier observationally tested way, equivalent to the vanishing derivative condition, but more precise. In fact, the use of D_A only to determine the position of the maximum would be problematic, as a large number of effects combine to smear out the profile of $D_A(z)$: the plateau at about z_M, the measurements of $D_A(z)$ from just a few redshift bins from a BAO survey, and the errors on the same measurements plus their intrinsic dispersion. The final consequence is the practical impossibility of determining the location of the maximum. However, Eq. (7) contains the solution for this problem, at least when we will have disentangled BAO modes measured by a future survey: having at our disposal separate measures of $y_t(z)$ and $y_r(z)$, we can, in principle, constrain the value of z_M with better precision because, instead of searching for the maximum in $D_A(z)$, one can search for the redshift where the condition $y_t(z_M) = y_r(z_M)$ holds. This is shown at the bottom panel of Fig. 1.

Anyway, even in this case, we can have a better measurement of z_M with respect to the single use of D_A, but it would still be far from useful to measure c_0 or some (possible) variation with enough accuracy. However, one can employ some cosmologically model-independent method to extract information from data. Literature about this topic is huge and is growing faster and larger, pulled by the need for disentangling dark energy models in a way which should be as free as possible from theoretical inputs, thus giving independent hints of a theory for further developments, confirmations, or a rebuttal. A nonexhaustive list of such literature is in [39]. We have finally decided to use Gaussian processes (GPs) [40,41], which are very well suited to our needs. We postpone a more detailed description of all of the details of our implementation of GPs; we just anticipate here that we have employed them to reconstruct $y_t(z)$ and $y_r(z)$ in order to find z_M. The application of GPs to the BAO modes yields $y_t(z)$ and $y_r(z)$ numerically reconstructed as smooth analytical functions, which can be evaluated at whatever redshift value one may need, and the sets of GP-reconstructed BAO modes can eventually be employed in a numerical algorithm to estimate z_M and its error. Finally, once z_M is known, it will be straightforward to check on whether $D_A(z_M) \cdot H(z_M) = c_0$ or not.

B. Varying speed of light theories

In a standard context where the speed of light is not expected to change, combining the errors on z_M with the errors on $D_A(z) \cdot H(z)$ will measure c_0 with some error, as follows from Eq. (4). Nowadays, the measurement of c_0 is assumed to be exact and is used as a ruler for the definition of the meter [1]. The best measurement for c_0, obtained with laser interferometry in a terrestrial laboratory, has a relative error of $\sim 10^{-9}$ [2]: this precision is largely out of the possibility of a cosmological measurement. However, if we assume a VSL, i.e., the existence of an—up to now unknown—function $c(z)$ [with the limit $c(z \to 0) \equiv c_0$], then we can recalculate the $\partial D_A/\partial z$ in this case, and we would find out that Eq. (4) would change to the more general expression,

$$D_A(z_M) \cdot H(z_M) = c(z_M), \tag{8}$$

where, possibly, $c(z_M) \neq c_0$ is the value of the speed of light at redshift z_M. Deviations from c_0—if any—defined from now on through the parameter $\Delta c \equiv c(z_M) - c_0$, can be of whatever order possible, not necessarily as small as 10^{-9}.

About the approach to follow, we have to advise that, so far, no definitive theoretical background exists for VSL. We have chosen to follow the approach summarized in [8,42], where a minimal coupling is assumed between matter and the field driving the change in the speed of light. More recent approaches are in [9], but we stress here that, for our needs, the choice of the approach is unimportant.

For our objectives, it is important to check on the modifications induced by a VSL approach to the Friedmann and continuity equations. In particular, following [8,42], the first Friedmann equation will look like

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2(t)}c^2(t), \tag{9}$$

while the continuity equation is

$$\dot{\rho}(t) + 3H(t)\left(\rho(t) + \frac{p(t)}{c^2(t)}\right) = \frac{3k}{4\pi G a^2(t)}c(t)\dot{c}(t), \tag{10}$$

where ρ and p are, respectively, the energy-mass density and the pressure of any fluid in the Universe, $a(t)$ is the scale factor, G is the universal gravitational constant, and the speed of light is expressed as a general function of time (or redshift), $c(t)$. What is interesting to note is that any change produced by a VSL is connected to the spatial curvature. Thus, in our case, where we are working assuming the condition of spatial flatness, e.g., $k = 0$, this implies that no effective change is working in the continuity equation and, consequently, in the first Friedmann equation, which, we emphasize, is directly connected to the observable quantity $H(z)$.

On the other hand, this is not the only change produced by a VSL; in fact, the speed of light enters all of the metric-derived terms like, for example, the expressions for cosmological distances, as D_A is, which involve integrals of the type

$$\int_{z_1}^{z_2} \frac{c_0}{H(z')} dz'; \tag{11}$$

a VSL modifies such integrals in this way:

$$\int_{z_1}^{z_2} \frac{c(z')}{H(z')} dz'. \tag{12}$$

Having clarified the VSL scenario we will work with (but again stressing that we need it only to produce some mock data which include a VSL; thus, the choice of one model over another is not important for our purposes), we have to show now that a general result, independent of the choice of $c(z)$, is that, still, even if we were assuming not negligible spatial curvature, the contribution of $k \neq 0$ to our Eq. (8) would be many orders smaller than a possible deviation of $c(z_M)$ from c_0. In fact, in VSL, Eq. (5) is generalized to

$$D_A(z) = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}(1+z)} \sinh\left(\sqrt{\Omega_k}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{D_C(z)}{1+z} & \text{for } \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}(1+z)} \sin\left(\sqrt{|\Omega_k|}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0, \end{cases} \tag{13}$$

where now the line-of-sight comoving distance is defined as $D_C(z) = D_H \int_0^z \Delta_c(z')/E(z')dz'$, and we have made use of the general ansatz $c(z) \equiv c_0 \Delta_c(z)$, with $\Delta_c(z) = 1$ for $z = 0$. From this set of equations, the condition for the maximum of D_A within VSL is

$$\frac{D_A(z_M)H(z_M)}{c(z_M)} = \begin{cases} \cosh\left(\sqrt{\Omega_k}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ 1 & \text{for } \Omega_k = 0 \\ \cos\left(\sqrt{|\Omega_k|}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0. \end{cases} \tag{14}$$

Unlike the case we considered in the previous section where the speed of light was constant, here we need to make some assumption on the function form of $c(z)$ in order to quantify the deviation between the nonzero curvature assumption and the formula that we used, Eq. (8). If we use for $c(z)$ the expression we will describe in the next section, i.e., Eq. (15), and consider the cases we are going to describe later and the value of the curvature from Planck that we used above, then we can easily find that, even in this case, the error we make in not considering the curvature contribution is $\lesssim 0.05\%$. One important point we have to stress here is that, in principle, some degeneracy could arise between VSL and curvature: the possible detection of a signal might be equally interpreted as "VSL + null curvature" or "constant $c(z)$ + curvature." However, this misleading interpretation has a reason to exist only if the VSL signal should result in one of the same order of curvature, i.e., $\sim 0.01\%$. Larger detections (if any), could be attributed to VSL only.

III. METHOD IMPLEMENTATION

Once we have defined all of the theoretical issues at the base of our test, we can now move on to giving more technical details about how we have built our algorithm and how we checked it to be working well.

A. Mock data sets

First of all, we have to face one problem: we do not now have any data concerning y_t and y_r, nor any BAO observations in the redshift range that we need. Thus, we will have to work with mock data. At this time, we have no clear and reliable phenomenological expression for $c(z)$ [10]; we have chosen to work with a general theoretically motivated expression given in [8], i.e.,

$$c(a) \propto c_0(1 + a/a_c)^n, \tag{15}$$

where, again, $a \equiv 1/(1+z)$ is the scale factor and a_c is the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \to c_0$ (at late times until now). Another possible ansatz could be $c \propto c_0 a^n$ [42], but it turned out to be less flexible in order to (qualitatively) match both early and late time observations, and it seems not to be consistent with experiments [8]. Of course, the choice of the functional form of $c(z)$ is only needed to simulate some mock observational data with some intrinsic variation of c, in order to test whether our method is able to detect it or not, and it has no influence at all on the final results.

We have decided to produce data based on three different cosmological models.

(i) $\Delta c/c_0 = 0\%$: the baseline Λ cold dark matter (ΛCDM) model from Planck 2015 release base_plikHM_TTTEEE_lowTEB_lensing_post_BAO. The current dimensionless matter density, inferred by this model, is $\Omega_m = 0.31$.

(ii) $\Delta c/c_0 \sim 0.1\%$[1] at $z_M \approx 1.55$–1.6: baseline ΛCDM model plus a $c(a)$ given by Eq. (15), with $a = 0.05$, $n = -0.001$.

(iii) $\Delta c/c_0 \sim 1\%$ at $z_M \approx 1.55$–1.6: baseline ΛCDM model plus a $c(a)$ given by Eq. (15), with $a = 0.05$, $n = -0.01$.

In order to make the global dynamics of the Universe within these two VSL scenarios compatible with present data, we have had to change the value of Ω_m, the current dimensionless matter density. This is expected because, as we have explained in previous sections, VSL can mimic the effects of a dark energy fluid, i.e., an accelerated expansion. A higher speed of light in the past can mimic the effects of a dark energy component, thus resulting in a lower value for Ω_{DE} (the current dimensionless dark energy density). Equivalently, when no spatial curvature is assumed, this gets converted to a larger value of Ω_m. In order to arrange for the above assessed variations in c, in the second model of VSL, we need $\Omega_m = 0.314$, and in the third we need $\Omega_m = 0.348$, while the value for the first case (thus, constant c_0) is $\Omega_m = 0.31$. We stress again that such values are not derived from a fitting procedure to present cosmological data, which is beyond the purpose of this work. We simply checked heuristically the values which could give a qualitatively good global description of the present data. As a proof of such goodness, in Table I we calculate all of the quantities of interest for all three models we have considered, and we compare them to the available measurements. The sound horizon at decoupling, $r_s(z_*)$, is derived from the same baseline model from Planck 2015, used to mimic data as described above; the BOSS Data Release 11 data are from [27]; the WiggleZ Dark Energy Survey data are from [26]; and the $H(z)$ data from cosmic chronometers are from [30]. We can easily check that the changes in the sound horizon are $\lesssim\sigma_s$ in all of the cases considered, where σ_s is

[1]Note that the effective variation of $c(z)$ at maximum redshift expected from the fiducial model is not exactly equal to 0.1%, but slightly less, ~0.08%. The same holds true also for the 1% case, which is more exactly ~0.8%. We used the 0.1% and 1% notation merely for an easier readability.

TABLE I. Qualitative comparison among data and models. The distances [sound horizon at decoupling, $r_s(z_*)$; angular diameter distances, D_A] are in Mpc; rate expansion data (Hubble function, H) are in km s^{-1} Mpc^{-1}.

	Data	ΛCDM	$\Delta c/c_0$ =0.1%	$\Delta c/c_0$ =1%
	Planck 2015			
$r_s(z_*)$	144.77 ± 0.24	144.70	144.67	144.75
	BOSS			
$D_A(z=0.57)$	1380 ± 23	1388	1385	1371
$H(z=0.57)$	93.1 ± 3.0	93.0	93.3	95.6
$D_A(z=2.34)$	1662 ± 96	1730	1725	1684
$H(z=2.34)$	222 ± 7	**237**	**238**	**250**
	WiggleZ			
$D_A(z=0.44)$	1204.9 ± 113.6	1196.2	1208.5	1198.0
$H(z=0.44)$	82.6 ± 7.8	88.0	86.2	88.0
$D_A(z=0.60)$	1380.1 ± 94.8	1400.5	1419.2	1403.4
$H(z=0.60)$	87.9 ± 6.1	**97.5**	**95.0**	**97.5**
$D_A(z=0.73)$	1533.7 ± 106.8	1517.0	1540.6	1520.6
$H(z=0.73)$	97.3 ± 7.0	**106.0**	102.9	**106.0**
	Cosmic Chronometers			
$H(z=0.070)$	69.0 ± 19.6	69.97	70.00	70.25
$H(z=0.090)$	69 ± 12	70.68	70.72	71.04
$H(z=0.120)$	68.6 ± 26.2	71.77	71.82	72.26
$H(z=0.170)$	83 ± 8	**73.69**	**73.76**	**74.40**
$H(z=0.179)$	75 ± 4	74.05	74.13	75.79
$H(z=0.199)$	75 ± 5	74.86	74.94	75.69
$H(z=0.200)$	72.9 ± 29.6	74.90	74.99	75.74
$H(z=0.270)$	77 ± 14	77.87	78.00	79.03
$H(z=0.280)$	88.8 ± 36.6	78.32	78.44	79.52
$H(z=0.352)$	83 ± 14	81.63	82.80	83.19
$H(z=0.400)$	95 ± 17	83.97	84.16	85.76
$H(z=0.480)$	97 ± 62	88.08	88.31	90.26
$H(z=0.593)$	104 ± 13	94.30	94.60	97.07
$H(z=0.680)$	92 ± 8	99.42	99.77	**102.64**
$H(z=0.781)$	105 ± 12	105.70	106.10	109.46
$H(z=0.875)$	125 ± 17	111.85	112.31	116.12
$H(z=0.880)$	90 ± 40	112.18	112.64	116.48
$H(z=1.037)$	154 ± 20	**123.08**	**123.64**	**128.24**
$H(z=1.300)$	168 ± 17	**142.90**	**143.61**	**149.54**
$H(z=1.363)$	160.0 ± 33.6	147.91	148.66	154.91
$H(z=1.430)$	177 ± 18	**153.35**	**154.14**	160.74
$H(z=1.530)$	140 ± 14	**161.66**	**162.52**	**169.64**
$H(z=1.750)$	202 ± 40	180.73	181.73	190.02
$H(z=1.965)$	186.5 ± 50.4	200.35	201.49	210.95

the error from the chosen Planck fiducial model. The same holds true for the angular diameter distance measurements, which are all consistent with data in the error confidence level, σ_{D_A}, we have from present surveys. For the rate expansion H we have some more tension with data (the bold numbers), but our proposed VSL models are still consistent with the theoretical ΛCDM model from the Planck results, which we use as fiducial in a "standard context."

In [43] many ongoing and future surveys are analyzed, among them BOSS, eBOSS, HETDEX, DESI, Euclid, and WFIRST-2.4. In particular, the authors focus on the constraints on D_A and H from a BAO analysis and conclude that the best results are from the ESA mission Euclid: in Table 6 of [43], they show the percentage errors on $D_A/r_s(z_*)$ and $H \cdot r_s(z_*)$ for 15 redshift bins (of width 0.1) in the redshift range [0.6;2.1] covered by Euclid. Once we have the fiducial mock data, $D_A^{\rm fid}/r_s^{\rm fid}(z_*)$ and $H^{\rm fid} \cdot r_s^{\rm fid}(z_*)$, derived from the three models described above, we can easily calculate the corresponding errors $\sigma_{D_A/r_s(z_*)}$ and $\sigma_{H \cdot r_s(z_*)}$ from columns 2 and 3 in Table 6 of [43].

Euclid will be considered like a sort of "pessimistic" scenario in our work because, at least using the available forecast estimation we have now, the SKA results [44] should be much better than Euclid's, even if in a smaller redshift range, but still reaching the values we need in order to determine the maximum redshift z_M (thus, up to $z \approx 1.8$). In [44] the percentage errors on D_A and H expected from this survey are shown in their Fig. 5; we can use them, once given the fiducial values for these quantities, to calculate their corresponding errors. In our work, results from SKA will be an "optimistic" scenario.

Anyway, we will not work directly on the fiducial model values. Instead, we will randomly pick up values of $D_A/r_s(z_*)$ and $H \cdot r_s(z_*)$ (or D_A and H) from a multivariate Gaussian centered on the fiducial values, and with a total covariance matrix built up from the errors we derived in the way previously described, and assuming an additional correlation factor between them, equal to $r \sim 0.4$, as derived in [45]. Such a procedure is needed in order to give to mock data an intrinsic dispersion closer to the real one. Finally, of course, we cannot rely on the results from only one single random run. Instead, we produce 10^3 random mock data sets, in the way just described, and we test our algorithm on each of them. Thus, our final results will then be a statistical output on an ensemble of possible universes observationally compatible with the starting fiducial model. We want to clarify that this (i.e., the making of mock cosmological data) is the only step in our work where we need to assume a cosmological model. This choice is quite unavoidable in order to have a reference point for establishing the goodness of our analysis, but it is a quite common procedure in forecast analysis. Moreover, the choice to test our method on a large number of data sets will greatly smear the effects of this initial input, which, anyway, is absolutely not in contrast with the (up to some limit, as clarified in the previous section) model independence of our method.

B. Gaussian processes

Once we have our set of data and related errors, we can apply GPs in order to reconstruct the observational quantities of interest (y_r and y_t) and calculate the position of the maximum (z_M). GPs are very helpful because they incorporate in a very natural and straightforward way correlations between data, even when expressed in the form of a nondiagonal covariance matrix, which is our case now. For all of the details about GPs, see the related literature [40,41]; here, we will discuss in more detail only some aspects of their implementation that are necessary for our purposes.

(i) In [11] we used a simple Gaussian as the covariance function relating two points at different redshifts, z and $\bar{z}$. In [41] it is shown that such a choice, for the quantities we are considering, can lead to an underestimation of the errors of the reconstruction. A more suitable choice, in this sense, would be the Matérn (9/2) function, given by

$$k(z,\bar{z}) = \sigma_f^2 \exp\left[-\frac{3(z-\bar{z})}{l}\right] \times \left(1 + \frac{3|z-\bar{z}|}{l} + \frac{27(z-\bar{z})^2}{7l^2} + \frac{18|z-\bar{z}|^3}{7l^3} + \frac{27(z-\bar{z})^4}{35l^4}\right), \tag{16}$$

where σ_f, the signal variance, and l, the characteristic length scale, are the hyperparameters of the proposed correlation.

(ii) For each one of the 10^3 mock data sets we have created, we employ a Markov chain Monte Carlo method in order to find the values of the hyperparameters which optimize the reconstruction of D_A and H, following [41].

(iii) Once we have found such optimized reconstruction parameters, we evaluate the GPs' output functions, i.e., y_t and y_r, on a $\Delta z = 0.01$ redshift grid, ten times finer than the Euclid and SKA forecasted bins.

(iv) Such a finer grid is useful for implementing a numerical algorithm to calculate z_M for each simulation. We finally have 10^3 sets of GP-reconstructed $(y_t^{GP}, \sigma_{y_t}^{GP})$'s and $(y_r^{GP}, \sigma_{y_r}^{GP})$'s and, using Eq. (7), we can estimate z_M for each of them.

Such a finer grid is useful for implementing a numerical algorithm to calculate z_M for each simulation.

(i) We have 10^3 sets of GP-reconstructed $(y_t^{GP}, \sigma_{y_t}^{GP})$'s and $(y_r^{GP}, \sigma_{y_r}^{GP})$'s: we randomly pick up ∼160 sets in the $[-4\sigma^{GP}, 4\sigma^{GP}]$ confidence level for each quantity, which we then combine, obtaining a total of $\sim 2.5 \times 10^4$ (y_t^{GP}, y_r^{GP}) pairs.

(ii) For each pair, we fit y_t^{GP} and y_r^{GP} with a high order polynomial in the redshift range [1., 2.] (for Euclid; the maximum redshift is 1.8 for SKA), and we find z_M numerically for each of them using Eq. (7).

Thus, we end with a set of $\sim 2.5 \times 10^4$ z_M's from which we can derive the mean value z_M for our statistical ensemble, and the related error σ_{z_M}.

C. Speed of light measured

Once we have (z_M, σ_{z_M}), we only need to calculate the quantity $D_A(z) \cdot H(z)/c_0$, using the GP-reconstructed data set and, using Eqs. (4) and (8), we can constrain the value of the speed of light. We choose to normalize the quantity $D_A(z) \cdot H(z)$ with c_0; thus, in the context of a constant speed of light, we expect to find $D_A(z_M) \cdot H(z_M)/c_0 \approx 1$ with some error, while in VSL theories it can be $\neq 1$.

One important question should be discussed at this point: a very useful relation for determining the speed of light at far cosmological epochs is Eq. (4); Eq. (7) is a completely equivalent way to write it, absolutely necessary for the determination of z_M, but quite useless for checking the constancy of the speed of light. Stated in another way, BAOs are necessary for the determination of z_M, using Eq. (7), but cannot be used to measure the speed of light, using Eq. (4). This is clearly understandable if we take a careful look at the way the radial mode, which can be measured from BAOs, is defined: the measured length, y_r, which is actually what one can see in the galaxy distribution, combines information from both the speed of light and the expansion rate H. In order to use Eq. (4), we need to determine H from what we see, but we cannot actually use the BAO radial model y_r because, in this case, we would need some assumption on the speed of light functional form, which is, of course, unknown to us (at least if one assumes it can be varying).

However, even if we cannot use BAOs from SKA, Euclid, or WFIRST-2.4, there is still a way these surveys (at least the optical ones, i.e., Euclid or WFIRST-2.4) can be useful to us: as mentioned in the Introduction, the same galaxies which are used to measure BAOs (or, better, the fraction of them corresponding to ETGs) can be used as cosmic chronometers, thus giving us direct measurements of $H(z)$, free of any degeneracy and/or assumption on the possible time variability of $c(z)$. Such a use is quite interesting because we can note down the close analogy of such probes with a laboratory experiment: here we would have D_A from BAOs, a length which plays the role of a standard (cosmological) ruler, and H^{-1} from cosmic chronometers, with the dimension of time, as a (cosmological) clock.

A preliminary study about the capability of future surveys in this context is discussed in [30], where a simulated Euclid-type survey gives a minimum $\sim 5\%$ error on H, when accounting for statistical errors only. Better performances should be obtained with WFIRST-2.4, which is going to observe more galaxies than Euclid. However, in the following, we will assume that the errors on H are those expected from BAO observations, which are smaller than this limit. This does not invalidate our method; in some ways, our analysis will give us a clear idea about how much can we expect from it, and it will give us an indication about the properties a survey should have in order to be sensitive to the signal we are searching for. If an already-planned survey is not going to reach such a limit, this does not exclude the possibility that, in the future, we can have such a detection by means of more advanced instruments.

IV. RESULTS AND DISCUSSION

All of the results from applying our method to the mock data that we have produced are summarized in Table II and depicted in Fig. 2, where we show, separately, expectations from assuming errors on D_A and H as they come from Euclid and SKA for the three cosmological scenarios we have considered. Note that the errors shown in the table are not the usual ones, except for those on z_M. Instead, given that we have been working on an ensemble of 10^3 possible observational sets, we give our results in the form average of the median from the ensemble $\pm$ average of the median

TABLE II. Results.

	Euclid				
$\Delta c/c_0$	z_M	$c(p>1)$	$c_{1\sigma}(p>1)$	$c_{2\sigma}(p>1)$	$c_{3\sigma}(p>1)$
1%	$1.559^{+0.054}_{-0.051}$	$1.00872^{+0.00003}_{-0.00003}$ (1)	$0.99993^{+0.00013}_{-0.00024}$ (0.32)	$0.99436^{+0.00023}_{-0.00041}$ (0)	$0.98879^{+0.00032}_{-0.00056}$ (0)
0.1%	$1.587^{+0.058}_{-0.052}$	$1.000880^{+0.000006}_{-0.000006}$ (0.98)	$0.99199^{+0.00014}_{-0.00024}$ (0.001)	$0.98636^{+0.00024}_{-0.00038}$ (0)	$0.98072^{+0.00034}_{-0.00053}$ (0)
	SKA				
$\Delta c/c_0$	z_M	$c(p>1)$	$c_{1\sigma}(p>1)$	$c_{2\sigma}(p>1)$	$c_{3\sigma}(p>1)$
0%	$1.593^{+0.018}_{-0.017}$	$1.^{+3\times10^{-7}}_{-4\times10^{-7}}$	$0.99708^{+0.00003}_{-0.00004}$	$0.99524^{+0.00006}_{-0.00007}$	$0.99339^{+0.00008}_{-0.00008}$
1%	$1.561^{+0.017}_{-0.017}$	$1.00873^{+0.00001}_{-0.00001}$ (1)	$1.00585^{+0.00003}_{-0.00003}$ (1)	$1.004036^{+0.00005}_{-0.00005}$ (1)	$1.00221^{+0.00008}_{-0.00009}$ (1)
0.1%	$1.590^{+0.018}_{-0.017}$	$1.000880^{+0.000001}_{-0.000001}$ (1)	$0.99797^{+0.00003}_{-0.00003}$ (0)	$0.99612^{+0.00006}_{-0.00006}$ (0)	$0.99428^{+0.00008}_{-0.00008}$ (0)
0.1% (err/3)	$1.590^{+0.006}_{-0.006}$	$1.0008800^{+0.0000001}_{-0.0000001}$ (1)	$0.999834^{+0.000009}_{-0.000009}$ (0)	$0.99917^{+0.00001}_{-0.00001}$ (0)	$0.998510^{+0.00002}_{-0.00002}$ (0)
0.1% (err/10)	$1.590^{+0.003}_{-0.003}$	$1.0008800^{+0.0000003}_{-0.0000002}$ (1)	$1.00032^{+0.00014}_{-0.00018}$ (0.94)	$0.99996^{+0.00023}_{-0.00029}$ (0.44)	$0.99961^{+0.00032}_{-0.00040}$ (0.10)

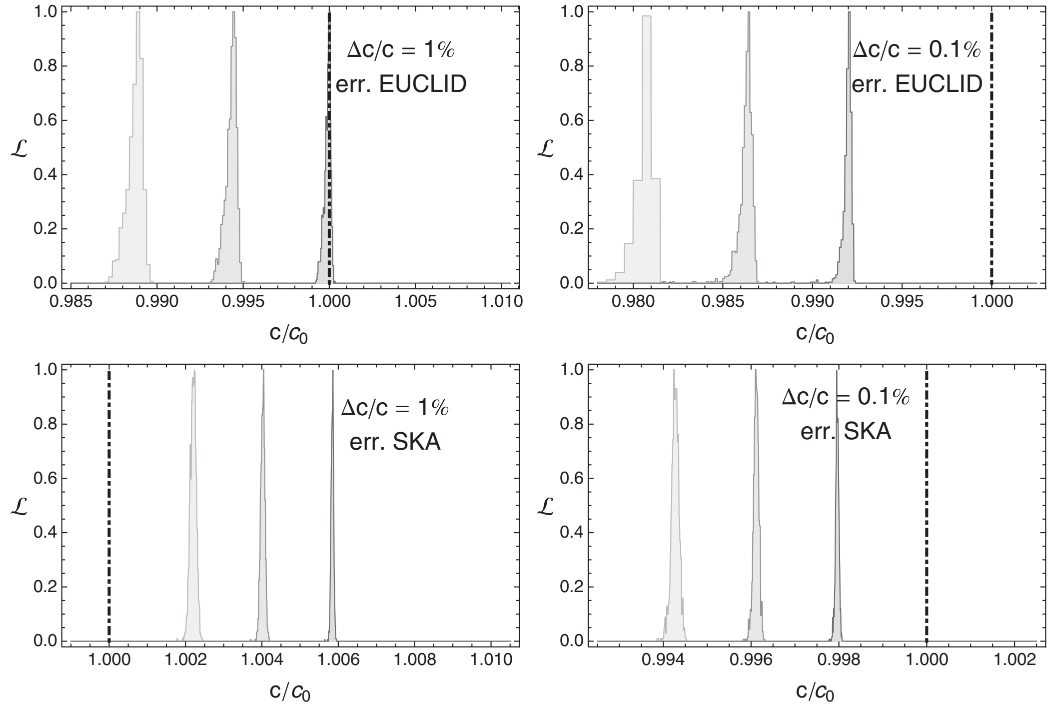

FIG. 2. Probability distribution of $c_{1\sigma}$, $c_{2\sigma}$, and $c_{3\sigma}$ (from dark to light grey) from 10^3 simulations in different survey configurations. The vertical black dot-dashed line is for $c(z_M) = c_0$.

of the standard deviation from the ensemble. We also have to specify the notation we used: c is the measured value from the experiment, i.e., $c(z_M)$, normalized to the present value c_0; $c_{1\sigma}$, $c_{2\sigma}$, $c_{3\sigma}$, are the lower limits at, respectively, the 1σ, 2σ, and 3σ confidence levels. We only consider the lower limits because, in all of the models, we have assumed that the speed of light in the past was greater than the present value; thus, any deviation from constancy is possibly detectable only if the lower limits are greater than c_0. The choice of models with a different trend would have been completely equivalent, implying only that we should have focused on the upper limits instead of the lower ones; however, all of the conclusions that we draw would have been completely equivalent to the present ones. The $p > 1$ number is the probability of having a $c(z_M) \neq c_0$ in our ensemble (e.g., the normalized number of simulations for which a clear nonconstant signal can be detected); higher values of $p > 1$ mean, of course, that the survey is more likely to observe a deviation from constancy of the speed of light.

As a preliminary check, we have tested our algorithm in the case of a nonvarying speed of light. The expected maximum from the fiducial model is $z_M = 1.589$, and we recover, in the case of SKA, a value that is highly consistent with this estimation. As expected, even the value of $c(z_M)$ is very consistent with the expected c_0, and there is a very small dispersion of the values from the 10^3 models we have considered. Thus, we can finally conclude that our method works quite well, as we are able to recover the input model with a very good accuracy.

The central point here is the lower limit detection: from the $\Delta c = 0$ case, we can see that the average 1σ limit is ≈ 0.003. So, the main question to be answered now is the following: is this accuracy enough to detect a possible VSL?

To answer this, we consider the VSL model with a 1% variation in c. First of all, again, the detection of z_M is good: the expected value is $z_M = 1.561$, and we recover 1.559 and 1.561, respectively, with Euclid and SKA, with both of the errors fully compatible with the expected input. What is interesting to note is the improvement in the determination of the maximum which is achieved when moving from one survey to another. In particular, the error on z_M from SKA is $\sim 30\%$ smaller than the same estimation from Euclid. This, of course, will also result in a better constraint on $c(z_M)$. Effectively, if we examine the 1σ lower limit, we can see how Euclid will be hardly able to detect such an order of variation in the speed of light, with only 32% of our simulations clearly detecting a deviation from 1 (i.e., from c_0 and, thus, constancy) at 1σ. On the other hand, SKA will be extremely useful, with a clear detection even at 3σ. We point out that in [11] we concluded that Euclid would have been able to detect such a variation at 2σ; here the signal is worsened by the change of the correlation function in the GPs. Using the Matérn $(9/2)$ function, instead of a Gaussian, as the GP correlation kernel made the errors much more realistic, but also larger than those that we obtained in [11], and this is reflected in such new results for Euclid.

Things go a little worse for smaller variations: it is clear that a 0.1% variation in c will hardly be very detectable, even with SKA: even if the corresponding values for z_M

and $c(z_M)$ can be recovered, the sensitivity will not be enough to discriminate between such a small deviation and the constancy. Thus, we have to assume that the sensitivity of at least these two already-planned surveys will not be enough to detect a VSL too much smaller than $\sim 1\%$.

For this reason, we have explored whether there is any chance for some future more extreme galaxy survey to perform better. Building a reliable galaxy survey in all possible details has many constructive difficulties and it is beyond the purpose of this work. We have thus carried out a naive "rule-of-thumb" analysis: we have assumed a SKA-style survey (i.e., with the same redshift range and bins as SKA), but with a better performance, quantified as smaller errors on D_A and H, and actually possible if the number of observed galaxies is increased. We have first considered the case where the errors on D_A and H are reduced by one third (this is approximately the same improvement one has when moving from Euclid to SKA). However, even in this case, the 0.1% variation is still beyond possibility. In order to find out something at the 1σ level, you need to reduce the errors by a factor of 10.

It is clear that to set a limit detection at 0.1% for a VSL might be problematic. We are not aware of any cosmological *direct or indirect* measurement of c which can be used as a comparison tool; in most cases, the speed of light is simply assumed to be constant and equal to the common value c_0. On the other hand, in the literature, there are many measurements of another quantity which is strictly related to c, namely, the fine-structure constant α, which is defined exactly as $\alpha \equiv e^2/(\hbar c)$, where e is the electron charge and $\hbar$ the reduced Planck constant. There are many observations which are compatible with a varying α [4,46], but these variations are always very small, at least $< 10^{-4}$; that is the reason why there is debate about whether they are really consistent or whether, instead, we should more correctly assume that α is constant. However, if we center on its definition, it is easy to check that, if the other parameters involved in its definition $(e, \hbar)$ are assumed to be constant, then $\Delta\alpha/\alpha = -\Delta c/c_0$. Thus, we would expect a variation for c of the same order, i.e., $< 10^{-4}$. The question here is more subtle, in any event. In fact, in principle, even a large variation in the speed of light could be compatible with such orders of magnitudes for the variation in α, if we admit that the other parameters can also vary. However, in this case we would have an unpleasant "fine-tuning" and would degenerate into a conspiracy plot from many different aspects of physics because, in order to accommodate such small variations in α, we would need *either* a larger variation from each of the other parameters to compensate for each other *or* roughly the same order of magnitude variation for each one of them.

Thus, assuming that such measurements of variation of α are correct, it is a conservative assumption to expect the same order of variation for c. Reading the literature, it is easy to check to see that the most used varying α detectors are quasars—in particular, their spectra and quasars are not *cosmological-scale* structures. What we are effectively measuring is a possible *local* variation of α. Instead, in our method, we are going to measure c from the cosmological-scale distribution of galaxies, which is many order of magnitudes larger than the quasar scale. The only similar probe which might be compared to our results is the CMB: from the first Planck release, a possible limit has been detected on the variation of α at a redshift $z \sim 10^3$ of the order of $\Delta\alpha/\alpha \lesssim 0.4\%$; see [47] and [48]. However, the constraint from [47] is plagued by a strong degeneracy with a cosmological parameter, the Hubble constant H_0, resulting in a tension which is only mildly reduced by joining CMB observations with some archival BAO data sets, and adding a prior on H_0. Moreover, it is worth pointing out that this is not a *direct* measurement of α: a variation is assumed, and then parametrized in a very simple, but arbitrary, way. From this point of view, we would like to stress that our "optimistic scenario" of VSL detection from SKA is highly competitive with CMB observations, and it would be obtained without any assumption on other possible cosmological parameters. It would be a direct measurement, and not indirectly inferred. Then, the constraint from [48] searches for an even more extreme variation: not only time but also spatial variation, for which the signal can be even smaller. Actually, their conclusion is that even if there is any variation, this is consistent with zero.

Finally, taking into account all such results and arguments, if we focus on our hypothesis about the sensitivity of futuristic surveys, one question which should be answered lastly is as follows: is it technically possible to achieve such small errors from galaxy surveys and thus be able to measure finer variations of c? From a quantitative point of view, the answer is not easy because, as we have said above, it would involve many technical problems. However, at least qualitatively, we feel confident enough that the 0.1% limit in VSL detection is within the reach of future observations. In [13] (Fig. 2.21) and in [43] (Fig. 3), the observational errors from many ongoing and planned future surveys are shown. As can be easily checked by a simple visual inspection, Euclid errors are expected to be about one tenth that of the errors obtained from already completed survey like the WiggleZ Dark Energy Survey. Thus, the level of improvement that we have considered should be possible. Moreover, it is also clear that any technical improvement will be useful: based on modern technology, ground based telescopes like DESI and SKA [44] are already almost as competitive as space ones like Euclid and WFIRST-2.4, with SKA errors that should be one third that of Euclid ones. Thus, we expect that in the future, even if still not planned, it will surely be possible to further improve space-based surveys and obtain even better constraints. One point we have to remember, however, is that the H measurements from such future galaxy surveys are strictly related to cosmic chronometers, whose errors are somewhat larger than the ones we have used here and which were estimated from a BAO analysis. This makes things more difficult, but not impossible.

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