Perturbative loop corrections and nonlocal gravity

Michele Maggiore

Département de Physique Théorique and Center for Astroparticle Physics, Université de Genève,
24 quai Ansermet, CH–1211 Genève 4, Switzerland

Nonlocal gravity has been shown to provide a phenomenologically viable infrared modification of general relativity. A natural question is whether the required nonlocality can emerge from perturbative quantum loop corrections due to light particles. We show that this is not the case. For the value of the mass scale of the nonlocal models required by cosmology, the perturbative form factors obtained from the loop corrections, in the present cosmological epoch, are in the regime where they are local. The mechanism behind the generation of the required nonlocality must be more complex, possibly related to strong infrared effects and nonperturbative mass generation for the conformal mode.

I. INTRODUCTION

In the last few years, together with various collaborators, we have proposed and developed a class of nonlocal infrared (IR) modifications of general relativity, which appear to have quite interesting cosmological consequences. The first successful model of this type was proposed in Ref. [1] (see also Refs. [2–6] for earlier related ideas), and is defined by the nonlocal equation of motion

$$G_{\mu\nu} - \frac{m^2}{3}(g_{\mu\nu}\Box^{-1}R)^{\mathrm{T}} = 8\pi G T_{\mu\nu}, \qquad (1)$$

where the superscript T denotes the operation of taking the transverse part of a tensor (which is itself a nonlocal operation). The mass m is a free parameter of the model, which replaces the cosmological constant in ΛCDM. A closed form of the action of this model is not currently known. A related model, defined at the level of the action, was introduced in Ref. [7], and is defined by

$$S_{\mathrm{RR}} = \frac{m_{\mathrm{Pl}}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{6}R\frac{1}{\Box^2}R\right], \qquad (2)$$

where $m_{\mathrm{Pl}}^2 = 1/(8\pi G)$. Both models, which we referred to as the RT and RR model, respectively, have a viable background evolution at the cosmological level displaying self-acceleration, i.e. the nonlocal term behaves as an effective dark energy density [1,7,8]. Their cosmological perturbations are well behaved [9,10] and fit well cosmic microwave background, supernovae, baryon acoustic oscillation (BAO) and structure formation data [9,11,12]. The cosmological perturbations have then been implemented in a Boltzmann code in Refs. [13,14]. This allowed us to perform Bayesian parameter estimation and a detailed quantitative comparison with ΛCDM. The result is that the RT model (1) fits the data at a level which is statistically indistinguishable from ΛCDM. In contrast, using the

Planck 2015 data and an extended set of BAO observations, we found in Ref. [14] that the RR model (2), even if it fits the data at a fully acceptable level on its own, in a Bayesian model comparison with ΛCDM or with the RT model is disfavored. The RT model can be considered as a nonlinear extension of the RR model, since the two models become the same when linearized over Minkowski space, so we expect that its action would contain further nonlinear terms with respect to the simpler action of the RR model. Since the observational data point toward the importance of these nonlinear terms, in Ref. [15] we have explored some other nonlinear extensions of the action (2), suggested by conformal symmetry. In particular, we found that the model defined by the action

$$S_{\mathrm{cRR}} = \frac{m_{\mathrm{Pl}}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{6}R\frac{1}{(-\Box + \frac{1}{6}R)^2}R\right] \quad (3)$$

appears to work quite well [see also Ref. [16] for a study with the more general operator $(-\Box + \xi R)^{-2}$]. Even if a full analysis of its cosmological perturbations has not yet been performed, from the equation of state of the effective dark energy we expect that its predictions will deviate from that of ΛCDM less than the predictions of the RT model (which in turn is closer to ΛCDM than the RR model), and therefore will be consistent with the data (and possibly difficult to distinguish from ΛCDM).

Another interesting aspect of these models is that they can be nicely connected with the Starobinski inflationary model, providing a simple model that describes both inflation in the early Universe and dark energy at late times. A unified model of this type was first proposed in Ref. [17] (see also Refs. [15,18,19]), where we suggested unifying the model (2) with the Starobinsky model, through an action of the form

$$S = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R + \frac{1}{6M_{\rm S}^2} R \left(1 - \frac{\Lambda_{\rm S}^4}{\Box^2}\right) R \right], \quad (4)$$

where $M_{\rm S} \simeq 10^{13}$ GeV is the mass scale of the Starobinski model and $\Lambda_{\rm S}^4 = M_{\rm S}^2 m^2$. The same can of course be done also for the model (3), considering the action [15]

$$S = \frac{m_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left[R + \frac{1}{6M_{\rm S}^2} R \left(1 - \frac{\Lambda_{\rm S}^4}{(-\Box + \frac{1}{6}R)^2}\right) R \right], \quad (5)$$

or for the RT model, combining the nonlocal contribution in Eq. (1) with the contribution to the equations of motion coming from the R^2 term in the Starobinski model. As discussed in Ref. [15], at early times the nonlocal term is irrelevant and we recover the standard inflationary evolution, while at late times the local R^2 term becomes irrelevant and we recover the evolution of the nonlocal models. This has also been recently confirmed in Ref. [19], through the explicit numerical integration of the equations of motion. Further work on these nonlocal models has been presented in Refs. [20–27].

II. PERTURBATIVE LOOP CORRECTIONS

Given that these nonlocal models are phenomenologically successful, the next question is whether nonlocalities of this form can emerge, at an effective level, from a fundamental local quantum field theory. In general, loops of massless or light particles induce nonlocal terms in the quantum effective action, so it is natural to ask whether such perturbative corrections can generate a nonlocal term such as that in Eq. (2), or in its nonlinear generalizations (1) or (3). In gravity the one-loop corrections induced by matter fields indeed produce nonlocal form factors associated to terms quadratic in the curvature, which have been computed in several classic papers using diagrammatic or heat-kernel techniques [28–33] (see also Refs. [34–36] for textbooks or reviews). The resulting quantum effective action has the general form

$$S = \int d^4x \sqrt{-g} \left[\frac{m_{\rm Pl}^2}{2} R - R k_R(\Box) R - C_{\mu\nu\rho\sigma} k_W(\Box) C^{\mu\nu\rho\sigma} \right], \quad (6)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, and we used as a basis for the quadratic term R^2, $C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$ and the Gauss-Bonnet term, which we have not written explicitly. For massless particles, the form factors $k_R(\Box)$ and $k_W(\Box)$ only contain logarithmic terms plus finite parts, i.e. $k_{R,W}(\Box) = c_{R,W} \log(\Box/\mu^2)$, where $\Box$ is the generally covariant d'Alembertian, μ is the renormalization point, and c_R, c_W are coefficients that depend on the number of matter species and on their spin. The form factors generated by loops of a massive particle with mass M are more

complicated. In the UV limit, i.e. at energies or curvatures such that $M^2/\Box$ can be treated as small, the form factors have an expansion of the general form

$$\begin{aligned} k_R\left(\frac{-\Box}{M^2}\right) &= \alpha \log\left(\frac{-\Box}{M^2}\right) + \beta\left(\frac{M^2}{-\Box}\right) \\ &+ \gamma\left(\frac{M^2}{-\Box}\right) \log\left(\frac{-\Box}{M^2}\right) + \delta\left(\frac{M^2}{-\Box}\right)^2 + \cdots, \end{aligned} \quad (7)$$

[and similarly for $k_W(-\Box/M^2)$], as discussed for instance in Ref. [37] using a covariant generalization of the effective field theory formalism of Ref. [38]. In Ref. [18] it was then observed that the logarithmic term, as well as the term $(M^2/\Box)$, have little effect on the cosmological evolution in the present epoch. This might leave as a dominant contribution the one due to $(M^2/\Box)^2$ which, as we know from Ref. [7], generates a phase of accelerated expansion in the recent epoch. In Ref. [18] it was then concluded that a nonlocal model such as Eq. (2) emerges naturally from the perturbative loop corrections.

The purpose of this short paper is to point out that, unfortunately, this is not the case, and the mechanism that generates these nonlocal cosmological models must be more complicated. The crucial point is that the expansion (7) only holds in the UV limit, where the operator $M^2/\Box$ can be treated as small. In a cosmological context, this means that $M^2/H^2 \ll 1$, where $H(t)$ is the Hubble parameter.[1]

To understand when this condition is satisfied, we observe that we can rewrite Eq. (2) as

$$S_{\rm RR} = \int d^4x \sqrt{-g} \left[\frac{1}{2} m_{\rm Pl}^2 R - R \frac{M^4}{\Box^2} R \right], \quad (8)$$

where

$$M^4 = \frac{1}{12} m_{\rm Pl}^2 m^2. \quad (9)$$

To obtain a viable cosmological evolution, with an accelerated expansion in the present epoch, we need $m = O(H_0)$, where H_0 is the present value of the Hubble parameter. This result was obtained in Refs. [1,7,8] from the explicit integration of the equations of motion, but of course the order of magnitude follows from simple dimensional considerations. The nonlocal term

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[1]More precisely $\Box^{-1}$ is a nonlocal operator, which depends on the whole past history. However, from the time evolution of the auxiliary fields $U = -\Box^{-1}R$ and $V = H_0^2\Box^{-2}R$ shown for instance in Fig. 1 of Ref. [9] we see that, up to the present epoch, the estimate $\Box^{-1} \sim 1/H^2(t)$ is correct, up to a factor of at most $\mathcal{O}(10)$, which will be irrelevant for the considerations below.

in Eq. (2) is suppressed, with respect to the Einstein-Hilbert term, by a factor of order $(m^2/\Box^2)R$. In Friedmann-Robertson-Walker, after radiation dominance, $R \sim H^2$ and $1/\Box \sim 1/H^2$, so $(m^2/\Box^2)R \sim m^2/H^2$. If we want the nonlocal term to become comparable to the Einstein-Hilbert term near the present epoch, we therefore need $m \sim H_0$. Setting $m \sim H_0$, Eq. (9) gives (apart from numerical factors of order one)

$$M \simeq (m_{\rm Pl}H_0)^{1/2}, \qquad (10)$$

which is huge compared to H_0. Indeed, numerically Eq. (10) gives $M = O(10^{-3})$ eV, while $H_0 = O(10^{-33})$ eV. This means that, for such a value of M, the UV expansion (7) is not valid near the present epoch, where we are rather in the opposite regime, $M^2 \gg -\Box$. The UV expansion is only valid for $M^2/H^2(t) \ll 1$ which, for the value of M given by Eq. (10), in terms of redshift means $z \gg 10^{15}$. The expansion (7) is therefore meaningful only in the very early Universe.

In the opposite (IR) limit $M^2 \gg -\Box$, a particle with mass M is actually heavy compared to the relevant curvature scale and it decouples, leaving only a local contribution. As an explicit example, for a massive scalar field with action

$$S_s = \frac{1}{2}\int d^4x g^{1/2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + M^2\phi^2 + \xi R\phi^2) \qquad (11)$$

the form factors $k_R(-\Box/M^2)$ and $k_W(-\Box/M^2)$ in Eq. (6) were computed in Refs. [32,33] in closed form, for $-\Box/M^2$ generic. The result is

$$k_W(\Box) = \frac{8A}{15a^4} + \frac{2}{45a^2} + \frac{1}{150}, \qquad (12)$$

$$k_R(\Box) = \bar\xi^2 A + \left(\frac{2A}{3a^2} - \frac{A}{6} + \frac{1}{18}\right)\bar\xi$$
$$+ A\left(\frac{1}{9a^4} - \frac{1}{18a^2} + \frac{1}{144}\right) + \frac{1}{108a^2} - \frac{7}{2160}, \qquad (13)$$

where $\bar\xi = \xi - (1/6)$ and

$$A = 1 - \frac{1}{a}\log\left(\frac{2+a}{2-a}\right), \qquad a^2 = \frac{4\Box}{\Box - 4M^2}. \qquad (14)$$

In the UV limit one recovers the expansion (7). However, in the opposite limit $-\Box/M^2 \ll 1$ one finds

$$k_W(\Box), \qquad k_R(\Box) = \mathcal{O}(\Box/M^2). \qquad (15)$$

Therefore in this limit the form factor is local, and small, corresponding to the decoupling of particles with a mass that is large compared to the momentum scale, which is explicit in the mass-dependent subtraction scheme used in

Refs. [32,33] (see also the discussion in Sec. 2.3.1 of Ref. [17]).

In conclusion, a particle with a mass $M \sim 10^{-3}$ eV (such as a neutrino), naively seems to give a contribution to the term $(M^2/\Box)^2$ in Eq. (7), of the right order of magnitude for reproducing the model (2) with $m \sim H_0$, as required by cosmology. However, for such a particle the expansion (7) is invalid near the present epoch. A neutrino is actually an extremely heavy particle compared to the scale H_0, and today it gives a *local* contribution of the form (15), suppressed by a factor $\mathcal{O}(\Box/M^2) \ll 1$. A nonlocal contribution proportional to $M^4/\Box^2$ at the present epoch could only be obtained from hypothetical massive particles with a mass $M \lesssim H_0 \sim O(10^{-33})$ eV. However, according to Eq. (9), this would produce a nonlocal term in Eq. (2) with a totally negligible value $m \sim H_0^2/m_{\rm Pl}$, rather than $m \sim H_0$.

It should also be observed that, at the level of terms quadratic in the curvature, logarithmic corrections involving graviton loops are not even well defined, since they depend on the gauge used, and one can even find gauges in which the corresponding divergences are absent, so that the theory is one-loop finite even off shell [39]. A related issue is that the particle creation due to these terms is a pure quantum noise, and the real effect of particle production only starts from terms of third order in the curvature [40].[2]

III. STRONG-COUPLING EFFECTS IN THE IR?

The above considerations stimulated us in Ref. [17] to look for less obvious mechanisms for the generation of the required nonlocalities. A possibility which is rather intriguing is that the scale M that appears in Eq. (8), rather than being identified with the mass of a particle running in quantum loops (which, as we have seen, is not a viable possibility) is actually generated dynamically by strong-coupling effects, much as $\Lambda_{\rm QCD}$ in QCD. To stress this different interpretation, in Ref. [17] we have indeed denoted the mass scale M as $\Lambda_{\rm RR}$. The idea of strong-coupling effects in gravity in the far infrared might seem difficult to implement. However, as discussed in Ref. [17], one can imagine mechanisms that leads to strong IR effects in general relativity (GR). One possibility is related to the running of the coupling constant associated to the R^2 term. If the running is such that the coupling is asymptotically free in the UV and grows in the IR, a strong-coupling regime could be reached at cosmological distances.

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[2]For this reason, it is interesting to study also the cosmological effects induced by nonlocal terms cubic in the curvature. The effect of adding to the Einstein-Hilbert action a term $R^2\Box^{-2}R$, or a term $R(\Box^{-1}R)^2$ was studied in [41]. In both cases, however, no viable cosmological model emerges already at the background level. This is due to the fact that, in both radiation and matter dominance, the Friedman equation is dominated by a term H^4, so we do not recover standard cosmology, see Eq. 4.1.6 of [41].

Another interesting possibility, again discussed in Ref. [17], is that the dynamics of the conformal mode could become strongly coupled at large distances. Indeed, restricting to the dynamics of the conformal mode σ, i.e. writing the metric as

$$g_{\mu\nu}(x) = e^{2\sigma(x)}\eta_{\mu\nu}, \qquad (16)$$

the quantum-loop corrections embodied in the anomaly-induced effective action generate a nontrivial kinetic term for the conformal mode [42,43],

$$S_{\text{anom}} = -\frac{Q^2}{16\pi^2} \int d^4x (\Box\sigma)^2, \qquad (17)$$

where Q depends on the number and type of conformal massless fields. Thus the conformal mode, which in classical GR is a constrained field, acquires a propagator $\propto 1/k^4$ because of quantum effects. The corresponding propagator in coordinate space grows logarithmically,

$$G(x,x') = -(2Q^2)^{-1} \log[\mu^2(x-x')^2]. \qquad (18)$$

This growth of the two-point correlation at large distances could in principle generate strong IR effects. The situation is quite similar to what happens in two dimensions, where a momentum-space propagator $\propto 1/k^2$ again generates a logarithmically growing propagator in coordinate space, often resulting in a rich infrared physics. A classic example

is the Berezinsky-Kosterlitz-Thouless phase transition where, changing the value of the parameter Q^2 in front of the propagator, a system can make a phase transition from an ordered phase to a disordered phase, with the generation of a mass gap. As discussed in Ref. [17], a nonlocal term of the form $R\Box^{-2}R$ (or its nonlinear generalizations) indeed describes a mass for the conformal mode. Indeed, in the metric (16) we have

$$R = -6\Box\sigma + \mathcal{O}(\sigma^2), \qquad (19)$$

and therefore, upon integration by parts,

$$m^2 R \frac{1}{\Box^2} R = 36m^2\sigma^2 + \mathcal{O}(\sigma^3). \qquad (20)$$

The nonlocal term $m^2 R\Box^{-2}R$ therefore provides a diffeomorphism-invariant way of giving a mass to the conformal mode. A mechanism of dynamical mass generation for the conformal mode would therefore naturally produce a term $m^2 R\Box^{-2}R$, or one of its nonlinear generalizations such as those in Eqs. (1) or (3).

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