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Test flavor SU(3) symmetry in exclusive Λ_c decays

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Flavor SU(3) symmetry is a powerful tool to analyze charmed baryon decays; however, its applicability remains to be experimentally validated. Since there is not much data on Ξ_c decays, various exclusive Λ_c decays especially the ones into a neutron state are essential for the test of flavor symmetry. These decay modes are also helpful to investigate final state interactions in charmed baryon decays. In this work, we discuss the explicit roles of Λ_c decays into a neutron in testing the flavor symmetry and exploring final state interactions. The involved decay modes include semileptonic decays, two-body and three-body non-leptonic decays, but all of them have not been experimentally observed to date.

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I. INTRODUCTION

Charmed baryon decays, in particular Λ_c and Ξ_c decays, are of great interest as they serve as a platform for the study of strong and weak interactions in heavy-to-light baryonic transitions. They can also provide the essential inputs for the Λ_b decay modes into a charmed baryon like Λ_c. On the experimental side, most available results on Λ_c decays were obtained using the old data until recently. In 2014, Belle Collaboration provided a measurement of the branching fraction with a very small uncertainty [1],

$$\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)_{\rm Belle} = (6.84 \pm 0.24^{+0.21}_{-0.27})\%, \qquad (1)$$

but the central value is much larger than the previous measurement by the CLEO-c Collaboration [2]:

$$\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)_{\rm CLEO} = (5.0 \pm 0.5 \pm 1.2)\%. \qquad (2)$$

Based on the large amount of data, Belle Collaboration also started to study the doubly Cabibbo-suppressed processes [3]. Making use of the data collected in the e^+e^- collision at the center-of-mass energy of $\sqrt{s} = 4.599$ GeV and adopting the double-tag technique, BES-III Collaboration has reported first measurements of absolute hadronic branching fractions of Cabibbo-favored decay modes [4]. In total, 12 Λ_c decay modes were observed with the significant improvement on the branching fraction, in particular, for the $\Lambda_c \to pK^-\pi^+$:

$$\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)_{\rm BESIII} = (5.84 \pm 0.27 \pm 0.23)\%. \qquad (3)$$

While the uncertainties are comparable with the Belle results in Eq. (1), its central value is much smaller and closer to the central value of the CLEO results in Eq. (2). We believe this difference will be clarified in the future since the experimental prospects on charmed baryon decays are very promising [5,6].

Theoretical description of charmed baryon decays is mostly based on the factorization assumption together with the analysis of some nonfactorizable contributions in nonperturbative explicit modes [7–10]. However, the factorization scheme does not seem to be supported by experiments, such as the observed large branching fraction for decays like $\Lambda_c \to \Sigma^+\pi^0/\Xi^0K^+$, which are forbidden in the factorization scheme [11]. An alternative and model-independent approach is to make use of the flavor SU(3) symmetry, which has been argued to work better in charmed baryon decays [12–17] and bottomed baryon decays [18–20].

As the experimental precision is gradually increasing, the time is ripe to validate or invalidate the applicability of the SU(3) symmetry to charmed baryon decays. The SU(3) transformation connects the Λ_c with the Ξ_c. But at this stage, and in the foreseeable future, there is no experiment which will focus on the study on Ξ_c decays. Thus, the Λ_c decays into various final states, especially the ones into a neutron, are of great value since they will be the only source for the test of the SU(3) symmetry in charmed-baryon

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decays. The motivation for this work is to discuss the roles of the Λ_c decays into a neutron into the test of SU(3) symmetry and the exploration of final state interactions, including semileptonic decays and two-body and three-body nonleptonic decays. None of these exclusive decay modes have been experimentally measured yet.

This paper is organized as follows. In Sec. II, the semileptonic Λ_c decays are studied. In Secs. III and IV, we will explore the two-body and three-body nonleptonic decays of the Λ_c, respectively. The last section contains our summary.

II. SEMILEPTONIC Λ_c DECAYS

We start with the semileptonic Λ_c decays. In the flavor SU(3) symmetry limit, the charmed baryons are classified according to the SU(3) irreducible representation, namely, as multiplets of the light-quark system: $3 \otimes 3 = \bar{3} \oplus 6$. The Λ_c and Ξ_c forms the charmed-baryon antitriplet in the initial state:

$$T^a = (\Xi_{c1}^0, -\Xi_{c1}^+, \Lambda_c^+). \tag{4}$$

For the light baryons, we focus on the SU(3) octet which is represented by the following matrix:

$$B_b^a = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{2/3}\Lambda^0 \end{pmatrix}. \tag{5}$$

The operator responsible for the transition $c \to q e^+ \bar{\nu}_e$ is $[\bar{q}\gamma^\mu(1-\gamma_5)c][\bar{\nu}_e\gamma_\mu(1-\gamma_5)e]$ with $q = d, s$, which forms an SU(3) antitriplet in the final state. Thus, the effective Hamiltonian at the hadron level is constructed as

$$H_{\rm eff} = a H_a(\bar{3}) T^b \bar{B}_b^a \bar{\nu}_e e, \tag{6}$$

with $H_1(\bar{3}) = 0$, $H_2(\bar{3}) = V_{cd}$, and $H_3(\bar{3}) = V_{cs}$. Here the coefficient a is a nonperturbative amplitude. An implication of the above Hamiltonian is obtained straightforwardly,

$$\mathcal{B}(\Lambda_c \to n e^+ \nu_e) = \frac{3}{2}\frac{|V_{cd}|^2}{|V_{cs}|^2}\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e), \tag{7}$$

where we have neglected the phase space difference due to the neutron and Λ masses. Measurements of the relevant branching fractions provide a most straightforward test of the flavor SU(3) symmetry in charmed baryon decays. With the most recent data from the BES-III Collaboration [21],

$$\mathcal{B}(\Lambda_c \to \Lambda e^+ \nu_e)_{\rm BESIII} = (3.65 \pm 0.38 \pm 0.20)\%, \tag{8}$$

we obtain the following result,

$$\mathcal{B}(\Lambda_c \to n e^+ \nu_e)_{\rm SU(3)} = (2.93 \pm 0.34) \times 10^{-3}, \tag{9}$$

which might be accessible for the BES-III and Belle-II Collaborations [5,6].

In semileptonic decays, the neutron can be produced together with a light pseudoscalar meson. The lowest-lying pseudoscalar meson can be written as

$$M_b^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{2/3}\eta \end{pmatrix}. \tag{10}$$

In this case, the effective hadronic interaction Hamiltonian is constructed as

$$H_{\rm eff} = a[T^a H_a(\bar{3})](\bar{B}_d^c M_c^d)\bar{\nu}_e e + b[T^a \bar{B}_a^b M_b^c H_c(\bar{3})]\bar{\nu}_e e + c[T^a M_a^b \bar{B}_b^c H_c(\bar{3})]\bar{\nu}_e e, \tag{11}$$

where the singlet contribution to η has been neglected. The a, b, c are nonperturbative coefficients. The above Hamiltonian leads to the expectation,

$$\mathcal{B}(\Lambda_c \to n\bar{K}^0 e^+ \nu_e) = \mathcal{B}(\Lambda_c \to p K^- e^+ \nu_e), \tag{12}$$

which is testable in the near future. In fact, the above identity holds in the isospin symmetry, whose breaking effect is much smaller in the charm decays than that of the flavor $SU(3)$ symmetry. In the semileptonic decays of $c \to s e^+ \nu_e$, the isospins do not change, $\Delta I = 0$. It should be stressed here that this identity is applicable to both resonant and nonresonant contributions. The resonant contribution can also be used to cross-check the results of $\Lambda_c \to \Lambda(1405) e^+ \nu \to \pi\Sigma e^+ \nu$ [22].

The branching fraction for the inclusive decay of the Λ_c into an electron has been measured as [11]

$$\mathcal{B}(\Lambda_c \to e^+ + X) = (4.5 \pm 1.7)\%. \tag{13}$$

Combining the results for the $\Lambda_c \to \Lambda e^+ \nu_e$ in (8), we may expect

$$\mathcal{B}(\Lambda_c \to n\bar{K}^0 e^+ \nu_e) = \mathcal{B}(\Lambda_c \to p K^- e^+ \nu_e) \sim \mathcal{O}(10^{-3}). \tag{14}$$

III. TWO-BODY NONLEPTONIC Λ_c DECAYS

For two-body nonleptonic decays of the Λ_c, there is no Cabibbo-allowed decay mode into a neutron. Two-body decays into a neutron are either singly Cabibbo suppressed,

$$\Lambda_c \to n\pi^+, \qquad \Lambda_c \to n\rho^+,$$

or doubly Cabibbo suppressed,

$$\Lambda_c \to nK^+, \qquad \Lambda_c \to nK^{*+}. \tag{15}$$

The nonleptonic Λ_c decays are induced by the operators $[\bar{s}c][\bar{u}d]$ for the Cabibbo-allowed mode and $[\bar{d}c][\bar{u}d]$ for the Cabibbo-suppressed mode. These operators can be decomposed into irreducible representations of flavor SU(3). For instance,

$$(\bar{s}c)(\bar{u}d) = \mathcal{O}_6 + \mathcal{O}_{\overline{15}}, \tag{16}$$

with

$$\mathcal{O}_6 = \frac{1}{2}[(\bar{s}c)(\bar{u}d) - (\bar{u}c)(\bar{s}d)],$$
$$\mathcal{O}_{\overline{15}} = \frac{1}{2}[(\bar{s}c)(\bar{u}d) + (\bar{u}c)(\bar{s}d)]. \tag{17}$$

Perturbative QCD corrections give rise to an enhancement of the coefficient for the $\mathcal{O}_6$ over the coefficient for the $\mathcal{O}_{\overline{15}}$ by [23,24]

$$\left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{18/23}\left[\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right]^{18/25} \sim 2.5. \tag{18}$$

If this is valid, then one has

$$H_{\rm eff} = eH^{ab}(6)T_{ac}\bar{B}^c_d M^d_b + fH^{ab}(6)T_{ac}M^c_d\bar{B}^d_b + gH^{ab}(6)\bar{B}^c_a M^d_b T_{cd}, \tag{19}$$

with $H^{22}(6) = 1$ for Cabibbo-allowed modes, $H^{23}(6) = H^{32}(6) = -2\sin(\theta_C)$ for singly Cabibbo-suppressed modes, and $H^{33}(6) = +2\sin(\theta_C)^2$ for doubly Cabibbo-suppressed modes, where θ_C is the Cabibbo angle, and

$$T_{ab} = \epsilon_{abc}T^c. \tag{20}$$

The coefficients e, f, g are the nonperturbative amplitudes.

Using Eq. (19), we find that, for the doubly Caibbo-suppressed modes,

$$\mathcal{B}(\Lambda_c \to nK^+) = \mathcal{B}(\Lambda_c \to pK^0). \tag{21}$$

For the singly Cabibbo-suppressed modes, we have the decay amplitudes,

$$\mathcal{A}(\Lambda_c \to n\pi^+) = \sqrt{2}\mathcal{A}(\Lambda_c \to p\pi^0) = (2f + 2g)\sin(\theta_C), \tag{22}$$

which implies the following relation:

$$\mathcal{B}(\Lambda_c \to n\pi^+) = 2\mathcal{B}(\Lambda_c \to p\pi^0). \tag{23}$$

Furthermore, we have the amplitudes for Cabibbo-allowed modes:

$$\mathcal{A}(\Lambda_c \to \Lambda\pi^+) = \frac{1}{\sqrt{6}}(-2e - 2f - 2g), \tag{24}$$

$$\mathcal{A}(\Lambda_c \to \Sigma^0\pi^+) = \frac{1}{\sqrt{2}}(-2e + 2f + 2g), \tag{25}$$

$$\mathcal{A}(\Lambda_c \to p\bar{K}^0) = -2e. \tag{26}$$

Thus, we can derive the sum rule that can be experimentally examined:

$$\mathcal{B}(\Lambda_c \to n\pi^+) = \sin^2(\theta_C)[3\mathcal{B}(\Lambda_c \to \Lambda\pi^+) + \mathcal{B}(\Lambda_c \to \Sigma^0\pi^+) - \mathcal{B}(\Lambda_c \to p\bar{K}^0)]. \tag{27}$$

The recent BES-III data [4] implies

$$\mathcal{B}(\Lambda_c \to n\pi^+) = \sin^2(\theta_C)[3\times 1.24\% + 1.27\% - 3.04\%] \sim 0.9\times 10^{-3}, \tag{28}$$

while the current PDG [11] give a larger result:

$$\mathcal{B}(\Lambda_c \to n\pi^+) = \sin^2(\theta_C)[3\times 1.46\% + 1.43\% - 3.21\%] \sim 1.3\times 10^{-3}. \tag{29}$$

Measurements in the future by BES-III will be able to validate or invalidate the dominance of the sextet assumption in the effective operator.

IV. THREE-BODY NONLEPTONIC Λ_c DECAYS

Compared to two-body decays, three-body Λ_c decays are more involved since, first, they can proceed via a quasi-two-body process and the nonresonant decays and, secondly, there are a number of independent amplitudes in SU(3) symmetry. Resonances can occur in any two-body pairs. For instance, in the $\pi N(\bar{K}N)$ system, both $I = 1/2$ and $3/2$ ($I = 0$ and 1) resonances contribute. Such contributions break the SU(3) symmetry, as one of the isospin components might be enhanced at some particular energy region. In the study of the Λ_b decay, where two pentaquark candidates decaying into the $J/\psi p$ were discovered by the LHCb Collaboration [25], a number of Λ^* resonance contributions are found to be important in the three-body decay $\Lambda_b \to J/\psi pK$. In the Λ_c decay, the $K^*(892)$, $\Delta(1232)$, and $\Lambda(1520)$ contributions have already been reported for the $\Lambda_c \to pK^-\pi^+$ channel [11]. In this paper, we will consider the nonresonant contributions in the SU(3) limit, and any deviation from the SU(3) prediction implies that it is mandatory to include the resonance contributions in these reactions. An example to explore the $\Lambda(1405)$ resonance contributions in Λ_c has been discussed in Ref. [26].

In the following, we consider the $NK\pi$ system in the isospin limit,

$$|p\bar{K}^0\pi^0\rangle = \left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle|10\rangle = |11\rangle|10\rangle = \frac{1}{\sqrt{2}}|21\rangle + \frac{1}{\sqrt{2}}|11\rangle^{(1)}, \tag{30}$$

$$|pK^-\pi^+\rangle = \left|\frac{1}{2}\frac{1}{2}\right\rangle\left|\frac{1}{2}-\frac{1}{2}\right\rangle|11\rangle = \left(\frac{1}{\sqrt{2}}|10\rangle + \frac{1}{\sqrt{2}}|00\rangle\right)|11\rangle = \frac{1}{2}|21\rangle - \frac{1}{2}|11\rangle^{(1)} + \frac{1}{\sqrt{2}}|11\rangle^{(2)}, \tag{31}$$

$$|n\bar{K}^0\pi^+\rangle = \left|\frac{1}{2}-\frac{1}{2}\right\rangle\left|\frac{1}{2}\frac{1}{2}\right\rangle|11\rangle = \left(\frac{1}{\sqrt{2}}|10\rangle - \frac{1}{\sqrt{2}}|00\rangle\right)|11\rangle = \frac{1}{2}|21\rangle - \frac{1}{2}|11\rangle^{(1)} - \frac{1}{\sqrt{2}}|11\rangle^{(2)}, \tag{32}$$

where the superscripts (1) and (2) are isospin states from $(1-1)$ and $(0-1)$ couplings, respectively, which are independent of each other. Since the Hamiltonian of the $c \to s\bar{d}u$ transition has $\Delta I = 1$, and the isospin of Λ_c is zero, we can derive the decay amplitudes from the above decompositions:

$$\mathcal{A}(\Lambda_c \to p\bar{K}^0\pi^0) = \frac{1}{\sqrt{2}}\mathcal{A}^{(1)},$$
$$\mathcal{A}(\Lambda_c \to pK^-\pi^+) = -\frac{1}{2}\mathcal{A}^{(1)} + \frac{1}{\sqrt{2}}\mathcal{A}^{(2)},$$
$$\mathcal{A}(\Lambda_c \to n\bar{K}^0\pi^+) = -\frac{1}{2}\mathcal{A}^{(1)} - \frac{1}{\sqrt{2}}\mathcal{A}^{(2)}. \tag{33}$$

The above amplitudes lead to the following sum rule:

$$\sqrt{2}\mathcal{A}(\Lambda_c \to p\bar{K}^0\pi^0) + \mathcal{A}(\Lambda_c \to pK^-\pi^+) + \mathcal{A}(\Lambda_c \to n\bar{K}^0\pi^+) = 0. \tag{34}$$

Note that the isospin amplitudes in Eq. (33) can be changed if we first couple the $K\pi$ states from Eqs. (30)–(32), but the sum rule in Eq. (34) still holds.

Measurements of branching ratios of the three channels are able to determine the two amplitudes and, in particular, investigate the relative strong phases between the two independent decay amplitudes. These phases arise from the final state interactions since, if factorization works, the two independent amplitudes are real with vanishing phases at leading order. These amplitudes, including phases, can provide the essential inputs for the analysis of nonleptonic decays into other baryons like Λ.

From Eq. (33), we define the relative strong phase, δ, between $\mathcal{A}^{(1)}$ and $\mathcal{A}^{(2)}$:

$$\frac{\mathcal{A}^{(2)}}{\mathcal{A}^{(1)}} = \left|\frac{\mathcal{A}^{(2)}}{\mathcal{A}^{(1)}}\right| e^{i\delta}. \tag{35}$$

Then the branching fractions can be expressed as

$$\mathcal{B}(\Lambda_c \to p\bar{K}^0\pi^0) = \frac{1}{2}|\mathcal{A}^{(1)}|^2,$$
$$\mathcal{B}(\Lambda_c \to pK^-\pi^+) = \frac{1}{4}|\mathcal{A}^{(1)}|^2 + \frac{1}{2}|\mathcal{A}^{(2)}|^2 - \frac{1}{\sqrt{2}}|\mathcal{A}^{(1)}||\mathcal{A}^{(2)}|\cos\delta,$$
$$\mathcal{B}(\Lambda_c \to n\bar{K}^0\pi^+) = \frac{1}{4}|\mathcal{A}^{(1)}|^2 + \frac{1}{2}|\mathcal{A}^{(2)}|^2 + \frac{1}{\sqrt{2}}|\mathcal{A}^{(1)}||\mathcal{A}^{(2)}|\cos\delta, \tag{36}$$

where we consider the relative strong phase to understand the final state interaction and neglect the phase spaces which are actually integrated in the three-body decays. Hence,

$$\cos\delta = \frac{\mathcal{B}(n\bar{K}^0\pi^+) - \mathcal{B}(pK^-\pi^+)}{2\sqrt{\mathcal{B}(p\bar{K}^0\pi^0)(\mathcal{B}(pK^-\pi^+) + \mathcal{B}(n\bar{K}^0\pi^+) - \mathcal{B}(p\bar{K}^0\pi^0))}}. \tag{37}$$

Defining

$$R_p = \frac{\mathcal{B}(\Lambda_c \to p\bar{K}^0\pi^0)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)}, \qquad R_n = \frac{\mathcal{B}(\Lambda_c \to n\bar{K}^0\pi^+)}{\mathcal{B}(\Lambda_c \to pK^-\pi^+)}, \tag{38}$$

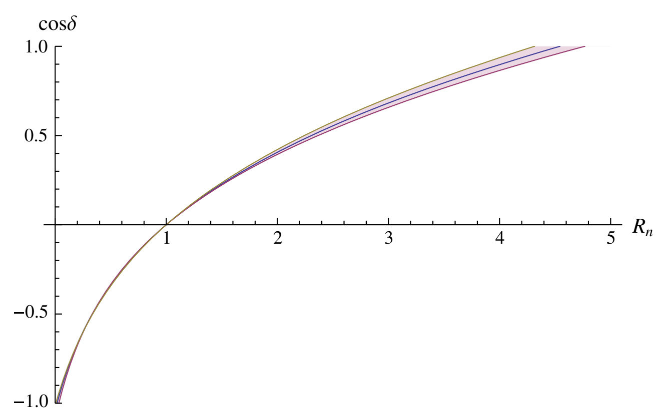

FIG. 1. Correlation between $\cos\delta$ and R_n, with δ as the strong phase difference in Eq. (35) and R_n as the ratio of branching fractions in Eq. (38).

we have

$$\cos\delta = \frac{R_n - 1}{2\sqrt{R_p(1 + R_n - R_p)}}. \tag{39}$$

From the recent measurement by BESIII [4], $R_p = 0.64 \pm 0.06$. Then $\cos\delta$ can be obtained once the R_n is measured. The relation between $\cos\delta$ and R_n is shown in Fig. 1. Since $-1 \le \cos\delta \le 1$, we have $0.017 \le R_n \le 4.54$, and then the branching fraction of $\Lambda_c \to n\bar{K}^0\pi^+$ is obtained as

$$0.04\% \le \mathcal{B}(\Lambda_c \to n\bar{K}^0\pi^+)_{\rm Belle} \le 33\%, \tag{40}$$

$$0.035\% \le \mathcal{B}(\Lambda_c \to n\bar{K}^0\pi^+)_{\rm BESIII} \le 28\%. \tag{41}$$

As we can see, this constraint is rather loose; thus, the experimental measurements are requested.

V. SUMMARY

Unlike the bottom hadron decays where the momentum transfer is typically large enough to ensure the perturbation theory in QCD, charmed meson and baryon decays are very difficult to understand. Due to the limited energy release, the factorization scheme based on the expansion of $1/m_c$ and $1/E$ is not always valid. Flavor SU(3) symmetry is a powerful tool to analyze the charmed baryon decays, which has been argued to work better than charmed meson decays; however, its validity has to be experimentally examined. Since there is not much data on Ξ_c decays, exclusive Λ_c decays into a neutron are essential for testing flavor symmetry and investigating final state interactions in charmed baryon decays.

In this work, we have discussed the roles of the exclusive Λ_c decays into a neutron in testing the flavor symmetry and final state interactions. We found that the semileptonic decays into a neutron provide the most straightforward way to explore the flavor SU(3) symmetry. Two-body nonleptonic decays allow us to examine the assumption of the sextet dominance mechanism, while three-body nonleptonic decays into a neutron are of great interest for exploring the final state interactions in Λ_c decays. None of these decay modes have been experimentally observed to date.

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