$B \to K^* l^+ l^-$: Zeros of angular observables as test of standard model

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We calculate the zeros of angular observables P'_4 and P'_5 of the angular distribution of 4-body decay $B \to K^*(\to K\pi)l^+l^-$ where LHCb, in its analysis of form-factor independent angular observables, has found deviations from the standard model predictions. In the large recoil region, we obtain relations between the zeros of P'_4, P'_5 and the zero ($\hat{s}_0$) of forward-backward asymmetry of lepton pair, A_{FB}. These relations are independent of hadronic uncertainties and depend only on the Wilson coefficients. We also construct a new observable, $\mathcal{O}_T^{L,R}$, whose zero in the standard model coincides with $\hat{s}_0$, but in the presence of new physics contributions will show different behavior. Moreover, the profile of the new observable, even within the standard model, is very different from A_{FB}. We point out that precise measurements of these zeros in the near future would provide a crucial test of the standard model and would be useful in distinguishing between different possible new physics contributions to the Wilson coefficients.

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I. INTRODUCTION

Rare B decays are mediated by flavor changing neutral current (FCNC) transitions (e.g. $b \to s$) which are absent in the standard model (SM) at tree level. The leading contributions come from one-loop diagrams. Being suppressed by Glashow-Iliopoulos-Maiani mechanism (GIM) and Cabibbo-Kobayashi-Maskawa (CKM) factors, their predictions in SM are very tiny. As these processes are very sensitive to heavy particles in the loops, any effect of new physics (NP) will show significant deviation from SM predictions. This makes these decays assets in probing NP. So far data collected on rare B-decays by dedicated experiments (LHCb, B-factories) are in excellent agreement with the predictions of SM. The data have been used to retrieve information on flavor structure of possible new physics and to put stringent constraints on beyond Standard Model (BSM) scenarios, but expectations of looking for any definitive hints of NP have not met with success. The results seem to be consistent with the Cabibbo-Kobayashi-Maskawa mechanism of the SM [1]. However, recent data on angular observables of 4-body distribution in the process $[B \to K^*(\to K\pi)l^+l^-]$ indicate a plausible change in this situation. LHCb has measured several angular observables as a binned function of the dilepton invariant mass squared (q^2). The data indicate some tension with the SM [2]. These discrepancies might be a result of statistical fluctuations or inevitable theoretical uncertainties inherent to the calculation of these observables [3]. One has to wait for more experimental data and a more careful analysis of theoretical uncertainties to clear the smoke. Assuming that these discrepancies are solely due to NP effects, there have been attempts in the literature to resolve this tension between theory and the experimental side (see for example [4]).

In this paper, we study some of the angular observables P'_4, P'_5, A_{FB} and a new observable, which we call $\mathcal{O}_T^{L,R}$, with a different approach. We look at the zeros of these observables. The expressions, under certain reasonable assumptions, are more or less independent of theoretical uncertainties, and depend solely on the short distance Wilson coefficients, and thus have very clean predictions in SM. Precise measurement of these quantities gives certain relations (experimentally testable) among the Wilson coefficients and therefore provides tests of short-distance physics. The most favored solutions to the present data explaining these deviations generally indicate towards new physics in the Wilson coefficient ($C_9^{\rm eff}$) of the semileptonic operator O_9 [5]. Since these zeros essentially probe new contributions to the Wilson coefficients, their experimental measurement in the near future can be worthwhile.

We proceed as follows. In the next section, we recall the effective Hamiltonian for $b \to sl^+l^-$. We discuss the 4-body angular distribution of $B \to K^*(\to K\pi)l^+l^-$ and various observables in the large energy recoil limit. In Sec. III, we calculate zeros of the observables P'_4, P'_5, $\mathcal{O}_T^{L,R}$ and obtain correlations among them. In Sec. IV, we give SM predictions for the zeros of the considered observables and discuss the implications of the zeros and their correlations in providing the new constraints on the BSM scenarios. The NP sensitivity of these zeros is discussed in detail. Finally, we summarize the results of this paper in Sec. V.

II. ANGULAR OBSERVABLES OF $B \to K^* l^+ l^-$ IN THE LARGE RECOIL LIMIT

The basic framework to study rare FCNC decays is that of the effective Hamiltonian which is obtained after

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integrating out the heavy degrees of freedom. The rare decay $B \to K^* l^+ l^-$ is governed by the effective Hamiltonian,

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i(\mu) O_i + C_i'(\mu) O_i') + \text{H.c.}, \quad (1)$$

where contribution of the term $\propto \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}$ is neglected. O_i are the effective local operators and $C_i(\mu)$ are called Wilson coefficients evaluated at scale μ. The factorization scale μ distinguishes between short distance physics (above scale μ) and long distance physics (below scale μ). The Wilson coefficients encode information about heavy degrees of freedom which have been integrated out while matrix elements of local operators O_i dictate the low energy dynamics (for a review, see [6]). The operators contributing significantly to the process $B \to K^* l^+ l^-$ in SM are

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2} m_b (\bar{s}_\alpha \sigma_{\mu\nu} R b_\alpha) F^{\mu\nu}, \\ O_9 &= \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma^\mu L b_\alpha)(\bar{l}\gamma_\mu l), \\ O_{10} &= \frac{e^2}{16\pi^2} (\bar{s}_\alpha \gamma^\mu L b_\alpha)(\bar{l}\gamma_\mu \gamma_5 l). \end{aligned} \quad (2)$$

Here, α, β are the color indices, L, $R = \frac{(1\mp\gamma_5)}{2}$ represent chiral projections and m_b is the b-quark mass. The primed operators come with flipped helicity. Their contribution within SM is either severely suppressed or not present. The effective coefficient of operator O_9 is given by $C_9^{\rm eff} = C_9 + Y(\hat{s})$. Here s is lepton invariant mass (q^2) and $\hat{s} = s/m_B^2$. $Y(\hat{s})$ contains contributions from one-loop matrix elements of operators $O_{1,2,3,4,5,6}$. The functional form of $Y(\hat{s})$ can be found in [7]. Due to $Y(\hat{s})$, $C_9^{\rm eff}$ is not real but has a small imaginary part. In the analytic relations below, $Y(\hat{s})$ is neglected and all the Wilson coefficients are assumed real, but for numerical calculations we include $Y(\hat{s})$ in $C_9^{\rm eff}$. As we will see, this turns out to be a good working approximation.

To calculate observables for the $B \to K^*$ process, one needs to calculate matrix elements of the local operators O_i's. These matrix elements are generally expressed in terms of seven form factors V, A_0, A_1, A_2, T_1, T_2 and T_3. These form factors are calculated via nonperturbative methods like QCD sum rules on the light cone [8]. Working in the QCD factorization framework and heavy quark and large recoil limit, all seven form factors can be written in terms of only two independent universal factors: $\xi_\perp$ and $\xi_\parallel$ [9]. The two sets of form factors are related to each other as (see for example [10])

$$\begin{aligned} \xi_\perp &= \frac{m_B}{m_B + m_{K^*}} V(q^2), \\ \xi_\parallel &= \frac{m_B + m_{K^*}}{2E} A_1(q^2) - \frac{m_B - m_{K^*}}{m_B} A_2(q^2). \end{aligned} \quad (3)$$

The angular distribution of $B \to K^*(\to K\pi) l^+ l^-$ offers experimentally accessible observables which are independent of form factors and hence theoretically cleaner. The fully differential decay distribution is given by [11]

$$\begin{aligned} \frac{d^4\Gamma(b \to K^*(\to K\pi) l^+ l^-)}{dq^2 d\cos\theta_{K^*} d\cos\theta_l d\phi} &= \frac{9}{32\pi} J(q^2, \theta_l, \theta_{K^*}, \phi) \\ &= J_1^s \sin^2\theta_{K^*} + J_1^c \cos^2\theta_{K^*} + (J_2^s \sin^2\theta_{K^*} + J_2^c \cos^2\theta_{K^*}) \cos 2\theta_l \\ &\quad + J_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi \\ &\quad + (J_6^s \sin^2\theta_{K^*} + J_6^c \cos^2\theta_{K^*}) \cos\theta_l + J_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi \\ &\quad + J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + J_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi, \\ &= \sum_i J_i(q^2) f(\theta_l, \theta_{K^*}, \phi) \end{aligned} \quad (4)$$

The angular coefficients $J_i(q^2)$ are expressed in terms of complex transversity amplitudes $A^{L,R}_{\perp,0,\parallel}$, A_t and A_s. For $m_l \neq 0$, we have [11]

$$\begin{aligned} J_1^s &= \frac{(2+\beta_l^2)}{4} [|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R)] + \frac{4m_l^2}{q^2} \text{Re}(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}), \\ J_1^c &= |A_0^L|^2 + |A_0^R|^2 + \frac{4m_l^2}{q^2} [|A_t|^2 + 2\text{Re}(A_0^L A_0^{R*})] + \beta_l^2 |A_s|^2, \\ J_2^s &= \frac{\beta_l^2}{4} [|A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R)], \\ J_2^c &= -\beta_l^2 [|A_0^L|^2 + (L \to R), \end{aligned}$$

$$
\begin{aligned}
J_3 &= \frac{1}{2}\beta_l^2[|A_\perp^L|^2 - |A_\parallel^L|^2 + (L \to R)],\\
J_4 &= \frac{\beta_l^2}{\sqrt{2}}[\mathrm{Re}(A_0^L A_\parallel^{L*}) + (L \to R)],\\
J_5 &= \sqrt{2}\beta_l\left[\mathrm{Re}(A_0^L A_\perp^{L*}) - (L \to R) - \frac{m_l}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_s^* + A_\parallel^R A_s^*)\right],\\
J_6^s &= 2\beta_l[\mathrm{Re}(A_\parallel^L A_\perp^{L*}) - (L \to R)],\\
J_6^c &= 4\beta_l\frac{m_l}{\sqrt{q^2}}\mathrm{Re}[A_0^L A_s^* + (L \to R)],\\
J_7 &= \sqrt{2}\beta_l\left[\mathrm{Im}\left(A_0^L A_\parallel^{L*} - (L \to R) + \frac{m_l}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_s^* + A_\perp^R A_s^*)\right)\right],\\
J_8 &= \frac{1}{\sqrt{2}}\beta_l^2[\mathrm{Im}(A_0^L A_\perp^{L*}) + (L \to R)],\\
J_9 &= \beta_l^2[\mathrm{Im}A_\parallel^{L*} A_\perp^L) + (L \to R)],
\end{aligned}
\tag{5}
$$

where

$$\beta_l = \sqrt{1 - \frac{4m_l^2}{q^2}}. \tag{6}$$

Note that A_s contributes only when scalar operators are taken into account. In this paper, we do not consider contributions from scalar operators. However, for the sake of generality, we have included A_s in the expressions of $J_i(q^2)$. Also, we have dropped the explicit q^2 dependence of the transversity amplitudes for notational simplicity. At the leading order in $1/m_b$ and α_s, the transversity amplitudes read

$$A_\perp^{L,R} = \sqrt{2}Nm_B(1-\hat{s})\left[(C_9^{\mathrm{eff}} + C_9^{\prime\mathrm{eff}}) \mp (C_{10} + C_{10}^\prime) + 2\frac{\hat{m}_b}{\hat{s}}(C_7^{\mathrm{eff}} + C_7^{\prime\mathrm{eff}})\right]\xi_\perp(E_{K^*}), \tag{7}$$

$$A_\parallel^{L,R} = -\sqrt{2}Nm_B(1-\hat{s})\left[(C_9^{\mathrm{eff}} - C_9^{\prime\mathrm{eff}}) \mp (C_{10} - C_{10}^\prime) + 2\frac{\hat{m}_b}{\hat{s}}(C_7^{\mathrm{eff}} - C_7^{\prime\mathrm{eff}})\right]\xi_\perp(E_{K^*}), \tag{8}$$

$$A_0^{L,R} = -\frac{Nm_b}{2\hat{m}_{K^*}\sqrt{\hat{s}}}(1-\hat{s})^2[(C_9^{\mathrm{eff}} - C_9^{\prime\mathrm{eff}}) \mp (C_{10} - C_{10}^\prime) + 2\hat{m}_b(C_7^{\mathrm{eff}} - C_7^{\prime\mathrm{eff}})]\xi_\parallel(E_{K^*}), \tag{9}$$

$$A_t = \frac{Nm_b}{\hat{m}_{K^*}\sqrt{\hat{s}}}(1-\hat{s})^2[C_{10} - C_{10}^\prime]\xi_\parallel(E_{K^*}). \tag{10}$$

In the above expressions,

$$N = \left[\frac{G_F^2\alpha^2}{3\times 2^{10}\pi^5 m_B^3}|V_{tb}V_{ts}^*|^2 q^2 \lambda^{1/2}\beta_l\right]^{1/2}. \tag{11}$$

Here, $\lambda = m_B^4 + m_{K^*}^4 + q^4 - 2(m_B^2 m_{K^*}^2 + m_{K^*}^2 q^2 + m_B^2 q^2)$, $\hat{m}_b = m_b/m_B$, and E_{K^*} is the energy of K^* meson. Terms of $\mathcal{O}(\hat{m}_{K^*}^2)$ have been neglected. However, it is worth mentioning that these relations hold only in the kinematic region $1 < q^2\ (\mathrm{GeV}^2) < 6$, which is precisely the region of interest. There are in total 24 angular coefficients [$J_i(q^2)$ and $\bar{J}_i(q^2)$]. The charge-parity (CP) conjugated coefficients $\bar{J}_i$ [corresponding to CP conjugate mode of $B \to K^*(\to K\pi)l^+l^-$] are given by J_i with the weak phases conjugated. The full angular analysis of $B \to K^*(\to K\pi)l^+l^-$ offers opportunities to construct observables which are insensitive to form factors as much as possible and therefore are theoretically cleaner and have high sensitivity to NP effects [11,12].

III. ZEROS OF ANGULAR OBSERVABLES AND RELATIONS IN SM

The zero crossing of the forward backward asymmetry of the lepton pair ($\hat{s}_0$) is known to be highly insensitive to form factors. This was first pointed out in [13] where a number of form-factor models were considered and was noted that the value of $\hat{s}_0$ is practically independent of hadronic form factors. Later Ali *et al.* [14] in their analysis showed that $\hat{s}_0$ depends on the Wilson coefficients and ratios of form factors and in the heavy quark limit and large $E_{K^*} \sim \mathcal{O}(m_B/2)$, the hadronic uncertainties in ratios of form factors drop out, and $\hat{s}_0$ essentially depends on a combination of short distance parameters only. This leads to a nearly model-independent relation between the Wilson coefficients. The position of the zero crossing is thus heralded as a test of SM.

In SM, $\hat{s}_0$ is given by [14]

$$\mathrm{Re}(C_9^{\mathrm{eff}}(\hat{s}_0)) = -2\frac{\hat{m}_b}{\hat{s}_0} C_7^{\mathrm{eff}} \frac{1-\hat{s}_0}{1+\hat{m}_{K^*}^2-\hat{s}_0} \sim -2\frac{\hat{m}_b}{\hat{s}_0} C_7^{\mathrm{eff}}. \tag{12}$$

Note that existence of zero from the above Eq. (12) necessarily requires the condition $\mathrm{Sign}[\mathrm{Re}(C_9^{\mathrm{eff}})C_7^{\mathrm{eff}}] = -1$ to be satisfied. For NP models where C_7^{eff} has the same sign as C_9^{eff}, there will then be no zero crossing. The LHCb collaboration [15][1] has measured the zero of forward-backward asymmetry of the lepton pair to be $q_0^2 = 4.9 \pm 0.9\ \mathrm{GeV}^2$ which, within errors, is consistent with SM predictions. The SM predictions for $\hat{s}_0$ typically lie in the range (3.7–4.3) GeV^2 which in units normalized by mass of B-meson ($\hat{s}=q^2/m_B^2$) translates to range (0.13–0.16) and have relative uncertainties below 10% level [10,17,18].

The value of zero $\hat{s}_0$ can be easily obtained from integrated q^2 angular observable, A_{FB}. In terms of the angular coefficients $(J_i(q^2))$, A_{FB} is defined as

$$A_{FB} = -\frac{3}{4}\frac{\int dq^2(J_{6s}+\bar{J}_{6s})}{\int dq^2(d\Gamma/dq^2 + d\bar{\Gamma}/dq^2)}. \tag{13}$$

To calculate $\hat{s}_0$, we use the expressions of the transversity amplitudes given in Eqs. (7)–(10), which are valid in the large recoil region. We retain contributions of helicity-flipped Wilson coefficients so that analysis done includes a subset of NP models involving primed Wilson coefficients.[2] We now discuss the angular variables of interest and work in the basis where SM operators are augmented with their helicity flipped counterparts. The expressions below clearly show the power of the zero-crossing point of these angular observables to probe different NP scenarios. The zero crossing of any observable is easily obtained by equating the numerator to zero. From Eq. (13), we obtain

$$\hat{s}_0 = -2\frac{(C_{10}C_7^{\mathrm{eff}} - C'_{10}C'_7)}{(C_{10}C_9^{\mathrm{eff}} - C'_{10}C'_9)}\hat{m}_b. \tag{14}$$

[1] The LHCb collaboration has recently updated its measured value: $q_0^2 = 3.7^{+0.8}_{-1.1}$ [16].

[2] We reiterate that in the analytic relations, we assume C_i's to be real but retain the complex nature in numerical analysis.

Within SM ($C'_i \to 0$), dependence on C_{10} cancels out and the expression reduces to Eq. (12), sensitive to the ratio of C_7^{eff} and C_9^{eff}.

The angular observables P'_5 and P'_4 both have zero-crossing point in their mass spectrum. The value of zero crossing for both lies in the "theoretically clean" low-q^2 region; interestingly the same region where LHCb has measured deviation from SM prediction for angular observables P'_5.

Observable P'_5 is related to angular coefficients J_5 through the following relation:

$$P'_5 = \frac{\int dq^2(J_5+\bar{J}_5)}{2\sqrt{-\int dq^2(J_{2s}+\bar{J}_{2s})\int dq^2(J_{2c}+\bar{J}_{2c})}}. \tag{15}$$

The numerator of P'_5 in the massless lepton limit is proportional to $[\mathrm{Re}(A_0^L A_\perp^{L*}) - (L\leftrightarrow R)]$. Then the zero of P'_5, in the low-recoil region, is given by the following combination of short-distance parameters:

$$\hat{s}_0^{P_5} = \frac{(C_7^{\mathrm{eff}}+C'_7)(C'_{10}-C_{10})}{[C_{10}C_9^{\mathrm{eff}} - C'_{10}C'_9 + (C_7^{\mathrm{eff}}-C'_7)(C_{10}+C'_{10})\hat{m}_b]}\hat{m}_b. \tag{16}$$

The zero of P'_5 turns out to be insensitive to hadronic form factors similar to the zero of A_{FB}. In the SM limit, C_{10} dependence disappears and the expression reduces to a very simple relation between the value of zero and the Wilson coefficient C_7^{eff} and C_9^{eff},

$$\hat{s}_0^{P_5,\mathrm{SM}} = -\frac{C_7^{\mathrm{eff}}}{C_9^{\mathrm{eff}} + C_7^{\mathrm{eff}}\hat{m}_b}\hat{m}_b. \tag{17}$$

Interestingly enough, we find that within SM, the zero of P'_5 can be written solely in terms of $\hat{s}_0$: zero of A_{FB}

$$\hat{s}_0^{P_5,\mathrm{SM}} = \frac{\hat{s}_0^{\mathrm{SM}}/2}{1-\hat{s}_0^{\mathrm{SM}}/2}. \tag{18}$$

We find this correlation between zero of A_{FB} and that of P'_5 an important result. Equation (18) can be expanded in a Taylor series and dropping out terms of order $\mathcal{O}((\hat{s}_0^{\mathrm{SM}}/2)^2)$ and higher, the relation predicts that zero of P'_5 is approximately half of the value of $\hat{s}_0$ in SM.

A similar analysis can also be done for observable P'_4. In terms of angular coefficients $J_i{}'s$, observable P'_4 is written as

$$P'_4 = \frac{\int dq^2(J_4+\bar{J}_4)}{\sqrt{-\int dq^2(J_{2s}+\bar{J}_{2s})\int dq^2(J_{2c}+\bar{J}_{2c})}}. \tag{19}$$

The numerator of P'_4 is $\propto [\mathrm{Re}(A_0^L A_\parallel^{L*}) + (L\leftrightarrow R)]$. Using expressions (8) and (9) for transversity amplitudes A_0^L and $A_\parallel^L$, we find zero of P'_4 to be

$$\hat{s}_0^{P_4} = -2\frac{(C_7^{\rm eff} - C_7')[C_9^{\rm eff} - C_9' + 2(C_7^{\rm eff} - C_7')\hat{m}_b]}{[(C_9^{\rm eff} - C_9')^2 + (C_{10} - C_{10}')^2 + 2(C_7^{\rm eff} - C_7')(C_9^{\rm eff} - C_9')\hat{m}_b]}\hat{m}_b. \tag{20}$$

The expression is again very "clean" and has a nontrivial dependence on short-distance parameters in the large recoil region. In the SM limit, this relation yields

$$\hat{s}_0^{P_4,\rm SM} = -2\frac{C_7^{\rm eff}C_9^{\rm eff} + 2(C_7^{\rm eff})^2\hat{m}_b}{C_{10}^2 + (C_9^{\rm eff})^2 + 2C_7^{\rm eff}C_9^{\rm eff}\hat{m}_b}\hat{m}_b. \tag{21}$$

The zero of P_4' can also be written in terms of $\hat{s}_0$ only as

$$\hat{s}_0^{P_4,\rm SM} = \frac{\hat{s}_0^{\rm SM}(1 - \hat{s}_0^{\rm SM})}{(2 - \hat{s}_0^{\rm SM})}. \tag{22}$$

Again using the fact that the value of $\hat{s}_0$ is very small compared to unity, we find the value of zero of P_4' to be approximately half of $\hat{s}_0$, similar to the case of P_5'. However, if we keep effects of higher order terms in $\hat{s}_0$, the value of zero of P_5' and that of P_4' turns out be a bit larger and smaller than $\hat{s}_0^{\rm SM}/2$ respectively and the leading effect is of order $(\hat{s}_0)^2$. From the experimental point of view, this accuracy is currently not there and therefore the effect can be safely neglected. The correlation between zeros of A_{FB}, P_4', P_5' is quite intriguing since in a chosen optimal basis of observables, A_{FB}, $P_5{}'$ and P_4' are independent observables and there is no *a priori* reason for their zero-crossing points to develop this dependence on each other.

With enough data available, one would be able to perform a full angular analysis of the final state distribution in the decay $B \to K^*(\to K\pi)\mathrm{l}^+\mathrm{l}^-$ and this would allow complete determination of the K^* spin amplitudes. Therefore one can use the spin amplitudes to design observables which are sensitive to specific NP and have relatively controlled theoretical uncertainties. With this in mind, we propose a new CP conserving observable which we call $\mathcal{O}_T^{L,R}$. It has the following form:

$$\mathcal{O}_T^{L,R} = \frac{|A_\perp^L|^2 + |A_\parallel^L|^2 - (L \leftrightarrow R)}{8(J_{2s} + \bar{J}_{2s})}. \tag{23}$$

This new observable is constructed out of both parallel and perpendicular spin amplitudes of K^* and has not been explored before in the literature. The ratio of amplitudes is chosen such that theoretical uncertainties due to the hadronic form factors cancel at the leading order. The profile of $\mathcal{O}_T^{L,R}$ also has a zero in the low-q^2 region. In a basis where SM operator structure is augmented with right-handed currents, the zero of $\mathcal{O}_T^{L,R}$ has NP sensitivity differently from that of A_{FB}. Its zero-crossing point occurs at

$$\hat{s}_0^{\mathcal{O}_T^{L,R}} = -2\frac{(C_{10}C_7^{\rm eff} + C_{10}'C_7')}{(C_{10}C_9^{\rm eff} + C_{10}'C_9')}\hat{m}_b. \tag{24}$$

The expressions $\hat{s}_0$ [Eq. (14)] and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ [Eq. (24)] have some interesting features. By definition, observables A_{FB} and $\mathcal{O}_T^{L,R}$ have nonidentical dependence on invariant mass $\hat{s}$ and therefore vary differently as a function of $\hat{s}$. But within SM, in spite of the different profiles, the values of zero crossings, $\hat{s}_0^{\rm SM}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R},\rm SM}$, are degenerate. However, in the presence of helicity flipped operators, the positions of zero-crossing shift in a dissimilar fashion and the degeneracy gets lifted. This rather utilitarian feature can be used to probe contributions from helicity flipped operators once the values of $\hat{s}_0$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ are known experimentally with good precision.

Let us remark that all the expressions and relations obtained above have been worked out under the hypothesis of no scalar and tensor contributions. Observables A_{FB}, P_4' and the proposed new observable $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ are blind to the presence of scalar/tensor contributions. Therefore, the expressions for zeros will remain unaltered even in the presence of these new contributions. Observable P_5', however, does receive contributions from the scalar component of K^*-spin amplitudes. But the sensitivity to this contribution is highly suppressed (m_μ^2/q^2 is the suppression factor) and in the limit of massless leptons limit, which we have entertained in this paper, these contributions vanish.

IV. CONSTRAINING NEW PHYSICS

All the Wilson coefficients are real in this analysis, i.e., NP does not introduce any new weak phase in the Wilson coefficients and we assume that the sign of C_7 is as in the SM. We will ignore NP scenarios where C_7 and C_9 have the same sign. The expressions of zeros of these observables depend only on the Wilson coefficients, practically independent of form factors, thereby leading to theoretically clean predictions. To calculate these zeros, we use $C_9 = 4.2297$, $C_{10} = -4.2068$, $C_7^{\rm eff} = -0.2974$ [19] at scale m_b. Other input parameters are $m_b^{\rm pole} = 4.80\,\mathrm{GeV}$, $G_F = 1.166 \times 10^{-5}$, $m_B = 5.280\,\mathrm{GeV}$, $m_{K^*} = 0.895$ GeV, $m_\mu = 0.106$ GeV, $\alpha = 1/129$, and $\alpha_s = 0.21$.

In Table I, we give the numerical values of zeros of the observables in the SM. The values in the second column are obtained using the relations in Eqs. (14), (18), (22), and (24). To compare with the exact predictions in the SM and to have a consistency check of these relations, we also calculate values of these zeros in the SM using the form factors and retaining $Y(\hat{s})$ in $C_9^{\rm eff}$, which we had ignored for obtaining analytic relations among the zeros. We use the form factors calculated in [8] using the light-cone sum rule and tabulate the results in the third column of Table I whereas in the last column we tabulate the same results

TABLE I. Zeros in the SM. In the second column, we quote the values calculated using Eqs. (14), (18), (22), and (24), while the third and fourth columns have entries predicted in the SM using form factors from [8,9], respectively.

	Value of zero	Exact values of zero crossings	
Observable	Using analytic relations	Using FFs from [8]	Using FFs from [9]
A_{FB}	0.128	0.122	0.125
P'_5	0.068	0.069	0.069
P'_4	0.059	0.054	0.056
$\mathcal{O}_T^{L,R}$	0.128	0.122	0.125

using form factors as in Beneke *et al.* [9]. As is evident, the two sets of form factors yield very similar values, thereby confirming that these zeros are (almost) independent of form factors. Clearly, the employed analytic relations yield values close to those when no approximations are made, showing the robustness of these relations. All the zeros lie in the low-q^2 region, where form factors are known with relatively greater precision. At leading order, soft form factors cancel precisely and predictions of zeros are clean. Largest corrections to the values of zeros come from form-factor uncertainties when next-to-leading order effects are included (as noted in [20] for the case of $\hat{s}_0$). The typical error on form factors is ∼10%–12% (see [8]). Assuming the size of errors in all the form factors of the same order, we find the relative uncertainties in our estimates of these zeros to be of order ∼30%. So far experimentally as well as theoretically only $\hat{s}_0$ has received attention. The experimental value of $\hat{s}_0$ has large relative uncertainties (of order 35%) [15,16]. Though we have ignored $\mathcal{O}(\alpha_s)$ contributions in favor of obtaining form-factor insensitive correlations among the zeros, our theoretical estimate of $\hat{s}_0$ is still competitive with the experimental value with current precision as discussed above. The zeros and the relations among them can be used to constrain the Wilson coefficients in the following ways:

(i) Under the hypothesis of no NP-induced right-handed currents and real Wilson coefficients, all the zeros including that of the new observable $\mathcal{O}_T^{L,R}$ are functions of $C_7^{\rm eff}$ and $C_9^{\rm eff}$ only. With the magnitude of $C_7^{\rm eff}$ stringently constrained from branching ratio of decay $B \to K^*\gamma$ (and $B \to X_s\gamma$), the zeros provides new information on $C_9^{\rm eff}$.

(ii) Some of the zero-crossing points are sensitive to the right-handed currents (more details below). These contributions can be probed once the precise measurements of zero crossings are made.

Global fits to recently updated data on angular analysis of the $B \to K^*\mu\mu$ indicate significant tension with the SM [5]. It has been suggested that solutions having a destructive NP contribution to C_9 or with $C_9^{\rm NP} = -C_{10}^{\rm NP} < 0$ are in very good agreement with the data. From this perspective, the measurement of these zero-crossing points would provide a very clean and good test of the hypothesis of the NP contribution to C_9. In Fig. 1, we show the constrained region in C_7 and C_9 plane in the SM-like operator basis. The most stringent bounds on C_7 come from decay $B \to X_s\gamma$. Then the precise measurement of $\hat{s}_0$ essentially determines the effective coefficient $C_9^{\rm eff}$. The recently measured value of $\hat{s}_0$ currently involves large errors (∼35%) [16]. Therefore, bounds on $C_9^{\rm eff}$ are not as stringent. But a qualitative analysis shows that $\hat{s}_0$ is compatible with models having NP contribution to C_9. We also provide a constrained region in the C_7–C_9 plane using bounds from the zero of P'_4 and P'_5. To this end, we employ derived relations between $\hat{s}_0$ and zeros of P'_4 and P'_5. Further, we use the experimentally measured value of $\hat{s}_0$ as an input to get constraints from zeros of P'_4 and P'_5. We find that the measurement of these zeros will provide equally efficient constraints on C_9 as drawn from $\hat{s}_0$. We

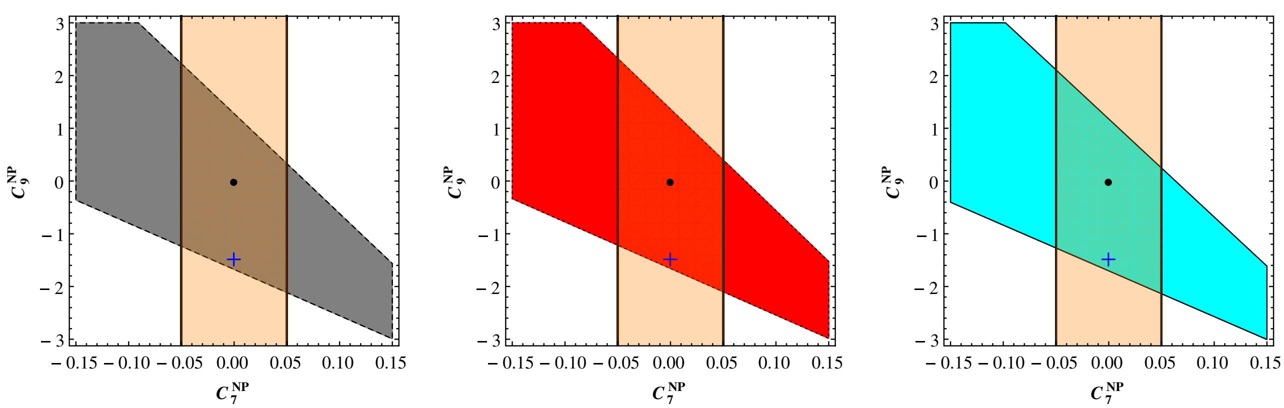

FIG. 1. Constraints on $C_7^{\rm NP} - C_9^{\rm NP}$ from zeros of observables A_{FB} (gray), P'_5 (red) and P'_4 (cyan) using analytic relations [Eqs. (14), (18), (22), and (24)]. The light orange band shows the constraints on the values of C_7 from $B \to X_s\gamma$. The black filled circle shows the SM point whereas the blue colored "+" in the plots corresponds to the simplest possible NP solution $C_9^{NP} = -1.5$ to explain the observed tension in the experimental data on $b \to s\mu^+\mu^-$. The NP solution $C_9^{NP} = -1.5$ corresponds to the "BSM1" scenario and has been discussed in detail later in the text.

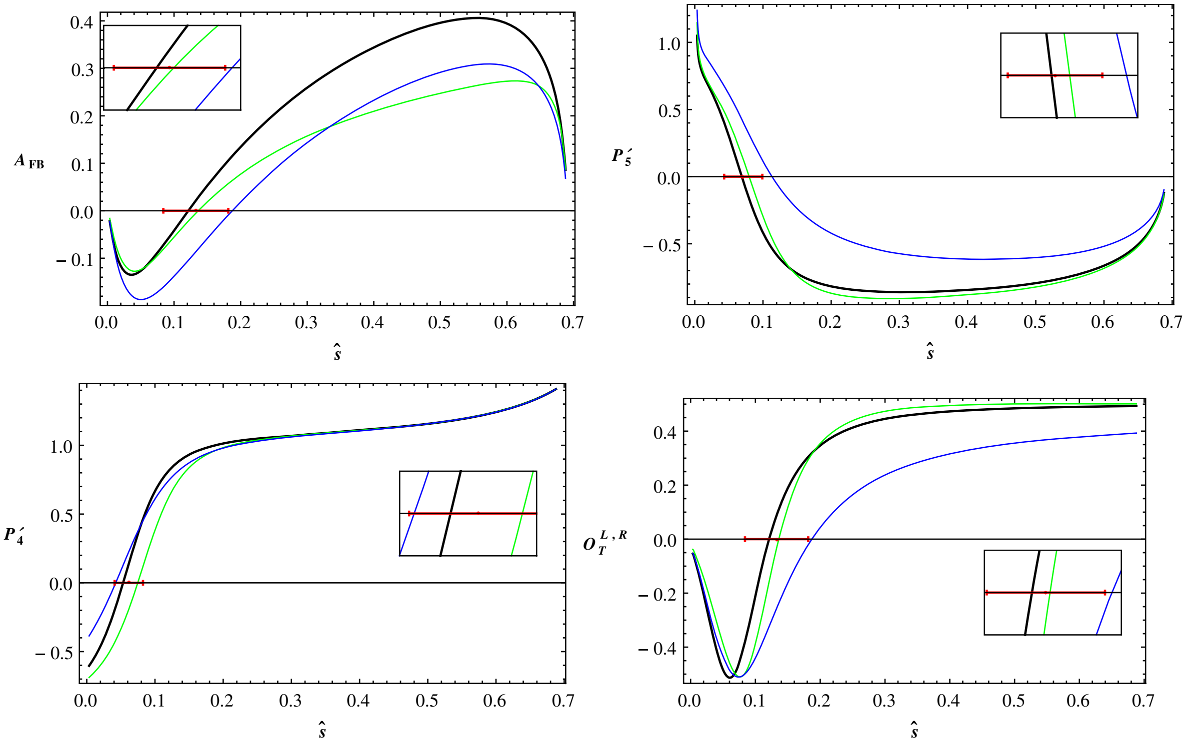

FIG. 2. The q^2 spectrum of observables A_{FB}, P'_5, P'_4 and $\mathcal{O}_T^{L,R}$ in SM (black curve) and two BSM scenarios: Z' motivated models (blue curve) and SUSY models (green curve). The Z' model [21,22] corresponds to $C_9^{NP} \sim -1.5$. The SUSY model (green curve) corresponds to large $\tan\beta$ with superpartners being heavy [23]. The red interval on the x-axis shows the experimentally allowed 1σ region. We use $\hat{s} = q^2/m_B^2$.

also note that zero-crossing points of these observables are rather sensitive to a slight change in the Wilson coefficient C_7 compared to a change in C_9 and C_{10} in the SM-like basis. For illustrative purposes, we individually varied C_7, C_9, and C_{10} by 15% with respect to their SM value. We find that the change in C_7 causes central values of $\hat{s}_0$, $\hat{s}_0^{P'_5}$, $\hat{s}_0^{P'_4}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ to shift by about 15% with respect to the SM value on the negative side, the change in C_9 causes relatively less shift (about 13%) in $\hat{s}_0$, $\hat{s}_0^{P'_5}$, $\hat{s}_0^{P'_4}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ and no shift in $\hat{s}_0^{P'_4}$ while the change in the Wilson coefficient C_{10} does not cause any modification in the SM value of the $\hat{s}_0$, $\hat{s}_0^{P'_5}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ but shifts the SM value of $\hat{s}_0^{P'_4}$ by a positive 15%.

In Fig. 2, we plot the q^2 spectrum of all four observables (A_{FB}, P'_5, P'_4 and $\mathcal{O}_T^{L,R}$) in different NP models along with SM. On the x-axis, the red interval shows the 1σ allowed region currently supported by experimental data on $\hat{s}_0$. In the plot A_{FB} vs $\hat{s}$, the red interval corresponds to experimental value $q_0^2 = 3.7^{+0.8}_{-1.1}$ GeV^2 [16]. Since at present measurements of zeros except A_{FB} are not available, we employ the correlations in Eqs. (14), (18), (22) and (24) and use the experimental value of $\hat{s}_0$ with associated errors to obtain the values and corresponding errors in the values of other zeros. As an illustration of how much these zeros can constrain the NP models, we include two scenarios of new physics in our analysis. First is the often discussed NP scenario which postulates a new $U(1)'$ gauge boson. These models, typically known as Z' models, have been shown to explain the observed anomalies in $B \to K^*\mu\mu$ [21,22]. We find that such models, which have NP contribution to $C_9^{NP} \sim -1.5$, are at 1.1σ tension with the current data on $\hat{s}_0$. The same tension translates to the zero of P'_4 as well. The theoretical value of $\hat{s}_0^{P'_5}$ in this model is at 1.5σ tension with the data while the value of $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ has 1.3σ tension with experimental data.[3] We also show the q^2 profile of all four observables with their zeros in the supersymmetric models (SUSY). The decays $B \to (K, K^*)ll$ are sensitive to the new contributions in these models and the invariant mass spectrum, forward-backward asymmetry, and lepton polarizations of these modes can constrain these models [23]. The variant of SUSY we have considered corresponds to large $\tan\beta$ with the masses of superpartners being relatively large. The details of the model can be found in [23]. Here we only show that zeros of all four observables in this model are consistent with the

[3]Let us remind again that since no actual data is available for the zeros if P'_4, P'_5, and $O_T^{L,R}$, what is meant by data in this specific context is the values obtained using correlations [Eqs. (18), (22) and (24)] with $\hat{s}_0$ as measured by LHCb as an input.

experimental data within 1σ. This good agreement between predictions in the discussed models and the measurement can be expected given the fact that substantial uncertainties are affecting the present experimental measurement of these zeros. Let us remark that the analysis in Fig. 2 for the cases of P'_5, P'_4, $\mathcal{O}_T^{L,R}$ is of qualitative nature since the zeros of these observables have not been measured so far (we again reiterate that we have used the experimental value of $\hat{s}_0$ to obtain the "experimentally allowed" red interval for these observables in Fig. 2). Our purpose here is just to illustrate that not only the q^2 profile, but the precise measurement of the zero-crossing points can also be used to discriminate various NP models. Once precise measurements of the zeros are available, the analysis can be done more precisely and the relations can certainly provide improved constraints on NP, especially on the $C_9^{\rm eff}$.

Finally, we investigate the BSM reach of these zeros by carrying out a numerical study of $\hat{s}_0^{P'_5}$, $\hat{s}_0^{P'_4}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ in Table II. In the SM, their values lie in the large recoil region and therefore these observables, like zero of A_{FB}, are expected to be very clean. These zeros also have sensitivity to BSM effects induced by right-handed currents. The BSM scenarios we have chosen in Table II are motivated from the analysis [5] of the updated data on $B \to K^* \mu\mu$ and are obtained by allowing variation in a single Wilson coefficient at a time. The case BSM1 is most favored while the cases BSM2 and BSM3 are less favorable. The three columns in Table II correspond to these scenarios as follows:

(i) The scenario BSM1 corresponds to a negative contribution of -1.5 to the SM value of C_9 (shown in Fig. 1 by the symbol "+"). This kind of scenario could, for example, be generated by a Z' boson which has vectorlike coupling to muons [24], where C_9 has a nonzero contribution while the NP contribution to the Wilson coefficient C_{10} vanishes.

(ii) The other two columns correspond to cases where NP enters in a correlated way in two Wilson coefficients. The second scenario, BSM2, has new physics in the $SU(2)_L$ invariant direction $C_9^{NP} = -C_{10}^{NP}$ and can be realized in Z' models with the Z' boson having coupling to left-handed muons [24]. A scalar leptoquark ϕ transforming as $(3,3)_{-1/3}$ under $(SU(3), SU(2))_{U(1)}$ with couplings to left-handed muons can also generate this scenario [25].

(iii) The third scenario stems from new contributions from helicity-flipped semileptonic operators O'_9 and O'_{10}. This case was specifically chosen to show the distinguishing features of these zeros when only right-handed currents have new physics contributions.

In each of the BSM scenarios, estimates of uncertainties are the same as discussed for the SM case. Our numerical analysis explicitly shows that the observables $\hat{s}_0^{P'_5}$, $\hat{s}_0^{P'_4}$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ along with $\hat{s}_0$ can certainly distinguish between the SM case (SM predictions for zeros are given in Table I) and different BSM hypotheses. An important point we would like to make here is that from Table II, it is clear that $\hat{s}_0$ has very similar values as $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ in all scenarios. This is true only when there is no contribution from right-handed currents (like the cases BSM1 and BSM2). The values of zero-crossing points would not be identical when right-handed currents are invoked (like in the case BSM3). However, the difference between $\hat{s}_0$ and $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ in the case BSM3 is arising only beyond the third decimal place and therefore, at present, can be neglected in favor of experimental errors. We would like to draw attention to the fact, as emphasized above also, that not just the position of the zero of an angular observable but also the complete profile as a function of $\hat{s}_0$ is a powerful tool at hand. This is illustrated in Fig. 2 where one can clearly see that, though the value of $\hat{s}_0^{\rm SM}$ coincides with $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ in the SM, the q^2 spectrums of A_{FB} and $\mathcal{O}_T^{L,R}$ are quite different.

We would be able to identify distinctions among different NP scenarios more accurately once these zeros are precisely measured. Experimentally, only $\hat{s}_0$ has received attention. We stress that the other zeros are equally important and should be measured or extracted experimentally, since this could already yield crucial information about NP, if present. Further, it may happen that some of the observable profiles (i.e. values in experimentally measured bins) turn out to be different from SM, as is the case say with P'_5. In such a situation, a further check would be the position of the zero. These two pieces of information put together will clearly point out to any NP present.

TABLE II. Values of zeros compared between different BSM scenarios. Only nonzero NP Wilson coefficients are shown in each scenario. The values in the parentheses correspond to beyond the third decimal place. See Table I for values in the SM.

	BSM1	BSM2	BSM3
Observable	$C_9^{NP} = -1.5$	$C_9^{NP} = -C_{10}^{NP} = -0.53$	$C'_9 = C'_{10} = -0.10$
$\hat{s}_0$	0.198	0.146	0.127(76)
$\hat{s}_0^{P'_5}$	0.109	0.078	0.067
$\hat{s}_0^{P'_4}$	0.050	0.067	0.061
$\hat{s}_0^{\mathcal{O}_T^{L,R}}$	0.198	0.146	0.127(91)

V. SUMMARY AND CONCLUSIONS

The radiative and semileptonic $b \to s$ decays have a potential sensitivity to effects beyond the SM. With LHCb's dedicated efforts to measure the decay $B \to K^* ll$ and associated angular observables extensively, the decay $B \to K^* ll$ seems to be a promising field to identify patterns of NP which can be provided by experimental data. Recent data shows some discrepancies in comparison to SM predictions but due to uncertainties inherent in the theoretical calculations of such processes, at present, it is difficult to infer the same in affirmation. Precise measurements of theoretically clean observables hold the best chance of unambiguously revealing the presence of physics beyond the SM, if any. The zero of forward-backward asymmetry ($\hat{s}_0$) is known to fall under this category of observables. But the current measurement is not precise enough to say anything definitive and is totally consistent with the SM. It may be useful to have more such observables measured with precision. In this paper, we point out that along with $\hat{s}_0$, the zeros of observables P'_5, P'_4 and $\mathcal{O}_T^{L,R}$ (a new angular observable proposed in this paper) are suitable candidates in this regard. The zeros of these observables, like the case of $\hat{s}_0$, have good theoretical control over hadronic uncertainties and can provide crucial tests of the SM. We note that there exist correlations among zeros of different observables within the SM and the position of all the zeros is essentially fixed by $\hat{s}_0$, up to small corrections. We further use these relations to model-independently constrain the $C_7^{NP} - C_9^{NP}$ plane. To this end, we define our framework by considering that NP enters in electromagnetic (O_7) and semileptonic operators (O_9, O_{10}), together with their chirally flipped counterparts. We have assumed the Wilson coefficients to be real, but generalization to complex coefficients is straightforward.

We studied the implications of these zeros on $C_7^{NP} - C_9^{NP}$ plane in the SM-like operator basis. The conservative bounds on C_7^{NP} are taken from $B \to X_s \gamma$ experimental data. Owing to the rather large uncertainties in the current measured value of $\hat{s}_0$, the constraints on the Wilson coefficient C_9 are rather weak and the deviations of up to ~ -1.5 in C_9 are compatible with experimental data within the 1σ range. Using relations between $\hat{s}_0$ and zeros of P'_5 and P'_4, we show that observables $\hat{s}_0^{P'_5}$, $\hat{s}_0^{P'_4}$ have equally good sensitivity to C_9 and C_7 as present in $\hat{s}_0$. In addition to the SM-like basis scenario, we further investigated the cases where operator basis is augmented by helicity-flipped operators. We note that these observables are quite sensitive to the effects stemming from BSM models. This can be observed from the numerical analysis we performed in Table II and Fig. 2. The analysis clearly shows that these observables have the capability to discriminate between different BSM models. Especially, the new proposed observable $\mathcal{O}_T^{L,R}$ and its zero are relatively more sensitive to the scenarios where one only includes the NP contribution to semileptonic vector operator O_9 (e.g. Z'-model). These scenarios are currently favored by data over SM (by 3.7σ for $C_9^{NP} \sim -1.1$) as noted in [5]. This sensitivity can be further exploited to test such scenarios once more precise data on this new observable as well as on the zeros of aforementioned observables become available. To date, only $\hat{s}_0$ has received attention but we have shown that zeros of other angular observables also carry important and complementary information on short-distance parameters. We thus hope that these observables will be measured precisely by the LHCb collaboration and data on these observables can certainly be used to put stern constraints on NP. The relations are obtained in the large recoil region in the large energy limit where theoretical uncertainties are supposed to be minimal. To the best of our knowledge, this is the first attempt to use such correlations as a stringent test of SM itself. A simultaneous accurate determination of these zeros will surely provide conclusive evidence of any NP present. Moreover, in a general setting, the zeros by themselves carry complementary information about the Wilson coefficients and their measurement together with the existing data can be used to pinpoint the class of NP scenarios which can give rise to such predictions. This is clearly evident from the position of $\hat{s}_0^{\mathcal{O}_T^{L,R}}$ which in the standard model limit yields the same value as $\hat{s}_0$ but when the helicity flipped operators are included, leads to complementary information on the Wilson coefficients compared to what was inferred from $\hat{s}_0$. We also hope that with more data, not just the position of various zeros, but also the complete profiles of angular observables will be known with high precision, which can be used further as a crucial test of the standard model.

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