Mass of heavy-light mesons in a constituent quark picture with partially restored chiral symmetry

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We probe effects of the partial chiral symmetry restoration to the mass of heavy-light mesons in a constituent quark model by changing the constituent quark mass of the light quark. Due to the competing effect between the quark mass and the linearly rising potential, whose contribution to the energy increases as the quark mass decreases, the heavy-light meson mass has a minimum value near the constituent quark mass typically used in the vacuum. Hence, the meson mass increases as one decreases the constituent quark mass consistent with recent QCD sum rule analyses, which show an increasing D meson mass as the chiral order parameter decreases.

I. INTRODUCTION

Investigating the relation between chiral symmetry breaking and physical observables has been the subject of great interest up to this date as these effects can be probed in heavy ion collisions and/or nuclear target experiments. The restoration of chiral symmetry has been linked to vector meson masses, σ meson masses, the quenching of f_π and even to the η' mass (see, e.g. Refs. [1–9]).

Of particular interest is the heavy-light quark meson as it embodies both chiral symmetry breaking and heavy quark symmetry. Recently, one of us in collaboration with other authors employed QCD sum rules to show that the D meson mass increases as the chiral symmetry is partially restored [10]. A qualitatively similar conclusion was obtained in Ref. [11] (see however Refs. [12–14] for other views). These results from QCD sum rules are consistent with the results obtained in analyses using effective chiral models in Refs. [15,16] which are based on the chiral partner structure proposed in Ref. [17]. At a first glance, an increasing D meson mass with a decreasing chiral parameter however seems counterintuitive. Within the naive heavy quark limit, the D meson can be thought of as a heavy quark playing the role of a color source, with a constituent quark around it, so that when chiral symmetry is partially restored, the light quark would become lighter making the D meson also light.

However, as we will see, if confinement persists, the decreasing light quark will allow the light quark to probe larger distances and hence a higher confining energy inducing a competition between the kinetic term and the confinement term. The combined effect produces a minimum energy value for a certain constituent quark mass so that when the quark mass decreases below the minimum point, the mass of the D meson will increase. In this article, we would like to highlight this effect and discuss why partial chiral symmetry restoration will not necessarily decrease the mass of the heavy-light system. We will first try to understand this effect with the help of a simple argument and then confirm the validity of our claim in a realistic and more quantitative constituent quark model calculation.

In the constituent quark model, the energy, or the mass, of a meson made of an anticharm quark and a light quark can be roughly expressed as

$$E = m_c + m_q + \frac{p^2}{2m_q} + \sigma r + C, \tag{1}$$

where m_c and m_q are masses of the charm quark and the light quark, respectively. p denotes a typical relative momentum between the two quarks, r stands for the typical size of the hadron, and σ is the string tension of the potential between the two quarks. Note that we omit the Coulomb part of the potential since it will not change the following naive analysis, as can be seen in the explicit quark model computation of the next section. C is a constant which is used to fit the total energy to the physical mass spectrum. Furthermore, we use m_q as the reduced

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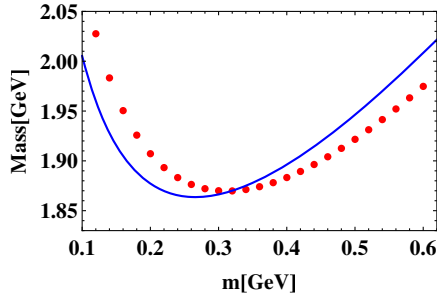[¶]diracdelta@hanmail.net

FIG. 1. The solid curve shows the m_q dependence of $E_{\min}$ according to Eq. (3) with $m_c + C = 0.8$ GeV. The red dots represent the m_q dependence of the D meson mass in the full constituent quark model.

mass in the kinetic energy term, since m_c is much larger than m_q.

For a rough estimation of the mass of the heavy-light meson, we first replace r in Eq. (1) with $1/p$, and minimize the energy with respect to the typical moment p. The minimization condition provides

$$p = (\sigma m_q)^{1/3}, \qquad (2)$$

which leads to the minimum energy as

$$E_{\min} = m_c + m_q + \frac{3}{2}\left(\frac{\sigma^2}{m_q}\right)^{1/3} + C. \qquad (3)$$

Using a conventional value of $\sigma = 0.2$ (GeV)^2, we plot the m_q dependence of $E_{\min}$ as a solid curve in Fig. 1. As can be observed in the figure, $E_{\min}$ takes a minimum value around $m_q \simeq 0.3$ GeV.

Suppose that $m_q \simeq 0.3$ GeV in the vacuum and that it reduces when the chiral symmetry is partially restored. It is natural to assume that σ and C does not change its value so much as it is related to the confinement effect and changes at most by less than 5% at normal nuclear matter density [2] or at finite temperatures around T_c [18]. As can be seen in Fig. 1, the mass of the heavy-light meson hence increases with a decreasing chiral condensate. The increase of the mass for smaller quark masses originates from the increase in the relative separation between the quark, as seen in Eq. (2), that leads to a larger contribution to the confining potential in Eq. (1). This result is consistent with that from QCD sum rules [10,11], and from effective chiral models [15,16].

II. CONSTITUENT QUARK MODEL CALCULATION

Let us next perform a more detailed analysis using a nonrelativistic constituent quark model. We start from the following nonrelativistic Hamiltonian:

$$H = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - \frac{3}{4}\sum_{i<j}^{n}\frac{\lambda_i^c}{2}\frac{\lambda_j^c}{2}(V_{ij}^C + V_{ij}^{SS}), \qquad (4)$$

where m_i and $\lambda_i^c/2$ are the quark masses and the color operators of the ith quark, and V_{ij}^C and V_{ij}^{SS} are the confinement and the hyperfine potential between quark i and j, respectively. We adopt the following form for the confinement and hyperfine potential [19]:

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D, \qquad (5)$$

$$V_{ij}^{SS} = \frac{\hbar^2 c^2 \kappa}{m_i m_j c^4}\frac{1}{(r_{0ij})^2 r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\sigma_i \cdot \sigma_j. \qquad (6)$$

Here, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between quark i and j, and (r_{0ij}) is chosen to depend on the respective masses as follows:

$$r_{0ij} = 1/\left(\alpha + \beta\frac{m_i m_j}{m_i + m_j}\right). \qquad (7)$$

The parameters are determined to give an overall fit to the meson systems in the light and heavy quark sector [20]. Here, β is introduced to fit the masses of both the light-light mesons as well as the heavy-light and heavy-heavy mesons. However, as will be shown later, we can also take $\beta = 0$ with the other parameters unchanged and obtain almost identical result for the heavy-light quark system. It should be noted that including the pion and sigma exchange potentials is important for a consistent description of three-quark and six-quark states as discussed in Refs. [21,22]. We use the variational method with a single Gaussian trial wave function and determine the parameters to reproduce the light-light and light-heavy meson masses. Using multiple Gaussian functions for the trial wave function leads to negligible change to the fitted masses [23]. The resulting parameter values are shown in Table I.

The D meson mass results of the full quark model calculation with a changing constituent quark mass are shown in Fig. 1 as red dots. It can be observed in this figure that the behavior of the naive formula of Eq. (3) is qualitatively reproduced by our more accurate calculation. In Fig. 2, the D meson (blue circles) and D^* mesons (red triangles) masses are given as a function of the constituent quark mass, while Fig. 3 shows the same for B and B^* mesons.

As can be seen in Figs. 2 and 3, the full constituent quark model calculation shows that starting from the fitted constituent quark mass of 324 MeV for the light quark, one finds that for both the D and B mesons, the meson masses increase when the constituent quark mass decreases. The main reason for this effect is that with a decreasing constituent quark mass, the kinetic term forces

TABLE I. Parameters fitted to the experimental meson masses using the variational method with a single Gaussian.

κ	a_0	D	α	β	m_q	m_c	m_b
0.48	5.43 GeV^{-2}	0.911 GeV	2.2 (fm)$^{-1}$	0.277	0.324 GeV	1.83 GeV	5.21 GeV

the relative momentum to become smaller and hence the light quark to probe larger distances. This will thus lead to a larger potential energy coming from the string tension.

This effect is well reflected in the wave function obtained for different constituent quark masses. The blue solid curve in Fig. 4 shows the wave function obtained with the vacuum constituent mass of 324 MeV, while the red dashed curve represents that with a mass of 160 MeV; the black dot-dashed curve shows the Coulomb and confining part of the potential. As can be seen in the figure, the wave function for the smaller mass spreads out more and thus probes higher values of the potential energy. Therefore, while a smaller quark mass will decrease the heavy-light meson mass, the contribution from probing larger confinement properties will increase its total mass.

The spread of the wave function is also responsible for keeping the mass splitting between D and D^* only weakly dependent on the constituent quark mass. As can be seen in Eq. (6), the spin-spin interaction responsible for the mass splitting between the D and D^* meson masses has an overall factor proportional to the inverse of the quark masses. However, when calculating the masses, the Gaussian potential will pick up the strength of the wave function at the origin. Therefore, while the overall constant will enhance the mass splitting between D and D^* as the constituent quark mass decreases, the wave function will be more spread out and reduce the strength at the origin keeping the mass splitting almost independent of the constituent quark mass. Actually, from a simple estimation of the size given in Eq. (2), one can estimate the square of the wave function at the origin is proportional to $|\Psi(0)|^2 \sim [(\sigma m_q)^{1/3}]^3 = \sigma m_q$. Then the splitting is just proportional to $1/m_c$ and independent of m_q.

A few comments are in order here. First, in this simple model we did not introduce any modification of the string

tension in relation to the (partial) restoration of chiral symmetry. This is so because in nuclear matter, while the chiral order parameter is expected to be reduced by more than 20% at normal nuclear matter density, the gluon condensate is expected to change by less than 5%. While there is no direct relation between confinement and gluon condensate, some connection can be made between the Wilson loop and gluon condensate via the operator product expansion (OPE) [24]. Moreover the abrupt (slow) change of the electric (magnetic) condensate across the phase transition can be associated with the critical (soft) change of the space-time (space-space) Wilson loop across the same phase transition temperature [25]. We furthermore note that the picture of unmodified confinement while chiral symmetry is partially restored has some resemblence to the "quarkyonic phase", proposed in [26]. Second, we did not consider any potential $D/\bar{D}$ splitting in medium. Such an effect could be introduced by including a vector meson exchange type interaction of the D mesons with the surrounding medium. Hence, the present mass change should be understood as the average mass shift of these mesons. Third, we expect that the present framework will break down for too small quark masses as the nonrelativistic approximation will become invalid. Fourth, the present argument will not be reliable for light-light meson systems as the hyperfine potential V^{SS} in Eq. (6) will become unrealistically large as the light quark masses decrease. V^{SS} in fact will increase at a $1/m_q^2$ rate with a decreasing constituent quark mass, eventually leading to a negative pion mass, which does not seem to be realistic.

As a last point, let us discuss the results of the D^* and B^* mesons, shown as red triangles in Figs. 2 and 3. It is seen in these figures that these mesons within our model behave almost exactly in the same way as their pseudoscalar counterparts. This means that our model predicts the D^*
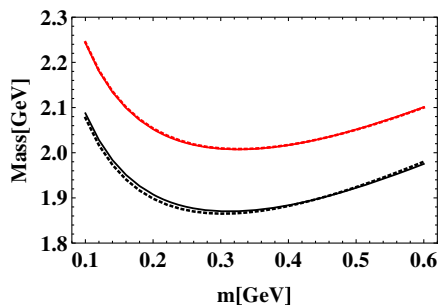
FIG. 2. m_q dependence of D meson (lower curves) and D^* meson (upper curves) masses in the constituent quark model. The dashed lines are the results with $\beta = 0$.
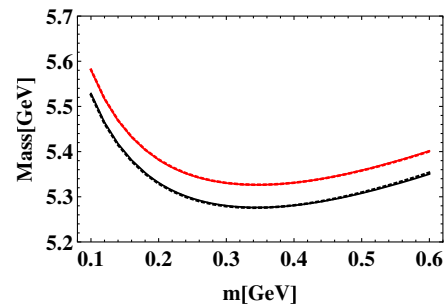
FIG. 3. m_q dependence of B meson (lower curves) and B^* meson (upper curves) masses in the constituent quark model. The dashed lines are the results with $\beta = 0$.
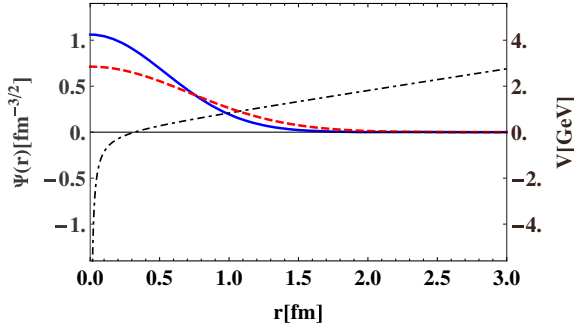
FIG. 4. r dependence of the D meson radial wave function with $m_q = 324$ MeV (blue solid line) and with $m_q = 160$ MeV (red dashed line). The black dot-dashed line represents the sum of the Coulomb and confining potential.

and B^* meson masses to increase as chiral symmetry is restored and the constituent quark mass decreases. The existence of the heavy quark symmetry implies the degeneracy between pseudoscalar and vector heavy-light mesons in the vacuum, which is expected to be intact in the dense matter.[1] Then, the analysis here is consistent with the heavy quark symmetry similarly to the result obtained in Ref. [16]. It is not yet clear whether this behavior will be consistent with the predictions of QCD sum rules, in which the operator product expansion (OPE) relates various order parameters of chiral symmetry to certain properties of the spectral function. In the D^* (or B^*) channel sum rules, the OPE term involving the quark condensate has the same sign as in the D (or B) case [14,28,29], which could indicate an increasing mass as chiral symmetry is restored, consistent with our model. There are, however, more terms present in the OPE, which could modify this naive expectation. Therefore, only a full QCD sum rule analysis done in the same way as in Refs. [10,11] will make a consistency check with our model possible. We leave this topic for future work.

III. SUMMARY

In this work, we have studied masses of heavy-light meson in a dense environment such as nuclear matter, in

which chiral symmetry is partially restored. For this purpose, we have made use of a simple quark model with one heavy and one constituent light quark. This constituent quark becomes lighter as chiral symmetry is restored and therefore likely changes the mass of the whole heavy-light meson. Naively, one would expect the heavy-light meson to decrease its mass as one of its constituents becomes lighter. We have however shown in this article that this is not necessarily the case. Assuming that the confining potential remains approximately constant at normal nuclear matter density, we have demonstrated that the wave function spreads out as the constituent quark mass decreases and therefore receives a larger potential energy due to the linearly rising confining potential. For constituent quark masses below about 0.3 GeV, this effect leads to an increase of the heavy-light meson, as shown for instance in Fig. 1. In our quark model calculations, we have examined states containing both a charm and bottom quarks and found the same behavior for both cases (see Figs. 2 and 3). Our results for D and B mesons are consistent with recent sum rule analyses which obtain increasing masses with increasing density [10,11] and with calculations based on the Skyrme crystal model [16]. A Polyakov-Nambu-Jona-Lasinio model calculation finds that the decrease in the constituent quark mass leads to an increase in the D^- mass due to the Pauli blocking effect [30]. Hence, for a more detailed study, it would be necessary to combine the Pauli blocking effect into our work and thereby also probe the splitting between D and $\bar{D}$ in the nuclear medium. Moreover, to consistently take into account the splitting between D and $\bar{D}$ as well as medium effects such as screening and Pauli blocking, a T-matrix approach, discussed for instance in Refs. [31,32], should be pursued in the future.

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