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Poincaré covariant pseudoscalar and scalar meson spectroscopy in Wigner-Weyl phase

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The coupled quark Dyson-Schwinger and meson Bethe-Salpeter equations in rainbow-ladder truncation for spin-0 mesons are solved in the Wigner-Weyl phase in the chiral limit and beyond, retaining only the ultraviolet finite terms of the phenomenologically most successful Maris-Tandy interaction. This allows one to reveal and discuss the scalar and pseudoscalar meson masses in a chirally symmetric setting without additional medium effects. Independent of the current-quark mass, the found solutions are spacelike, i.e., have negative squared masses. The current-quark mass dependence of meson masses, leptonic decay constants and chiral condensate are illustrated in the Wigner-Weyl phase.

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I. INTRODUCTION

The major mass fraction of visible matter in the universe originates in the nonperturbative momentum regime of quantum chromodynamics (QCD). There is consensus that dynamical chiral symmetry breaking (DχSB) and/or confinement dynamically generate hadron masses that are several orders of magnitude larger than the current masses of the underlying valence constituents. However, the interrelation of DχSB and confinement is nowhere near understood and has recently been discussed in [1–4] within a lattice-regularized QCD (LQCD) approach. While nonobservable color charges and the nondegeneracy of chirality partners are clearly associated with confinement and DχSB,[1] respectively, dynamically generated hadron masses are attributed to either the first or the second. If confinement and DχSB are tantamount and equivalent to each other, such a distinction is, of course, meaningless, and an underlying mechanism causing both might be conceivable. In any case, investigating the origin of mass is one way to approach this issue.

Therefore, future and existing large-scale experiments aim at studying the properties of visible matter under extreme conditions, i.e., large temperatures and/or densities, e.g., [5,6], also in order to reveal if and how DχSB and confinement are related to each other and if the mechanisms that are believed to generate the hadron properties are compatible with the physics observed there. Here a phase transition or crossover toward the chirally symmetric and deconfined phase is anticipated. In turn the properties of matter under extreme conditions, in particular under chiral symmetry restoration, are also theoretically widely discussed. An interesting aspect of this endeavor is the question, what would be the properties of matter under the restoration of chiral symmetry isolated from density or temperature effects?

Poincaré covariant and symmetry preserving calculations of the hadron spectrum within a combined Dyson-Schwinger (DS) equation and Bethe-Salpeter (BS) equation approach [7–9] generate the crucial nonperturbative quark mass dressing that suffices to explain the nonexotic [10,10–21] and study the exotic [22–25] hadron spectrum. Though this quark mass dressing is not an observable quantity, it is a valuable intuition building ingredient. Here the axial-vector Ward-Takahashi identity (AVWTI) ensures compliance with chiral symmetry and its dynamical breakdown at all energy scales [26–28]. For a reliable hadron phenomenology it is thus a crucial constraint. In view of this, investigating hadron properties in a world that is initially chirally symmetric within the DS-BS approach seems natural. A chirally symmetric scenario without density and temperature might be considered as the simplest approximation to the complicated dynamics governing the QCD phase diagram, which still exhibits a phase transition. In particular, such a scenario might show if restoration of chiral symmetry is sufficient for deconfinement.[2] Deviations from the results and predictions of such a scenario are then attributed to medium effects. Furthermore, the presented approach allows one to study the effect of explicit chiral symmetry breaking and to compare it to DχSB effects.

In the chiral quark mass limit, the pion would be massless in the Nambu-Goldstone (NG) phase of chiral symmetry. Its finite mass stems from the small explicit current-quark masses. It is much smaller and clearly separated from the next heavier meson, the ρ meson. While the mass of the latter can intuitively be understood by the large finite dynamically generated quark masses, the pion, with the same quark content, is so light due to its

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[1] These are the defining phenomena.

[2] Conversely, confinement would be sufficient for DχSB.

nature as a pseudo-Goldstone boson. The latter must necessarily exist in the spectrum if chiral symmetry is spontaneously broken. From this perspective it is by no means clear what the pion mass would be in the Wigner-Weyl (WW) phase of chiral symmetry, where neither significant dynamical quark masses are generated nor the need for massless Goldstone bosons arises.

Experimentally, the pion dynamics and the pion mass in a WW phase realization without additional medium effects is not known and, most probably, will never be known. Nevertheless, in view of the unclear interrelation of confinement and DχSB, in particular qualitative statements that are not interfered with additional medium effects, such as collisional broadening, are most enlightening and guide expectations. Calculations within the Nambu–Jona-Lasinio model at nonzero temperature and baryon density [29,30] point to a (monotonically) increasing pion mass. In [31] the question about the ρ meson properties in a scenario that is chirally symmetric in vacuum has been posed and answered for the first time within QCD sum rules (QSRs).

Similar investigations regarding the spectrum of spin-1 and 2 mesons, as well as nucleons, in such a scenario have recently been performed in [32–34], where the authors observed a new SU(4) symmetry in the hadron spectrum. Within these studies, no spin-0 mesons have been found in the meson spectrum. In the same spirit one may employ the phenomenologically successful, symmetry preserving, coupled DS-BS approach to probe the effect of exclusive chiral symmetry restoration on hadronic properties as such and, in particular, disentangled from many-body effects. In particular, this approach is genuinely Poincaré covariant and, thus, correctly reflects the related phenomena in the hadron spectrum, whatever these might be. As the WW spectrum is experimentally unknown and guidance is missing from this side, such a Poincaré covariant approach is well justified. Early studies with the same scope have also been done in [35], and references therein, for simplified confining models. This is feasible because the coupled DS-BS approach naturally entails solutions in the WW phase. Note that the WW phase solution to the rainbow ladder (RL) truncated DS-BS equation approach is as valid and consistent as the NG solution. In this spirit, it has the advantage that it does not require a deformation of the theory contrary to current QSR or LQCD based approaches. On the other hand, one is forced to employ a model interaction rather than a first principle QCD calculation. Nevertheless, because the Maris-Tandy (MT) model, on which the employed Alkofer-Watson-Weigel (AWW) model [12] is based, is phenomenologically very successful with a solid and well-founded relation to QCD, enlightening results are found.

In Secs. II and III the quark DS and meson BS equations in RL truncation together with the employed AWW model interaction are presented. Results are presented and discussed in Sec. IV with conclusions in Sec. V.

II. QUARK DS EQUATION

The employed DS equation in rainbow truncation for the nonperturbative quark propagator reads

$$S(p)^{-1} = Z_2(i\gamma\cdot p + Z_4 m_q) + \Sigma(p), \tag{1a}$$

$$\Sigma(p) = C_\mathrm{F}\int_q^\Lambda \mathcal{G}((p-q)^2) D^\mathrm{f}_{\mu\nu}(p-q)\gamma_\mu S(q)\gamma_\nu, \tag{1b}$$

where the Casimir color factor is given as $C_\mathrm{F} = (N_\mathrm{c}^2-1)/2N_\mathrm{c}$ and the number of color degrees of freedom is $N_\mathrm{c}=3$. $\Sigma(p)$ is called the quark self-energy or mass shift operator, and $D^\mathrm{f}_{\mu\nu}(l) = (\delta_{\mu\nu} - l_\mu l_\nu/l^2)$ is the transversal projector part of the free gluon propagator in the Landau gauge. The model interaction $l^2\mathcal{G}(l^2)$ algebraically replaces the gluon propagator dressing function and is intended to imitate the combined effects of omitted quark-gluon vertex terms, gluon propagator dressing function, and running coupling. $\int_q^\Lambda = \int^\Lambda \frac{\mathrm{d}^4 q}{(2\pi)^4}$ is a translationally invariant regularized integration measure with regularization scale Λ [26]. Z_2 and Z_4 are quark wave function and quark mass renormalization constants, whereas the quark gluon-vertex renormalization constant is absorbed in the model interaction $\mathcal{G}((p-q)^2)$. The current-quark mass is denoted by m_q.

With the decompositions

$$\begin{aligned} S(p)^{-1} &= i\gamma\cdot pA(p^2) + B(p^2) = Z(p^2)(i\gamma\cdot p + M(p^2)) \\ &= [-i\gamma\cdot p\sigma_\mathrm{V}(p^2) + \sigma_\mathrm{S}(p^2)]^{-1}, \end{aligned} \tag{2}$$

Eq. (1) defines a system of inhomogeneous, nonlinear, singular, coupled Fredholm integral equations of the second kind for the propagator dressing functions A and B. Depending upon details of the specific interaction, e. g., strength, multiple solutions exist. Most commonly two different solution strategies are employed: fixed-point iteration and optimization algorithms.

Solutions in the chiral limit with nonzero chiral quark condensate are identified as solutions in the NG phase. Those with zero chiral condensate are assumed to be solutions in the WW phase.[3] Furthermore, we assume that the transition from one solution in a certain phase of chiral symmetry to another solution in the same phase but for a different current-quark mass is continuous in the current-quark mass m_q. This allows one to identify solutions in the WW phase beyond the chiral limit. Finally it is assumed that the gluon and quark-gluon dynamics is not affected by a transition to the chirally symmetric phase or that the corresponding corrections are at least sufficiently small to

[3]Note that in general a zero chiral condensate is necessary but not sufficient for chiral symmetry realizations. See also the discussion in Sec. IV.

give reasonable qualitative results when neglected. Clearly, because of the coupling of the gluon propagator and quark-gluon vertex to the quark propagator (see, e.g., [36,37] or [38,39] for an intuitive model), this is an approximation. Up to now, the gluon propagator in WW phase is unknown.

The phenomenologically most successful model within the RL truncated DS-BS approach to the meson spectrum is the MT model developed in [11] with the most recent applications to a comprehensive meson phenomenology in [10,18,19,22]. It consists of an infrared part and an ultraviolet part. The latter is crucial to ensure the proper perturbative limit of the running coupling in QCD. However, it is of minor qualitative importance for meson spectroscopy [12,40]. In particular, it can be neglected when mainly qualitative aspects are investigated, as in the scope of the current investigation.

The AWW parametrization [12] of the interaction reads

$$\mathcal{G}(q^2) = 4\pi^2 D \frac{q^2}{\omega^2} e^{-\frac{q^2}{\omega^2}}. \tag{3}$$

It is ultraviolet finite, all renormalization constants are equal to one, and the limit $\Lambda \to \infty$ can be taken initially. In particular, it obeys a chirally symmetric solution of Eq. (1) if $m_q = 0$, which can easily be obtained by employing $B_0(p) = 0$ as the initial function for a fixed-point iteration. It features $M(p) = 0$ with $A(p) \neq 0$; cf. Fig. 1 for a comparison to the chiral NG phase solution and the employed parameters. Both dressing functions have the same asymptotic behavior in the WW phase as in the NG phase. While $M(p)$ in the WW phase significantly deviates from its NG phase, $A(p)$ differs only below ≈1 GeV.

Among others, the Newton-Krylov root finding method [41], which is suitable for large scale optimizations, can be used to find solutions in the WW phase beyond the chiral limit as well [42,43]. Figure 1 depicts the WW solution for different bare current-quark masses m_q up to a critical mass $m_q^{\mathrm{cr}} = 31$ MeV, above which no solution in the WW phase has been found for the employed set of parameters in the present setup. The quark mass functions $M(p)$ resemble roots at ≈1 GeV and deviate significantly from each other. Again, the propagator functions $A(p)$ only deviate significantly below ≈1 GeV from each other. The characteristic excess of this function at ≈1 GeV remains unaffected. Consequently, even in the chiral limit and WW phase a simple constituent quark picture or the modeling of quark bound states by virtue of free quark propagators is not applicable or justified in the light quark sector. Finally, both figures resemble the relevance of the 1 GeV scale for DχSB and its restoration. However, the rigorous interrelation of the characteristic DχSB scale and the critical current-quark mass m_q^{cr} within this model remains unknown.

III. MESON BS EQUATION

To respect the fundamental symmetries of the underlying interaction, the kernels of DS and BS equations must satisfy the AVWTI. This can be shown to be true for the RL truncated DS-BS approach. The RL truncated homogeneous meson BS equation reads

$$\Gamma(p;P) = -C_{\mathrm{F}} Z_2^2 \int_q^\Lambda \mathcal{G}((p-q)^2) D^{\mathrm{f}}_{\mu\nu}(p-q) \times \gamma_\mu S_1(q_+) \Gamma(q;P) S_2(q_-) \gamma_\nu, \tag{4}$$

with $q_\pm = q \pm [1/2 \pm (\eta - 1/2)]P$ and $\Gamma(p;P)$ is the Bethe-Salpeter amplitude (BSA). In what follows, the momentum partitioning parameter is set to $\eta = 1/2$.

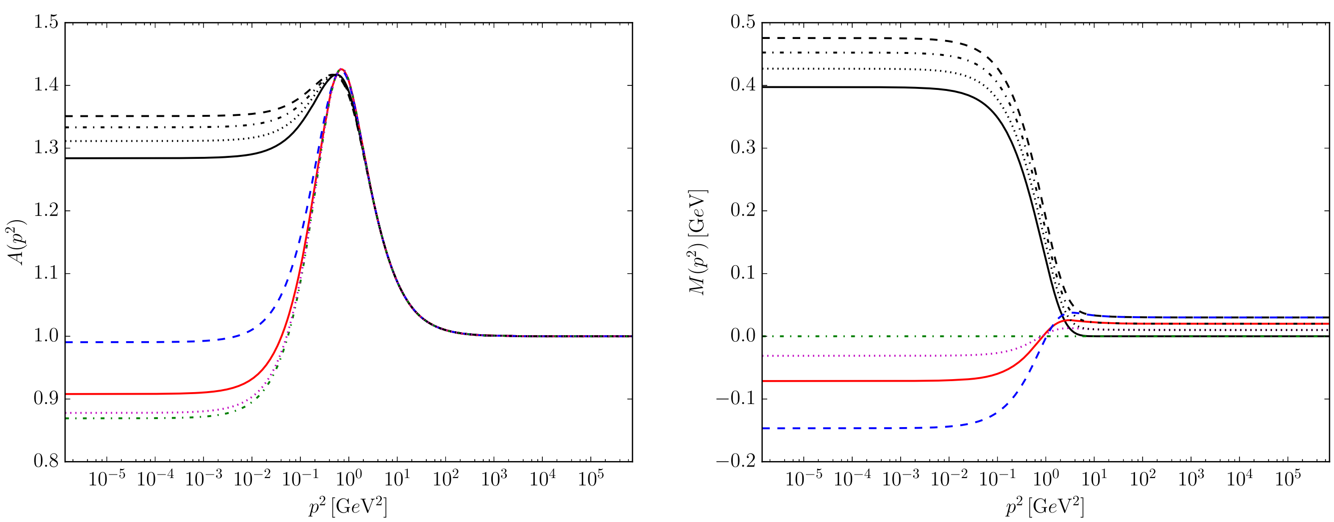

FIG. 1. Propagator functions $A(p^2)$ (left panel) and $M(p^2)$ (right panel) along the real axis obtained from Newton-Krylov optimization for different initial functions $B_0(p^2)$ at $\omega = 0.5$ GeV and $D = 16.0$ GeV^2 in the WW phase for $m_q = 0$ (solid red curve), $m_q = 10$ MeV (dashed blue curve), $m_q = 20$ MeV (dash-dotted green curve), and $m_q = 30$ MeV (dotted magenta curve). Solutions in the NG phase are depicted in black with line styles that correspond to the WW phase.

The solution strategy and covariant basis follow Refs. [44–47]. The BS amplitude is expanded in a finite set of covariants $T_n(p,P,\gamma)$, which specify the quantum numbers J and P, according to

$$\Gamma(p;P) = \sum_n \Gamma_n(p^2, \cos\sphericalangle(P,p); P^2) T_n(p,P,\gamma), \quad (5)$$

with $\Gamma_n(p^2, \cos\sphericalangle(P,p); P^2)$ being the partial amplitudes. Canonical normalization, leptonic decay constants, residual, and (generalized) Gell-Mann–Oakes–Renner (GMOR) relation are evaluated according to [11].

Having the propagator functions in the WW phase for zero current-quark mass at disposal, one can solve the BS equation in the WW phase and obtain an explicit value of the bound state mass. As in [35], the lowest bound state mass where the $J=0$-BS equation (4) in the chiral limit and the parameters of Fig. 1 can be solved is found at spacelike masses $M^2 = -0.1172\ \mathrm{GeV}^2 = -(342.1\ \mathrm{MeV})^2$, which are called tachyonic solutions. For timelike bound state momenta, the BS equation integration domains of the quark propagators extend to the complex plane, and the analytical structure must be accounted for [45,48]. For spacelike momenta, the integration domain is limited to the positive real axis where no nonanalytical behavior has been observed in the WW phase and the solution of the coupled DS-BS system is straightforward.

As argued in [49] the WW (tachyonic) solution corresponds to a maximum of the effective action. Therefore, the squared mass must be negative and signals the instability of the chirally symmetric ground state. An arbitrary small disturbance drives the system from the WW realization to the NG realization. Conversely, if a stable chirally symmetric phase is to be realized at high densities and/or temperatures, the here neglected medium effects must, thus, eliminate all tachyonic solutions.

For completeness it is noted that the result for the NG pseudoscalar bound-state mass is $M_\pi = 137$ MeV with $f_\pi = 94.1$ MeV, and the NG scalar bound-state mass is $M_\sigma = 645$ MeV for $m_q = 5$ MeV and the parameters as in Fig. 1.

IV. RESULTS AND DISCUSSION

The analytic properties of the quark propagator may be analyzed as in [45] by Cauchy's argument principle or utilizing a Newton-Krylov root finding method. In the NG phase, the quark propagator has a tower of complex conjugated poles off the real axis [45]. In the WW phase, the pole that is closest to the origin, i.e., the first relevant pole for timelike bound states, can be found at $q^2 \approx -0.225 \times 10^{-3}\ \mathrm{GeV}^2$, i.e., on the real axis and rather close to the origin. It corresponds to a maximal accessible quarkonia bound-state mass of $M \approx 30$ MeV if the pole has to be kept outside of the integration domain in Eq. (4). Within the momentum region $-1\ \mathrm{GeV}^2 \leq \mathrm{Re}p^2 \leq 0$, $|\mathrm{Im}p^2| \leq 1\ \mathrm{GeV}^2$ no complex conjugated poles (off the real axis) have been found. Furthermore, apart from σ_V, all propagator functions for $m_q = 5$ MeV [cf. Eq. (2)] have inflection points below 3 GeV^2 in the WW phase. Such inflection points have been argued in, e.g., [50], to be sufficient for confinement. Clearly, for $m_q = 0$ the propagator functions $B(p^2)$, $M(p^2)$, and $\sigma_S(p^2)$ have no inflection points in the WW phase.

The fact that the chiral limit WW solution is compatible with a realization of chiral symmetry, i.e., Eq. (4) gives identical BS matrices and BS amplitudes for chiral partner mesons and degenerate mass spectra, can be seen in a twofold way.

First, the chiral condensate, which is given within the employed model (3) for the gluon propagator by [51,52]

$$\langle :\bar{q}q: \rangle = -\frac{3}{4\pi^2} \int_0^\infty \mathrm{d}^2 l l^2 \sigma_\mathrm{S}(l^2), \quad (6)$$

is zero, because $B(p) = 0$. In the NG phase a value of $\langle :\bar{q}q: \rangle = (-251\ \mathrm{MeV})^3$ is obtained for the employed set of parameters, which is in agreement with the (traditional) GMOR relation

$$f_\pi^2 M_\pi^2 = -2m_q \langle :\bar{q}q: \rangle. \quad (7)$$

The leptonic decay constant of the pion within the AWW model is

$$\frac{f_\pi}{3} = \int_q \frac{\mathrm{Tr}[\gamma_5 \gamma \cdot P \chi^{0^-}(q;P)]}{\sqrt{2} P^2} \bigg|_{P^2=-M_\pi^2}, \quad (8)$$

with

$$\chi^{0^-}(q;P) \equiv S_1(q_+) \Gamma^{0^-}(q;P) S_2(q_-)$$

and $\Gamma^{0^-}(q;P)$ the pion's BSA. Equation (7) can be extended to the above mentioned generalized GMOR relation [26–28]

$$f_\pi M_\pi^2 = 2m_q r_\pi, \quad (9)$$

where the residue of the pion mass pole in the pseudoscalar vertex of the AWW model is given by [26,27]

$$\mathrm{i} r_\pi = \int_q \frac{\mathrm{Tr}[\gamma_5 \chi^{0^-}(q;P)]}{\sqrt{2}} \bigg|_{P^2=-M_\pi^2}. \quad (10)$$

Equation (9) is valid for all current-quark masses and pseudoscalar states. Thus, it provides a natural definition of the chiral condensate that is valid beyond the chiral limit [28,53],

$$\langle :\bar{q}q: \rangle \equiv -f_\pi r_\pi. \quad (11)$$

As the chiral condensate transforms nontrivially under the chiral transformation group, the vanishing of the chiral

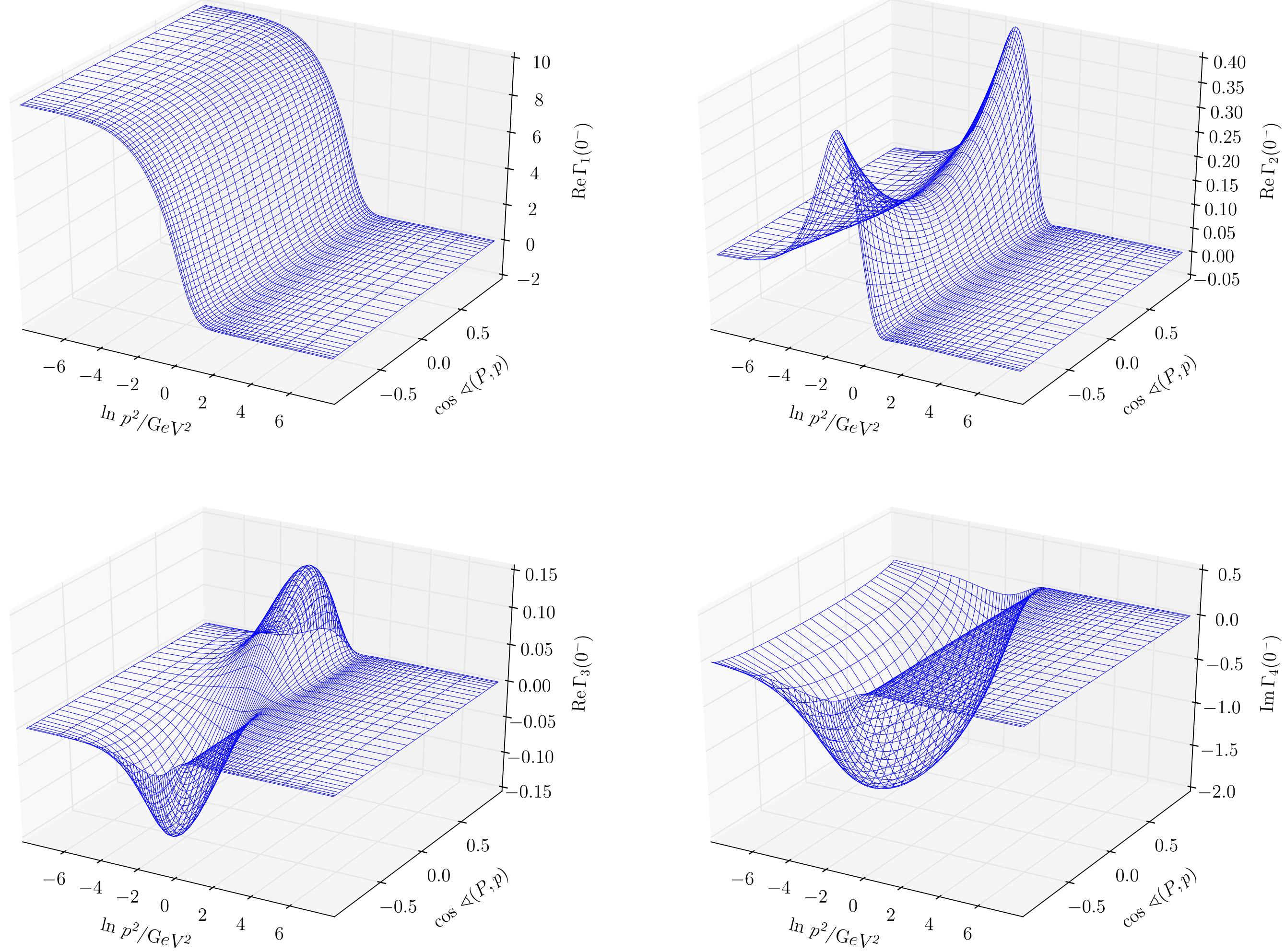

FIG. 2. Nonzero parts of the pseudoscalar BS partial amplitudes in the WW phase for $m_q = 5$ MeV, $\omega = 0.5$ GeV, and $D = 16.0\ \mathrm{GeV}^2$. All other parts are $\lessapprox 10^{-14}$.

condensate is a necessary requirement for restoration of the symmetry. However, strictly speaking the vanishing of an order parameter, which is qualified as such solely by means of its transformation properties, is merely a necessary but not sufficient requirement for the realization of a symmetry. The realization of other symmetries or complicated medium effects may lead to vanishing condensates as well, which has been discussed in some detail for four-quark condensates in [54,55]. A rather drastic example has recently been discussed in [56,57]. In this spirit, determination of the phase transition temperature and/or density by virtue of the vanishing chiral condensate (6) only gives a lower bound as all nontrivially transforming condensates must be zero in the chirally symmetric phase. In view of this, an accidental simultaneous vanishing of all such condensates with increasing temperature and/or density would indicate a systematic interrelation among these condensates which is up to now not known. Within the context of open flavor chiral partner QSRs [58–61], considering a zero chiral condensate as a sufficient condition for chiral symmetry restoration, in the sense that no nontrivially transforming condensate points to a larger restoration temperature/density, corresponds to the claim that the lowest spectral moment of chiral partner spectra is the last one to vanish with respect to increasing temperature/density.[4] There is no proof of such a claim up to now.

Second, it can be seen by degeneracy of the solutions to the BS equation for chiral partners simply by the fact that the quark mass functions are zero. In RL truncation it can be shown that the BS equation integral kernels for scalar and pseudoscalar mesons only differ by terms proportional to quark mass functions $M(p)$ [44]. Since the chiral limit BS equations for scalar and pseudoscalar mesons are, therefore, identical in the WW phase, the BS amplitudes and, hence, any observables are, too. Strictly speaking degeneracy of the spin-0, or any other subset of the meson

[4] As the chiral condensate is the nontrivially transforming condensate with the lowest mass dimension, such a scenario seems indeed natural or at least tempting. However, scenarios with DχSB and zero chiral condensate have been discussed some time ago in [62–64] and recently again in [65].

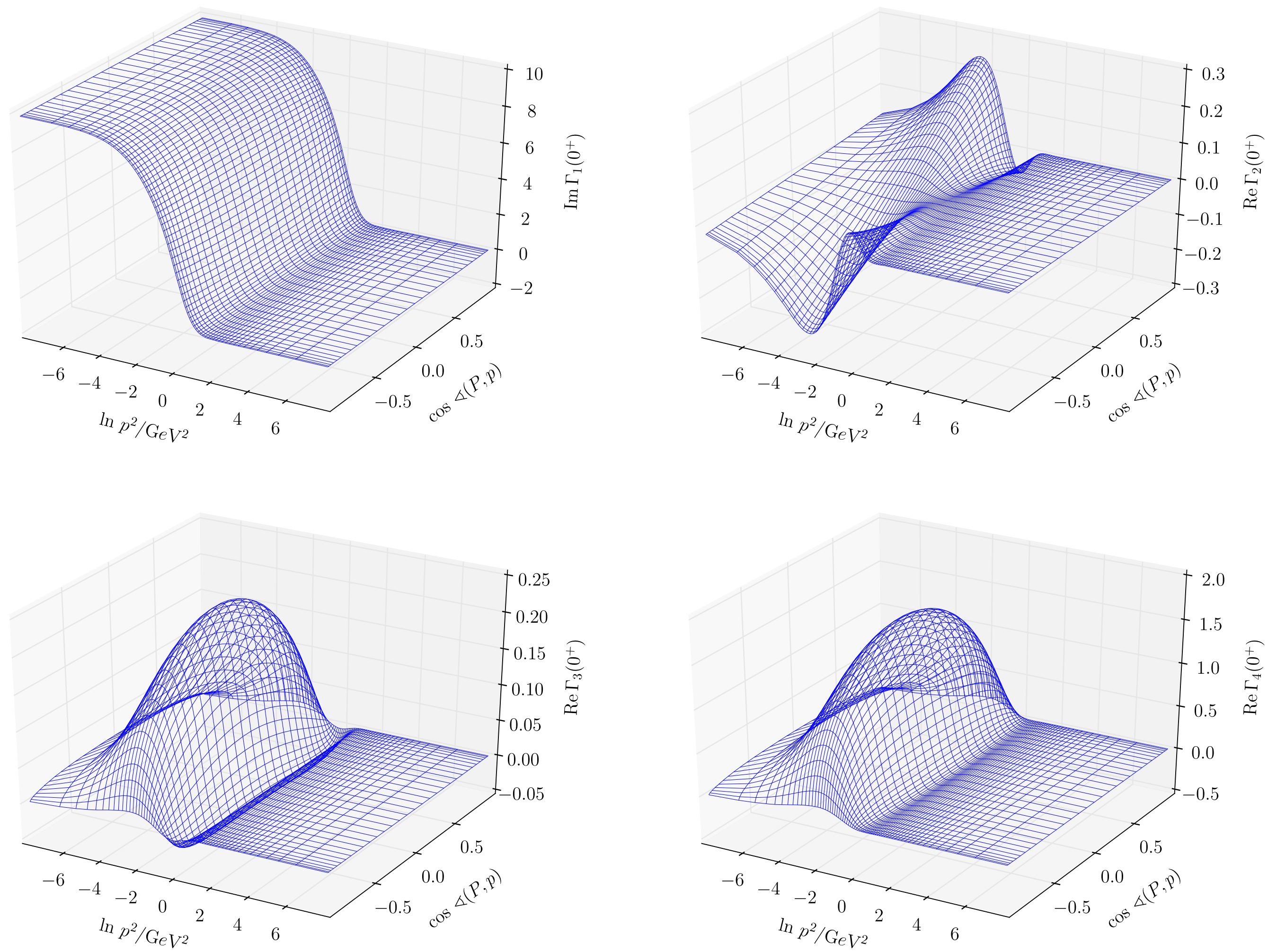

FIG. 3. Nonzero parts of the scalar BS partial amplitudes in the WW phase for $m_q = 5$ MeV, $\omega = 0.5$ GeV, and $D = 16.0$ GeV^2. All other parts are $\lessapprox 10^{-15}$.

spectrum, alone is again, as in the case of nontrivially transforming condensates, not a sufficient but only a necessary requirement for chiral symmetry. All chiral partner spectral densities must be degenerate. However, as can be seen from chiral partner QSRs [61,66] the spectra of chiral partners can be degenerate only if a complete hierarchy of nontrivially transforming condensates is zero. In particular, it has been explicitly demonstrated that if, e.g., scalar and pseudoscalar open flavor mesons are degenerate within the scope of QSRs, the same holds true for vector and axial-vector open flavor mesons [61].

In Figs. 2 and 3 the nonzero parts of the complex canonically normalized partial amplitudes for scalar and pseudoscalar mesons in the WW phase at $m_q = 5$ MeV are exhibited. The major nonzero amplitudes, Γ_1, are very similar in both channels. The other amplitudes drastically differ. Note that imaginary and real parts of some partial amplitudes interchange their roles; i.e., what is zero in one channel is nonzero in the other. This is related to the particular choice of covariants and P^2 being spacelike. For comparison, the nonzero parts of the complex canonically normalized partial amplitudes in the NG phase at $m_q = 5$ MeV for scalar and pseudoscalar mesons are exhibited in Figs. 4 and 5. The changing pattern is similar but not identical. For example, imaginary and real parts of all partial amplitudes interchange their roles. Turning from the NG to the WW phase in the scalar channel, all but the fourth partial amplitude switch from vanishing real to vanishing imaginary part and vice versa. In contrast, in the pseudoscalar channel, only the first partial amplitude switches. Furthermore, up to a sign change in $\Gamma_4^{0^-}$, all amplitudes show the same qualitative behavior, in particular the same symmetries. In the scalar channel, there is no sign change when passing over from one phase to the other.

In [26] it has been shown that

$$f_\pi \Gamma_1^{0^-}(p^2, \cos\sphericalangle(P,p); P^2)|_{P^2=0} = 2B(p^2) \qquad (12)$$

follows from the chiral limit AVWTI. As $B(p^2)$ is zero in the chiral limit WW phase solution and the partial

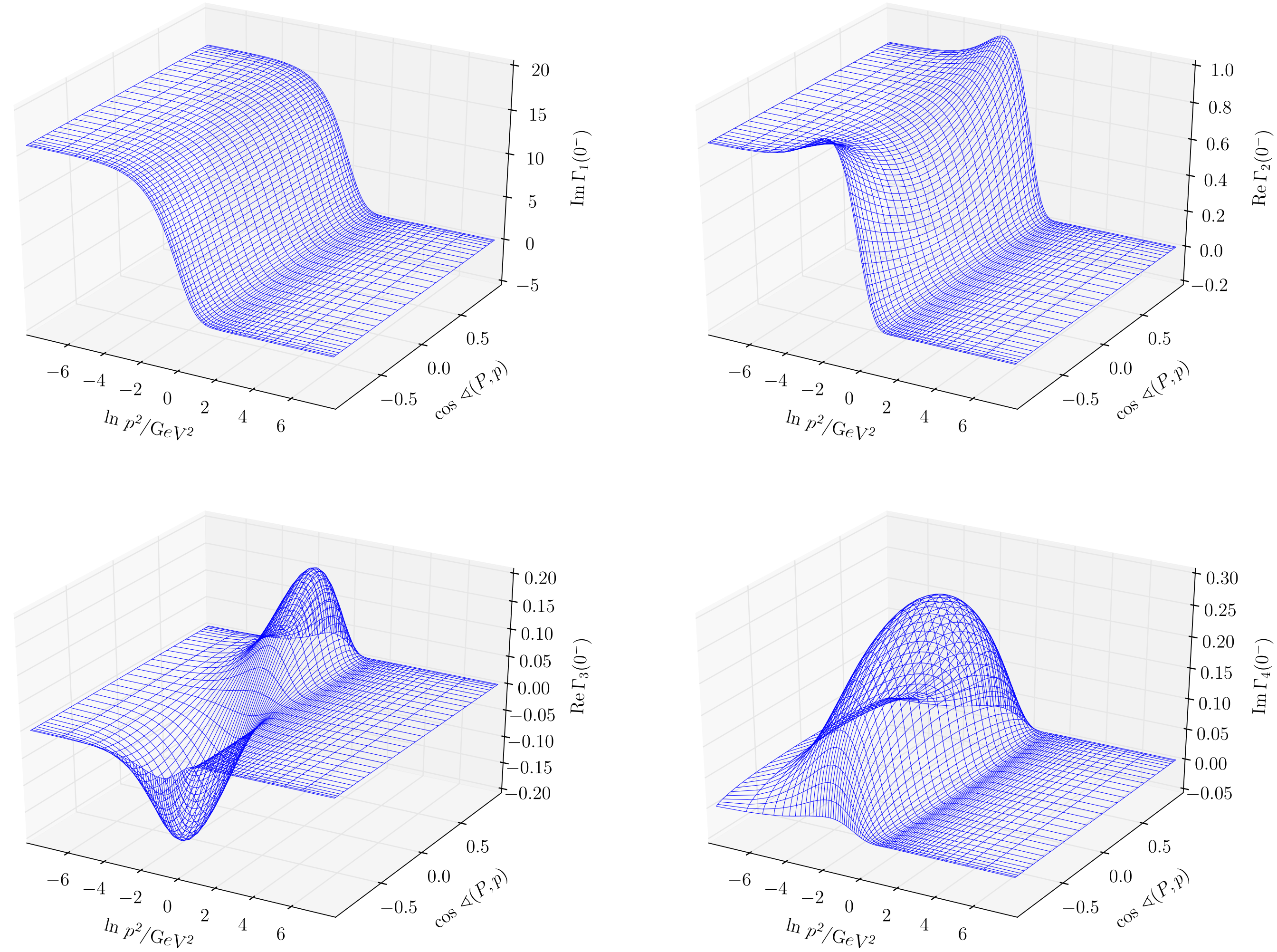

FIG. 4. Nonzero parts of the pseudoscalar BS partial amplitudes in the NG phase (pion) for $m_q = 5$ MeV, $\omega = 0.5$ GeV, and $D = 16.0\ \mathrm{GeV}^2$. All other parts are $\lessapprox 10^{-14}$.

amplitude $\Gamma_1^{0^-}$, as well as all other amplitudes, are not, $f_\pi = 0$ must hold in order not to violate the AVWTI. Indeed, for $m_q = 0$ one has $f_\pi \lessapprox 10^{-14}$ GeV within this study, which numerically confirms the above conclusion.

In [43,67–69] the WW solution and so-called noded solutions[5] to the DS equation have been used to discuss the chiral condensate beyond the chiral limit. Linear combinations of the quark propagators have been introduced, which all generate identical condensates in the chiral limit. However, because of the nonlinearity of the DS equation, a linear combination of solutions cannot fulfill the corresponding DS equation and, therefore, does not represent a self-consistent solution to the given DS equation.

[5]Solutions of the DS equation in the NG phase do not have roots along the positive real axis. Solutions in the WW phase have one root (node). Other solutions have more than one node and are, therefore, dubbed noded solutions. In analogy to vibrating strings, they are sometimes referred to as excited solutions [70].

Having WW solutions for the quark DS equation beyond the chiral limit at disposal, one is able to study a scenario only with explicit symmetry breaking in the scope of a coupled DS-BS approach. Such a scenario reveals the amount of mass splitting in the parity doublet caused by finite quark masses, shows the effect of explicit symmetry breaking on order parameters, and allows for qualitative discussions related to DχSB, its restoration, and the relation to confinement. For the scalar and pseudoscalar channels the masses are shown in Fig. 6. They are spacelike over the whole current-quark mass region. Moreover, the pseudoscalar squared bound state mass M^2 is even decreasing with increasing current-quark mass. While the modulus of the pion mass scarcely changes by 20 MeV for a change of the current-quark mass of 30 MeV and stabilizes toward m_q^{cr}, the (imaginary) scalar mass decreases by more than 120 MeV with a still increasing slope modulus. As expected, the chiral limit behavior in the WW phase qualitatively differs significantly from the limit in the NG phase. In the NG phase a strong current-quark mass

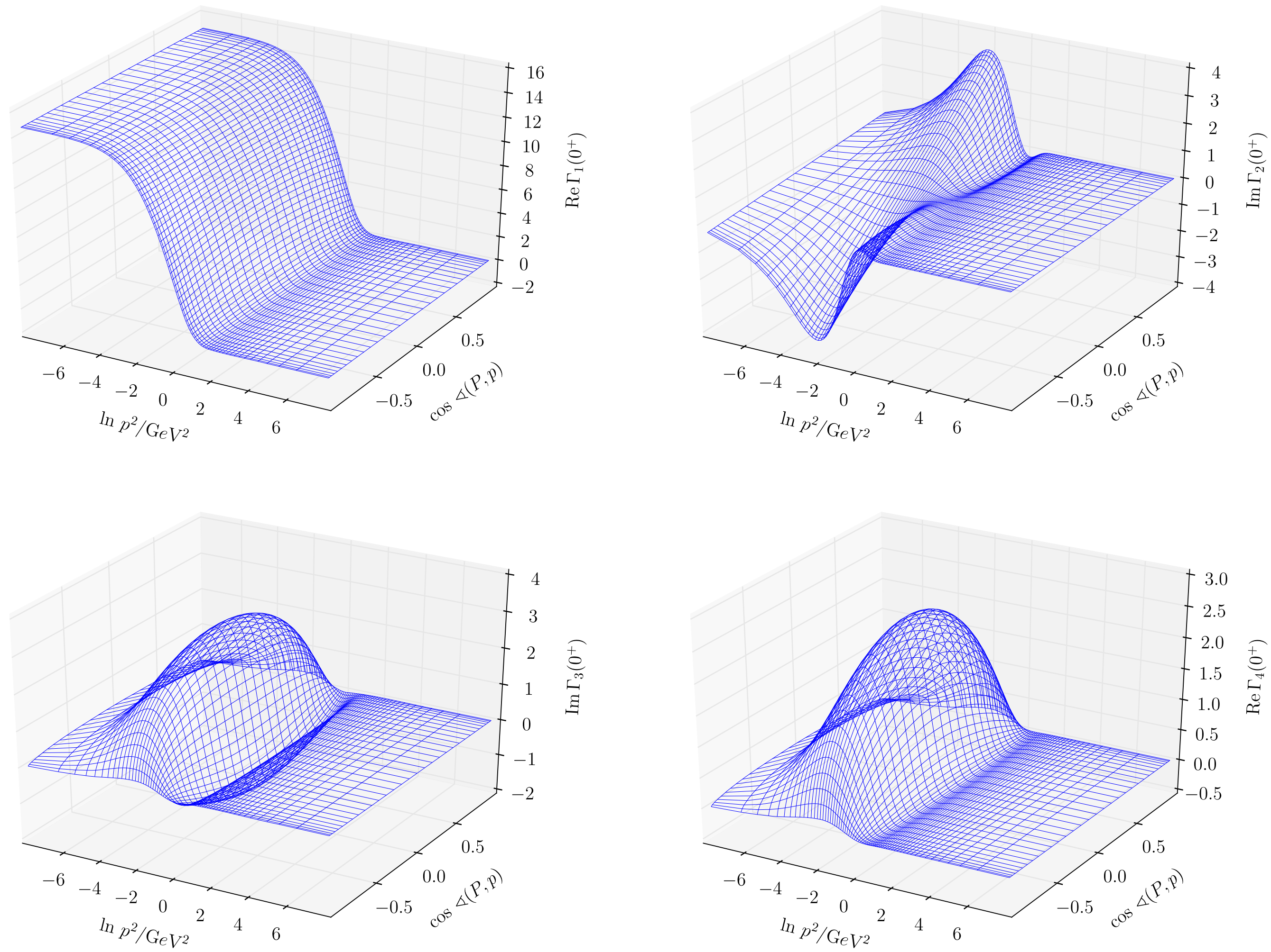

FIG. 5. Nonzero parts of the scalar BS partial amplitudes in the NG phase (sigma) for $m_q = 5$ MeV, $\omega = 0.5$ GeV, and $D = 16.0\ \mathrm{GeV}^2$. All other parts are $\lessapprox 10^{-14}$.

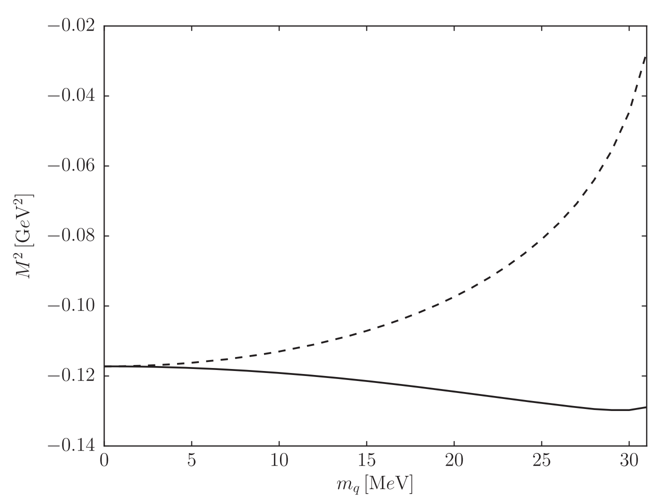

FIG. 6. Bound state masses in the WW phase for equal quarks in the pseudoscalar (solid curve) and scalar (dashed curve) channel up to the critical current-quark mass $m_q^{\mathrm{cr}} = 31$ MeV, $\omega = 0.5$ GeV, and $D = 16.0\ \mathrm{GeV}^2$.

dependence of the scalar and, even more, the pseudoscalar bound state mass is observed, whereas the slope of the bound state mass curve in the WW phase approaches zero in the chiral limit. Evaluating the formal splitting of scalar and pseudoscalar mesons at a current-quark mass of $m_q = 5$ MeV yields $|\Delta M| = 1.6$ MeV, which is tiny as compared to the splitting of chirality partners due to DχSB (roughly $|\Delta M| \approx 350, \ldots, 450$ MeV). However, it is of the order of the mass splitting in the isospin multiplet.

The leptonic decay constant, f_π, in the WW phase is depicted in Fig. 7. Apparently, f_π rises linearly with m_q. Hence, the current-quark mass dependence in the WW phase qualitatively differs from the NG phase [28]. Interestingly, the absolute change of f_π of ≈ 50 MeV is larger than the corresponding change in the NG phase. For $m_q = 5$ MeV, $f_\pi \approx 8.5$ MeV, which is $\approx 9\%$ of its NG phase value. Note that over the whole current-quark mass region employed within this study, deviations from the generalized GMOR, Eq. (9), are of the order 10^{-3} or below.

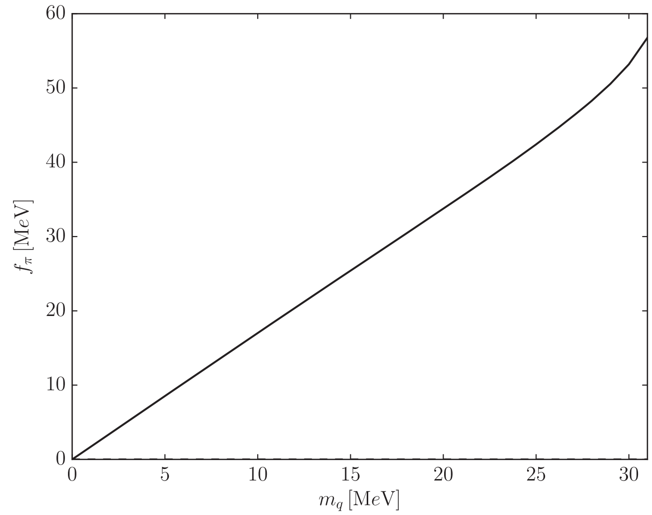

FIG. 7. Leptonic decay constant in the WW phase for equal quarks in the pseudoscalar channel up to the critical current-quark mass $m_q^{\rm cr} = 31$ MeV, $\omega = 0.5$ GeV, and $D = 16.0\ {\rm GeV}^2$. The scalar leptonic decay constant is numerically zero, $f_\sigma \lesssim 10^{-14}$ GeV, for all current-quark masses as in the NG phase [71].

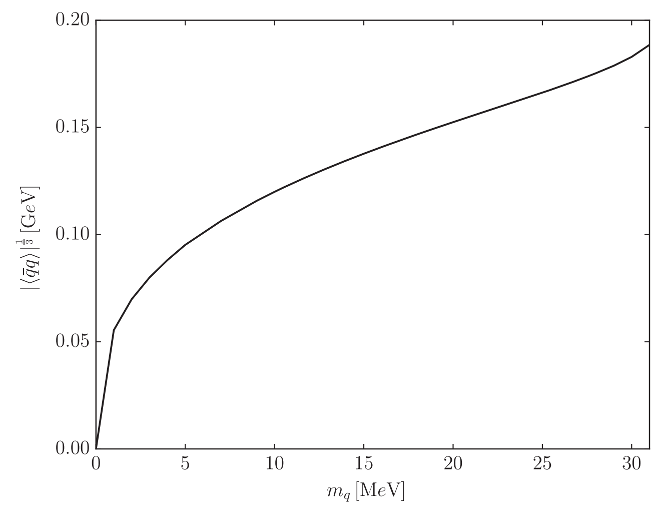

FIG. 8. Chiral condensate in WW phase up to the critical current-quark mass $m_q^{\rm cr} = 31$ MeV, $\omega = 0.5$ GeV and $D = 16.0\ {\rm GeV}^2$.

Finally, in Fig. 8 the chiral condensate in the WW phase is depicted beyond the chiral limit [cf. Eq. (11)] and reveals the impact of explicit chiral symmetry breaking only. Similar to f_π, it rises linearly with the current-quark mass up to $m_q \approx 25$ MeV. Explicit chiral symmetry breaking by current-quark masses of $m_q \approx 30$ MeV mimics roughly 40%–50% of the NG phase values for the leptonic decay constant and chiral condensate. For $m_q = 5$ MeV, $|\langle :\bar{q}q: \rangle| \approx (95\ {\rm MeV})^3$, which is less than 6% of its NG phase value $|\langle :\bar{q}q: \rangle| \approx (256\ {\rm MeV})^3$.

V. CONCLUSIONS AND OUTLOOK

The bound state masses of pseudoscalar and scalar mesons within the Poincaré covariant, symmetry preserving, RL truncated DS-BS approach with a model closely related to the phenomenologically successful MT interaction have been studied in the chiral limit and beyond. The validity of the AVWTI in the WW phase has been confirmed. It has been found that the spin-0 states disappear from the timelike spectrum and become spacelike over the whole accessible current-quark mass range. Furthermore, the mass splitting between chirality partners due to explicit current-quark masses in the WW phase has been quantified to be of the same order as the experimental splitting in the NG phase isospin multiplet. Finally, the current-quark mass dependence of the pion leptonic decay constant and the chiral condensate in the WW phase has been revealed. A strong linear dependence of both on the current-quark mass m_q has been found, driving f_π and $\langle :\bar{q}q: \rangle$ in the WW phase at the critical current-quark mass $m_q^{\rm cr}$ above 40%–50% of their NG phase values at $m_q = 5$ MeV. Whereas the difference between the NG and the WW phase propagators is qualitatively clearly visible in the quark mass function, $M(p^2)$ [or $B(p^2)$], the difference in $\langle :\bar{q}q: \rangle$ or the observable quantity f_π is less pronounced when allowing for larger current-quark mass in the WW phase.

In view of the investigation of [31] the extension of the above presented analysis to excited states, where the effects of DχSB are suppressed, the spin-1 channel, and beyond, is in order. It is not expected that higher spin states have spacelike solutions, either. Similarly, the presented approach allows for the evaluation of decay properties on the same footing. Furthermore, based on a phenomenological interaction which successfully describes the hadronic spectrum, the solutions of the DS equation in the WW phase may be used to determine condensates [72–74], in particular the symmetric four-quark condensate that dominates the chirally symmetric QSRs for the ρ meson. This provides a reliable implicit relation between changes of chirally symmetric and chirally odd condensates, which may be employed in a QSR calculation as in [31], and allows for a more sophisticated analysis.

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