

Novel method to deal with off-shell particles in cascade decaysJoydeep Chakraborty,^{1,*} Anirban Kundu,^{2,†} and Tripurari Srivastava^{1,‡}¹*Department of Physics, Indian Institute of Technology, Kanpur 208016, Uttar Pradesh, India*²*Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, West Bengal, India*

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We propose a novel algorithm to compute the width of any generic n -body decay involving multiple off-shell particles having zero and nonzero spins. Starting from a toy example, we show the computations for three different processes that contain spin-0, $-\frac{1}{2}$, and -1 off-shell particles. We check that our results match with the existing results at the analytical level. This proposal can be automatized and should be useful to compute the phase space for long cascade decays, without any Monte Carlo sampling.

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I. INTRODUCTION

A standard problem in quantum field theory is to calculate the decay width Γ of a parent particle A to n number of daughter particles. As is well known, the physics resides in the spin-averaged matrix element squared $|\overline{\mathcal{M}}|^2$ for the transition, but there are two kinematic factors also, namely, the initial flux (which is given by the mass of A if it is at rest) and the n -body phase space. While the calculation of Γ for $1 \rightarrow 2$ processes is an undergraduate exercise, the phase space integration gets complicated for $n \geq 3$, and even more so if all the particle masses are kept in the calculation. Often, this has to be done by some Monte Carlo sampling [1], introducing further uncertainties and also taking a lot of computer time. Depending on the complexity of the phase space, one has to compromise between the accuracy and the computer time needed, even more so if a huge number of events are to be generated.

In this paper we would like to propose an algorithmic approach to deal with the $1 \rightarrow n$ cascade decays, mediated by *virtual* particles. The algorithm does not work if one or more of the intermediate particles are on shell; one must apply the algorithm separately for different cascade branches. The algorithm consists of the following steps. Several examples are provided later on, as well as estimates of numerical accuracy of the approach.

1. Cut each and every off-shell propagator into two pieces such that the full cascade can be decomposed in terms of multiple $1 \rightarrow 2$ decays. This is, of course, not the usual prescription of Cutkosky [2] of cutting an on-shell propagator to get the absorptive part of the amplitude. Rather, this is an artificial cut, and one must remember that the cut propagator is still off shell. Thus, (i) the spin sum over the off-shell leg

cannot be done at this stage, and (ii) the phase space becomes imaginary.

2. Assign spin (for fermions) and polarization (for gauge bosons) indices for all particles, both on and off shell. Our convention for the following examples will be to use the latin (greek) alphabet for fermion (gauge boson) polarization indices. We will call both of them spin, as there should not be any chance of confusion.
3. While squaring the amplitude, one must not sum over the external leg spins that are off shell, as mentioned in rule 1. The indices that appear in the vertices are to be summed over as usual. Following this prescription, we calculate a few quantities (examples are provided later) which are analogous to the scalar quantities like $|\overline{\mathcal{M}}|^2$ or Γ that one usually computes. However, because of the floating indices, they are not scalars in our case; rather, they are tensors in spin indices.
4. Once we compute all such variables necessary for the full cascade decay, we will club them according to their appearance in the cascade such that all the spin indices are contracted leading to the trace of the full matrix in spin space. The final trace is a scalar quantity. For an off-shell scalar propagator, the entire trace can be decomposed into the product of two traces.
5. The most important part is to write the $1 \rightarrow 2$ phase space function in terms of the invariant masses of the off-shell particles and then integrate over all possible values of the invariant mass.

For n off-shell particles there will be n such integrals. This integral takes into account the off-shell propagator too. This is the crux of the algorithm and should better be followed by the following examples.

6. All the intermediate “partial decay widths” $\tilde{\Gamma}$ are to be defined in the prescribed way. Their dimensions need not be that of mass.

*joydeep@iitk.ac.in
 †anirban.kundu.cu@gmail.com
 ‡tripurar@iitk.ac.in

7. Finally, for identical particles in the final state, we need to incorporate the symmetry factor in the form of $(1 - \frac{1}{2}\delta_{IJ})$ for the decay $A^{(*)} \rightarrow IJ$. Thus, the width for the decay $A \rightarrow B^*C^*$, $B^* \rightarrow DE$, $C^* \rightarrow FG$ should typically be of the form

$$\Gamma = \frac{1}{m_A} \int \left[\frac{1}{\pi(m_{DE}^2 - m_B^2)^2} \right] \left[\frac{1}{\pi(m_{FG}^2 - m_C^2)^2} \right] \times \tilde{\Gamma}(A \rightarrow BC) \tilde{\Gamma}(B \rightarrow DE) \tilde{\Gamma}(C \rightarrow FG), \quad (1)$$

where, for example,

$$\tilde{\Gamma}(B \rightarrow DE) = \frac{1}{2} \int d_{PS}^{B \rightarrow DE} |\mathcal{M}(B \rightarrow DE)|^2, \quad (2)$$

which is easy to evaluate; the only thing to keep in mind is to use m_{DE} instead of m_B because B is off shell. The factor of $1/m_A$ is the flux factor evaluated in the rest frame of the parent particle. If the intermediate state B has a large width, we should replace

$$(m_{DE}^2 - m_B^2)^2 \rightarrow (m_{DE}^2 - m_B^2)^2 + m_B^2 \Gamma_B^2, \quad (3)$$

where Γ_B is the decay width of B . Here we will work in the narrow width approximation.

Why is this proposal working? An intuitive justification is that when one integrates over the invariant masses, the invariant mass can effectively be used in place of the physical mass for the parent particle. Using the invariant mass has the extra advantage that the phase space is always real. We are effectively decomposing the full phase space of the entire cascade into several parts, writing each of them in terms of trivial $1 \rightarrow 2$ phase spaces. Now these individual subdiagrams have been computed using the standard techniques of quantum field theory, so there is no ambiguity. The essential part of our proposal is to provide the prescription to join those contributions, maintaining the flow of polarizations through off-shell propagators. Thus this method can be applied to any tree level cascade decay, irrespective of the spin of the intermediate propagators, and the number of such propagators. However, at this present form, it cannot be applied to calculate loop integrals, unless they can be reduced to some effective operators.

A few examples will now follow. We will, however, not show the detailed evaluation of $|\mathcal{M}|^2$, which is an undergraduate exercise.

The paper is arranged as follows. In Sec. II, we will provide a ‘‘toy’’ example with an off-shell scalar propagator. In Sec. III, more examples will be provided, including numerical checks with the existing software. We conclude in Sec. IV.

II. A ‘‘TOY’’ EXAMPLE

The first example follows from Ref. [3] where an outline of the algorithm was given for scalar propagators only. Consider the decay of a heavy lepton ℓ_0 to three leptons ℓ_1 , ℓ_2 , and ℓ_3 mediated by scalars which we will call Δ . The coupling of Δ with ℓ_i and ℓ_j will be denoted by y_{ij} . Suppose the decay chain is $\ell_0 \rightarrow \ell_1 \Delta^*$, $\Delta^* \rightarrow \ell_2 \ell_3$. According to our proposal, the virtual decay width of $\Delta^* \rightarrow \ell_2 \ell_3$ is given by

$$\begin{aligned} \tilde{\Gamma}_{\ell_2 \ell_3}^{\Delta^*} &= \left(1 - \frac{1}{2}\delta_{\ell_2 \ell_3}\right) \int d_{PS}^{\Delta^* \rightarrow \ell_2 \ell_3} \frac{|\mathcal{M}(\Delta^* \rightarrow \ell_2 \ell_3)|^2}{2} \\ &= \left(1 - \frac{1}{2}\delta_{\ell_2 \ell_3}\right) \frac{\lambda^{1/2}(m_{23}^2, m_{\ell_2}^2, m_{\ell_3}^2)}{16\pi m_{23}^2} \\ &\quad \times |y_{23}|^2 (m_{23}^2 - m_{\ell_2}^2 - m_{\ell_3}^2), \end{aligned} \quad (4)$$

where m_{23} is the momentum transfer through Δ ; note the use of the invariant mass m_{23} in this step. The decay width, therefore, is

$$\begin{aligned} \Gamma_{123} &= \frac{1}{2m_{\ell_0}} \int \frac{dm_{23}^2}{\pi(m_{23}^2 - m_{\Delta}^2)^2} \\ &\quad \times \int d_{PS}^{\ell_0 \rightarrow \ell_1 \Delta^*} |\mathcal{M}(\ell_0 \rightarrow \ell_1 \Delta^*)|^2 \tilde{\Gamma}_{\ell_2 \ell_3}^{\Delta^*} \\ &= \left(1 - \frac{1}{2}\delta_{\ell_2 \ell_3}\right) \int_{(m_{\ell_2} + m_{\ell_3})^2}^{(m_{\ell_0} - m_{\ell_1})^2} \frac{dm_{23}^2}{\pi(m_{23}^2 - m_{\Delta}^2)^2} \\ &\quad \times \left[\frac{\lambda^{1/2}(m_{\ell_0}^2, m_{23}^2, m_{\ell_1}^2)}{16\pi m_{\ell_0}^3} |y_{01}|^2 (m_{\ell_0}^2 + m_{\ell_1}^2 - m_{23}^2) \right] \\ &\quad \times \left[\frac{\lambda^{1/2}(m_{23}^2, m_{\ell_2}^2, m_{\ell_3}^2)}{16\pi m_{23}^2} |y_{23}|^2 (m_{23}^2 - m_{\ell_2}^2 - m_{\ell_3}^2) \right]. \end{aligned} \quad (5)$$

Note again the integration over m_{23} over its entire range. The integration may have to be done numerically if all lepton masses are kept.

III. FURTHER EXAMPLES WITH FERMION AND GAUGE PROPAGATORS

A. $\mu \rightarrow \nu_\mu W^*$, $W^* \rightarrow e\bar{\nu}_e$

Muon decay is instructive because the intermediate W propagator has spin indices. Cutting it as shown in Fig. 1, we get

$$\begin{aligned} \Gamma_{(\mu \rightarrow e\bar{\nu}_e \nu_\mu)} &= \frac{1}{m_\mu} \int \left[\frac{1}{\pi} \left(\frac{dm_{12}^2}{(m_{12}^2 - m_W^2)^2} \right) \right] \\ &\quad \times \text{Tr}[\tilde{\Gamma}(\mu \rightarrow \nu_\mu W^*) \tilde{\Gamma}(W^* \rightarrow e\bar{\nu}_e)]. \end{aligned} \quad (6)$$

The virtual decay width for $(\mu \rightarrow \nu_\mu W^*)$ is

$$[\tilde{\Gamma}(\mu \rightarrow \nu_\mu W^*)]_\nu^\alpha = \frac{1}{2} \int d_{PS}^{\mu \rightarrow \nu_\mu W^*} [|\mathcal{M}_1(\mu \rightarrow \nu_\mu W^*)|^2]_\nu^\alpha, \quad (7)$$

where, following the momentum convention shown in Fig. 1, and using fermion spin summation and the trace identities,

$$\begin{aligned} [|\mathcal{M}_1|^2]_\nu^\alpha &= \frac{g^2}{8} \epsilon_\lambda^\alpha(p_2) \epsilon_\lambda^{\mu*}(p_2) [\bar{u}^s(p_3) \gamma_\mu (1 - \gamma^5) v^{s'}(p_4)] [\bar{v}^{s'}(p_4) \gamma_\nu (1 - \gamma^5) u^s(p_3)] \\ &= \frac{g^2}{8} \epsilon_\lambda^\alpha(p_2) \epsilon_\lambda^{\mu*}(p_2) \text{Tr}[(\not{p}_3 - m_e) \gamma_\mu (1 - \gamma_5) (\not{p}_4) \gamma_\nu (1 - \gamma_5)] \\ &= \frac{g^2}{2} \epsilon_\lambda^\alpha(p_2) \epsilon_\lambda^{\mu*}(p_2) (p_{3\mu} p_{4\nu} + p_{3\nu} p_{4\mu} - g_{\mu\nu} (p_3 \cdot p_4) - i \epsilon^{\mu\rho\nu\sigma} p_3^\rho p_4^\sigma). \end{aligned} \quad (8)$$

Similarly,

$$[\tilde{\Gamma}(W^* \rightarrow e \bar{\nu}_e)]_\alpha^\nu = \frac{1}{2} \int d_{PS}^{W^* \rightarrow e \bar{\nu}_e} [|\mathcal{M}_2(W^* \rightarrow e \bar{\nu}_e)|^2]_\alpha^\nu, \quad (9)$$

where

$$\begin{aligned} [|\mathcal{M}_2|^2]_\alpha^\nu &= \frac{g^2}{8} \epsilon_{\alpha\lambda'}^*(p_2) \epsilon_{\beta\lambda'}(p_2) \text{Tr}[\not{p}_{\nu_\mu} \gamma^\beta (1 - \gamma_5) \\ &\quad \times (\not{p}_1 + m_\mu) \gamma^\nu (1 - \gamma_5)]. \end{aligned} \quad (10)$$

Next, we sum over the W spin, $\sum_\lambda \epsilon_\lambda^{\mu*}(p) \epsilon_\lambda^\nu(p) = -g^{\mu\nu}$ (the $p_{2\mu} p_{2\nu}$ term gives zero with massless fermions in the final state), and evaluate the scalar trace, $\text{Tr}[|\mathcal{M}_1|^2 |\mathcal{M}_2|^2] = 4g^4 [(p_1 \cdot p_4) (p_{\nu_\mu} \cdot p_3)]$, and average over the initial μ spin, to get

$$\begin{aligned} \Gamma_{(\mu \rightarrow e \bar{\nu}_e \nu_\mu)} &= \frac{1}{2m_\mu} \int \left[\frac{1}{\pi} \frac{dm_{12}^2}{(m_{12}^2 - m_W^2)^2} \int d_{PS}^{\mu \rightarrow \nu_\mu W^*} \right. \\ &\quad \left. \times \int d_{PS}^{W^* \rightarrow e \bar{\nu}_e} \{g^4 (p_1 \cdot p_4) (p_{\nu_\mu} \cdot p_3)\} \right]. \end{aligned} \quad (11)$$

Again, note the same logic: two $1 \rightarrow 2$ phase space integrals, and an integration over the free variable m_{12} .

In the rest frame of the muon, the decay width can be written (after neglecting the electron mass) as

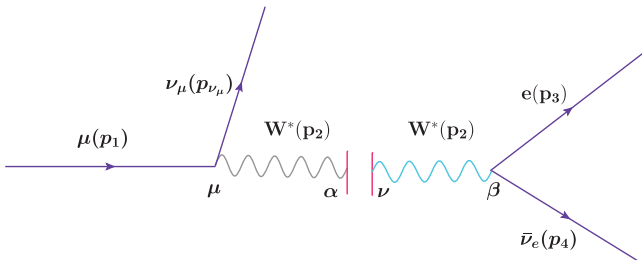


FIG. 1. Feynman diagram for $\mu \rightarrow \nu_\mu W^* \rightarrow \nu_\mu \bar{\nu}_e e$.

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{g^4 m_\mu^5}{6144 \pi^3 m_W^4}. \quad (12)$$

Keeping the electron mass, a numerical integration gives $\Gamma = 6.91095 \times 10^{-11} g^4 / m_W^4$ (in GeV). Both the results are in complete agreement with that in the literature.

B. $H \rightarrow W^- W^{+*}, W^{+*} \rightarrow t \bar{b}$

The next example is the decay $H \rightarrow W^- W^{+*}, W^{+*} \rightarrow t \bar{b}$, with the momenta as shown in Fig. 2. The masses m_H and m_t are kept as free parameters. The decay width is given by

$$\begin{aligned} \Gamma(H \rightarrow W^- t \bar{b}) &= \frac{1}{m_H} \int \left[\frac{1}{\pi} \frac{dm_{12}^2}{(m_{12}^2 - m_W^2)^2} \right] \\ &\quad \times \text{Tr}[\tilde{\Gamma}(H \rightarrow W^- W^{+*}) \tilde{\Gamma}(W^{+*} \rightarrow t \bar{b})]. \end{aligned} \quad (13)$$

The virtual decay widths are

$$\begin{aligned} [\tilde{\Gamma}(H \rightarrow W W^*)]_\nu^\alpha &= \int \frac{d_{PS}^{H \rightarrow W W^*}}{2} [|\mathcal{M}_1(H \rightarrow W W^*)|^2]_\nu^\alpha, \\ [\tilde{\Gamma}(W^* \rightarrow t \bar{b})]_\alpha^\nu &= \int \frac{d_{PS}^{W^* \rightarrow t \bar{b}}}{2} [|\mathcal{M}_2(W^* \rightarrow t \bar{b})|^2]_\alpha^\nu, \end{aligned} \quad (14)$$

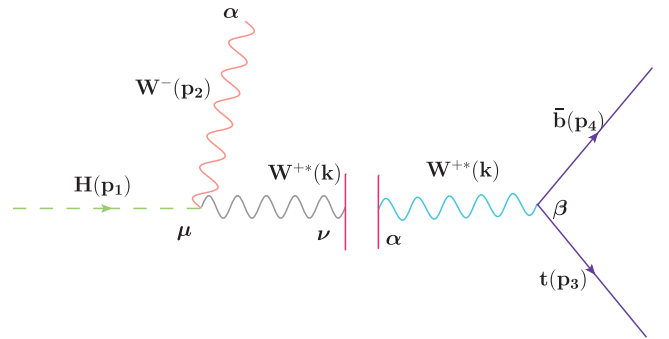


FIG. 2. Feynman diagram for the cascade decay: $H \rightarrow W^- W^{+*} \rightarrow W^- t \bar{b}$.

TABLE I. Decay widths for different values of m_t and m_H . Central values from CALCHEP v.3.6.25 [1] with a typical error of 10% are shown in parentheses. We have taken $m_b = 0$ and $V_{tb} = 1$.

m_t (GeV)	m_H (GeV)	Γ (GeV)
0	82	$2.962(2.942) \times 10^{-11}$
	125	$2.771(2.739) \times 10^{-4}$
	150	$4.007(3.954) \times 10^{-3}$
25	106	$2.595(2.687) \times 10^{-10}$
	120	$1.989(1.916) \times 10^{-5}$
	130	$1.478(1.419) \times 10^{-4}$
50	132	$2.846(2.869) \times 10^{-8}$
	140	$1.901(1.833) \times 10^{-5}$
	150	$4.040(3.859) \times 10^{-4}$

and

$$\begin{aligned}
[|\mathcal{M}_1|^2]_\nu^\alpha &= \left(\frac{2m_W^2}{v}\right)^2 [\epsilon_\lambda^\alpha(p_2)\epsilon_\lambda^{\mu*}(p_2)\epsilon_{\mu\lambda'}(k)\epsilon_{\nu\lambda'}^*(k)], \\
[|\mathcal{M}_2|^2]_\alpha^\nu &= \frac{g^2|V_{tb}|^2}{8} \text{Tr}[(\not{p}_3 - m_t)\gamma^\nu(1 - \gamma_5)(\not{p}_4 + m_b) \\
&\quad \times \gamma^\beta(1 - \gamma_5)]\epsilon_\alpha^{\lambda''}(k)\epsilon_\beta^{\lambda''*}(k). \quad (15)
\end{aligned}$$

We have kept only the spin indices of the cut propagator as free and used

$$\sum_\lambda \epsilon_\lambda^{\mu*}(k)\epsilon_\lambda^\nu(k) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_W^2}. \quad (16)$$

$$\begin{aligned}
\mathcal{F} &= \frac{16}{m_W^4} [2m_W^2(p_2 \cdot p_3)(p_2 \cdot p_4) + 4m_W^2(k \cdot p_3)(k \cdot p_4) - 2m_W^2 m_{12}^2(p_3 \cdot p_4) - 2(k \cdot p_2)(k \cdot p_3)(p_2 \cdot p_4) \\
&\quad - 2(k \cdot p_2)(k \cdot p_4)(p_2 \cdot p_3) + 2(k \cdot p_2)^2(p_3 \cdot p_4) + m_{12}^4(p_3 \cdot p_4) - 2m_{12}^2(k \cdot p_3)(k \cdot p_4) \\
&\quad + 2m_W^{-2}(k \cdot p_2)^2(k \cdot p_3)(k \cdot p_4) - m_W^{-2} m_{12}^2(p_3 \cdot p_4)(k \cdot p_2)^2 + m_W^4(p_4 \cdot p_3)]. \quad (18)
\end{aligned}$$

A comparison of our method, where the integration is done numerically by MATHEMATICA [4], and the result from CALCHEP v.3.6.25 [1], is shown in Table I. The typical uncertainty in the evaluation of the decay width through Monte Carlo sampling is about 10%, and our method is in complete agreement.

C. $H \rightarrow t\bar{t}^*, \bar{t}^* \rightarrow \bar{b}W^-$

This is an analogous case with a fictitious heavy Higgs boson and involving an off-shell fermion. The Feynman diagram including the spin and momentum labels is shown in Fig. 3. Thus

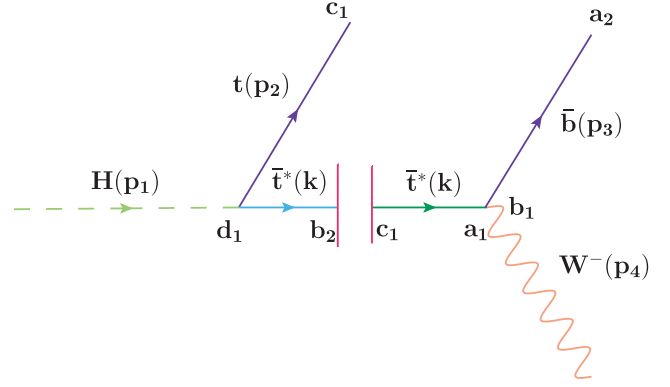


FIG. 3. Feynman diagram for $H \rightarrow t\bar{t}^*, \bar{t}^* \rightarrow \bar{b}W^-$.

Taking the parent H to be at rest and using $k^2 = m_{12}^2$, we get

$$\begin{aligned}
\Gamma(H \rightarrow Wt\bar{b}) &= \frac{N_c g^2 m_W^4}{4m_H v^2} |V_{tb}|^2 \\
&\quad \times \int \left[\frac{1}{\pi} \frac{dm_{12}^2}{(m_{12}^2 - m_W^2)^2} \right] \\
&\quad \times \int d_{PS}^{W^* \rightarrow t\bar{b}} \int d_{PS}^{H \rightarrow WW^*} \mathcal{F}, \quad (17)
\end{aligned}$$

where \mathcal{F} evaluated from the spin sum and the trace is

$$\begin{aligned}
\Gamma(H \rightarrow t\bar{b}W) &= \frac{1}{m_H} \int \frac{1}{\pi} \frac{dm_{12}^2}{(m_{12}^2 - m_t^2)^2} \\
&\quad \times \text{Tr}[\tilde{\Gamma}(H \rightarrow t\bar{t}^*)\tilde{\Gamma}(\bar{t}^* \rightarrow \bar{b}W)]. \quad (19)
\end{aligned}$$

The virtual decay widths are

$$\begin{aligned}
[\tilde{\Gamma}(H \rightarrow t\bar{t}^*)]_{c_1 b_2} &= \int \frac{d_{PS}^{H \rightarrow t\bar{t}^*}}{2} [|\mathcal{M}_1(H \rightarrow t\bar{t}^*)|^2]_{c_1 b_2}, \\
[\tilde{\Gamma}(\bar{t}^* \rightarrow \bar{b}W)]_{b_2 c_1} &= \int \frac{d_{PS}^{\bar{t}^* \rightarrow \bar{b}W}}{2} [|\mathcal{M}_2(\bar{t}^* \rightarrow \bar{b}W)|^2]_{b_2 c_1}, \quad (20)
\end{aligned}$$

and the amplitudes can be written as

$$\begin{aligned} [|\mathcal{M}_1|^2]_{c_1 b_2} &= \frac{g^2 m_t^2}{4m_W^2} \{ [\bar{u}^{s_1}(p_2)]_{c_1} [u^{s_1}(p_2)]_{d_1} [\bar{v}^{s_2}(k)]_{d_1} [v^{s_2}(k)]_{b_2} \}, \\ [|\mathcal{M}_2|^2]_{b_2 c_1} &= \frac{g^2 |V_{tb}|^2}{8} \{ [\bar{v}^{s_3}(k)]_{a_1} [\gamma^\mu (1 - \gamma_5)]_{a_1 a_2} [v^{s_4}(p_3)]_{a_2} \\ &\quad \times [\bar{v}^{s_4}(p_3)]_{b_1} [\gamma^\nu (1 - \gamma_5)]_{b_1 b_2} [v^{s_3}(k)]_{c_1} \} [\epsilon_\mu^\lambda(p_4) \epsilon_\nu^{\lambda*}(p_4)]. \end{aligned} \quad (21)$$

Neglecting the mass of the bottom quark, one gets

$$\begin{aligned} \Gamma(H \rightarrow t\bar{b}W) &= \left(\frac{1}{4m_H} \right) \frac{g^4 m_t^2 N_c |V_{tb}|^2}{32m_W^2} \\ &\quad \times \int \left[\frac{1}{\pi} \left(\frac{dm_{12}^2}{(m_{12}^2 - m_t^2)^2} \right) \right] \\ &\quad \times \int d_{ps}^{H \rightarrow t\bar{t}^*} \int d_{ps}^{\bar{t}^* \rightarrow \bar{b}W} \mathcal{F}, \end{aligned} \quad (22)$$

where

$$\mathcal{F} = (16(p_2 \cdot k) - 16m_t m_{12}) \left((k \cdot p_3) + \frac{2(k \cdot p_3)(p_3 \cdot p_4)}{m_W^2} \right) \quad (23)$$

obtained after performing the spin sum and trace.

The expressions for $|\mathcal{M}|^2$ match with the standard expressions in the literature [5]. Our final results also match with those using other formalisms for evaluating the three-body phase space [6,7].

For $m_t = 174$ GeV and $m_H = 260(280, 300, 320)$ GeV, we find $\Gamma = 9.327 \times 10^{-9}(4.979 \times 10^{-6}, 8.484 \times 10^{-5}, 6.924 \times 10^{-4})$ GeV respectively.

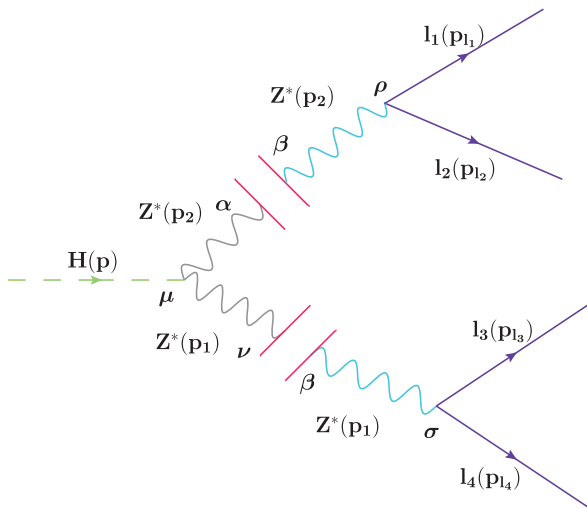


FIG. 4. Representative figure of cascade decay: $H \rightarrow Z^* Z^* \rightarrow l_1 l_2 l_3 l_4$.

D. $H \rightarrow Z^* Z^*, Z^* \rightarrow l_1 l_2, Z^* \rightarrow l_3 l_4$

For this process, as depicted in Fig. 4, the decay width can be written as

$$\begin{aligned} \Gamma(H \rightarrow l_1 l_2 l_3 l_4) &= \frac{1}{2} \frac{1}{m_H} \int \left[\frac{1}{\pi} \left(\frac{dm_{12}^2}{(m_{12}^2 - m_Z^2)^2} \right) \right] \\ &\quad \times \int \left[\frac{1}{\pi} \left(\frac{dm_{34}^2}{(m_{34}^2 - m_Z^2)^2} \right) \right] \\ &\quad \times \text{Tr}[\tilde{\Gamma}(H \rightarrow Z^* Z^*) \tilde{\Gamma}(Z^* \rightarrow l_1 l_2) \\ &\quad \times \tilde{\Gamma}(Z^* \rightarrow l_3 l_4)], \end{aligned} \quad (24)$$

where $\frac{1}{2} = (1 - \delta_{ZZ}/2)$ is the symmetry factor, and

$$\begin{aligned} [\tilde{\Gamma}(H \rightarrow Z^* Z^*)]_\nu^\alpha &= \int \frac{d_{PS}^{H \rightarrow Z^* Z^*}}{2} [|\mathcal{M}_1(H \rightarrow Z^* Z^*)|^2]_\nu^\alpha, \\ [\tilde{\Gamma}(Z^* \rightarrow l_i l_j)]_\beta^\nu &= \int \frac{d_{PS}^{Z^* \rightarrow l_i l_j}}{2} [|\mathcal{M}_2(Z^* \rightarrow l_i l_j)|^2]_\beta^\nu, \end{aligned} \quad (25)$$

$$\begin{aligned} [|\mathcal{M}_1|^2]_\nu^\alpha &= \frac{g^2 m_Z^2}{\cos^2 \theta_W} \epsilon^\alpha(p_2) \epsilon^{*\mu}(p_2) \epsilon_\mu(p_1) \epsilon_\nu^*(p_1), \\ [|\mathcal{M}_2|^2]_\beta^\nu &= \frac{g^2}{16 \cos^2 \theta_W} \epsilon_\lambda^\nu(p_1) \epsilon_\lambda^{*\sigma}(p_1) \\ &\quad \times \text{Tr}[(\not{p}_{l_i} - m_{l_i}) \gamma_\beta (c_v + \gamma_5) (\not{p}_{l_j} + m_{l_j}) \gamma_\sigma (c_v + \gamma_5)], \end{aligned} \quad (26)$$

where $c_v = -1 + 4 \sin^2 \theta_W$.

Neglecting the lepton masses and using the spin sum $\sum_\lambda \epsilon_\lambda^{*\mu}(k) \epsilon_\lambda^\nu(k) = -g^{\mu\nu}$, the decay width can be written as

$$\begin{aligned} \Gamma(H \rightarrow 4l) &= \frac{1}{16m_H} \int \left[\frac{1}{\pi} \left(\frac{dm_{12}^2}{(m_{12}^2 - m_Z^2)^2} \right) \right] \\ &\quad \times \int \left[\frac{1}{\pi} \left(\frac{dm_{34}^2}{(m_{34}^2 - m_Z^2)^2} \right) \right] \\ &\quad \times \int d_{PS}^{H \rightarrow Z^* Z^*} \int d_{PS}^{Z^* \rightarrow l_1 l_2} \int d_{PS}^{Z^* \rightarrow l_3 l_4} \mathcal{F}, \end{aligned} \quad (27)$$

where

$$\mathcal{F} = \left(\frac{g^6 m_Z^2}{256 \cos^6 \theta_W} \right) (16(c_v^2 + 1)^2 \times [2(p_{l_3} \cdot p_{l_2})(p_{l_1} \cdot p_{l_4}) + 2(p_{l_3} \cdot p_{l_1})(p_{l_4} \cdot p_{l_2})]). \quad (28)$$

One can further simplify the decay width after writing the phase spaces explicitly. The similar decay processes are discussed in [8–10]. One can easily accommodate the interfering contribution by adding an extra term (if there is any), $\Gamma(l_1 \leftrightarrow l_3, l_2 \leftrightarrow l_4)$, in the above equation.

IV. CONCLUSIONS

In this paper we have proposed an algorithm to treat the n -body phase space analytically, as a product of several virtual two-body phase spaces. Compared to the standard Monte Carlo sampling, this method is not only time saving, particularly when a huge number of events are to be

generated, but also has comparable or even better accuracy. We have discussed the algorithm with several examples involving off-shell scalars, fermions, and gauge bosons, and cross-checked our results with those available in the literature or with standard software like CALCHEP. Implementation of this algorithm in an easy-to-use software is also under progress.

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