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Axial-vector dominance predictions in quasielastic neutrino-nucleus scattering

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The axial form factor plays a crucial role in quasielastic neutrino-nucleus scattering, but the error of the theoretical cross section due to uncertainties of G_A remains to be established. Conversely, the extraction of G_A from the neutrino nucleus cross section suffers from large systematic errors due to nuclear model dependencies, while the use of single-parameter dipole fits underestimates the errors and prevents an identification of the relevant kinematics for this determination. We propose to use a generalized axial-vector-meson dominance in conjunction with large-N_c and high-energy QCD constraints to model the nucleon axial form factor, as well as the half-width rule as an *a priori* uncertainty estimate. The minimal hadronic ansatz comprises the sum of two monopoles corresponding to the lightest axial-vector mesons being coupled to the axial current. The parameters of the resulting axial form factor are the masses and widths of the two axial mesons as obtained from the averaged Particle Data Group values. By applying the half-width rule in a Monte Carlo simulation, a distribution of theoretical predictions can then be generated for the neutrino-nucleus quasielastic cross section. We test the model by applying it to the (ν_μ, μ) quasielastic cross section from ^{12}C for the kinematics of the MiniBooNE experiment. The resulting predictions have no free parameters. We find that the relativistic Fermi gas model globally reproduces the experimental data, giving $\chi^2/\#\text{bins} = 0.81$. A Q^2-dependent error analysis of the neutrino data shows that the uncertainties in the axial form factor $G_A(Q^2)$ are comparable to the ones induced by the *a priori* half-width rule. We identify the most sensitive region to be in the range $0.2 \lesssim Q^2 \lesssim 0.6\ \text{GeV}^2$.

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I. INTRODUCTION

Since the first measurement of the muon neutrino charged-current quasielastic double differential cross section [1–3] many attempts have been made to characterize an effective axial-vector form factor of the nucleon [4–7]. This is often made in terms of a dipole axial mass M_A, assuming a dipole form [8,9], for $Q^2 > 0$,

$$G_A^{\text{dipole}}(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2}. \quad (1)$$

The world average value of the nucleon dipole axial mass is $M_A \sim 1$ GeV [10] (see, e.g., Ref. [11] for a review and references therein) which is obtained as a weighted sum of different oncoming dipole fits to independent experiments. However, this does not mean that the full spread of axial form factors can be described by a single dipole mass with a given uncertainty in a statistically significant way. Actually, there is a great variety of often largely incompatible experimental data from different processes which for illustration can be seen in Fig. 1. The correct discrimination and selection of these mutually compatible data is a complicated problem in data analysis (which requires proper weighting of experimental ranges and falsifiable reliable theoretical input and not just parametrizations) which awaits resolution and will not be addressed here. Nonetheless, given the present rather confusing state of affairs and the lack of further qualified information on what data on the axial form factor one should objectively prefer, we will face the problem from a different and somewhat unconventional perspective where the traditional fitting strategy is sidestepped by the use of a theoretically based axial form factor with an inherent error band.

The MiniBooNE cross section data values are too large compared to the theoretical models of quasielastic neutrino scattering in the impulse approximation, unless a significant larger value of $M_A \sim 1.35$ GeV is employed in the nuclear axial current. Microscopic explanations of the large value of M_A have been proposed based on ingredients involving nucleon spectral functions and multinucleon emission induced by short-range correlations and meson-exchange currents [8,21,22]. Recently, studies with a monopole parametrization have been performed in Ref. [23] as well as nucleon mean-field effective mass analyses of blurred electron scattering data [24]. In the absence of reliable theoretical uncertainty estimates, the disparate values obtained upon consideration of different nuclear effects can so far be regarded as a genuine source of systematic errors. Those turn out to be much larger than the alleged statistical

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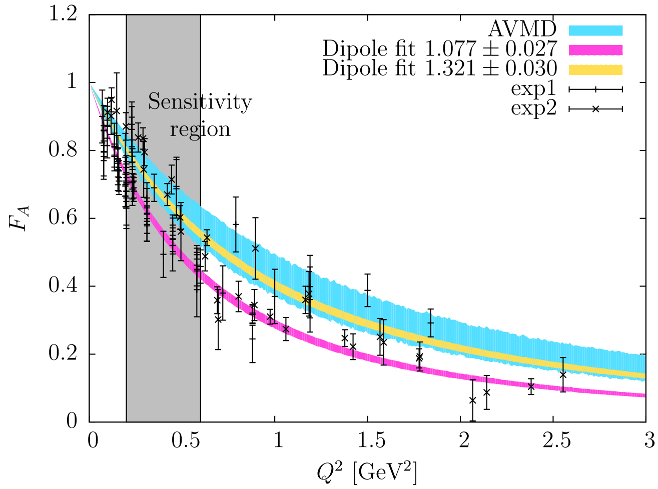

FIG. 1. The axial-meson dominance band prediction is compared with the experimental data from the nucleon and with the dipole prediction bands of Ref. [8] using the Fermi gas approximation and some relevant nuclear and reaction effects thereof. Experimental data are from Refs. [12–16] (exp1) and Refs. [17–20] (exp2). The axial form factor and the data are normalized to $F_A(0) = 1$.

uncertainties which, if taken literally, would lead to the most precise determination of the axial form factor to date in the range $Q^2 \lesssim 2$ GeV^2. A careful statistical analysis has been undertaken more recently [25] and some tension among different data in different models has been reported.

The popular dipole form factor parametrization enjoys the perturbative QCD (pQCD) result [26] asymptotically, $G_A \sim 1/Q^4$, but despite the phenomenological success for separate and independent experiments, it finds no further theoretical support at finite Q^2, nor does it describe *all* experiments globally with an acceptable χ^2 value. Moreover, a one-parameter fit such as the dipole form introduces an artificial bias linking high and low energies unnaturally and tightly; it is unclear if the statistical fluctuations inherited from the uncertainties in experimental neutrino-nucleus scattering data are faithfully represented by the corresponding fluctuations in the dipole mass. This is a well-known issue in the statistical analysis of data since the goodness of fit and the parameter confidence level are based on estimating the probability that the proposed parametrization is the correct one, and this implies a mapping between data fluctuations and the fitting parameter fluctuations. To overcome this limitation a model-independent analysis of the axial form factor using dispersion relations under definite convergence assumptions and based on neutrino scattering was performed [27], with the expected finding that errors inferred from a dipole ansatz analysis may be underestimated. A duality-based parametrization has been proposed searching for significant deviations to the widely used dipole form [28]. To be fair one should say that the neutrino scattering vs nucleon axial form factor is a kind of red herring; the significance of nuclear effects is claimed *after* a fit of the dipolar mass to the data is undertaken, in which case astonishingly precise values for the dipolar mass are inferred (see, e.g., Ref. [8] where extremely accurate values for M_A are quoted). Since neutrino-based determinations often imply certain and sometimes questionable assumptions, it is instructive to review other sources of information which at least do not rest on the same assumptions.

On a fundamental level, *ab initio* calculations allow a direct evaluation of the axial-current matrix elements. The first lattice QCD determination of the axial form factor [29] provided $M_A = 1.03(5)$ GeV in agreement with world average neutrino data at the time, $M_A = 1.032(36)$ GeV. However, subsequent calculations [30] yielded $M_A = 1.5$ GeV, a number which has recently been confirmed [31] for unphysical pion masses (about twice the physical value); the corresponding dipolar axial mass is larger than the experimental one, although there is some trend towards agreement as the pion mass approaches the physical value. The role of excited states has been analyzed in a more recent lattice analysis [32] confirming these results. In addition, light-cone QCD sum rules also overestimates the experimental dipole fit by 30% [33] in the range $1 < Q^2 < 4$ GeV^2, a trend checked by subsequent analyses [34,35] and that also agrees with lattice calculations. While these QCD calculations are still subjected to many improvements, one should also recognize that they generate a family of axial nucleon form factors which fall within the experimental band which is wide enough to pose again the pertinent question of which are the correct ones within uncertainties. This situation makes an interesting case of lifting the conventional fitting strategy based on *ad hoc* parametrizations and incompatible data in favor of assuming a theoretically founded axial nucleon form factor with a credible uncertainty band generated by independent fluctuations and not directly based on the neutrino-nucleus data under discussion. In this paper we propose a simple scheme furnishing these requirements (see Sec. II), and provide a framework where the significance of different nuclear effects might be addressed.

On the more phenomenological hadronic level, the algebra of fields [36] which yields field-current identities [37] implies a generalized meson dominance which has proven as a convenient tool to analyze many important hadronic properties, most notably generalized vertex functions and hadronic form factors [38]. In the particular case of conserved currents, and more specifically axial-vector currents the general form of the form factor is expected to be a sum of infinitely many monopoles with isovector *axial* meson masses, whereas the pQCD result [26] yields $G_A \sim 1/Q^4$. The goodness of the axial-vector-meson dominance (AVMD) for the axial nucleon form factor was posed in Ref. [39] by including the strong vertex corrections. However, meson dominance implies exchange of resonances which have a mass spectrum characterized by a mass and a width. Amazingly there is a theoretical limit where meson dominance with narrow resonances is realized in QCD, namely the large N_c-limit introduced by 't Hooft and Witten long ago [40,41]; within the large-N_c expansion mesons become stable particles. Their width-to-mass ratio is $\Gamma_R/M_R = \mathcal{O}(1/N_c) \sim 0.33$ which turns out to give the correct order of magnitude of the average experimental value 0.12(8) [42]. The phenomenological implications of meson dominance within a large-N_c approach have been analyzed in Ref. [43] and, after natural uncertainty estimates based on the resonance width, a good description of experimental data and lattice results was achieved, with competitive accuracy.

Motivated by these theoretical insights, in the present paper we explore the large-N_c-inspired parametrization of the nucleon axial form factor [43] (see Sec. II), and explore the consequences of axial-vector dominance directly. We apply these findings to neutrino-nucleus scattering by starting with the simplest nuclear model, i.e., the relativistic Fermi gas (see Sec. III), which can be performed analytically. We do this without fitting any neutrino data in Sec. IV. This simple approach allows us to address more clearly some important issues from a statistical point of view, and in particular to pin down the region of Q^2 values where the axial nucleon form factor fits are more sensitive to the existing neutrino-nucleus scattering data (see Sec. V). We finally summarize our results in Sec. VI.

II. AXIAL-VECTOR-MESON DOMINANCE AND THE AXIAL FORM FACTOR

AVMD was first introduced by Lee and Zumino [37] into particle physics as a very natural generalization of the successful realization that vector-meson dominance explained the bulk of electromagnetic form factors. It simply states that the axial-vector current is given by the current field identity, which for just u, d quarks reads

$$\vec{J}^{\mu}_A = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5\vec{\tau}q = \sum_A f_A \partial^\nu \vec{A}_{\mu\nu} + \sum_P f_P \partial_\mu \vec{P}, \qquad (2)$$

where f_A and f_P are the decay amplitudes of the axial-vector $A = a_1, a_1', \ldots$ and pseudoscalar mesons $P = \pi, \pi', \ldots$, respectively, and $\vec{A}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu$ is the corresponding field-strength tensor of the axial meson. This equation yields a generalized partially conserved axial current (PCAC), which implies in turn a generalization [44–49] of the celebrated Goldberger-Treimann relation.

As a consequence the axial form factor of the nucleon can be written as a sum of monopole form factors,

$$G_A(Q^2) = g_A \sum_n c_{n,a} \frac{m_{n,a}^2}{m_{n,a}^2 + Q^2}, \qquad (3)$$

where $c_{n,a} = f_{n,a} g_{n,aNN}/g_A$, and $f_{n,a}$, $g_{n,aNN}$, and $m_{n,a}$ are the vacuum amplitude, the coupling to the nucleon, and the mass (respectively) of the corresponding isovector-axial-vector meson n. From $G_A(0) = g_A$ we have the normalization condition

$$1 = \sum_n c_{n,a}. \qquad (4)$$

The asymptotic pQCD result [26], $G_A \sim 1/Q^4$, requires

$$0 = \sum_n c_{n,a} m_{n,a}^2. \qquad (5)$$

In this paper we use the minimal hadronic ansatz for the axial nucleon form factor furnishing meson dominance and proper pQCD behavior,

$$G_A(Q^2) = g_A F_A(Q^2) = g_A \frac{m_{a_1}^2 m_{a_1'}^2}{(m_{a_1}^2 + Q^2)(m_{a_1'}^2 + Q^2)}, \qquad (6)$$

with $g_A = 1.267$, and where the axial-meson masses are $m_{a_1} = 1.230$ GeV, $m_{a_1'} = 1.647$ GeV. As noted in Ref. [43], one of the problems with this ansatz is that generally the interpolating fields are resonances which have a mass and a width, and we stand by the solution proposed there to use the width as a genuine uncertainty of the meson-dominance ansatz. This generates a full band of predictions which provide an uncertainty range for a formula of the form of Eq. (6). The experimental widths are $\Gamma_{a_1} = 0.425$ GeV and $\Gamma_{a_1'} = 0.254$ GeV as listed in the Particle Data Group (PDG) compilation [50].[1] The masses are only the central values of the axial-mesons' spectra. We use the half-width rule to generate random values for m_{a_1} and $m_{a_1'}$ following Gaussian distributions with variances $\Gamma_{a_1}/2$ and $\Gamma_{a_1'}/2$, respectively. This provides a distribution band for the axial form factor [43] which is slightly above the bulk of the abundant and incompatible G_A data (see Fig. 1) but agrees well with the lattice [30–32] and light-cone QCD sum rules estimates [33–35].

It might be useful to provide a parametrization of the uncertainty bands of the AVMD form factor as $F_A^{\text{lower}}(Q^2) \leq F_A(Q^2) \leq F_A^{\text{upper}}(Q^2)$. The lower and upper axial form factors are defined as the boundaries of the usual 1σ 68% confidence level region. They can be parametrized as a product of two monopoles, similarly to Eq. (6), as

[1] Of course this ansatz provides a value for the mean square axial radius, $\langle r^2 \rangle_A = 6/m_{a_1}^2 + 6/m_{a_1'}^2$. One can add a further axial state fixing the radius to its precise value, and comply to the pQCD short-distance constraint but the effect is not large. This way, one might take into account the tiny and predictable differences between axial radii determined by either electroproduction or neutrino scattering.

$$F_A^\alpha(Q^2) = \frac{\Lambda_\alpha^2 \Lambda_{\alpha'}^2}{(\Lambda_\alpha^2 + Q^2)(\Lambda_{\alpha'}^2 + Q^2)}, \qquad (7)$$

with $\alpha = \text{upper, lower}$. In the range $0 \le Q^2 \le 3\ \mathrm{GeV}^2$, corresponding to the blue band of Fig. 1, the cutoff parameters turn out to be $(\Lambda_{\mathrm{lower}}, \Lambda_{\mathrm{lower}'}) = (0.97486, 1.73345)$ and $(\Lambda_{\mathrm{upper}}, \Lambda_{\mathrm{upper}'}) = (1.5436, 1.54194)$, respectively (in GeV). These values illustrate the fact that the fluctuations of the form factor do not necessarily correspond to a single dipole mass fluctuation.

We will thus apply this axial form factor band for the neutrino cross section theoretical predictions. Our point of view is that given the many effects which might contribute to neutrino-nucleus scattering it may be sensible to use a credible form factor with an error estimate based on a different source of data, without resting on a specific fit to the neutrino data. It was found in Ref. [43] that the large-N_c meson-dominated form factors with pQCD constraints and supplemented with the half-width rule for an uncertainty estimate worked well also for other form factors, such as electromagnetic, scalar, and gravitational form factors. As a general rule uncertainties turned out to be comparable or smaller than lattice QCD predictions but larger than experimental data.

After presenting our main results, for completeness we will also analyze the conventional approach of fitting Eq. (6) to the MiniBooNE data. We want to investigate the traditional point of view of *assuming* certain nuclear effects before undertaking a fit of the axial form factor of the nucleon. For example, in Ref. [8] a model with nucleon spectral functions, RPA correlations, and meson exchange currents has been considered and a fit to the axial form factor has been undertaken assuming a fixed $\Delta - N$-transition form factor. We want to understand why in these studies one can extract more accurate information on the axial form factor than on the nuclear model response functions.

Anticipating some of the results to be discussed below and for a comparison, we depict also in Fig. 1 the results found in the analysis of Ref. [8] where the role of nuclear effects beyond the local Fermi gas have been addressed when a dipole form factor is fitted to the neutrino scattering data. As can be deduced from Fig. 1 the statistical errors are comparable when including the additional nuclear effects and the large and quite visible systematic change between the two fits is comparable to the spread generated by our AVMD form factor. Following the scheme of Ref. [28] we also plot in Fig. 2 the ratio between the AVMD form factor and the dipole form factor, Eq. (1), with $M_A = 1.014$ GeV in terms of the dimensionless variable ξ defined there. This quotient was fitted in Ref. [28] by including an interpolating polynomial without discarding any of the compiled data which, as we have mentioned, are incompatible as a whole.[2] As we can see that the AVMD model produces a spread compatible with the spread of the region covered by the form factor data.

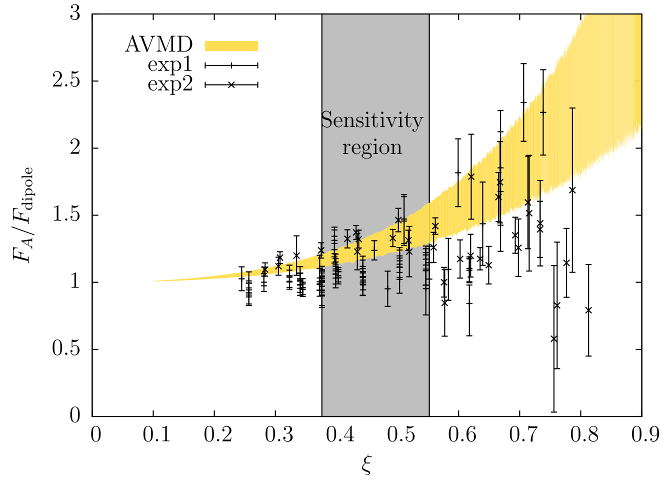

FIG. 2. The axial-meson dominance band prediction divided by the dipole parametrization for $M_A = 1.014$ MeV as a function of the dimensionless variable $\xi = 2/(1 + \sqrt{1 + 4m_N^2/Q^2})$ defined in Ref. [28]. Experimental data are from Refs. [12–16] (exp1) and Refs. [17–20] (exp2). The form factors and data are normalized to one for $Q^2 = 0$.

In both Fig. 1 and Fig. 2 we also highlight in shaded gray the main Q^2 region where fluctuations in the axial form factor have a sizable impact in the MiniBooNE data, as will be discussed below in Sec. V. Thus, our AVMD-motivated axial form factor describes reasonably well the known data spread in the Q^2 region relevant for the MiniBooNE experiment.

III. QUASIELASTIC NEUTRINO SCATTERING

In this paper we are interested in the charged-current quasielastic (CCQE) reactions in nuclei induced by neutrinos. In particular, we compute the (ν_μ, μ^-) cross section. The total energies of the incident neutrino and detected muon are $\epsilon = E_\nu$, $\epsilon' = m_\mu + T_\mu$, and their momenta are $\mathbf{k}, \mathbf{k}'$. The four-momentum transfer is $k^\mu - k'^\mu = (\omega, \mathbf{q})$, with $Q^2 = q^2 - \omega^2 > 0$. If the lepton scattering angle is θ, the double-differential cross section can be written as [51,52]

$$\frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu) = \left(\frac{M_W^2}{M_W^2 + Q^2}\right)^2 \frac{G^2 \cos^2\theta_c}{4\pi} \frac{k'}{\epsilon} v_0 S_\pm. \qquad (8)$$

[2] If would be interesting to check if the rather small uncertainties obtained in Ref. [28] are triggered by the inevitable large χ^2 values which are usually obtained when fitting mutually incompatible data and by the stiffness against fitting parameter variations. Unfortunately, no χ^2 value has been quoted and it is difficult to asses the goodness of fit.

Here $G = 1.166 \times 10^{-11}$ MeV$^{-2} \sim 10^{-5}/m_p^2$ is the Fermi constant, θ_c is the Cabibbo angle, $\cos\theta_c = 0.975$, and the kinematical factor $v_0 = (\epsilon + \epsilon')^2 - q^2$.

The nuclear structure function $S_\pm$ is defined as a linear combination of the five nuclear response functions (+ is for neutrinos and − is for antineutrinos)

$$S_\pm = V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm 2V_{T'}R_{T'}, \tag{9}$$

where the V_K coefficients depend only on the neutrino and muon kinematics and do not depend on the details of the nuclear target,

$$V_{CC} = 1 - \delta^2 \frac{Q^2}{v_0}, \tag{10}$$

$$V_{CL} = \frac{\omega}{q} + \frac{\delta^2}{\rho'}\frac{Q^2}{v_0}, \tag{11}$$

$$V_{LL} = \frac{\omega^2}{q^2} + \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right)\delta^2 \frac{Q^2}{v_0}, \tag{12}$$

$$V_T = \frac{Q^2}{v_0} + \frac{\rho}{2} - \frac{\delta^2}{\rho'}\left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right)\frac{Q^2}{v_0}, \tag{13}$$

$$V_{T'} = \frac{1}{\rho'}\left(1 - \frac{\omega\rho'}{q}\delta^2\right)\frac{Q^2}{v_0}, \tag{14}$$

where we have defined the dimensionless factors $\delta = m'/\sqrt{Q^2}$, proportional to the muon mass m', $\rho = Q^2/q^2$, and $\rho' = q/(\epsilon + \epsilon')$.

We evaluate the five nuclear response functions R_K, $K = CC, CL, LL, T, T'$ (C = Coulomb, L = longitudinal, T = transverse). following the simplest approach that treats exactly relativity, gauge invariance, and translational invariance, which is the relativistic Fermi gas model (RFG) [51,52]. The single nucleons are described by plane-wave spinors and the response functions are analytical. It is a remarkable result that the nuclear response function R_K of the RFG is proportional to a single-nucleon response function U_K times the so-called scaling function $f(\psi)$,

$$R_K = \frac{N\xi_F}{m_N \eta_F^3 \kappa} U_K f(\psi), \tag{15}$$

where N is the neutron number, $\eta_F = k_F/m_N$, and $\xi_F = \sqrt{1 + \eta_F^2} - 1$. The scaling function is defined as

$$f(\psi) = \frac{3}{4}(1 - \psi^2)\theta(1 - \psi^2), \tag{16}$$

where θ is the Heaviside step function and ψ is the scaling variable

$$\psi^2 = \frac{1}{\xi_F}\max\left\{\kappa\sqrt{1 + \frac{1}{\tau}} - \lambda - 1, \xi_F - 2\lambda\right\}, \tag{17}$$

where $\lambda = \omega/(2m_N)$, $\kappa = q/(2m_N)$, and $\tau = \kappa^2 - \lambda^2$.

Finally, we give the single-nucleon responses U_K. For $K = CC$ it is the sum of the vector and axial-vector responses, in turn is written as the sum of conserved (c.) plus nonconserved (n.c.) parts,

$$U_{CC} = U_{CC}^V + (U_{CC}^A)_{\text{c.}} + (U_{CC}^A)_{\text{n.c.}}. \tag{18}$$

For the vector CC response we have

$$U_{CC}^V = \frac{\kappa^2}{\tau}\left[(2G_E^V)^2 + \frac{(2G_E^V)^2 + \tau(2G_M^V)^2}{1 + \tau}\Delta\right], \tag{19}$$

where G_E^V and G_M^V are the isovector electric and magnetic nucleon form factors (we use Galster's parametrization), and

$$\Delta = \frac{\tau}{\kappa^2}\xi_F(1 - \psi^2)\left[\kappa\sqrt{1 + \frac{1}{\tau}} + \frac{\xi_F}{3}(1 - \psi^2)\right]. \tag{20}$$

The axial-vector CC response is the sum of conserved (c.) plus nonconserved (n.c.) parts,

$$(U_{CC}^A)_{\text{c.}} = \frac{\kappa^2}{\tau}G_A^2\Delta, \tag{21}$$

$$(U_{CC}^A)_{\text{n.c.}} = \frac{\lambda^2}{\tau}(G_A - \tau G_P)^2, \tag{22}$$

where G_A is the nucleon axial-vector form factor and G_P is the pseudoscalar axial form factor. From PCAC the pseudoscalar form factor is

$$G_P = \frac{4m_N^2}{m_\pi^2 + Q^2}G_A. \tag{23}$$

Similarly, for $K = CL, LL$ we have

$$U_{CL} = U_{CL}^V + (U_{CL}^A)_{\text{c.}} + (U_{CL}^A)_{\text{n.c.}}, \tag{24}$$

$$U_{LL} = U_{LL}^V + (U_{LL}^A)_{\text{c.}} + (U_{LL}^A)_{\text{n.c.}}. \tag{25}$$

The vector and conserved axial-vector parts are determined by current conservation,

$$U_{CL}^V = -\frac{\lambda}{\kappa}U_{CC}^V, \tag{26}$$

$$(U_{CL}^A)_{\text{c.}} = -\frac{\lambda}{\kappa}(U_{CC}^A)_{\text{c.}}, \tag{27}$$

$$U_{LL}^V = \frac{\lambda^2}{\kappa^2}U_{CC}^V, \tag{28}$$

$$(U^A_{LL})_{\text{c.}} = \frac{\lambda^2}{\kappa^2}(U^A_{CC})_{\text{c.}}, \tag{29}$$

while the n.c. parts are

$$(U^A_{CL})_{\text{n.c.}} = -\frac{\lambda\kappa}{\tau}(G_A - \tau G_P)^2, \tag{30}$$

$$(U^A_{LL})_{\text{n.c.}} = \frac{\kappa^2}{\tau}(G_A - \tau G_P)^2. \tag{31}$$

Finally, the transverse responses are given by

$$U_T = U^V_T + U^A_T, \tag{32}$$

$$U^V_T = 2\tau(2G^V_M)^2 + \frac{(2G^V_E)^2 + \tau(2G^V_M)^2}{1+\tau}\Delta, \tag{33}$$

$$U^A_T = 2(1+\tau)G^2_A + G^2_A\Delta, \tag{34}$$

$$U_{T'} = 2G_A(2G^V_M)\sqrt{\tau(1+\tau)}[1+\tilde{\Delta}], \tag{35}$$

with

$$\tilde{\Delta} = \sqrt{\frac{\tau}{1+\tau}}\frac{\xi_F(1-\psi^2)}{2\kappa}. \tag{36}$$

IV. NUMERICAL RESULTS

In Fig. 3 we show the AVMD predictions for the total integrated CCQE cross section

$$\sigma(E_\nu) = \int dT_\mu \int d\cos\theta \frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu). \tag{37}$$

The theoretical uncertainties represented by the displayed band have been computed by a Monte Carlo calculation assuming a Gaussian distribution for the axial-meson mass distributions. For comparison we show also the results obtained with a dipole axial form factor with $M_A = 1$ GeV. The MiniBooNE data are compatible with the axial-meson-dominance predictions. Note that no attempts to fit the experimental data have been made. The only parameter of the RFG model is the Fermi momentum $k_F = 225$ MeV.

The MiniBooNE unfolded energy-dependent cross section is model dependent based on a reconstruction of the neutrino energies assuming a quasielastic interaction with a neutron at rest. These data suffer from uncertainties driven by the model dependence of the neutrino energy reconstruction. For proper and useful comparisons, the flux-averaged doubly differential cross section should be used. We compute this cross section as

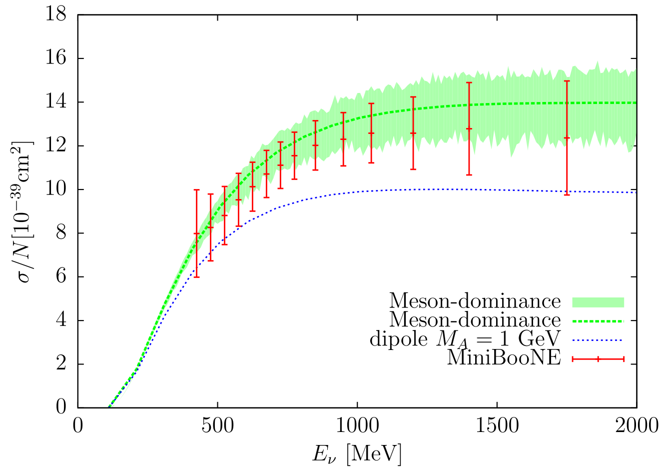

FIG. 3. Integrated quasielastic neutrino cross section of ^{12}C. The axial-meson-dominance band prediction is centered around the axial-meson masses, and it is compared to the dipole parametrization with dipolar axial mass $M_A = 1$ GeV. The experimental data are from the MiniBooNE experiment.

$$\frac{d^2\sigma}{dT_\mu d\cos\theta} = \frac{\int dE_\nu \phi(E_\nu)\frac{d^2\sigma}{dT_\mu d\cos\theta}(E_\nu)}{\int dE_\nu \phi(E_\nu)}, \tag{38}$$

where $\phi(E_\mu)$ is the incident neutrino flux.

In Fig. 4 we show results for the flux-averaged doubly differential CCQE cross section as a function of the muon kinetic energy. The continuous bands are the axial-meson-dominance model predictions for fixed values of $\cos\theta$ at the center of the experimental bins.

The MiniBooNE ν_μ CCQE flux-integrated double differential cross section is provided in bins (t_i, t_{i+1}) of T_μ and bins (c_j, c_{j+1}) of $\cos\theta$. The size of the bins are $c_{j+1} - c_j = \Delta c = \Delta\cos\theta_\mu = 0.1$ and $t_{i+1} - t_i = \Delta t = \Delta T_\mu = 0.1$ GeV.

For a meaningful comparison with the experimental data we have computed the averaged cross section for each bin, by integrating the doubly differential cross section over each discrete bin,

$$\Sigma_{ij} = \frac{1}{\Delta t \Delta c}\int_{t_i}^{t_{i+1}} dT_\mu \int_{c_j}^{c_{j+1}} d\cos\theta \frac{d^2\sigma}{dT_\mu d\cos\theta}. \tag{39}$$

The axial-vector-dominance predictions for the averaged cross section Σ_{ij} are also shown in Fig. 4, where they are compared to the experimental data. The theoretical errors are again computed by assuming a Gaussian distribution of the axial-meson masses. Note that the averaged cross section for low scattering angles (the bin $\cos\theta = 0.9 - 1.0$) is quite different from the cross section at the central value $\cos\theta = 0.95$. This is due to the strong angular dependence of the differential cross section for small angles, as can be seen in Fig. 5. Therefore the integration of the cross section over the bin is crucial to get the correct average. Note also

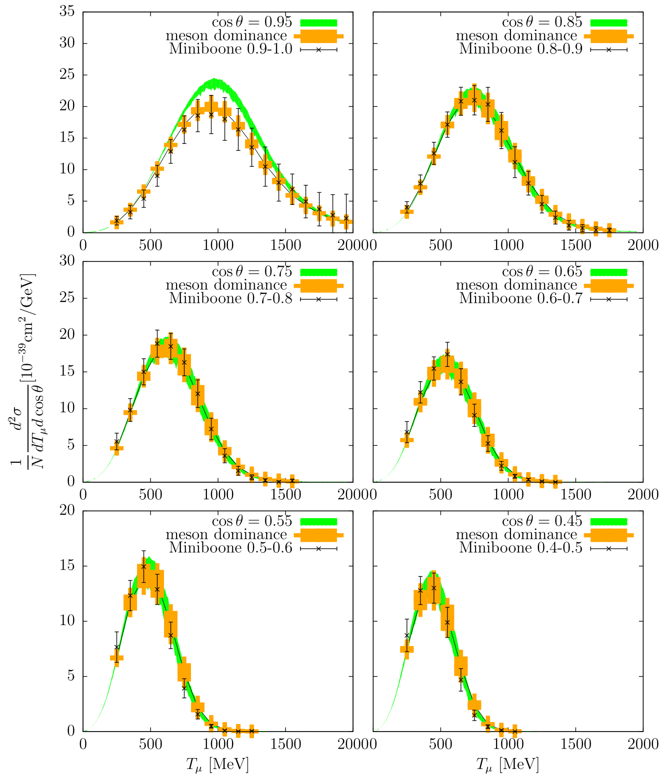

FIG. 4. Flux-averaged doubly differential CCQE cross section as a function of the muon kinetic energy. The continuous band predictions (green) have been computed for fixed values of $\cos\theta$ at the center of the experimental bins. The discrete meson-dominance predictions have been computed by integrating the doubly differential cross section over each discrete bin.

that in this region the momentum transfer takes the smallest values compatible with energy transfer, and one expects that the model dependence of the results is maximized. As a matter of fact, according to Ref. [53] the shell structure

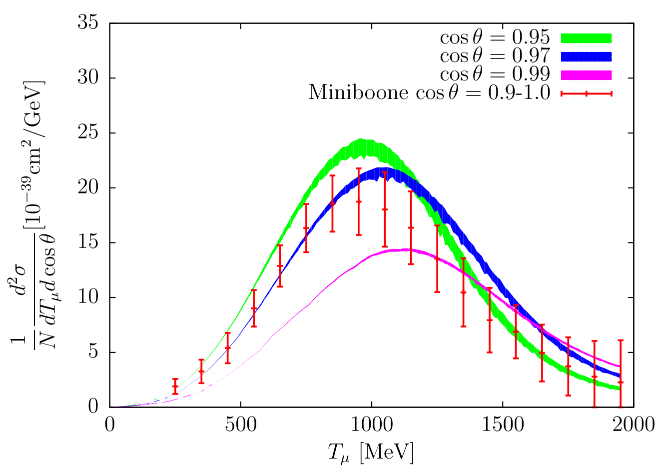

FIG. 5. Flux-averaged doubly differential CCQE cross section for several values of $\cos\theta$, and for low scattering angles, compared with the experimental data for the bin $\cos\theta = 0.9 - 1.0$.

effects for both discrete and continuum excitations are softened after integration, approaching the results of the RFG. For larger scattering angles, the angular dependence is mild, and the value of the cross section at the center of the bin is closer to the average [Eq. (39)], as can be seen in Fig. 4.

V. GOODNESS OF THE MODEL

To get a global measure of the goodness of the theoretical model in describing the experimental data requires including both theoretical and experimental uncertainties. We thus compute the distance between theory and data as given by a χ^2 metric, defined as $\chi^2 = \sum_{i,j}\chi_{ij}^2$. The χ_{ij} matrix provides the distance between theory and experiment within each bin (i, j), in units of the total uncertainty. It is defined as

$$\chi_{ij} = \frac{\Sigma_{ij}^{(\mathrm{th})} - \Sigma_{ij}^{(\mathrm{exp})}}{\sqrt{(\Delta\Sigma_{ij}^{(\mathrm{th})})^2 + (\Delta\Sigma_{ij}^{(\mathrm{exp})})^2}}, \tag{40}$$

where $\Delta\Sigma_{ij}^{(\mathrm{exp})}$ is the experimental error. and $\Delta\Sigma_{ij}^{(\mathrm{th})}$ is the theoretical uncertainty due to the physical widths of the axial mesons. In Fig. 6 we show the matrix values χ_{ij} computed for all the bins of the MiniBooNE CCQE neutrino experiment. We obtain $\chi^2 = 111$, so that dividing by the number of bins $N = 137$, we get $\chi^2/N = 0.81$. Globally the model agrees remarkably well with data taking into account that we do not minimize any χ^2 and we just compute it.

While the χ^2 value seems to be acceptable, let us analyze the assumptions underlying the comparison and its statistical significance in some more detail. We are just testing that the difference between the theory and the data should behave as a random variable, namely a standardized normal distribution. However, a look at Fig. 6 reveals that the level

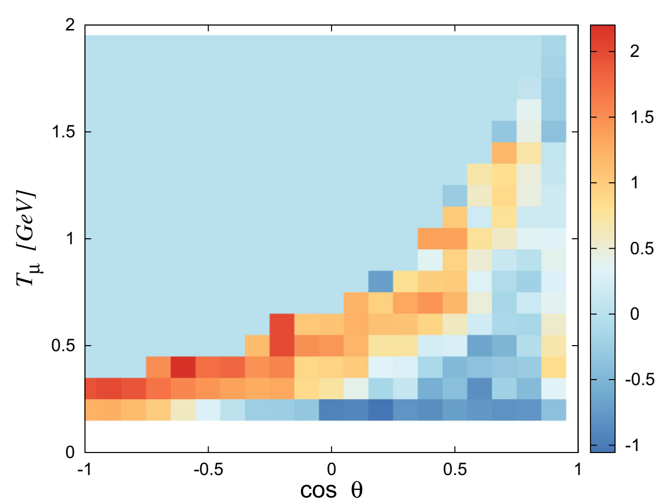

FIG. 6. The χ_{ij} computed values for each bin pair are represented in the $(\cos\theta_\mu, T_\mu)$ plane as a color image.

of disagreement is located at the edges of the plot, while we should expect a more uniform pattern globally if the χ_{ij} were distributed randomly. Through a statistical analysis, we found that there is a strong asymmetry in the residuals χ_{ij}, indicating gross systematic differences. Thus, we believe that these large discrepancies are possibly beyond the applicability of the RFG. At the same time one should also admit that the double binning procedures—which are essential for a proper comparison with the data—tend to wash out nuclear effects.

As we anticipated in our discussion around Fig. 1 some fits to the dipolar axial mass do generate rather good χ^2 values and unprecedented accuracy for the axial form factor [8]. Let us recall that a too low value is as bad as a too high value, since the χ^2 distribution for a large number of degrees of freedom $\nu = N - P$ behaves as a Gaussian distribution and thus it must be $\chi^2/\nu = 1 \pm \sqrt{2/\nu}$ within the 1σ confidence level. For instance, in Ref. [8] a value of $\chi^2/\nu = 33/(137 - 2) = 0.24$ was obtained which is outside the expected confidence level by 6σ. This suggests that experimental errors may be too large, and the question is whether errors can be reduced without destroying the Gaussian nature of the fluctuations. Moreover, let us recall that the statistical approach based on χ^2 fits deals with testing the validity of a given functional form for the *true* form factor, while despite the much extended popularity there is no field-theoretical support for a dipole form factor.

In order to understand those results we have performed a conventional χ^2 fit with two axial masses as minimization parameters. For this fit we include only the experimental errors in the denominator of Eq. (40). As in Ref. [8], we normalize the data by a factor $\lambda = 0.96$ and subtract a constant Q value $Q_b = 17$ MeV from the energies of the particle-hole excitations (note that while this modification by hand of the RFG energies improves the fit, the gauge invariance of the model is broken). We find the minimum at $m_{a_1} = m_{a_1'} = 1293$ with $\chi^2/\nu = 0.31$. We have tested the normality of residuals, and we find that they very likely correspond to a Gaussian distribution. This indicates that the experimental errors should probably be rescaled by a factor less than $1/2$, i.e., the fit would be acceptable if the errors were twice as small as those stated in the experiment. This observation concerns all previous determinations of the dipolar axial mass from these neutrino data, based on fits trying to minimize the discrepancies with the experiment. Note that we *are not* disputing the existence of certain well-known important nuclear effects. The total uncertainty on the theoretical neutrino-nucleus cross section can be due to uncertainties on both the nuclear effects and on the axial form factor. Here we focus on the size of the axial form factor uncertainties since they are obviously not small.

In Fig. 7 we plot the χ^2/ν values as a function of the two axial masses, showing that they are highly correlated. While the dipole form factor (two equal axial masses) is contained in the confidence region around the minimum, it is not the only allowed solution as two different axial masses also provide acceptable fits.

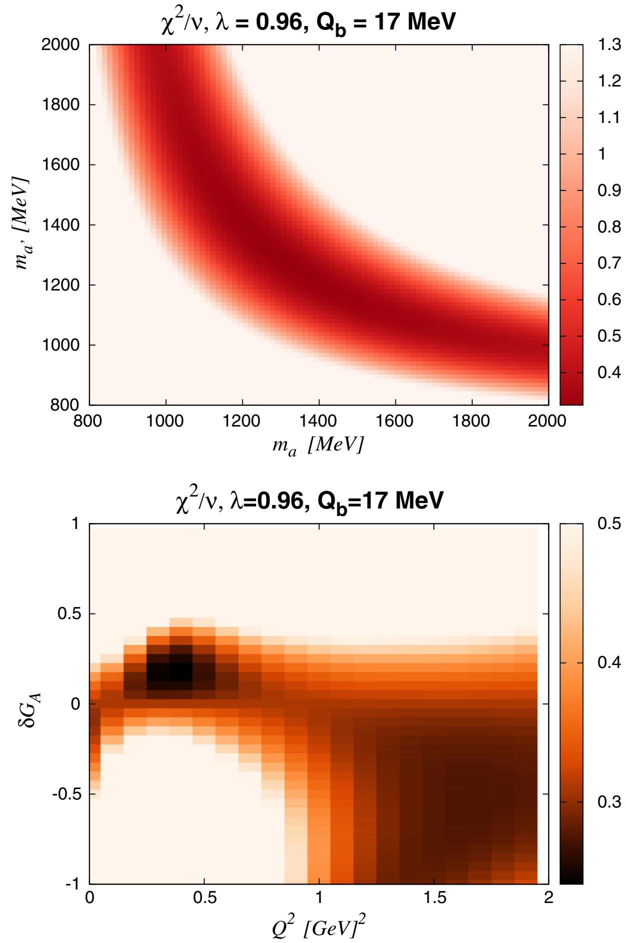

FIG. 7. Top panel: χ^2 contour plot for the fitted axial masses. Bottom panel: χ^2 contour plots for the errors in the axial form factor $\delta G_A(Q^2)$ when the Q^2 values are binned with $\Delta Q^2 = 0.1\ \mathrm{GeV}^2$ in the range $0.1\ \mathrm{GeV}^2 \leq Q^2 \leq 2\ \mathrm{GeV}^2$.

In order to study the sensibility of the results against general variations of the form factor, we also show in Fig. 7 the χ^2/ν contour plots for the errors in the axial form factor $\delta G_A(Q^2)$ when the Q^2 values are binned with $\Delta Q^2 = 0.1\ \mathrm{GeV}^2$ in the range $Q^2 \leq 2\ \mathrm{GeV}^2$. The χ^2 for each value of δG_A in a Q^2 bin has been computed by adding the specified value of δG_A to the form factor at the Q^2 values of the corresponding bin only. This shows that the value of χ^2 can be lowered further for more general variations of the axial form factor. To explore this issue deeper we have performed simultaneous variations of $\delta G_A(Q^2)$ in 20 Q^2 bins. A new minimum was found giving $\chi^2 = 23.8$ and $\chi^2/\nu = 0.17$. As seen in Fig. 7—and verified by our minimization—the data seems to favor a larger form factor around $Q^2 = 0.4\,\mathrm{GeV}^2$ and a smaller one around $1.2 - 2\ \mathrm{GeV}^2$.

Note that, while our analysis here is focused on the quasielastic neutrino-nucleus scattering, similar meson-dominance ideas for the nucleon and Δ resonance have been discussed in previous work by Dominguez and collaborators [54,55] suggesting a possible extension to $N - \Delta$ transition form factors.

VI. CONCLUSIONS AND OUTLOOK

Most of the previous analyses of the axial form factor assuming a dipolar form and fitting MiniBooNE neutrino-nucleus scattering data provide unprecedented accurate but incompatible determinations of the dipolar axial mass, regardless of the assumed nuclear model. This suggests that while the proposed dipolar parametrization minimizes the mean squared distance between theory and data, it does not account properly for the experimental data fluctuations, introducing a systematic bias and invalidating the conventional least-squares fitting strategy assumptions. Besides, the large spread of the many experimental data for the axial form factor is not sharpened by the currently available lattice QCD calculations or QCD sum rules estimates, where a direct determination of the axial-current matrix element has been undertaken. As a consequence, the validation of known nuclear effects in neutrino-nucleus scattering is hampered by the many contradicting determinations of the axial form factor already in the quasielastic region.

Here we have taken a different perspective by admitting from the beginning the existence of an uncertainty band in the axial form factor. We assumed a theoretically based axial form factor with an *a priori* uncertainty estimate, regardless of the neutrino data we intend to describe. Namely, we used the minimal AVMD compatible with pQCD, an ansatz motivated by quite general large-N_c features and which requires just the a_1 and a_1' mesons to be saturated. As it has been done in previous determinations of other hadronic and generalized form factors, we have taken as an educated guess the half-width rule for the axial-vector isovector masses. The produced spread is fairly consistent with the current experimental and lattice spread of values.

Most remarkably, the errors in the axial form factor determined by the axial-vector dominance and using the half-width rule, while quite generous, do not generally produce larger uncertainties in the neutrino-nucleus scattering than the experimental differential cross sections reported by the MiniBooNE Collaboration. We have also provided evidence that the region of the axial form factor having the most impact in the MiniBooNE data is in the range $0.2 \lesssim Q^2 \lesssim 0.6\ \mathrm{GeV}^2$, whereas fluctuations outside this regime tend to be marginal. We stress that these features cannot be captured by the conventional dipole parametrization.

Of course the minimal hadronic ansatz could be improved by adding other poles from the PDG axial-mesons compilation. In the case of three poles, unlike the present case additional unknown information such as, e.g., the coupling of the third meson to the nucleon is needed. One could expect that future neutrino data might be accurate enough to pin down this extra parameter.

Our analysis has been carried out using the RFG model which for the quasielastic region does not seem to miss effects which are larger than the present uncertainty in the axial form factor using the AVMD ansatz supplemented with the half-width rule. The role of additional nuclear effects improving the present model will be presented in further work. The role played at higher energies by the equivalent AVMD form factors and further nuclear mechanisms remains to be seen.

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