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Integrability of a D1-brane on a group manifold with mixed three-form flux

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We consider a D1-brane as a natural probe of the group manifold with mixed three-form fluxes. We determine the Lax connection for a given theory. Then we switch to the canonical analysis and calculate the Poisson brackets between spatial components of Lax connections, and we argue for the integrability of a given theory.

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I. INTRODUCTION AND SUMMARY

The integrability in the AdS/CFT correspondence is fundamental for calculations beyond the perturbative theory. A famous example is the duality between $\mathcal{N} = 4$ super-Yang-Mills theory in four dimensions and type-IIB theory on $AdS_5 \times S^5$ with the Ramond-Ramond (RR) flux, where the exact string spectrum and the spectrum of anomalous dimensions in the super-Yang-Mills theory can be described by Bethe-ansatz equations.[1] The integrability on the string theory side of the correspondence is based on the existence of the Lax connection that implies the existence of an infinite number of conserved charges [5]. However, this is only a necessary condition since the integrability of the theory also requires that these conserved charges are in involution, as was stressed in Ref. [6].

It is well known that integrability can be applied for group manifolds with nontrivial RR and Neveu-Schwarz Neveu-Schwarz (NSNS) fluxes. Such a famous example is string theory on an AdS_3 background with nontrivial RR and NSNS fluxes. It turns out that in the case of pure NSNS flux the string theory can be quantized using world-sheet conformal field theory techniques [7–12]. On the other hand, the RR AdS_3 backgrounds have a more complicated CFT description [13], while these backgrounds are integrable as well [14,15].

On the other hand, the case of a mixed RR-NSNS AdS_3 background is much more challenging either from the CFT perspective or from the integrability point of view. One possibility is to consider small derivations from the pure NSNS point using the conformal perturbative theory [16]. Another possibility was suggested in Ref. [17], where the starting point was a pure RR background with a new WZ term that represents the coupling to the NSNS flux. This beautiful construction leads to rapid progress in understanding of role integrability in the theory with mixed fluxes; for related works, see [18–23].

It is well known that the AdS_3 backgrounds with different fluxes are related by U-duality transformations. For example, type IIB S-duality relates $AdS_3 \times S^3 \times T^4$ backgrounds supported by different three-form fluxes: the pure RR flux background arises as the near-horizon limit of a D1-D5-brane system, while a background supported by mixed three-form flux involves the near-horizon limit of NS5-branes and fundamental strings in addition to the D1- and D5-branes. At the same time, the fundamental string transforms under S-duality to the bound state of a D1-brane and fundamental string. Then one can ask the question whether a D1-brane could be considered as another probe in string theory that naturally incorporates the coupling between NSNS and RR fields. In fact, the low-energy description of the D1-brane is given by Dirac-Born-Infeld action together with a Chern-Simons term with explicit coupling to RR and NSNS two-forms. We demonstrated in our previous paper [24] that a D1-brane on the group manifold with nontrivial NSNS flux is integrable. In this paper, we extend the given analysis to the most general background including a dilaton, Ramond-Ramond zero-form $C^{(0)}$, and Ramond-Ramond two-form $C^{(2)}$ together with the three-forms $F = dC^{(2)}$ and $H = dB$ that can be expressed using the structure constants of the group that defines the group manifold on which the D1-brane propagates. We find that this D1-brane is integrable on the condition that the dilaton and Ramond-Ramond zero-form are constants. Then we perform a canonical analysis of the given theory and calculate the Poisson brackets between spatial components of Lax connections. We show that this Poisson bracket has the form that ensures that the conserved charges are in involutions up to the well-known problems with terms containing a derivative of delta functions that need special regularizations [25–27]. Then we consider a concrete example, which is a D1-brane on an $AdS_3 \times S^3$ background with mixed RR-NS flux. We first show that the equation of motion for this D1-brane can be expressed as the equation of conservation of a specific current which is, however, nonlinear due to the specific form of D-brane action. Then, introducing an auxiliary

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[1]For review, see [1–4].

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metric and corresponding constraint, we can rewrite this current to the manifestly linear form.[2] Then fixing the gauge and for certain backgrounds, we can find currents whose conservation law is special and that is an analogue of the holomorphic and antiholomorphic currents in the Wess-Zumino-Witten (WZW) model [28]. Explicitly, we find that this occurs in the case of a D1-brane in the near-horizon limit of a D1-D5-brane background with zero electric flux. Surprisingly, we also find that the same situation occurs in the case of the background with nonzero RR and NSNS fluxes that arises from a D1-D5-brane background through specific $SL(2,Z)$ transformation. This is a very interesting result that suggests the possibility that for these values of fluxes the D1-brane theory can be treated with the help of powerful techniques of the two-dimensional conformal field theory.

Let us outline our results. We show that a D1-brane can be considered as a natural probe of backgrounds with mixed flux. We mean that the given idea is very attractive and should be elaborated further. For example, it would be nice to explicitly determine a world-sheet S matrix for a given theory in the AdS_3 background with mixed flux. It would also be nice to analyze classical solutions on the world volume of a given theory corresponding to possible magnon solutions and compare them with the string solutions. We hope to return to these problems in the future. It would also be interesting to try to extend the given analysis to the supersymmetric D1-brane theory. A further question that deserves detailed treatment is the question of the conformal field theory description of a D1-brane with electric flux on $\mathrm{AdS}_3\times S^3$ with specific values of fluxes. We hope to return to all these problems in the future.

This paper is organized as follows. In Sec. II, we introduce a D1-brane on the group manifold background with nontrivial NSNS and two RR forms. We analyze under which conditions the world-sheet theory is integrable. Then, in Sec. III, we perform a Hamiltonian analysis of the given theory and calculate the Poisson brackets between spatial components of the Lax connection. Finally, in Sec. IV, we consider a D1-brane on various $\mathrm{AdS}_3\times S^3$ backgrounds with three-form fluxes.

II. D1-BRANE ON GROUP MANIFOLD

In this section, we introduce D1-brane action that governs the dynamics of the D1-brane on a general background. Recall that the given action is the sum of Dirac-Born-Infeld and Chern-Simons terms and has the form

$$\begin{aligned}S&=-T_{D1}\int d\tau d\sigma e^{-\Phi}\sqrt{-\det\mathbf{A}}\\&\quad+T_{D1}\int d\tau d\sigma((b_{\tau\sigma}+2\pi\alpha'\mathcal{F}_{\tau\sigma})C^{(0)}+c_{\tau\sigma}),\\ \mathbf{A}_{\alpha\beta}&=G_{MN}\partial_\alpha x^M\partial_\beta x^N+2\pi\alpha'\mathcal{F}_{\alpha\beta}+B_{MN}\partial_\alpha x^M\partial_\beta x^N,\\ \mathcal{F}_{\alpha\beta}&=\partial_\alpha A_\beta-\partial_\beta A_\alpha,\end{aligned}\tag{1}$$

where $x^M, M, N=0,1,\ldots,D$ are embedding coordinates of a D1-brane in the background that is specified by the metric $G_{MN}(X)$ and NSNS two-form $B_{MN}=-B_{NM}$ together with Ramond-Ramond two-form $C^{(2)}_{MN}=-C^{(2)}_{NM}$. We further consider a background with nontrivial dilaton Φ and RR zero-form $C^{(0)}$. Furthermore, $\sigma^\alpha=(\tau,\sigma)$ are world-sheet coordinates of the D1-brane, and $b_{\tau\sigma}, c_{\tau\sigma}$ are pullbacks of B_{MN} and C_{MN} to the world volume of the D1-brane. Explicitly,

$$b_{\alpha\beta}\equiv B_{MN}\partial_\alpha x^M\partial_\beta x^N=-b_{\beta\alpha},\qquad c_{\tau\sigma}=C^{(2)}_{MN}\partial_\tau x^M\partial_\sigma x^N.\tag{2}$$

Finally, $T_{D1}=\frac{1}{2\pi\alpha'}$ is D1-brane tension, and A_α, $\alpha=\tau,\sigma$ is a two-dimensional gauge field that propagates on the world sheet of the D1-brane.

It is useful to rewrite the action (1) into the form

$$\begin{aligned}S&=-T_{D1}\int d\tau d\sigma e^{-\Phi}\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}\\&\quad+T_{D1}\int d\tau d\sigma((b_{\tau\sigma}+2\pi\alpha'\mathcal{F}_{\tau\sigma})C^{(0)}+c_{\tau\sigma}),\end{aligned}\tag{3}$$

where $g_{\alpha\beta}=G_{MN}\partial_\alpha x^M\partial_\beta x^N$, $\det g=g_{\tau\tau}g_{\sigma\sigma}-(g_{\tau\sigma})^2$. From (3), we obtain the equations of motion for x^M:

$$\begin{aligned}&\partial_M[\Phi]e^{-\Phi}\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}-\partial_\alpha\left[\frac{G_{MN}\partial_\beta x^N g^{\beta\alpha}\det g}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}\right]+\frac{\partial_M G_{KL}\partial_\alpha x^K\partial_\beta x^L g^{\beta\alpha}\det g}{2\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}\\&\quad+\frac{(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}\partial_M B_{KL}\partial_\tau x^K\partial_\sigma x^L-\partial_\tau\left[\frac{B_{MN}\partial_\sigma x^N(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}\right]\\&\quad+\partial_\sigma\left[\frac{B_{MN}\partial_\tau x^N(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}\right]+\partial_M C^{(0)}(b_{\tau\sigma}+2\pi\alpha'\mathcal{F}_{\tau\sigma})+C^{(0)}\partial_M b_{KL}\partial_\tau x^K\partial_\sigma x^L\\&\quad-\partial_\tau[C^{(0)}b_{MK}\partial_\sigma x^K]-\partial_\sigma[C^{(0)}b_{KM}\partial_\tau x^K]+\partial_M C^{(2)}_{KL}\partial_\tau x^K\partial_\sigma x^L-\partial_\tau[C^{(2)}_{MK}\partial_\sigma x^K]-\partial_\sigma[C^{(2)}_{KM}\partial_\tau x^K]=0,\end{aligned}\tag{4}$$

[2] This is a similar situation as in the case of the equivalence between Nambu-Goto and Polyakov action for a bosonic string.

while the equations of motion for A_τ, A_σ take the form

$$\partial_\tau\left[e^{-\Phi}\frac{(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}+C^{(0)}\right]=0,$$
$$\partial_\sigma\left[e^{-\Phi}\frac{(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}+C^{(0)}\right]=0. \quad (5)$$

The last two equations imply the existence of a constant electric flux

$$\frac{e^{-\Phi}(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}}+C^{(0)}=\Pi, \qquad \Pi=\text{const.} \quad (6)$$

With the help of this constant, we can express $2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma}$ as

$$2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma}=\frac{(\Pi-C^{(0)})\sqrt{-\det g}}{\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}}, \quad (7)$$

so that the equations of motion (4) simplify considerably:

$$-\partial_M\left[\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]\sqrt{-\det g}$$
$$+\partial_\alpha\left[G_{MN}\partial_\beta x^N g^{\beta\alpha}\sqrt{-\det g}\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]$$
$$-\frac{1}{2}\partial_M G_{KL}\partial_\alpha x^K\partial_\beta x^L\sqrt{-\det g}\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}$$
$$+\Pi H_{MKN}\partial_\tau x^K\partial_\sigma x^N+F_{MKN}\partial_\tau x^K\partial_\sigma x^N=0, \quad (8)$$

where

$$H_{MNK}=\partial_M B_{NK}+\partial_N B_{KM}+\partial_K B_{MN},$$
$$F_{MNK}=\partial_M C^{(2)}_{NK}+\partial_N C^{(2)}_{KM}+\partial_K C^{(2)}_{MN}. \quad (9)$$

Now we are going to be more specific about the background. When we consider group manifold G, we presume that the metric G_{MN} can be expressed as

$$G_{MN}=E_M{}^A E_N{}^B K_{AB}, \quad (10)$$

where for the group element $g\in G$ we have

$$J\equiv g^{-1}dg=E_M{}^A T_A dx^M, \quad (11)$$

where T_A is the basis of Lie algebra $\mathcal{G}$ of the group G. Note that $K_{AB}=\mathrm{Tr}(T_A T_B)$. Furthermore, from the definition (11), we obtain

$$dJ+J\wedge J=0 \quad (12)$$

that implies an important relation

$$\partial_M E_N{}^A-\partial_N E_M{}^A+f^A{}_{BC}E_M{}^B E_N{}^C=0, \quad (13)$$

where

$$[T_B,T_C]=T_A f^A{}_{BC}. \quad (14)$$

In the case of the fluxes F_{KLM}, H_{KLM}, we presume the following relations between them and the structure constants f_{ABC} of the Lie algebra $\mathcal{G}$:

$$H_{MNK}E^M{}_A E^N{}_B E^K{}_C=\kappa f_{ABC},$$
$$F_{MNK}E^M{}_A E^N{}_B E^K{}_C=\omega f_{ABC}, \quad (15)$$

where κ and ω are constants. The first formula is a well-known relation that defines the Wess-Zumino term when we describe the motion of string on a group space with B flux.[3] In the case of Ramond-Ramond flux, we introduce this relation in order to preserve symmetry between NS-NS and RR fluxes. At this place we will not discuss the problem of whether background fields define a consistent string theory background, and, hence, we can consider κ and ω as free parameters. On the other hand, it is important to stress that when we discuss a D1-brane on $\mathrm{AdS}_3\times S^3$ with mixed fluxes these coefficients κ and ω have concrete values in order to define a consistent string theory background. We will discuss this case in more detail in Sec. IV.

With the help of (15), we can write

$$E^M{}_C H_{MKL}\partial_\tau x^K\partial_\sigma x^L=\kappa f_{CAB}J^A_\tau J^B_\sigma,$$
$$E^M{}_C F_{MKL}\partial_\tau x^K\partial_\sigma x^L=\omega f_{CAB}J^A_\tau J^B_\sigma. \quad (16)$$

Note that $E^M{}_A$ is inverse to $E_M{}^B$ defined as

$$E^M{}_A E_M{}^B=\delta^B_A, \qquad E^M{}_A E_N{}^A=\delta^M_N. \quad (17)$$

Now with the help of (13) and (16), we can rewrite the equations of motion (8) to the form that contains the current $J^A_\alpha=E_M{}^A\partial_\alpha x^M$:

$$-E^M{}_C\partial_M\left[\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]\sqrt{-\det g}$$
$$+K_{CB}\partial_\alpha\left[J^B_\beta g^{\beta\alpha}\sqrt{-\det g}\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]$$
$$+\Pi\kappa f_{CAB}J^A_\tau J^B_\sigma+\omega f_{CAB}J^A_\tau J^B_\sigma=0. \quad (18)$$

Now we are ready to analyze the integrability of the given theory. Let us consider the following current:

[3] See, for example, [29] for a nice discussion and calculations of the Poisson brackets of various currents.

$$L_\tau^A = AJ_\tau^A + B\sqrt{-g}g^{\sigma\alpha}\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}J_\alpha^A,$$
$$L_\sigma^A = AJ_\sigma^A - B\sqrt{-g}g^{\tau\alpha}\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}J_\alpha^A, \quad (19)$$

where A and B are coefficients that will be determined by the requirement that the current L_α^A is flat. First of all, we calculate

$$\begin{aligned}&\partial_\tau L_\sigma^A - \partial_\sigma L_\tau^A\\ &= -AJ_\tau^B J_\sigma^C f^A{}_{BC} + B(\Pi\kappa+\omega)f^A{}_{BC}J_\tau^B J_\sigma^C\\ &\quad - K^{AB}E^M{}_B\partial_M\left[\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]\sqrt{-\det g},\end{aligned} \quad (20)$$

where we used the equations of motion (18) together with the condition (13). As the next step, we calculate

$$f^A{}_{BC}L_\tau^B L_\sigma^C = (A^2 - B^2[e^{-2\Phi}+(\Pi-C^{(0)})^2])f^A{}_{BC}J_\tau^B J_\sigma^C. \quad (21)$$

Collecting these two results together, we obtain

$$\begin{aligned}&\partial_\tau L_\sigma^A - \partial_\sigma L_\tau^A + f^A_{BC}L_\tau^B L_\sigma^C\\ &= (-A + B(\Pi\kappa+\omega) + A^2\\ &\quad - B^2[e^{-2\Phi}+(\Pi-C^{(0)})^2])f^A{}_{BC}J_\tau^B J_\sigma^C\\ &\quad - K^{AB}E^M{}_B\partial_M\left[\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}\right]\sqrt{-\det g}.\end{aligned} \quad (22)$$

Let us now discuss the result derived above. The expression on the third line is proportional to the currents, while the expression on the fourth line contains derivatives of the background fields $C^{(0)}$ and Φ. Then clearly the expressions on the second and third lines have to vanish separately in order for L_α^A to be flat. The expression on the third line vanishes when we require that $\sqrt{e^{-2\Phi}+(\Pi-C^{(0)})^2}$ is constant. This can be ensured for nonzero electric flux Π when Φ and $C^{(0)}$ are constant. Then we have to demand that the expression on the second line in (22) vanishes. If we consider the ansatz $B=-\Lambda A$, we find the solutions in the form

$$A = \frac{1}{1-\Lambda^2(e^{-2\Phi}+(\Pi-C^{(0)})^2)}(1+(\Pi\kappa+\omega)\Lambda),$$
$$B = -\frac{\Lambda}{1-\Lambda^2(e^{-2\Phi}+(\Pi-C^{(0)})^2)}(1+(\Pi\kappa+\omega)\Lambda), \quad (23)$$

where Λ is a spectral parameter. Finally, we should mention that this is an on-shell condition. On the other hand, if we calculate the Poisson bracket between these currents, we have to express A and B given in (23) using an off-shell form of the combinations $e^{-2\Phi}+(\Pi-C^{(0)})^2$ and Π. Explicitly, from (7), we obtain

$$e^{-2\Phi}+(\Pi-C^{(0)})^2 = \frac{e^{-2\Phi}\det g}{\det g+(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2},$$
$$\Pi = \frac{e^{-\Phi}(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha'\mathcal{F}_{\tau\sigma}+b_{\tau\sigma})^2}} + C^{(0)}. \quad (24)$$

Inserting (23) and (24) into (19), we find an off-shell formulation of the flat current. In the next section, we express the spatial components of the flat current using canonical variables and calculate a Poisson bracket between them.

Let us summarize the results derived in this section. We studied the dynamics of a D1-brane on the group manifold with nontrivial NSNS and RR two-form fluxes and together with a dilaton and RR zero-form. We argued that it is possible to define the Lax connection for this theory, and we showed that this Lax connection is flat on the condition that the dilaton and RR zero-form are constant. The existence of the Lax connection is a necessary condition of integrability. The additional condition is that corresponding conserved charges are in involution, which can be seen from the form of the Poisson bracket between spatial components of the Lax connection. The calculation of this Poisson bracket will be performed in the next section.

III. POISSON BRACKETS OF THE LAX CONNECTION

In this section, we calculate the Poisson brackets between spatial components of the Lax connection. To do this, we have to develop the Hamiltonian formalism for $D1$-brane action in a general background. We start with the action (3) and find the corresponding conjugate momenta

$$\begin{aligned}p_M &= \frac{\delta L}{\delta\partial_\tau x^M} = T_{D1}\frac{e^{-\Phi}}{\sqrt{-\det g-(2\pi\alpha' F_{\tau\sigma}+b_{\tau\sigma})^2}}\\ &\quad\times(G_{MN}\partial_\alpha x^N g^{\alpha\tau}\det g\\ &\quad + (2\pi\alpha' F_{\tau\sigma}+b_{\tau\sigma})B_{MN}\partial_\sigma x^N)\\ &\quad + T_{D1}(C^{(0)}B_{MN}\partial_\sigma x^N + C^{(2)}_{MN}\partial_\sigma x^N),\\ \pi^\sigma &= \frac{\delta L}{\delta\partial_\tau A_\sigma} = \frac{e^{-\Phi}(2\pi\alpha' F_{\tau\sigma}+b_{\tau\sigma})}{\sqrt{-\det g-(2\pi\alpha' F_{\tau\sigma}+b_{\tau\sigma})^2}} + C^{(0)},\\ \pi^\tau &= \frac{\delta L}{\delta\partial_\tau A_\tau} \approx 0,\end{aligned} \quad (25)$$

and hence

$$\Pi_M \equiv p_M - \frac{\pi^\sigma}{(2\pi\alpha')} B_{MN}\partial_\sigma x^N - T_{D1}(C^{(0)}B_{MN}\partial_\sigma x^N + C^{(2)}_{MN}\partial_\sigma x^N) = T_{D1}\frac{e^{-\Phi}}{\sqrt{-\det g - (2\pi\alpha' F_{\tau\sigma} + b_{\tau\sigma})^2}} G_{MN}\partial_\alpha x^N g^{\alpha\tau}\det g. \tag{26}$$

Using these relations, it is easy to see that the bare Hamiltonian is equal to

$$H_B = \int d\sigma(p_M\partial_\tau x^M + \pi^\sigma\partial_\tau A_\sigma - \mathcal{L}) = \int d\sigma \pi^\sigma \partial_\sigma A_\tau, \tag{27}$$

while we have three primary constraints:

$$\pi^\tau \approx 0, \qquad \mathcal{H}_\sigma \equiv p_M\partial_\sigma x^M \approx 0,$$
$$\mathcal{H}_\tau \equiv \frac{1}{T_{D1}}\Pi_M G^{MN}\Pi_N + T_{D1}(e^{-2\Phi} + (\pi^\sigma - C^{(0)})^2)g_{\sigma\sigma} \approx 0. \tag{28}$$

Including these primary constraints to the definition of the Hamiltonian, we obtain an extended Hamiltonian in the form

$$H = \int d\sigma(\lambda_\tau\mathcal{H}_\tau + \lambda_\sigma\mathcal{H}_\sigma - A_\tau\partial_\sigma\pi^\sigma + v_\tau\pi^\tau), \tag{29}$$

where λ_τ, λ_σ, and v_τ are Lagrange multipliers corresponding to the primary constraints $\mathcal{H}_\tau \approx 0$, $\mathcal{H}_\sigma \approx 0$, and $\pi^\tau \approx 0$, respectively. Now we have to check the stability of all constraints. The requirement of the preservation of the primary constraint $\pi^\tau \approx 0$ implies the secondary constraint

$$\mathcal{G} = \partial_\sigma\pi^\sigma \approx 0. \tag{30}$$

In the case of the constraints $\mathcal{H}_\tau$ and $\mathcal{H}_\sigma$, we can easily show in the same way as in Ref. [24] that the constraints $\mathcal{H}_\tau$ and $\mathcal{H}_\sigma$ are first-class constraints and hence they are preserved during the time evolution.

Now we are ready to proceed to the calculations of the Poisson brackets between spatial components of the flat current L^A_σ for different spectral parameters Λ and Γ:

$$\{L^A_\sigma(\Lambda,\sigma), L^B_\sigma(\Gamma,\sigma')\}. \tag{31}$$

Recall that these are currents that define the monodromy matrix and hence corresponding conserved charges. Using (24) and (25), we find that the spatial component of the current L^A_σ expressed using canonical variables has the form

$$L^A_\sigma = \frac{1+\Lambda(\pi^\sigma\kappa + \omega)}{1-\Lambda^2(e^{-2\Phi} + (\pi^\sigma - C^{(0)})^2)} \times \left(E_M{}^A\partial_\sigma x^M - \frac{\Lambda}{T_{D1}}E^M{}_B K^{AB}\Pi_M\right). \tag{32}$$

In order to calculate (31), we need the following Poisson brackets:

$$\{x^M(\sigma), \Pi_N(\sigma')\} = \delta^M_N\delta(\sigma-\sigma'),$$
$$\{E_M{}^A(\sigma), \Pi_N(\sigma')\} = \partial_N E_M{}^A\delta(\sigma-\sigma'),$$
$$\{E^M{}_A(\sigma), \Pi_N(\sigma')\} = \partial_N E^M{}_A\delta(\sigma-\sigma'), \tag{33}$$

and also

$$\{\Pi_M(\sigma), \Pi_N(\sigma')\} = \frac{1}{2\pi\alpha'}(\pi^\sigma + C^{(0)})H_{MNK}\partial_\sigma x^K\delta(\sigma-\sigma') + \frac{1}{2\pi\alpha'}F_{MNK}\partial_\sigma x^K\delta(\sigma-\sigma') + \frac{1}{2\pi\alpha'}\mathcal{G}B_{MN}\delta(\sigma-\sigma'), \tag{34}$$

and finally

$$\{E^M{}_A\Pi_M(\sigma), E^N{}_B\Pi_N(\sigma')\} = -E^M{}_D f^D{}_{AB}\Pi_M\delta(\sigma-\sigma') + E^M{}_A\left(\frac{1}{2\pi\alpha'}(\pi^\sigma + C^{(0)})H_{MNK}\partial_\sigma x^K + \frac{1}{2\pi\alpha'}F_{MNK}\partial_\sigma x^K + \frac{1}{2\pi\alpha'}\mathcal{G}B_{MN}\right)E^N{}_B\delta(\sigma-\sigma'). \tag{35}$$

With the help of these results, we obtain

$$\begin{aligned}\{L^A_\sigma(\Lambda,\sigma), L^B_\sigma(\Gamma,\sigma')\} = &-\frac{1}{T_{D1}}f(\Lambda)f(\Gamma)(\Gamma+\Lambda)K^{AB}\partial_\sigma\delta(\sigma-\sigma') - \frac{1}{T_{D1}}K^{AB}\left[\Gamma\frac{df}{d\pi^\sigma}(\Lambda,\sigma) + \Lambda\frac{df}{d\pi^\sigma}(\Gamma,\sigma)\right]\mathcal{G}\delta(\sigma-\sigma')\\ &-\frac{1}{T_{D1}}f(\Lambda)f(\Gamma)\left(\Lambda+\Gamma-\frac{1}{T_{D1}}\Lambda\Gamma[(\pi^\sigma + C^{(0)})\kappa+\omega]\right)K^{AC}f^B{}_{CE}E^E{}_M\partial_\sigma x^M\delta(\sigma-\sigma')\\ &+\frac{\Lambda\Gamma}{T^2_{D1}}f(\Lambda)f(\Gamma)K^{AC}f^B{}_{CD}K^{\mathrm{DE}}E^M{}_E\Pi_M\delta(\sigma-\sigma'),\end{aligned} \tag{36}$$

where we introduced the function $f(\Lambda,\sigma)$,

$$f(\Lambda,\sigma)=\frac{1+\Lambda(\pi^\sigma(\sigma)\kappa+\omega)}{1-\Lambda^2(e^{-2\Phi}+(\pi^\sigma(\sigma)-C^{(0)})^2)}, \tag{37}$$

and used the fact that

$$\partial_\sigma f(\Lambda,\sigma)=\frac{df(\Lambda,\sigma)}{d\pi^\sigma}\partial_\sigma\pi^\sigma=\frac{df(\Lambda,\sigma)}{d\pi^\sigma}\mathcal{G}. \tag{38}$$

Now we demand that the expression proportional to the δ function is equal to

$$\begin{aligned}-K^{AD}f^B{}_{DC}(XL_\sigma^C(\Lambda)-YL_\sigma^C(\Gamma))={}&-K^{AD}f^B{}_{DC}(Xf(\Lambda)-Yf(\Gamma))E_M{}^C\partial_\sigma x^M\\&+\frac{1}{T_{D1}}K^{AD}f^B{}_{DC}(Af(\Lambda)\Lambda-Bf(\Gamma)\Gamma)E^M{}_EK^{CE}\Pi_M,\end{aligned} \tag{39}$$

where X and Y are unknown functions. Comparing (36) with (39), we derive the following equations for X and Y:

$$\begin{aligned}\frac{1}{T_{D1}}f(\Lambda)f(\Gamma)\left(\Lambda+\Gamma-\frac{1}{T_{D1}}\Lambda\Gamma((\pi^\sigma+C^{(0)})\kappa+\omega)\right)&=Xf(\Lambda)-Yf(\Gamma),\\ \frac{\Lambda\Gamma}{T_{D1}}f(\Lambda)f(\Gamma)&=Xf(\Lambda)\Lambda-Yf(\Gamma)\Gamma.\end{aligned} \tag{40}$$

These equations have the following solutions:

$$\begin{aligned}X&=\frac{\Lambda^2}{\Gamma-\Lambda}\frac{f(\Lambda)}{T_{D1}}[1-\Gamma((\pi^\sigma+C^{(0)})\kappa+\omega)],\\ Y&=\frac{\Gamma^2}{\Gamma-\Lambda}\frac{f(\Gamma)}{T_{D1}}[1-\Lambda((\pi^\sigma+C^{(0)})\kappa+\omega)]\end{aligned} \tag{41}$$

that are a generalization of the solutions found in Ref. [24] to the case of nontrivial Ramond-Ramond flux. In summary, we obtain the final result

$$\begin{aligned}\{L_\sigma^A(\Lambda,\sigma),L_\sigma^B(\Gamma,\sigma')\}={}&-\frac{1}{T_{D1}}f(\Lambda)f(\Gamma)(\Gamma+\Lambda)K^{AB}\partial_\sigma\delta(\sigma-\sigma')-\frac{1}{T_{D1}}K^{AB}\left(\Gamma\frac{df(\Lambda)}{d\pi^\sigma}+\Lambda\frac{df(\Gamma)}{d\pi^\sigma}\right)\mathcal{G}\delta(\sigma-\sigma')\\&-\frac{1}{T_{D1}(\Gamma-\Lambda)}K^{AD}f^B{}_{DC}(\Gamma^2f(\Gamma)[1-\Lambda((\pi^\sigma+C^{(0)})\kappa+\omega)]L_\sigma^C(\Lambda)\\&-\Lambda^2f(\Lambda)[1-\Gamma((\pi^\sigma+C^{(0)})\kappa+\omega)]L_\sigma^C(\Gamma))\delta(\sigma-\sigma').\end{aligned} \tag{42}$$

We see that the expression proportional to $\mathcal{G}\approx 0$ vanishes on the constraint surface. We also see that there is still a term proportional to the derivative of the delta function that needs an appropriate regularization. Then the terms proportional to the delta functions are a natural generalization of the Poisson brackets of a flat connection of the principal chiral model with the Wess-Zumino term to the background with a RR background two-form. Note also that the form of the expression proportional to the delta functions implies that corresponding conservative charges are in involution, which is the condition for the integrability of the given theory [6].

IV. EXPLICIT EXAMPLE: D1-BRANE ON $AdS_3\times S^3$ WITH THREE-FORM FLUXES

In this section, we will analyze a D1-brane on $AdS_3\times S^3$ with three-form fluxes. Before we proceed to the analysis of this specific background, we still consider an arbitrary group manifold with nontrivial fluxes but with a constant dilaton and RR zero-form. Then note that, with the help of the flat condition, we can rewrite the equation of motion into the form

$$\partial_\alpha\hat{J}^{A\alpha}=0, \tag{43}$$

where we introduced the current

$$\hat{J}^{A\alpha} = T_{D1}\Big[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2}\sqrt{-g}g^{\alpha\beta}J^A_\beta + (\Pi\kappa + \omega)\epsilon^{\alpha\beta}J^A_\beta\Big], \quad (44)$$

where $\epsilon^{\tau\sigma} = -\epsilon^{\sigma\tau} = 1$. We see that the current $\hat{J}^{A\alpha}$ is conserved. On the other hand, we see that the current $\hat{J}^A$ is nonlinear and there is nothing more to say about it. We can make the given system more tractable when we introduce an auxiliary metric $\gamma_{\alpha\beta}$ that obeys the equation

$$T_{\alpha\beta} \equiv \frac{1}{2}\gamma_{\alpha\beta}\gamma^{\mu\nu}g_{\mu\nu} - g_{\alpha\beta} = 0. \quad (45)$$

It is easy to see that this equation has a solution $\gamma_{\alpha\beta} = g_{\alpha\beta}$. If we further introduce light-cone coordinates

$$\sigma^+ = \frac{1}{2}(\tau + \sigma), \qquad \sigma^- = \frac{1}{2}(\tau - \sigma), \quad (46)$$

we can rewrite Eq. (43) into the form

$$\partial_+\hat{J}^{A+} + \partial_-\hat{J}^{A-} = 0, \qquad \partial_\pm = \frac{\partial}{\partial\sigma^\pm}, \quad (47)$$

where

$$\begin{aligned}\hat{J}^{A+} &= \frac{1}{2}(\hat{J}^{A\tau} + \hat{J}^{A\sigma})\\ &= \frac{T_{D1}}{2}\Big[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2}\sqrt{-\gamma}(\gamma^{\tau\alpha}J^A_\alpha + \gamma^{\sigma\alpha}J^A_\alpha) + (\Pi\kappa + \omega)(J^A_\sigma - J^A_\tau)\Big],\\ \hat{J}^{A-} &= \frac{1}{2}(\hat{J}^{A\tau} - \hat{J}^{A\sigma})\\ &= \frac{T_{D1}}{2}\Big[\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2}\sqrt{-\gamma}(\gamma^{\tau\alpha}J^A_\alpha - \gamma^{\sigma\alpha}J^A_\alpha) + (\Pi\kappa + \omega)(J^A_\sigma + J^A_\tau)\Big].\end{aligned} \quad (48)$$

As the next step, we fix the auxiliary metric to have the form $\gamma_{\alpha\beta} = \eta_{\alpha\beta}, \eta_{\mu\nu} = \mathrm{diag}(-1, 1)$, keeping in mind that currents still have to obey Eq. (45). In this gauge, $\hat{J}^A_\pm$ simplify considerably, and we obtain

$$\begin{aligned}\hat{J}^{A+} &= -\frac{1}{2}\hat{J}^A_- = \frac{T_{D1}}{2}\Big[J^A_\sigma\Big(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} + (\Pi\kappa + \omega)\Big) - J^A_\tau\Big(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} + (\Pi\kappa + \omega)\Big)\Big],\\ \hat{J}^{A-} &= -\frac{1}{2}\hat{J}^A_+ = -\frac{T_{D1}}{2}\Big[J^A_\tau\Big(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} - (\Pi\kappa + \omega)\Big)\\ &\quad + J^A_\sigma\Big(\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} - (\Pi\kappa + \omega)\Big)\Big],\end{aligned} \quad (49)$$

where we introduced the light-cone metric with $\eta_{+-} = \eta_{-+} = -2, \eta^{+-} = \eta^{-+} = -\frac{1}{2}$ so that $\hat{J}^{A+} = \eta^{+-}\hat{J}^A_- = -\frac{1}{2}\hat{J}^A_-$, $\hat{J}^{A-} = \eta^{-+}\hat{J}^A_- = -\frac{1}{2}\hat{J}^A_+$. We see that for

$$\Pi\kappa + \omega = \sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2} \quad (50)$$

the current $\hat{J}^A_+$ vanishes identically and Eq. (47) gives

$$\partial_+\hat{J}^A_- = 0, \qquad \hat{J}^A_- = 2T_{D1}\sqrt{e^{-2\Phi_0} + (C^{(0)} - \Pi)^2}(J^A_\tau - J^A_\sigma). \quad (51)$$

Note that we can write $\hat{J}_- = \hat{J}^A_- T_A = 2g^{-1}\partial_- g$. Then from (51) we obtain

$$\frac{1}{2}\partial_+\hat{J}_- = -g^{-1}\partial_+ g g^{-1}\partial_- g + g^{-1}\partial_-\partial_+ g g^{-1} g = g^{-1}\partial_-[\partial_+ g g^{-1}]g = 0, \quad (52)$$

so that there is a second current $\hat{J}_+ = \partial_+ g g^{-1}$ that obeys the equation

$$\partial_-\hat{J}_+ = 0. \quad (53)$$

Equations (51) and (53) strongly resemble the conservations of currents in the WZW model.

The previous analysis is valid for any group manifold with NSNS and RR fluxes and for a constant dilaton and RR zero-form. Now we would like to see whether the condition (50) can be realized in a consistent string background. As the first case, we consider an $AdS_3 \times S^3 \times M$ background with a pure RR flux where M is four torus T^4 of four-volume $V_M = (2\pi)^4 v\alpha'^2$ in the metric ds^2_M that implies that each x^i is identified with the period $2\pi v^{1/4}\alpha'^{1/2}$. The background has the form [30]

$$ds^2 = r_1 r_5 (ds^2_{\mathrm{AdS}_3} + ds^2_{S^3}) + \frac{r_1}{r_5} ds^2_M,$$
$$F = \frac{2r_5^2}{g}(\epsilon + *_6\epsilon_3),$$
$$e^{-\Phi} = \frac{1}{g}\frac{r_5}{r_1}, \qquad r_5 = \sqrt{gQ_5\alpha'}, \qquad r_1 = \frac{4\pi^2\alpha'}{\sqrt{V_M}}\sqrt{gQ_1\alpha'}, \tag{54}$$

where $ds^2_{\mathrm{AdS}_3}$ and $ds^2_{S^3}$ are line elements defined with the group elements from $SL(2,R)$ and $SU(2)$, respectively, that define the currents J^A_α. Furthermore, ds^2_M is a Ricci-flat metric on M with volume V_M and where Q_1 and Q_5 are the $D1$- and $D5$-brane charge, respectively. Finally, ϵ is a volume element of AdS_3, and $*_6\epsilon$ is a volume element of S^3, where $*_6$ is the Hodge dual in six dimensions. Using (54), we obtain

$$\hat{J}^A_- = -\frac{T_{D1}r_5^2}{g}\left[J^A_\sigma\left(\sqrt{1+\Pi^2\frac{g^2r_1^2}{r_5^2}}+1\right) - J^A_\tau\left(\sqrt{1+\Pi^2\frac{g^2r_1^2}{r_5^2}}+1\right)\right],$$
$$\hat{J}^A_+ = \frac{T_{D1}r_5^2}{g}\left[J^A_\sigma\left(\sqrt{1+\Pi^2\frac{g^2r_1^2}{r_5^2}}-1\right) + J^A_\tau\left(\sqrt{1+\Pi^2\frac{g^2r_1^2}{r_5^2}}-1\right)\right]. \tag{55}$$

We see that $\hat{J}^A_+$ vanishes identically in the case when $\Pi = 0$, while $\hat{J}^A_-$ is equal to

$$\hat{J}^A_- = \frac{Q_5}{\pi}(J^A_\tau - J^A_\sigma), \qquad \partial_+\hat{J}^A_- = 0. \tag{56}$$

This is the expected result, since in this case we have a D1-brane in the near-horizon limit of a D1-D5-brane system which is S-dual to the configuration of a probe fundamental string in the near-horizon limit of the background NS-branes and fundamental strings. These models are known as WZW models [28] and can be analyzed using powerful conformal field techniques.

Let us now consider a D1-brane in this background. Recall that the type-IIB theory has nonperturbative $SL(2,Z)$ symmetry:

$$\hat{G}_{MN} = e^{\frac{1}{2}(\hat{\Phi}-\Phi)}G_{MN}, \qquad \hat{\tau} = \frac{a\tau + b}{c\tau + d},$$
$$\hat{B}_{MN} = cC^{(2)}_{MN} + dB_{MN}, \qquad \hat{C}^{(2)}_{MN} = aC^{(2)}_{MN} + bB_{MN}, \tag{57}$$

where $\tau = C^{(0)} + ie^{-\Phi}$ and where $ad - bc = 1$. Note that an S-duality transformation corresponds to the following values of the parameters: $a = 0$, $b = 1$, $c = -1$, and $d = 0$. Then we find that the S-dual background has the form

$$e^{-2\hat{\Phi}} = \frac{g^2r_1^2}{r_5^2} = \frac{g^2}{v}\frac{Q_1}{Q_5},$$
$$d\hat{s}^2 = e^{-\Phi}ds^2 = \frac{1}{g}r_5^2(ds^2_{\mathrm{AdS}_3} + ds^2_{S^3}) + gds^2_M = Q_5\alpha'(ds^2_{\mathrm{AdS}_3} + ds^2_{S^3}) + gds^2_M,$$
$$H = 2Q_5\alpha'(\epsilon_3 + *_6\epsilon_3), \tag{58}$$

so that it is easy to see that the currents $\hat{J}^A$ have the form

$$\hat{J}^A_- = -T_{D1}\alpha'Q_5\left[J^A_\sigma\left(\sqrt{\frac{g^2Q_1}{vQ_5}+\Pi^2}+\Pi\right) - J^A_\tau\left(\sqrt{\frac{g^2Q_1}{vQ_5}+\Pi^2}+\Pi\right)\right],$$
$$\hat{J}^A_+ = T_{D1}\alpha'Q_5\left[J^A_\sigma\left(\sqrt{\frac{g^2Q_1}{vQ_5}+\Pi^2}-\Pi\right) + J^A_\tau\left(\sqrt{\frac{g^2Q_1}{vQ_5}+\Pi^2}-\Pi\right)\right]. \tag{59}$$

It is clear that $\hat{J}_+^A$ does not vanish for finite values of the parameters. On the other hand, we easily see that $\hat{J}_+^A$ vanishes identically when we consider the formal limit $g \to 0$. Physically, this is the situation when a D1-brane becomes infinite heavy and decouples so that the probe can be considered as the collection of Π fundamental strings. In this case, the model corresponds to the ΠQ_5 level WZW model that can be studied by conventional conformal field theory techniques. However, it is important to stress that this is not possible in the case of a finite value of the string coupling constant.

Finally, we consider a more general case when we perform the $SL(2,Z)$ duality transformation from the near-horizon limit of a D1-D5-brane background. We begin with the observation [7] that the near-horizon limit and S-duality commute. Then for the general form of the $SL(2,Z)$ transformation (with $C^0 = 0$), we obtain (using $\tau^* = -\tau$)

$$\hat{C}^{(0)} = \frac{ace^{-2\Phi} + bd}{c^2 e^{-2\Phi} + d^2}, \qquad e^{-\hat{\Phi}} = \frac{e^{-\Phi}}{c^2 e^{-2\Phi} + d^2}.$$
$$d\hat{s}^2 = \sqrt{c^2 e^{-2\Phi} + d^2}\, ds^2, \qquad \hat{B} = cC^{(2)}, \qquad \hat{C}^{(2)} = aC^{(2)}. \tag{60}$$

Let us start with the symmetric flux background that corresponds to the following values of parameters a, b, c, and d:

$$a = 1, \qquad b = 0, \qquad c = 1, \qquad d = 1. \tag{61}$$

It turns out that in this case the current $\hat{J}_+^A$ vanishes identically in the case when $\Pi = 0$, while $\hat{J}_-^A$ is equal to

$$\hat{J}_-^A = \frac{Q_5}{\pi}(J_\tau^A - J_\sigma^A), \qquad \partial_+ \hat{J}_-^A = 0. \tag{62}$$

In fact, this remarkable result is valid whenever the parameter b is equal to zero. Explicitly, when $b = 0$, we find from the condition $ad - bc = 1$ that $a = d = 1$, which implies

$$\hat{C}^{(0)} = \frac{ce^{-2\Phi}}{c^2 e^{-2\Phi} + d^2}, \qquad e^{-\hat{\Phi}} = \frac{e^{-\Phi}}{c^2 e^{-2\Phi} + d^2}. \tag{63}$$

Then for $\Pi = 0$ and for the background given above, we obtain that the currents (49) have the form

$$\hat{J}_-^A = -\frac{T_{D1} r_5^2}{g}(J_\sigma^A - J_\tau^A) \times \left(\frac{g r_1}{r_5} \sqrt{c^2 e^{-2\Phi} + 1} \sqrt{\frac{e^{-2\Phi}}{c^2 e^{-2\Phi} + 1}} + 1 \right),$$
$$\hat{J}_+^A = \frac{T_{D1} r_5^2}{g}(J_\sigma^A + J_\tau^A) \times \left(\frac{g r_1}{r_5} \sqrt{c^2 e^{-2\Phi} + 1} \sqrt{\frac{e^{-2\Phi}}{c^2 e^{-2\Phi} + 1}} - 1 \right), \tag{64}$$

where the first square root $\sqrt{c^2 e^{-2\Phi} + 1}$ follows from the definition of dual line element (60) and the second one from the fact that $e^{-2\hat{\Phi}} + (C^{(0)})^2 = \frac{e^{-2\Phi}}{c^2 e^{-2\Phi} + 1}$. We immediately see that the currents $\hat{J}^A$ have the same form as in the case of the original near-horizon limit of a D1-D5-brane background where $\hat{J}_+^A$ vanishes identically.

Let us outline the results derived in this section. We analyzed the conditions under which we can find holomorphic or antiholomorphic currents for a D1-brane in the background $AdS_3 \times S^3$ with different combinations of NSNS and RR fluxes. While the D1-brane is integrable for any values of fluxes and world-volume electric flux, it possesses two holomorphic and antiholomorphic currents that allow a more powerful conformal field theory analysis in the near-horizon limit of a D1-D5-brane background on the condition that the electric flux is zero. We also showed that this holds in the case of $AdS_3 \times S^3$ with mixed fluxes that is related to the original D1-D5-brane system by the $SL(2,Z)$ duality transformation.

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