Branestrahlung: Radiation in the particle-brane collision

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We calculate the radiation accompanying the gravitational collision of a domain wall and a point particle in five-dimensional spacetime. This process—which can be regarded as brane-particle bremsstrahlung, here called *branestrahlung*—has unusual features. Since the brane has intrinsic dynamics, it gets excited in the course of collision, and, in particular, at the moment of perforation the shock branon wave is generated, which then expands with the velocity of light. Therefore, apart from the time-like source whose radiation can be computed in a standard way, the total radiation source contains a light-like part whose retarded field is quite nontrivial, exhibiting interesting retardation and memory effects. We analyze this field in detail, showing that—contrary to the claims that the light-like sources should not radiate at all—the radiation is nonzero and has a classically divergent spectrum. We estimate the total radiation power introducing appropriate cutoffs. In passing, we explain how the sum of the nonlocal (with the support inside the light cone) and the local (supported on the cone) singular parts of the Green's function of the five-dimensional d'Alembert equation together defines a regular functional.

I. INTRODUCTION

The gravitational interaction of domain walls with matter was extensively discussed in the past in the context of four-dimensional cosmology [1–3]. More recently, the general development of the theory of branes in the superstring setting stimulated investigation of other aspects of this interaction, such as collisions of domain walls and other branes with black holes [4–6]. Especially interesting is the process of perforation, which was suggested as a new mechanism of domain-wall destruction in the early Universe [4,7,8]. Other applications refer to the braneworld models of the Randall-Sundrum type [9–12], in particular, the problem of escape of black holes from branes [13–15].

In most of the literature the brane-black hole problem is treated by considering one object as creating the fixed background, and another as moving in this background. But genuine two-body features of their interaction including the backreaction are very difficult to describe in the full nonlinear setting. In an attempt to explore such aspects of the interaction, we consider a simplified model in which the second body is the point-like mass living in the bulk. Recently we have investigated the perforation of a domain wall by a point mass in a particularly simple setting of the linearized gravity [16,17]. Due to the continuous nature of the gravitational field of the domain wall in its location, one can describe the perforation of the brane in terms of distributions. This approach allows one to consider one novel aspect that is difficult to analyze in the full nonlinear theory, namely, the excitation of the internal degrees of

freedom of the domain wall in the course of collision. The domain wall gets excited after the perforation in the form of the spherical shock branon wave propagating outwards along the brane with the speed of light. This wave is a reaction of the wall to the change of the particle's acceleration by a finite amount at the moment of piercing. Thus, contrary to the two-particle case, the collision becomes classically nonlocal. The energy-momentum balance in this process can therefore be defined only instantaneously, as it takes into account the contribution of the branon wave [17].

Here we calculate the radiation accompanying the particle–domain-wall collision which is somewhat similar to bremsstrahlung. Actually we are interested in gravitational radiation, but in order to simplify the computation we consider the emission of the scalar waves, introducing the interaction of the massless scalar field with the trace of the brane energy-momentum tensor. We realize that the spectral distribution of radiation is essentially dependent on its spin, so the scalar case cannot fully reproduce gravitational radiation. However, such aspects as the propagation of the wave fronts and causality structure of the radiation field should be similar for all spins, and in this paper we are interested mainly with this. The novel feature of this radiation problem is the presence of the light-like source due to the branon as a part of the total source. Recently, the retarded solutions of the d'Alembert equation with light-like sources were extensively discussed in connection with the radiation problem [18,19], the memory effect [20–24], and the Bondi-Metzner-Sachs asymptotic symmetries [25]. Our radiation problem provides a novel interesting setting for these studies.

As far as the problem of radiation from massless sources is concerned, the arguments were presented that radiation in this case is totally absent [26,27]. Indeed, within the classical

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theory the retarded potentials from massless point sources diverge on a line parallel to the velocity, and various ways to regularize these divergences—which also have quantum counterparts as collinear divergences [28–31]—were suggested. An ultimate result of such procedures was sometimes claimed to be zero. Meanwhile the relativistic Liénard formula for the total radiation power from an accelerated point charge diverges in the massless limit. This controversy was resolved in Ref. [32], where it was shown that radiation from the massless charge is nonzero and finite, but (being essentially quantum) it is described by classical theory only in the IR part of the spectra. We show that *branestrahlung*—which always contains a contribution from the massless component of the source—also has a similar property requiring appropriate cutoffs.

The computation of radiation depends on the spacetime dimension in a nontrivial way, being different, in particular, in the case of even and odd dimensions [33]. Here we restrict ourselves to the case of the five-dimensional bulk, appealing primarily to the Randall-Sundrum II (RS II) kind of setting. Radiation in five dimensions does not satisfy Huygens' principle which leads to additional new features as compared with the four-dimensional case. In particular, we discuss the structure of the retarded Green's function of the five-dimensional d'Alembert operator whose treatment is often confusing in the literature, emphasizing the importance of its purely local part as a regulator.

II. THE SETUP

We consider a 3-brane embedded in the 5-dimensional spacetime (the bulk) with the metric g_{MN}, where $M, N = 0, 1, 2, 3, 4$. The brane worldvolume $\mathcal{V}_{3+1}$ defined by the embedding equations $x^M = X^M(\sigma_\mu)$ is parametrized by the coordinates σ_μ, $(\mu = 0, \dots, 3)$ on $\mathcal{V}_{3+1}$. The brane gravitationally interacts with a point mass m moving in the bulk. In addition, the brane (and not the particle) is coupled with the bulk scalar field φ, being endowed with the charge density f. Therefore the particle-brane interaction is purely gravitational, while the scalar radiation is generated only by the brane. This simplifies the problem considerably, while still keeping the main features of the realistic gravitational radiation.

The corresponding action reads

$$
S = -\frac{\mu}{2} \int [X_\mu^M X_\nu^N g_{MN} \gamma^{\mu\nu} - 2]\left(1 + \frac{f}{\mu}\varphi\right)\sqrt{-\gamma}d^4\sigma
$$
$$
- \frac{1}{2} \int \left(e g_{MN}\dot{z}^M \dot{z}^N + \frac{m^2}{e}\right)d\tau
$$
$$
- \frac{1}{\varkappa_5^2} \int R\sqrt{g}d^5x - \frac{1}{2}\int g_{MN}\nabla^M\varphi\nabla^N\varphi\sqrt{g}d^5x. \quad (2.1)
$$

Here μ is the brane mass density (tension), f is the brane scalar density, $X_\mu^M = \partial X^M/\partial\sigma^\mu$ are the brane tangent vectors, $g = \det g_{MN}$, and $\gamma^{\mu\nu}$ is the inverse metric on

$\mathcal{V}_{3+1}$, with $\gamma = \det\gamma_{\mu\nu}$. Also $\varkappa^2 \equiv 16\pi G_5$, $e(\tau)$ is the einbein on the particle worldline, the metric signature is $(+ - - - -)$, and we use the Landau-Lifshitz convention for the curvature. Note that due to the choice of the signature, in $D = 5$ one has $g > 0$, while $\gamma < 0$.

It is worth noting that by taking the interaction of the scalar field with gravity into account we introduce a nonlinearity similar to that described by the three-graviton vertex in the genuine gravitational problem. Due to this interaction, the effective radiation source term will include the stress part in the same way that gravitational stresses contribute to the gravitational radiation amplitude as prescribed by the Bianchi identity.

The first term in Eq. (2.1), representing the action of the brane, has to be varied independently over X^M and $\gamma^{\mu\nu}$ to get the equations of motion and the constraint. Similarly, the particle term in Eq. (2.1) has to be varied over z^M and e, while the terms containing the bulk fields φ and g_{MN}, before being varied over them, have to be extended to the bulk integrals by inserting the appropriate delta functions.

The variation of Eq. (2.1) with respect to X^M and $\gamma^{\mu\nu}$ gives the brane equation of motion in the covariant form

$$
\nabla_\mu(X_\nu^N g_{MN}\gamma^{\mu\nu}\sqrt{-\gamma}(\mu + f\varphi)) = \sqrt{-\gamma}f\varphi_{,M}, \quad (2.2)
$$

with $\nabla_\mu \equiv X_\mu^M\nabla_M$, and the constraint equation

$$
\gamma_{\mu\nu} = X_\mu^M X_\nu^N g_{MN}|_{x=X}, \quad (2.3)
$$

where we define $\gamma_{\mu\nu}$ as the induced metric on $\mathcal{V}_4$. Varying S with respect to $z^M(\tau)$ and $e(\tau)$, one obtains the corresponding system for the particle:

$$
\frac{d}{d\tau}(e\dot{z}^N g_{MN}) = \frac{e}{2}g_{NP,M}\dot{z}^N\dot{z}^P, \quad (2.4)
$$

$$
e^2 g_{MN}\dot{z}^M\dot{z}^N = m^2. \quad (2.5)
$$

Finally, the variation over φ leads to the scalar field equation

$$
g_{MN}\nabla^M\nabla^N\varphi = \rho, \quad (2.6)
$$

with the source

$$
\rho = \frac{q}{2}\int [X_\mu^M X_\nu^N g_{MN}\gamma^{\mu\nu} - 2]\sqrt{-\gamma}\frac{\delta^5(x^M - X^M(\sigma^\nu))}{\sqrt{g(x)}}d^4\sigma, \quad (2.7)
$$

or, substituting Eq. (2.3),

$$
\rho(x) = f\int \sqrt{-\gamma}\frac{\delta^5(x^M - X^M(\sigma^\nu))}{\sqrt{g(x)}}d^4\sigma. \quad (2.8)
$$

A. Iteration scheme

Next we present the bulk metric as the perturbed Minkowski metric η_{MN},

$$g_{MN} = \eta_{MN} + \varkappa H_{MN}, \qquad (2.9)$$

and expand all quantities in powers of H_{MN}, using η_{MN} to raise and lower the indices. As usual, we impose the flat-space harmonic gauge

$$\partial_N H^{MN} = \frac{1}{2} \partial^M H, \qquad H \equiv \eta^{LR} H_{LR}. \quad (2.10)$$

Our technique consists in solving the equations for the bulk metric, the embedding functions $z^M(\tau), X^M(\sigma^\mu)$, the Lagrange multiplier e, the worldvolume metric $\gamma_{\mu\nu}$, and the scalar φ by iteratively expanding them in terms of the couplings $\varkappa, f$. When doing this we have to keep only the mutual interaction terms and omit the self-action. The goal is to compute the scalar radiation in the lowest reliable order in both couplings.

The zero-order solution is trivial. It describes the free flat unperturbed brane and the particle moving with constant velocity normally to the brane $u^M = \gamma(1, 0, 0, 0, v)$, where $\gamma = 1/\sqrt{1 - v^2}$. Correspondingly, the trajectory $^0z^M = (^0t(\tau), 0, 0, 0, ^0z(\tau))$ is the straight line $^0z^M(\tau) = u^M\tau$. The einbein is chosen equal to the particle mass $^0e = m$, so that the trajectory is parametrized by the proper time and the velocity satisfies the normalization $\eta_{MN}u^M u^N = 1$.

In the zeroth order in $\varkappa$ the brane is assumed to be unexcited and to fill the plane $z = 0$,

$$^0X^M(\sigma) = \Sigma_\mu^M \sigma^\mu, \qquad \Sigma_\mu^M \Sigma_\nu^N \eta_{MN} = \eta_{\mu\nu}, \quad (2.11)$$

so the internal coordinates on the brane coincide with the corresponding bulk coordinates and the induced metric on $\mathcal{V}_4$ is flat: $\gamma_{\mu\nu} = \eta_{\mu\nu}$. Obviously, this is a solution to Eq. (2.3) for $\varkappa = 0$, and it is convenient to fix $\Sigma_\mu^M = \delta_\mu^M$ without loss of generality. In other terms, we choose the Lorentz frame where the unperturbed brane is at rest.

In the first order the metric deviation is the sum of the contributions of the brane and the particle,

$$^1H_{MN} = h_{MN} + \bar{h}_{MN}, \quad (2.12)$$

which satisfy

$$\Box h^{MN} = -\varkappa\left(^0T^{MN} - \frac{1}{3}{}^0T\eta^{MN}\right),$$
$$\Box \bar{h}^{MN} = -\varkappa\left(^0\bar{T}^{MN} - \frac{1}{3}{}^0\bar{T}\eta^{MN}\right), \quad (2.13)$$

with $\Box \equiv \partial_M \partial^M$ being the flat five-dimensional d'Alembert operator and $^0T \equiv {}^0T^{MN}\eta_{MN}$. The source terms read

$$^0T^{MN} = \mu \int \Sigma_\mu^M \Sigma_\nu^N \eta^{\mu\nu} \delta^5(x - {}^0X(\sigma))d^4\sigma, \quad (2.14)$$

$$^0\bar{T}^{MN} = m \int u^M u^N \delta^5(x - {}^0z(\tau))d\tau. \quad (2.15)$$

Using h^{MN} and the zeroth-order quantities in Eqs. (2.5) and (2.4) one obtains for ^1e and $^1z^M$ the equations[1];

$$^1e = -\frac{m}{2}(\varkappa h_{MN}u^M u^N + 2\eta_{MN}u^{M1}\dot{z}^N), \quad (2.16)$$

and Eq. (2.4) (upon elimination of ^1e) gives for $^1z^M$

$$\frac{d}{d\tau}{}^1\dot{z}_M = -\varkappa\left(h_{PM,Q} - \frac{1}{2}h_{PQ,M}\right)u^P u^Q. \quad (2.17)$$

Substituting the first-order metric deviation $\bar{h}_{MN}$ due to the particle (to be determined below) and the first-order brane perturbations into Eq. (2.8), and computing the perturbation of the induced metric (2.3), $\gamma_{\mu\nu} = \eta_{\mu\nu} + {}^1\gamma_{\mu\nu} + \mathcal{O}(\varkappa^2 h^2)$,

$$^1\gamma_{\mu\nu} = 2\delta_{(\mu}^{M1}X_{\nu)}^N \eta_{MN} + \varkappa\bar{h}_{\mu\nu},$$
$$^1\sqrt{-\gamma} = \frac{1}{2}(2\delta_\mu^{M1}X_\nu^N \eta_{MN}\eta^{\mu\nu} + \varkappa\bar{h}_\lambda^\lambda), \quad (2.18)$$

one gets

$$\Pi_{MN}\Box_{D-1}{}^1X^N = \Pi_{MN}J^N,$$
$$\Pi^{MN} \equiv \eta^{MN} - \Sigma_\mu^M \Sigma_\nu^N \eta^{\mu\nu}, \quad (2.19)$$

where $\Box_{D-1} \equiv \partial_\mu \partial^\mu$ and Π^{MN} is the projector onto the (one-dimensional) subspace orthogonal to $\mathcal{V}_{D-1}$. The source term in Eq. (2.19) reads

$$J^N = \varkappa\Sigma_P^\mu \Sigma_Q^\nu \eta_{\mu\nu}\left(\frac{1}{2}\bar{h}^{PQ,N} - \bar{h}^{NP,Q}\right)_{z=0}, \quad (2.20)$$

where $\Sigma_M^\alpha \equiv \Sigma_\nu^N \eta^{\nu\alpha}\eta_{MN}$.

Finally, the equation for $^1\varphi$ is given by the leading order of Eq. (2.6):

$$\Box^1\varphi = {}^0\rho = f \int \delta^4(x^\mu - \sigma^\mu)\delta(z)d^4\sigma = f\delta(z), \quad (2.21)$$

with $^0\rho$ being the leading order of Eq. (2.8).

Equations (2.13), (2.16), and (2.17), together with an equation for the brane perturbations, form a complete set of equations to this order. The gauge fixing condition (2.10) for $^1\psi^{MN}$ is a consequence of the conservation of $^0T^{MN}$.

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[1] Our gauge condition is $g_{MN}\dot{z}^M\dot{z}^N = 1$. To this order it reduces to $^1e = 0$.

To solve the d'Alembert equation for the field variables we will use the Fourier transforms defined as

$$\varphi(x) = \frac{1}{(2\pi)^5} \int \varphi(k)e^{-ikx}d^5k,$$

$$\varphi(k) = \int \varphi(x)e^{ikx}d^5x, \qquad (2.22)$$

where $kx \equiv k_M x^M$.

B. Radiation

We will compute the scalar radiation generated in the particle-brane collision as the simplified version of the gravitational radiation. Recall that the particle is assumed to not carry the scalar charge, so the interaction between the brane and the particle is purely gravitational. Gravity also interacts with the scalar field, which is generated by the brane and lives in the bulk. The radiative component of the scalar field, which is detected by the nonzero Fourier transform $\varphi(k)$ on the mass shell $k^2 = 0$ (five-dimensional square), appears at the lowest order in the second iteration, namely, the first in $\varkappa$ and the first in f. Expanding Eq. (2.6) in $\varkappa$, we obtain in this order

$$\Box^2\varphi = \varkappa j, \qquad (2.23)$$

with the source term consisting of two contributions,

$$j = \tilde{\rho} + S, \qquad S \equiv -\partial_M\left(\frac{1}{2}h\partial^M - h^{MN}\partial_N\right)^1\varphi, \qquad (2.24)$$

while $\tilde{\rho} \equiv {}^1(\rho\sqrt{g})$ is the direct, or *local*, current,

$$\tilde{\rho} = \frac{f}{2}\int[\varkappa(\bar{h}^\lambda_\lambda - \bar{h}^L_L) + 2^1X^L_\lambda\Sigma^\lambda_L$$
$$- 2^1X^L\partial_L]\delta^5(x^A - \Sigma^A_\alpha\sigma^\alpha)d^4\sigma, \qquad (2.25)$$

whereas the nonlocal term S (which depends on the bulk gravitational field) is due to the expansion of the curved-space d'Alembert operator in $\varkappa$.

The total momentum loss (for a more detailed analysis of the momentum balance in our problem see Ref. [17]) in the collision is computed in the standard way and is expressed in terms of the Fourier transform of the current as follows:

$$\Delta P^M = \frac{1}{(2\pi)^4}\int\theta(k^0)k^M\delta(k^2)|j(k)|^2d^5k. \qquad (2.26)$$

Finally, by introducing the frequency $\omega = k^0$ and integrating over $|\mathbf{k}|$, for $E_{\text{rad}} = \Delta P^0$ we arrive at

$$E_{\text{rad}} = \frac{1}{2(2\pi)^4}\int_0^\infty\omega^3 d\omega\int_{S^3}d\Omega|j(k)|^2, \qquad (2.27)$$

where the wave five-vector k is null. We will parametrize it by the frequency, the normal to the brane projection k^z, and the three-dimensional vector $\mathbf{k}_\perp$ parallel to the brane:

$$k^2 = k^Mk_M = \omega^2 - (k^z)^2 - (k_\perp)^2 = 0. \qquad (2.28)$$

III. THE FIRST ORDER

The first-order solutions are obtained straightforwardly by passing to the momentum space, so we just list the corresponding results.

A. Linearized fields

The brane gravitational field $h_{MN}(q)$ is obtained by taking the Fourier transform of the energy-momentum tensor (2.14) and dividing by the box operator:

$$h_{MN}(q) = \frac{(2\pi)^4\varkappa\mu\delta^4(q^\mu)}{q^2}\left(\delta^\mu_M\delta^\nu_N\eta_{\mu\nu} - \frac{4}{3}\eta_{MN}\right). \qquad (3.1)$$

In the coordinate representation it reads

$$h_{MN}(x) = \frac{\varkappa\mu}{2}\left(\delta^\mu_M\delta^\nu_N\eta_{\mu\nu} - \frac{4}{3}\eta_{MN}\right)|z|$$
$$= \frac{\varkappa\mu|z|}{6}\text{diag}(-1, 1, 1, 1, 4), \qquad (3.2)$$

with the trace being

$$h = -\frac{4}{3}\varkappa\mu|z|.$$

The solution of the linearized equation (2.21) for the scalar field in the momentum representation is

$${}^1\varphi(q) = -\frac{(2\pi)^4f\delta^4(q^\mu)}{q^2}. \qquad (3.3)$$

The corresponding coordinate-space solution

$${}^1\varphi(x) = -\frac{1}{2}f|z| \qquad (3.4)$$

is proportional to the trace of the gravitational perturbation. Similarly, for the particle's metric perturbation we get

$$\bar{h}_{MN}(q) = \frac{2\pi\varkappa m\delta(qu)}{q^2 + i\varepsilon q^0}\left(u_Mu_N - \frac{1}{3}\eta_{MN}\right), \qquad (3.5)$$

or, in the coordinate representation,

$$\bar{h}_{MN}(x) = -\frac{\varkappa m}{4\pi^2}\left(u_Mu_N - \frac{1}{3}\eta_{MN}\right)\frac{1}{\gamma^2(z-vt)^2 + r^2}, \qquad (3.6)$$

where $r^2 = \sum_{i=1}^{n}(\sigma^i)^2$. This is nothing but the Lorentz-contracted D-dimensional Newton gravitational field of a uniformly moving point particle.

B. Branon

The picture of the collision looks as follows. The particle impinges on the brane normally and perforates it at the moment $t = 0$. The particle's gravitational field causes the perturbation of the brane worldvolume, which is somewhat nontrivial to analyze (the details of which can be found in Refs. [16,17][2]); here, we just present the results. Only the perturbation of X^N transverse to the brane is physical; the longitudinal ones can be gauged away by the coordinate transformation in the worldvolume. Denoting this component as another scalar (branon) $\Phi(x^\mu) = {}^1X^z$ and linearizing the Nambu-Goto equation, we obtain for the branon the four-dimensional d'Alembert equation

$$\Box_4 \Phi(\sigma^\mu) = J(\sigma^\mu), \qquad (3.7)$$

with the source term

$$J(\sigma) = \varkappa \left[\frac{1}{2}\eta_{\mu\nu}\bar{h}^{\mu\nu,z} - \bar{h}^{z0,0} \right]_{z=0} = -\frac{\lambda vt}{[\gamma^2 v^2 t^2 + r^2]^2},$$
$$\lambda = \frac{\varkappa^2 m \gamma^2}{4\pi^2}\left(\gamma^2 v^2 + \frac{1}{3}\right). \qquad (3.8)$$

In the momentum space the retarded solution to this d'Alembert equation reads

$$\Phi(q^\mu) = -\frac{J(q^\mu)}{q_\nu q^\nu + 2i\epsilon q^0},$$
$$J(q^\mu) = -\frac{2\pi^2\lambda}{\gamma}\frac{iq^0}{\gamma^2 v^2 \mathbf{q}^2 + (q^0)^2}. \qquad (3.9)$$

Expanding the double pole into the sum of two simple poles, we get the decomposition $\Phi \equiv \Phi_a + \Phi_b$, where

$$\Phi_a(q^\mu) = -\frac{2\pi^2\lambda i}{q^0}\frac{\gamma^2 v^2}{\gamma^2 v^2 \mathbf{q}^2 + (q^0)^2},$$
$$\Phi_b(q^\mu) = \frac{2\pi^2\lambda i}{q^0}\frac{1}{q^2 + 2i\epsilon q^0}, \qquad (3.10)$$

which have different physical meanings. The component Φ_a, which in the coordinate representation reads

$$\Phi_a = -\frac{\lambda}{2\gamma^3 r}\arctan\frac{r}{\gamma vt}, \qquad (3.11)$$

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[2]The transverse coordinate of the brane can be viewed as the Nambu-Goldstone boson (branon) which appears as a result of spontaneous breaking of the translational symmetry [34]. In the braneworld setting it is coupled to bulk gravity and matter on the brane via the induced metric [35].

is *antisymmetric* in time. Recall that the gravitational force between the brane and a particle is repulsive and it changes sign at the moment of piercing. The brane gets deformed in the direction opposite to the instantaneous location of particles, and this deformation (described by Φ_a) is rigidly tight to the particle motion, without retardation. In the momentum space it is due to the pole on the imaginary axis.

The second component Φ_b is the shock branon wave which is excited just at the moment of perforation. It arises from the pole on the real axis and corresponds to a (quasi) free branon wave propagating with the velocity of light outwards from the center $r = 0$ where the particle perforates the brane:

$$\Phi_b = \frac{\pi\lambda}{2r\gamma^3}\theta(t)\theta(r - t). \qquad (3.12)$$

What is more surprising is that it is only quasifree, since by acting with the d'Alembert operator one finds the derivative of the delta function:

$$\Box_4 \Phi_b = \frac{\lambda\pi}{2\gamma^3 r}\delta'(t). \qquad (3.13)$$

The peculiar nature of this source is explained as follows. The force between the particle and the brane at the moment of perforation is nonzero, and it changes sign. It acts as the singular local flush which causes the excitation of the shock branon wave. It is important that the shock wave now becomes a source of radiation, i.e., we have to explore the retarded solution of the bulk d'Alembert equation with such a light-like source. This is a rather nontrivial problem, which we solve in the next section and the Appendix. It leads to a complicated structure of fields propagating in the bulk.

The "rigid" part of the deformation of the brane (3.11) satisfies the following equation:

$$\Box_4 \Phi_a = -\frac{\lambda\pi}{2\gamma^3 r}\delta'(t) - \frac{\lambda vt}{[\gamma^2 v^2 t^2 + r^2]^2}, \qquad (3.14)$$

so in the sum of both contributions the delta-function sources cancel, as expected.

In the ultrarelativistic limit the shock wave dominates. The second term on the right-hand side of Eq. (3.14) in this case behaves like the regularized derivative of the delta function as $\gamma \to \infty$; an accurate calculation shows that it exactly cancels the first term. So, the direct action of the particle upon the brane vanishes for a light-like particle. This is not surprising since the infinite boost causes the gravitational field of the particle to become a shock wave acting solely at the moment of perforation. Hence the total branon solution caused by the light-like particle is determined by Φ_b only. In this case one has to use as a parameter the particle energy $\mathcal{E}$, arriving at

$$\Phi = \frac{\varkappa^2 \mathcal{E}}{8\pi}\frac{\theta(t)\theta(r - t)}{r}.$$

IV. RETARDED FIELD OF THE LIGHT-LIKE BRANON SOURCE

Before proceeding with the calculation of the total radiation amplitude $^2\varphi$ generated by the current $j(k)$, we consider its relativistic component ψ generated by the shock branon part of the source term. As we have seen, this part becomes an exact field in the case of a particle of zero mass. The construction of the retarded solution of the d'Alembert equation for the light-like source is nontrivial, especially in odd spacetime dimensions since the Green's function is not localized on the light cone.

Thus we seek the retarded solution of the following five-dimensional wave equation:

$$\Box\psi = -\alpha\frac{\theta(t)\theta(r-t)}{r}\delta'(z),\qquad(4.1)$$

where we defined $\alpha = \pi\lambda f/2\gamma^3$. We will abbreviate the product of two Heaviside functions as

$$\Theta_x(a,b) = \theta(x-a)\theta(b-x)$$
$$= \begin{cases} 0, & x < a, \\ 1, & a \le x < b, \quad\text{for } b > a. \\ 0, & x \ge b, \end{cases}$$

In this notation $\theta(t)\theta(r-t) = \Theta_t(0,r)$. This double Heaviside function restricts the domain of integration to a finite interval: $\int \Theta_x(a,b)f(x,\ldots)dx = \int_a^b f(x,\ldots)dx$.

A. Regularization of the Green's function in five dimensions

The retarded Green's function in five dimensions satisfying the equation $\Box G = -\delta^5(x-x')$ is obtained by differentiation of the three-dimensional Green's function localized inside the light cone [33] (recall again that Huygens' principle does not hold in odd spacetime dimensions):

$$G^{\rm ret}(X) = -\frac{\theta(T)}{2\pi^2}\frac{d}{dX^2}\frac{\theta(X^2)}{(X^2)^{1/2}},$$
$$X^M = x^M - x'^M.\qquad(4.2)$$

One obtains therefore the sum of the nonlocal ($\sim\theta$) and the local ($\sim\delta$) terms:

$$G^{\rm ret}(X) = \frac{1}{4\pi^2}\left(\frac{\theta(T-R)}{(T^2-R^2)^{3/2}} - \frac{\delta(T-R)}{R(T^2-R^2)^{1/2}}\right),$$
$$R^2 \equiv (\mathbf{r}-\mathbf{r}')^2 + (z-z')^2.\qquad(4.3)$$

As $\chi \equiv T \pm R \to +0$ these terms contain nonintegrable singularities $\chi^{-3/2}$ and $\delta(\chi)/\chi^{1/2}$, respectively. Nonetheless, their sum represents the *regular* functional. To see this we

apply the regularization $T \to T - \epsilon$ assuming that the limit $\epsilon \to +0$ has to be taken after summing up both contributions. The regularization shifts the support of the Green's function from the interior of the future light cone $T^2 - R^2 = 0$ into the half of the time-like hyperboloid $T^2 - R^2 = \epsilon^2$ with an apex at x^M. The distance between the cone and the hyperboloid measured along the T axis is ϵ. With this regularization,

$$G_\epsilon^{\rm ret}(X) = \frac{1}{4\pi^2}\lim_{\epsilon\to+0}\left(\frac{\theta(T-R-\epsilon)}{(T^2-R^2)^{3/2}} - \frac{\delta(T-R-\epsilon)}{R(T^2-R^2)^{1/2}}\right),$$
$$(4.4)$$

where $T > 0$ and $R > 0$ are assumed. We are going to show that the divergent parts of the two terms mutually cancel and after removing the regularization we are left with the finite result.

Thereby, we have to compute

$$\psi = \frac{1}{4\pi^2}\lim_{\epsilon\to+0}\int\left(\frac{\theta(T-R-\epsilon)}{(T^2-R^2)^{3/2}} - \frac{\delta(T-R-\epsilon)}{R(T^2-R^2)^{1/2}}\right)$$
$$\times\frac{\theta(t')\theta(r'-t')}{r'}\delta'(z')d^5x'.\qquad(4.5)$$

Integrating over z' with the derivative of the delta function, one can pass to a differentiation over z since the integrand depends on z' and z through the difference $z - z'$, obtaining

$$\psi = -\alpha\frac{\partial J}{\partial z},\qquad(4.6)$$

where

$$J \equiv \lim_{\epsilon\to+0}\int\left(\frac{\theta(T-R-\epsilon)}{(T^2-R^2)^{3/2}} - \frac{\delta(T-R-\epsilon)}{R(T^2-R^2)^{1/2}}\right)\frac{\Theta_{t'}(0,r')}{r'}d^4x'$$
$$(4.7)$$

and, from now on, $R \equiv \sqrt{(\mathbf{r}-\mathbf{r}')^2 + z^2}$. Since the source is nonzero only for $t' > 0$ and $R \ge 0$, it follows that the solution is nonzero at $t > 0$, as expected.

First we consider the contribution of the local part of the Green's function (4.7) by integrating over t' with the delta function. If $t - R$ lies outside the interval $(0, r')$, the integral vanishes; otherwise,

$$J_{\rm loc} = -\int\frac{\theta(t-R)\theta(r'-t+R)}{R(2R+\epsilon)^{1/2}r'\sqrt{\epsilon}}d^3\mathbf{r}',\qquad(4.8)$$

which diverges as $\epsilon \to +0$.

The nonlocal term in Eq. (4.7) is more complicated. Using the identity

$$\theta(r'-t')\theta(t-t'-R-\epsilon)$$
$$= \theta(r'-t')\theta(t-R-r')$$
$$+ \theta(t-t'-R-\epsilon)\theta(r'-t+R),\qquad(4.9)$$

we split it into two parts, obtaining after integration over t'

$$J_{\text{nloc}} = \int [(F(r') - F(0))\theta(t - R - r') + (F(t - R - \epsilon)$$

$$- F(0))\theta(r' - t + R)\theta(t - R)]\frac{d^3\mathbf{r}'}{r'}, \quad (4.10)$$

where the function

$$F(x) = \frac{t - x}{R^2\sqrt{(t - x)^2 - R^2}} \quad (4.11)$$

is an antiderivative of $1/[(t - x)^2 - R^2]^{3/2}$. Consider now the term $F(t - R - \epsilon)$ in Eq. (4.10) whose contribution to J_{nloc} (denoted as J_{reg}) reads

$$J_{\text{reg}} = \int \frac{R + \epsilon}{R^2\sqrt{\epsilon(2R + \epsilon)}} \frac{\theta(r' - t + R)\theta(t - R)}{r'} d^3\mathbf{r}'. \quad (4.12)$$

This differs from the local part (4.8) by $\mathcal{O}(\epsilon^{1/2})$ and thereby the divergent parts indeed mutually cancel:

$$\lim_{\epsilon \to +0}(J_{\text{loc}} + J_{\text{reg}}) = 0. \quad (4.13)$$

B. Finite part

The remaining part of Eq. (4.10) does not depend on ϵ and therefore it is regular in the limit $\epsilon \to 0$:

$$J = \int [(F(r') - F(0))\theta(t - R - r')$$

$$- F(0)\theta(r' - t + R)\theta(t - R)]\frac{d^3\mathbf{r}'}{r'}. \quad (4.14)$$

Integration over $\mathbf{r}'$ is performed in spherical coordinates with the polar angle ϑ between $\mathbf{r}$ and $\mathbf{r}'$ and the cyclic azimuthal angle: $d^3\mathbf{r}' = 2\pi r'^2 \sin\vartheta dr' d\vartheta$. First we integrate over r'. In order to reveal the limits of integration, one has to resolve the inequalities $t - R > 0$ and $t - R - r' <> 0$ with respect to r'. The support of $\theta(t - R)$ is determined by the solution of the equation $t = R$,

$$r'^2 - 2rr'\cos\vartheta + r^2 + z^2 - t^2 = 0, \quad (4.15)$$

with respect to r'. The discriminant of this quadratic equation with respect to r' is

$$D \equiv r^2\cos^2\vartheta + t^2 - r^2 - z^2. \quad (4.16)$$

Denoting

$$\Delta \equiv t^2 - r^2 - z^2, \quad (4.17)$$

we are led to the description of the domains of integration over r', ϑ according to different signs of Δ and D. This analysis is relegated to the Appendix. After careful specification of the corresponding limits in the double integrals, one can perform both integrations in terms of antiderivatives. The final result reads

$$J = \frac{2\pi\theta(t)}{r}\left[z\theta(\Delta)\left(\arctan\frac{z\sqrt{\Delta}}{t^2 - tr - z^2} - \arctan\frac{z\sqrt{\Delta}}{t^2 + tr - z^2}\right.\right.$$

$$+ \pi\text{sgn}(z)\theta(tr + z^2 - t^2) - \frac{2t}{z}\arctan\frac{r}{\sqrt{\Delta}}\Big)$$

$$- \pi\theta(-\Delta)\theta(t^2 - z^2)(t - |z|)\Big] \equiv J_{+\Delta} + J_{-\Delta}. \quad (4.18)$$

According to this formula the retarded solution contains several sectors divided by the hypersurfaces $\Delta = 0$ and $z = \pm t$ splitting the spacetime into three regions, as shown on Fig. 1.

In the sector $t < |z|$, J is zero. For $z^2 < t^2 < r^2 + z^2$ it is determined by the second line of Eq. (4.18), while for $t^2 > r^2 + z^2$ it is determined by the first line. It is easy to check that J is continuous everywhere at $t > 0$:

(i)   The jump of $\pi\theta(tr + z^2 - t^2)$ in the region $\Delta > 0$ is compensated by the negative jump of the function $\arctan\frac{z\sqrt{\Delta}}{t^2 - tr - z^2}$.

(ii) For $t \to |z|$, $J_{-\Delta}$ tends to zero, so J is continuous on the hypersurface $t^2 = z^2$.

(iii) For $t \to \sqrt{r^2 + z^2}$, $J_{-\Delta}$ behaves as

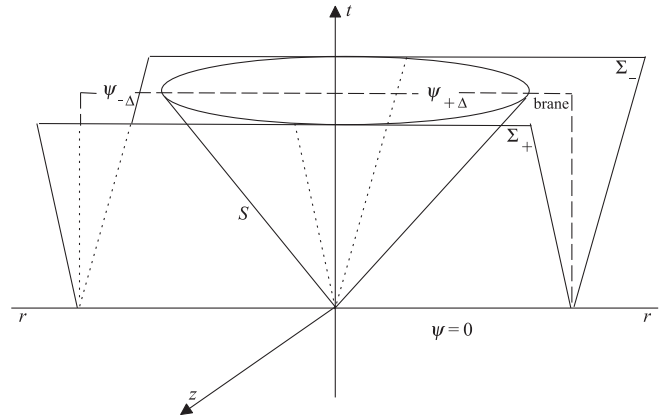$$\lim_{\Delta \to -0} J_{-\Delta} = -2\pi^2\theta(t)\frac{t - |z|}{r},$$

FIG. 1.   Spacetime diagram of the spherical (the light cone) and the plane-wave fronts in the bulk. Inside the light cone S defined by $t^2 - r^2 - z^2 = 0$, the solution is $\psi_{+\Delta}$; outside the light-cone between the $\Sigma_{\pm}$ (defined by $t = \pm z$) the solution is $\psi_{-\Delta}$; outside the interior of two planes and at $t < 0$ the solution is zero. The function ψ is discontinuous at the light cone, but continuous on the plane fronts.

while in $J_{+\Delta}$ the first two arctan terms in Eq. (4.18) vanish, and the third one has the limit $\arctan(r/\sqrt{\Delta}) \to +\pi/2$. Thus

$$\lim_{\Delta \to +0} J_{+\Delta} = \frac{2\pi^2\theta(t)}{r}z\left[\mathrm{sgn}(z)\theta(r(t-r)) - \frac{t}{z}\right]$$

$$= \frac{2\pi^2\theta(t)}{r}(|z| - t),$$

compensating the above.

We will also see that, when the derivatives over z are computed, there are no delta functions with supports on these boundaries.

C. Physical properties of the solution

Differentiating Eq. (4.18) over z with the help of Eq. (A18), the solution splits as $\psi = \psi_{-\Delta} + \psi_{+\Delta}$, where

$$\psi_{-\Delta} = \frac{\alpha}{4r}\theta(t)\theta(t^2 - z^2)\theta(-\Delta)\mathrm{sgn}z, \qquad (4.19)$$

$$\psi_{+\Delta} = \frac{\alpha}{4\pi}\frac{\theta(t)\theta(\Delta)}{r}\left[\arctan\frac{z\sqrt{\Delta}}{t^2 - tr - z^2} - \arctan\frac{z\sqrt{\Delta}}{t^2 + tr - z^2}\right.$$

$$\left. + \pi\mathrm{sgn}(z)\theta(-t^2 + tr + z^2)\right], \qquad (4.20)$$

respectively. One can observe the following.

(i) The solution contains different regions divided by two characteristic four-dimensional hypersurfaces: (i) the light cone S, $t^2 - r^2 - z^2 = 0$, at which the solution is continuous, and (ii) two planes $\Sigma_\pm$, $t \pm z = 0$, at which the solution has jumps. S and $\Sigma_\pm$ touch along the lines $z = \pm t$, $r = 0$ (Fig. 1). Three terms in $\psi_{+\Delta}$ defined inside the cone $t^2 < r^2 + z^2$ together have no discontinuities on the hypersurface $t^2 - tr - z^2 = 0$.

(ii) The solution has support only in the bulk; its restriction on the domain wall vanishes. In other words, the spherical branon shock wave localized on the brane generates the *bulk* spherical shock wave and two *planar* shock waves. All of them start at the moment of perforation $t = 0$, propagate in the positive and negative z directions, and for $f > 0$ carry the positively and negatively defined scalar fields, respectively. The structure of the characteristic spatial surfaces at fixed $t > 0$ is presented on Fig. 2.

(iii) At a given point r in the bulk one first sees a planar shock wave $\Sigma_\pm$ arriving at $t = |z|$, i.e., the jump of ψ form zero to the value $\pm\alpha/4r$ for positive/negative z, respectively (the whole solution is odd in z). These values are memorized until the arrival of the spherical wave S, when they start to gradually decay to zero (Fig. 3). In other words, the perforation of the
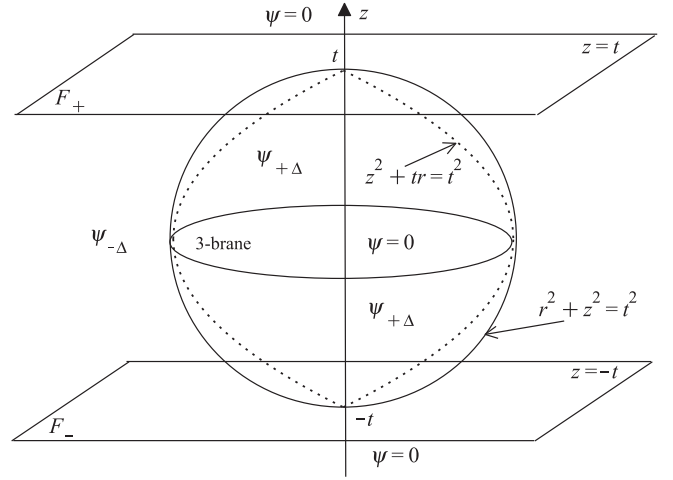

FIG. 2. The spatial section of the solution at fixed time $t > 0$. The planes $F_\pm$ are the $t = \mathrm{const}$ sections of $\Sigma_\pm$. The surface $z^2 + rt = t^2$ (dotted line) is a solid of revolution with the axis z and the parabola with axis "r" as a generatrix inside the sphere $r^2 + z^2 = t^2$.
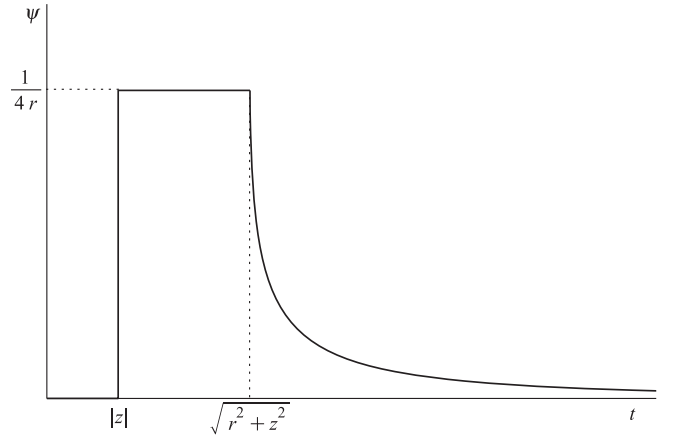

FIG. 3. The retarded field ψ as a function of time at a fixed observation point $(r > 0, z > 0)$ in the units $\alpha = 1$. The moment $t = |z|$ corresponds to the passage of the planar shock wave leaving as the memory of the constant value $\psi = 1/4r$, which then starts to decay after the passage of the spherical shock front at the moment $t = \sqrt{r^2 + z^2}$.

brane leaves the *temporary memory* in the bulk.[3] The front of the spherical shock wave S has no discontinuity. For fixed r and z after the passage of the wave front, the solution relaxes with time as

$$\psi_{+\Delta} \sim \frac{\alpha}{2\pi}\frac{z}{t^2},$$

which does not depend upon r. Therefore for large enough $t \gg \sqrt{r^2 + z^2}$ the field on the spatial plane

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[3]This feature is interesting to compare with various memory effects in asymptotics of the solutions to d'Alembert equations with light-like sources discussed recently [20–25].

$z = $ const is *constant* at fixed moment t. The plot is presented on Fig. 3.

(iv) The planar shock waves have the structure $\psi \sim \theta(t \pm |z|)/r$ which differs from the Aichelburg-Sexl metric [16,36] $h_{MN} \sim c_M c_N \delta(t \mp z)/r$, where $c^M = (1, 0, 0, 0, \pm 1)$ by the degree of discontinuity: the Heaviside function versus the delta function. Our shock waves therefore are "softer."

Recall that in this section we have considered the part of the retarded scalar field due to the light-like part of the source. For any relative velocity of the collision, this part is dominant in the region of small k^z [Eq. (5.3)] in the momentum space. For ultrarelativistic velocities it is dominant for all momenta.

V. RADIATION AMPLITUDE

To select the radiative part of the retarded potential, one has to pass to the momentum representation of the full current (2.24) consisting of the local and the stress terms. The first-order perturbation of the brane stress tensor in the coordinate representation is given by Eq. (2.25). Substituting there the first-order quantity $^1X^N = \Phi(\sigma)\delta_z^N$, we find

$$\tilde{\rho}(k^M) = \frac{f}{4\pi} \int [\varkappa \bar{h}_{zz}(q) - 2ik^z \Phi(q)] \delta^4(k^\mu - q^\mu) d^5 q, \quad (5.1)$$

where

$$\Phi(q^M) = -\frac{i\pi\varkappa^2 m}{\gamma} \frac{q^z \delta(q^0 - vq^z)}{q^2(q_\mu q^\mu + 2i\epsilon q^0)} \left(\gamma^2 v^2 + \frac{1}{3}\right). \quad (5.2)$$

The integration over q is fulfilled by using delta functions, resulting in

$$\tilde{\rho}(k^M) = \frac{\varkappa^2 fm}{2} \frac{\gamma v}{\omega^2 + k_\perp^2 \gamma^2 v^2} \left(\gamma^2 v^2 + \frac{1}{3}\right)$$
$$\times \left[\frac{k^0 k^z}{v(k_\mu k^\mu + 2i\epsilon k^0)} - 1\right], \quad (5.3)$$

where $k_\perp^2 \equiv \delta_{ij} k^i k^j$.

The stress part of the second-order current is given by Eq. (2.24) where only the metric perturbation due to the particle has to be retained:

$$S(x^M) = \partial_M \left(\bar{h}^{MN}\partial_N - \frac{1}{2}\bar{h}\partial^M\right)^1\varphi. \quad (5.4)$$

Representing the products of fields as a convolution in the momentum space, and integrating using delta functions,

$$\int \frac{\delta(ku - qu)\delta^4(q^\mu)}{q^2(k-q)^2} d^5 q \bigg|_{k^2=0} = \frac{\gamma^3 v^3}{(ku)^3(\bar{k}u)},$$
$$\bar{k}^M = (k^0, -\mathbf{k}, -k_z), \quad (5.5)$$

one obtains

$$S(k^M) = \frac{\varkappa^2 fm\gamma^3 v^3}{(ku)(\bar{k}u)}. \quad (5.6)$$

We now consider some particular cases.

A. Ultrarelativistic case

Denoting the ratio of the component k^z (orthogonal to the domain wall) to the total energy of the emitted quantum as $\cos\chi = k^z/k^0$, we obtain in the limit $v \to 1$, $\gamma \to \infty$ the local contribution (5.3) in the form

$$\tilde{\rho}(k^M) = \frac{\varkappa^2 f\mathcal{E}}{4\omega^2} \frac{1}{\cos\chi\cos^2(\chi/2)}, \quad (5.7)$$

while the stress term (5.6) reads

$$S(k^M) = \frac{\varkappa^2 f\mathcal{E}}{\omega^2(1 - v^2\cos^2\chi)}. \quad (5.8)$$

One observes the following.

(i) At $\chi \approx \pi/2$ the brane contribution dominates and blows up at $\chi \to \pi/2$.

(ii) At $\chi < \gamma^{-1}$ in the forward/backward direction the stress contribution has apparent maxima and blows up at $\chi \to 0$ in the massless limit.

(iii) At $\chi \approx \pi$ the brane contribution also blows up due to $\cos^2(\chi/2)$ in the denominator.

Considering the last case more closely, we introduce $\chi' = \pi - \chi$ and find that the local amplitude is negative and diverges quadratically as $\chi' \to 0$:

$$\tilde{\rho}(k^M) = -\frac{\varkappa^2 f\mathcal{E}}{4\omega^2} \frac{1}{\sin^2(\chi'/2)} \simeq -\frac{\varkappa^2 f\mathcal{E}}{\omega^2\chi'^2}. \quad (5.9)$$

The stress term (5.6) is positive and is equal to minus Eq. (5.9):

$$S(k^M) = \frac{\varkappa^2 f\mathcal{E}}{\omega^2\sin^2\chi} \simeq \frac{\varkappa^2 f\mathcal{E}}{\omega^2\chi'^2}. \quad (5.10)$$

Thus, as could be expected, radiation is asymmetric with respect to the particle motion and there is no radiation in the backward direction.

To get the total radiation power, we note that (only) the stress amplitude is dominant and is beamed around the forward direction. So to get an estimate of the total power we substitute Eq. (5.8) in Eq. (2.27) and integrate over angles *for finite* γ. The remaining integral over the spectrum diverges both in the low and high frequencies requiring the IR and the UV cutoffs:

$$E_{\text{rad}} = \frac{\varkappa^4 f^2 m^2 \gamma^2}{(2\pi)^3} \int_0^\infty \frac{d\omega}{\omega} \int_0^{\pi/2} \frac{\sin^2\chi \, d\chi}{(1 - v^2\cos^2\chi)^2}$$

$$\simeq \frac{\varkappa^4 f^2 \mathcal{E}^2 \gamma}{8(2\pi)^2} \ln \frac{\omega_{\max}}{\omega_{\min}}. \tag{5.11}$$

Note that this expression diverges in the massless limit $\gamma \to 0$, indicating that the interaction of the brane with massless particles appeals to quantum theory.

B. Nonrelativistic collision

Now consider the opposite case: $v \ll 1$. Then the stress part (5.6) is small, so the total amplitude $j(k)$ is given by the brane contribution (5.3) only:

$$j(k) = \frac{\varkappa^2 f m}{6\omega} \frac{k^z}{k_\mu k^\mu + 2i\epsilon k^0}, \tag{5.12}$$

where $\mu = 0 \ldots 3$ and the five-dimensional wave vector in null, so $k_\mu k^\mu = (k^z)^2$, $\omega^2 = (k^z)^2 + k_\perp^2$. This can be rewritten as

$$j(k) = \frac{\varkappa^2 f m}{6} \frac{1}{k_z \sqrt{(k^z)^2 + k_\perp^2}}, \tag{5.13}$$

so we see that the integration over k^z is dominated in the infrared, and with logarithmic accuracy we can set $(k^z)^2 + k_\perp^2 \sim k_\perp^2$, obtaining

$$E_{\text{rad}} \simeq \frac{\varkappa^4 f^2 m^2}{36(2\pi)^3} \int \frac{dk^z}{(k_z)^2} \int dk_\perp \sim \frac{\varkappa^4 f^2 m^2}{18(2\pi)^3} \frac{k_\perp^{\max}}{k_{\min}^z}, \tag{5.14}$$

where the nature of the cutoff has to be clarified. Actually, the divergence of the amplitude in the directions of the emitted quanta along the brane takes place for any velocity, and its origin is worth discussing when going to the coordinate representation.

C. Emission along the brane

The general expression for the five-momentum loss under collision in the coordinate representation is

$$\Delta P^M = \int \nabla_N T^{MN} \sqrt{-g} d^5 x = \int \varphi^{,M} \rho \, d^5 x. \tag{5.15}$$

Since the source on the right-hand side of Eq. (4.1) has support only at $r > t > 0$, under the multiplication by ψ only the $\psi_{-\Delta}$p art of ψ will contribute. Computing the time derivative of ψ [Eqs. (4.19) and (4.20)], we multiply by a source term (4.1), obtaining the estimate

$$E_{\text{rad}} \sim \frac{(\lambda f)^2}{\gamma^6} \frac{r_{\max}}{z_{\min}}, \tag{5.16}$$

where for consistency and independence of the integration sequence, the formal cutoffs for r and z are introduced.

Meanwhile, the validity of our iteration scheme has some restrictions. The gravitational constant in five dimensions has the dimension of $(\text{length})^3$. Combining it with the particle energy $\mathcal{E}$ [dimension of $(\text{length})^{-1}$] and the brane tension μ [dimension of $(\text{length})^{-2}$], we have two length parameters: $l = \varkappa^2\mu$, $r_S = \varkappa\sqrt{\mathcal{E}}$, with the first corresponding to the curvature radius of the bulk in the RS II setup, and the second to the gravitational radius of the energy $\mathcal{E}$.[4] To keep contact with the RS II model we have to consider distances that are small with respect to l, while to justify the linearization of the metric for the particle we have to consider distances that are large with respect to r_S. So to apply the linearized theory to both objects we have to assume $l \gg r_S$, or $\mathcal{E} \ll \varkappa^2\mu^2$. With this motivation we take the maximal cutoff parameters to be

$$r_{\max} \sim |z|_{\max} \sim [\varkappa^2\mu]^{-1}. \tag{5.17}$$

The minimal values of r, z can be estimated from the assumed convergence of the iterative solution. For the second and the first terms of the scalar field, we expect $|^2\varphi| \ll |^1\varphi|$. Comparing $^2\varphi(\psi)$ [Eq. (4.19)] with $^1\varphi$ [Eq. (3.4)] for sufficiently small r, one finds

$$r_S^2 < r|z|. \tag{5.18}$$

On the other hand, we need $r > r_S$ in order to use the linearized gravity and the concept of a point-like particle. Combining these arguments, we conclude that for

$$r_{\min} \sim z_{\min} \sim r_S, \tag{5.19}$$

the perturbation theory converges.

The momentum-space formula (5.14) matches with Eq. (5.16) if

$$k_{\min}^z \sim 1/z_{\max}, \qquad k_\perp^{\max} \sim 1/r_{\min}, \tag{5.20}$$

assuming that the bulk coordinate z is restricted by the same curvature effect as the brane coordinate r.

With these cutoffs the total radiation loss becomes

$$E_{\text{rad}} \sim \frac{\lambda^2 f^2}{\gamma^6} \frac{1}{\varkappa^3\mu\sqrt{\mathcal{E}}} \sim \frac{\varkappa\mathcal{E}^{3/2} f^2}{\mu}, \tag{5.21}$$

with the last estimate being valid in the massless-particle limit.

Finally, from the above restrictions we obtain the frequency cutoffs:

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[4]Similarly, $1/\varkappa f$ will be the scalar analogue of the curvature length parameter.

$$\omega_{\min} \sim \varkappa^2 \mu, \qquad \omega_{\max} \sim 1/r_S. \qquad (5.22)$$

D. Radiation normal to the brane

Now we come back to the massless limit by taking into account the existence of $r_{\min}$ preventing the angle χ from approaching zero:

$$\chi_{\min} \sim \arctan \frac{r_{\min}}{z_{\max}} \simeq \frac{r_{\min}}{z_{\max}} \ll 1.$$

In the relativistic case we thus have to integrate the angular distribution (5.11) from $\chi_{\min}$ to $\pi/2$. Since the integrand is beamed inside the cone $0 < \chi \lesssim 1/\gamma$, the final result depends upon the relation between $1/\gamma$ and $\chi_{\min}$. Namely, by expanding the numerator and the denominator in Eq. (5.11) in $\chi \ll 1$,

$$1 - v^2 \cos^2 \chi \simeq \chi^2 + \gamma^{-2},$$

we are led to consider two cases.

Case A: $\chi_{\min} > 1/\gamma$. Then, $1 - v^2 \cos^2 \chi \simeq \chi^2$, so

$$
\begin{aligned}
E_{\mathrm{rad}} &= \frac{\varkappa^4 f^2 m^2 \gamma^2}{(2\pi)^3} \ln \frac{\omega_{\max}}{\omega_{\min}} \int_{\chi_{\min}}^{\pi/2} \frac{\sin^2 \chi \, d\chi}{(1 - v^2 \cos^2 \chi)^2} \\
&\simeq \varkappa^4 f^2 \mathcal{E}^2 \frac{r_{\max}}{r_{\min}} \ln \frac{r_{\max}}{r_{\min}}.
\end{aligned} \qquad (5.23)
$$

The radiation efficiency $\epsilon = E_{\mathrm{rad}}/\mathcal{E}$ then reads

$$
\begin{aligned}
\epsilon &\sim \varkappa^4 f^2 \mathcal{E} \frac{r_{\max}}{r_{\min}} \ln \frac{r_{\max}}{r_{\min}} \to \varkappa^2 \mu^2 \mathcal{E} \frac{r_{\max}}{r_{\min}} \ln \frac{r_{\max}}{r_{\min}} \\
&\sim \frac{r_{\min}}{r_{\max}} \ln \frac{r_{\max}}{r_{\min}} < 1,
\end{aligned} \qquad (5.24)
$$

since the function $\ln x / x$ does not exceed 1 for $x > 1$. Hence there is no efficiency catastrophe in our model.

Case B: $\chi_{\min} < 1/\gamma$. Now $1 - v^2 \cos^2 \chi \simeq \gamma^{-2}$. Then, Eq. (5.11) holds, but according to the above restrictions,

$$E_{\mathrm{rad}} \sim \varkappa^4 f^2 \mathcal{E}^2 \gamma \ln \frac{r_{\max}}{r_{\min}} < \frac{r_{\max}}{r_{\min}} \ln \frac{r_{\max}}{r_{\min}}, \qquad (5.25)$$

so the emitted radiation is smaller than in case A. But now radiation is beamed inside the characteristic cone in the forward direction $\chi \lesssim 1/\gamma$.

VI. CONCLUSIONS

In this paper we have considered radiation in the collision of a point particle with an extended object—namely, a domain wall—possessing an internal dynamics. The interaction between them is assumed to be purely gravitational, while the radiation is scalar and it is generated solely by the domain wall. The main new feature of this process is the creation of the shock spherical branon wave, which propagates freely along the wall with the velocity of light. This branon constitutes the part of the source of the scalar radiation. We have carefully calculated the retarded solution of the bulk d'Alembert equation with such a light-like source, revealing a sophisticated structure of the solution. This solution dominates in the case of the ultra-relativistic collision. We also computed the full radiation by taking into account other relevant source terms.

Performing these calculations, we encountered classical singularities in the solutions of the five-dimensional d'Alembert equation which are absent in four dimensions. Namely, the local and the nonlocal parts of the Green's functions generate two singular parts of the full solution[5] which require regularization. We have then proved that the sum of these singular parts remains finite when the regularization is removed. A similar picture holds in higher odd spacetime dimensions, so our regularity proof may help in other dimensions too.

The retarded potential of the branon contains two shock plane waves $\Sigma_{\pm}$ which have theta-like behavior on the front, differing from the well-known Aichelburg-Sexl solution [36] which has a δ singularity there. Correspondingly, our plane waves have no point-like sources located at the centers of the wave fronts. The theta-like nature of the planar waves means that after the passage of the wave front the field remains constant, imitating the memory effects [20,21]. After the subsequent arrival of the spherical wave, this value starts to decrease and asymptotically disappears. We realize, however, that this memory is basically due to the nonlocal nature of the retarded Green's function in five dimensions.

Finally, we would like to draw attention to one peculiarity of the radiation amplitude: it exhibits *backward* destructive interference between the local and nonlocal contributions, contrary to the *forward* destructive interference of the particle-particle bremsstrahlung [38,39] and synchrotron radiation [32,40]. In an attempt to explain the difference, one can observe that i) the brane gravity is repulsive, and ii) the source of radiation is the brane, which moves in the particle rest frame in the backward direction.

We expect that basic features of the scalar branestrahlung will survive in the case of the true gravitational radiation.

ACKNOWLEDGMENTS

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[5]An ambiguity seems to exist in the literature concerning the local term of the five-dimensional Green's function; compare, e.g., with [37], Eq. 5).

APPENDIX: DERIVATION OF THE RETARDED SOLUTION (4.14)

Here we give details of the calculation of the retarded field generated by the light-like branon source (4.14).

Integration domains: The limits of integration in r', ϑ are restricted by several Heaviside functions in the integrand depending on the signs of the parameters Δ, D defined in Eqs. (4.16) and (4.17).

$\Delta > 0$. If $\Delta > 0$, then $t > r$, $D > 0$, and the roots of Eq. (4.15)

$$r'_{\pm} = r\cos\vartheta \pm \sqrt{r^2\cos^2\vartheta + \Delta} \qquad (A1)$$

(positive and negative, respectively) exist for all angles ϑ. For fixed t, r, z, and ϑ, the quadratic form $R^2 - t^2$ as a function of r' goes to $+\infty$ if $r' \to \pm\infty$, and hence the support of $\theta(t - R)$ lies *between* the roots. Thus $\theta(t - R)\theta(\Delta) = \Theta_{r'}(r'_-, r'_+)\theta(\Delta)$. Finally, since $r'_- < 0$, by assuming $r' > 0$ one obtains $\theta(t - R)\theta(\Delta)\theta(r') = \Theta_{r'}(0, r'_+)\theta(\Delta)$.

The next restricting inequality for the second term of Eq. (4.14) is $R > t - r'$. Being squared, this is equivalent to $2r'(t - r\cos\vartheta) > \Delta$. If $\Delta > 0$, then $t > r > r\cos\vartheta$, so

$$r' > \frac{1}{2}\frac{\Delta}{t - r\cos\vartheta} \equiv \underline{r}', \qquad \underline{r}' > 0,$$
$$\theta(r' - t + R) = \theta(r' - \underline{r}'). \qquad (A2)$$

Substituting $r' = \underline{r}'$ into the form $R^2 - t^2$ makes it negative. Indeed, being a root of the equation $r' - t + R(r') = 0$, $\underline{r}'$ satisfies $\underline{r}' - t + \underline{R} = 0$, where $\underline{R} \equiv R(r')|_{r'=\underline{r}'}$. Thus $\underline{R}^2 - t^2 = (\underline{R} - t)(\underline{R} + t) = -\underline{r}'(\underline{R} + t) < 0$ by virtue of the positiveness of $\underline{r}'$, t, and R. It follows that $r'_- < \underline{r}' < r'_+$. To, summarize: if $t^2 - r^2 - z^2 > 0$, then

$$r'_- < 0 < \underline{r}' < r'_+\theta(r' - t + R)\theta(t - R)\theta(\Delta)\theta(r')$$
$$= \Theta_{r'}(\underline{r}', r'_+)\theta(\Delta). \qquad (A3)$$

Next consider the first term in Eq. (4.14): according to Eq. (A2) one obtains

$$\theta(t - r' - R) = \theta(\underline{r}' - r'),$$
$$\theta(t - r' - R)\theta(\Delta)\theta(r') = \Theta_{r'}(0, \underline{r}')\theta(\Delta). \qquad (A4)$$

Thereby making use of $\Theta_{r'}(0, \underline{r}') + \Theta_{r'}(\underline{r}', r'_+) = \Theta_{r'}(0, r'_+)$, one concludes that the $\theta(\Delta)$ contribution becomes

$$J_{+\Delta} = 2\pi\theta(\Delta)\int[F(r')\Theta_{r'}(0, \underline{r}')$$
$$- F(0)\Theta_{r'}(0, r'_+)]r'dr'\sin\vartheta d\vartheta. \qquad (A5)$$

$\Delta < 0$, $D > 0$. This happens if $r^2 > t^2 - z^2 > r^2\sin^2\vartheta$. Here the equation $t = R$ also has two real roots given by Eq. (A1) which have the same sign as $\cos\theta$. Thus if $\cos\theta > 0$, then $\vartheta < \arcsin(\sqrt{t^2 - z^2}/r)$, and $0 < r'_- < r'_+$; hence,

$$\theta(r')\theta(t - R) = \theta(\cos\vartheta)\Theta_{r'}(r'_-, r'_+). \qquad (A6)$$

In the case $\cos\vartheta < 0$ one has $r'_- < r'_+ < 0$, so by restoring $\theta(r')$, the indicator $\theta(r')\theta(t - R) = 0$, so that the second term in the integrand of Eq. (4.14) vanishes.

Now consider the support of $\theta(t - r' - R)$. The limiting value, $\underline{r}'$ is still determined by Eq. (A2), but the sign depends upon the sign of $t - r\cos\vartheta$. If $t > r\cos\vartheta$, then $\underline{r}' < 0$ by virtue of $\Delta < 0$. Here $\theta(t - r' - R) = \theta(\underline{r}' - r')$ but

$$\theta(r')\theta(t - r' - R) = 0 \qquad (A7)$$

since $\underline{r}'$ is negative. Thus the first term in the integrand of Eq. (4.14) vanishes, while the integration range for the second one is determined by $\theta(t - R)$ and reads $r'_- < r' < r'_+$.

If $t < r\cos\theta$, then $\underline{r}'$ [given as before by the Eq. (A2)] becomes positive, but this root is *false* and represents an artifact of the inequality squaring in Eq. (A2). Indeed, the formal resolution of $(t - r')^2 = R^2$ gives $r' > \underline{r}' = (r^2 + z^2 - t^2)/[2(r\cos\vartheta - t)]$ and $\underline{r}' > t$ by virtue of the inequality chain $t^2 + r^2 + z^2 \geq t^2 + r^2 \geq 2rt > 2rt\cos\vartheta$, which is equivalent to $r^2 + z^2 - t^2 > 2t(r\cos\vartheta - t)$. Meanwhile, $t > r' + R > r'$, in contradiction with the latter. In what follows, the property (A7) is valid for all cases of the sign of $(t - r\cos\vartheta)$.

To conclude: in all cases with $\Delta < 0$, $D > 0$ the first term in Eq. (4.14) vanishes, while the second is to be integrated over r' from r'_- to r'_+ with an additional angular restriction $0 \leq \vartheta \leq \pi/2$.

$D < 0$. This restriction is equivalent to $t^2 - z^2 < r^2\sin^2\vartheta$ and implies $\Delta < 0$. The equation $t = R$ has no real roots in this case, so the second term in the integrand of Eq. (4.14) vanishes completely. The restrictions on $\theta(t - r' - R)$ are the same as in the previous case, so $\theta(r')\theta(t - r' - R) = 0$. Thus one concludes that both terms in the integrand of Eq. (4.14) have no support, and hence the contribution $J_{D<0}$ to the total J vanishes. Combining the two cases $D > 0$ and $D < 0$ of $\Delta < 0$, one finds

$$J_{-\Delta} = -2\pi\theta(-\Delta)$$
$$\times \int F(0)\Theta_{r'}(r'_-, r'_+)r'dr'\Theta_\vartheta(0, \pi/2)\theta(D)\sin\vartheta d\vartheta. \qquad (A8)$$

Integration over r': We have to integrate $J_{-\Delta}$, given by Eq. (A8), and $J_{+\Delta}$ from Eq. (A5). First consider $J_{-\Delta}$: by integrating Eq. (A8) from r'_- to r'_+ with help of the table integral

$$\int \frac{(\alpha x + \beta)dx}{(x^2 + b^2)\sqrt{a^2 - x^2}} = \frac{1}{\sqrt{a^2 + b^2}} \left[\frac{\beta}{b} \arctan \frac{x\sqrt{a^2 + b^2}}{b\sqrt{a^2 - x^2}} \right.$$

$$\left. - \alpha \arctan \frac{\sqrt{a^2 - x^2}}{\sqrt{a^2 + b^2}} \right], \qquad (A9)$$

one arrives at

$$J_{-\Delta} = -2\pi^2 r\theta(t)\theta(-\Delta) \int \frac{\theta(D)\cos\vartheta \sin\vartheta}{\sqrt{z^2 + r^2 \sin^2 \vartheta}} d\vartheta, \quad (A10)$$

with the remaining integral over ϑ.

The contribution $J_{+\Delta}$ consists of two parts: $J_{+\Delta} \equiv Q_1 + Q_2$. The one containing $F(0)$ is of the type (A9), so by integrating it from 0 to r'_+ we obtain

$$Q_2 = -\frac{1}{2} \ln \frac{t + \sqrt{\Delta}}{t - \sqrt{\Delta}} - \frac{r\cos\vartheta}{\sqrt{z^2 + r^2 \sin^2 \vartheta}}$$

$$\times \left(\arctan \frac{rt\cos\vartheta}{\sqrt{\Delta(z^2 + r^2 \sin^2 \vartheta)}} + \frac{\pi}{2} \right). \qquad (A11)$$

The first contribution in Eq. (A5) coming from the $F(r')$ term contains the integrals of type

$$\int \frac{(\alpha x^2 + \beta x + \gamma)dx}{(x^2 + b^2)\sqrt{c - x}}. \qquad (A12)$$

A routine calculation gives

$$Q_1 = \frac{1}{2} \ln \frac{t + \sqrt{\Delta}}{t - \sqrt{\Delta}} - \frac{\sqrt{\Delta}}{t - r\cos\vartheta} + \frac{r\cos\vartheta}{\sqrt{z^2 + r^2 \sin^2 \vartheta}}$$

$$\times \left(\arctan \frac{t - r\cos\vartheta + \sqrt{\Delta}}{\sqrt{z^2 + r^2 \sin^2 \vartheta}} - \arctan \frac{t - r\cos\vartheta - \sqrt{\Delta}}{\sqrt{z^2 + r^2 \sin^2 \vartheta}} \right).$$

Combining it with Eq. (A11) and using the identity

$$\arctan x - \arctan y = \arctan \frac{x - y}{1 + xy} + \pi\theta(-1 - xy)\mathrm{sgn}x,$$

$$(A13)$$

we get

$$J_{+\Delta} = 2\pi\theta(\Delta) \int \left[-\frac{\sqrt{\Delta}}{t - r\cos\vartheta} + \frac{r\cos\vartheta}{\sqrt{z^2 + r^2 \sin^2 \vartheta}} \right.$$

$$\times \left(\arctan \frac{\sqrt{\Delta}\sqrt{z^2 + r^2 \sin^2 \vartheta}}{r^2 + z^2 - rt\cos\vartheta} + \frac{\pi}{2}\mathrm{sgn}(rt\cos\vartheta - z^2 - r^2) \right.$$

$$\left. \left. - \arctan \frac{rt\cos\vartheta}{\sqrt{\Delta(z^2 + r^2 \sin^2 \vartheta)}} \right) \right] \sin\vartheta d\vartheta. \qquad (A14)$$

Integration over ϑ: The condition $D > 0$ is equivalent to $t^2 - z^2 > r^2 \sin^2 \vartheta > 0$ in addition to $\theta(-\Delta)$. Thus one integrates over ϑ from zero to $\arcsin(\sqrt{t^2 - z^2}/r)$. A thorough analysis of the characteristic domains gives in this case a simple overall condition $t > |z|$:

$$\theta(-\Delta)\theta(D) = \theta(-\Delta)\theta(t^2 - z^2)\Theta_\vartheta \left(0, \arcsin \frac{\sqrt{t^2 - z^2}}{r} \right). \qquad (A15)$$

Using this in the angular integral in Eq. (A10), we obtain

$$J_{-\Delta} = -2\pi^2 \theta(t)\theta(-\Delta)\theta(t^2 - z^2)\frac{t - |z|}{r}. \qquad (A16)$$

For the region $\Delta > 0$ one has to integrate Eq. (A14). The first term in the brackets yields

$$-\int_0^\pi \frac{\sqrt{\Delta} \sin\vartheta}{t - r\cos\vartheta} d\vartheta = \frac{\sqrt{\Delta}}{r} \ln \frac{t - r}{t + r}. \qquad (A17)$$

The second one is integrated by parts, taking into account the jump of the arctangent:

$$\frac{d}{dx} \arctan \frac{1}{x} = -\frac{1}{1 + x^2} + \pi\delta(x). \qquad (A18)$$

One obtains

$$J_{+\Delta} = \frac{2\pi z\theta(t)\theta(\Delta)}{r} \left[\arctan \frac{z\sqrt{\Delta}}{t^2 - tr - z^2} - \arctan \frac{z\sqrt{\Delta}}{t^2 + tr - z^2} \right.$$

$$\left. + \pi\mathrm{sgn}(z)\theta(tr + z^2 - t^2) - \frac{2t}{z} \arctan \frac{r}{\sqrt{\Delta}} \right].$$

Combining this with Eq. (A16), we find the result (4.18) presented in the main text.

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