Quantum critical behavior of semisimple gauge theories

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(Received 17 December 2015; published 9 February 2016)

We study the perturbative phase diagram of semisimple fermionic gauge theories resembling the Standard Model. We investigate an SU(N) gauge theory with M Dirac flavors where we gauge first an $SU(M)_L$ and then an $SU(2)_L \subset SU(M)_L$ of the original global symmetry $SU(M)_L \times SU(M)_R \times U(1)$ of the theory. To avoid gauge anomalies we add leptonlike particles. At the two-loop level an intriguing phase diagram appears. We uncover phases in which one, two or three fixed points exist and discuss the associated flows of the coupling constants. We discover a phase featuring complete asymptotic freedom and simultaneously an interacting infrared fixed point in both couplings. The analysis further reveals special renormalization group trajectories along which one coupling displays asymptotic freedom and the other asymptotic safety, while both flowing in the infrared to an interacting fixed point. These are *safety free* trajectories. We briefly sketch out possible phenomenological implications, among which an independent way to generate near-conformal dynamics $\hat{a} \, la$ walking is investigated.

DOI: 10.1103/PhysRevD.93.045009

I. INTRODUCTION

The Standard Model is an example of a semisimple gauge theory with two non-Abelian and a single Abelian gauge group. It is in addition a nonsupersymmetric gauge-Yukawa theory that contains spin-0, spin- $\frac{1}{2}$ and spin-1 particles. Whereas the literature contains investigations of the phase diagram of semisimple and simple super-symmetric gauge theories [1–3], vectorlike fermionic gauge theories [4–12], chiral gauge theories [13–15], Yukawa theories [16–20] and scalar theories [21] only very little has been done concerning the more general class of theories in which the Standard Model falls [22].

Recently however much interest has been given to gauge-Yukawa theories featuring a single gauge group [23]. The reason for such an interest is that this theory was found to flow to a nontrivial ultraviolet stable fixed point in a completely controllable manner [23,24]. The matter sector contains a set of N_f Dirac fermions and $N_f \times$ N_f scalar mesons. The result shows that no additional symmetry principles, such as space-time supersymmetry [25], are required to ensure well-defined and predictive ultraviolet theories. The ultraviolet fixed point arises dynamically through renormalizable interactions between non-Abelian gauge fields, fermions, and scalars, and in a regime where asymptotic freedom is absent. Furthermore the dangerous growth of the gauge coupling towards the ultraviolet is countered by Yukawa interactions, while the Yukawa and scalar couplings are tamed by the fluctuations of gauge and fermion fields. This has led to the discovery of complete asymptotic safety, meaning an interacting ultraviolet fixed point in all couplings [23]. This phenomenon is quite distinct from the conventional setup of *complete* asymptotic freedom [26–28], where the ultraviolet dynamics of Yukawa and scalar interactions is tamed by asymptotically free gauge fields; see [29,30] for recent studies. It is, indeed, very important that it is only via the combined analysis of the gauge, Yukawa and scalar self-couplings that one observes the existence of the ultraviolet fixed point. For instance it is not a feature of the gauge theory with either pure fermionic or scalar matter. Neither does the ultraviolet fixed point exist for the supersymmetrized version [31,32]. It is also straightforward to engineer OCD-like IR behavior including confinement and chiral symmetry breaking. In practice one needs to decouple, at some intermediate energies, the unwanted fermions by adding mass terms or via spontaneous symmetry breaking in such a way that at lower energies the running of the gauge coupling mimics QCD [33]. Tantalizing indications that ultraviolet interacting fixed points may exist nonperturbatively, and without the need for elementary scalars, appeared in [8], and they were further explored in [23,34]. Nonperturbative techniques are needed to establish the existence of such a fixed point when the number of colors and flavors is taken to be three and the number of UV light flavors is large but finite. Asymptotic safety was originally introduced by Weinberg [35] to address quantum aspects of gravity [36-43].

These observations should make it clear that further studies of gauge-Yukawa theories are to be carried out. Here we take one step further and study nonsupersymmetric semisimple gauge theories with fermionic matter. This is the logical next step given that a *trait d'union* of the majority of the extensions of the Standard Model (and the Standard Model itself) is the presence of multiple gauge couplings. Furthermore the theories we study are chosen such as to resemble the Standard Model in a most natural

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way and candidate theories of fermion mass generation [44–48] in elementary [49,50] or composite extensions of the Standard Model [51–54].

To construct our case study we begin with an SU(N) gauge theory with M Dirac fermions in the fundamental representation. We then gauge a subgroup of the global symmetry $SU(M)_L \times SU(M)_R \times U(1)$ and study the flow of both gauge couplings at the two-loop level.

Since we gauge a subgroup of the global symmetry we have to worry about potential anomalies. In order for our theory to be consistent we therefore add a set of leptonlike fermions. We then consider two different scenarios: one in which we gauge $SU(M)_L$ and one in which we only gauge a subgroup $SU(2)_L \subset SU(M)_L$.

We discover a rich phase diagram that contains one, two or three fixed points and determine the associated renormalization group flow. A phase featuring complete asymptotic freedom and simultaneously a complete (in both couplings) interacting infrared fixed point emerges. Intriguingly we further expose the emergence of special renormalization group trajectories along which one coupling displays asymptotic freedom and the other asymptotic safety, while both flow in the infrared to an interacting fixed point. We term these trajectories *safety free*.

We then present the actual running of the couplings for potentially phenomenologically interesting renormalization group trajectories that hint at new avenues for model building. Among these also are new ways to generate near-conformal dynamics $\hat{a} \, la$ walking.

The paper is organized as follows. In Sec. II we study the phase diagram for the $SU(N) \times SU(M)_L$ gauge theory while in Sec. III we study the theory with $SU(2)_L \subset SU(M)_L$ gauged. We sketch out phenomenological implications of the quantum critical behavior of the theories investigated here in Sec. IV. Finally we present our conclusions in Sec. V.

II. THE QUANTUM CRITICAL BEHAVIOR OF THE $SU(N) \times SU(M)_L$ THEORY

We start out by considering the simple gauge theory with an SU(N) gauge group and two sets of M Weyl fermions q and \tilde{q} in the fundamental and antifundamental representations respectively. This theory has an $SU(M)_L \times$ $SU(M)_R \times U(1)_V$ anomaly free global symmetry. At the classical level there is an additional Abelian U(1) symmetry but this is broken by an anomaly at the quantum level. This theory is QCD with N colors and M Dirac flavors. This theory is asymptotically free if $M < \frac{11}{2}N$ and



FIG. 1. Triangle diagrams that potentially can give rise to anomalies.

possesses an infrared fixed point for a number of flavors just below this critical value.

We now gauge the $SU(M)_L$ part of the global flavor symmetry so that the theory has the semisimple gauge group $SU(N) \times SU(M)_L$. The Weyl fermions q are charged under $SU(N) \times SU(M)_L$ while the fermions \tilde{q} are charged only under SU(N). For M > 2 this therefore induces an $SU(M)_L^3$ gauge anomaly which we need to avoid. To cancel the anomaly we add N Weyl fermions L in the antifundamental representation of $SU(M)_L$ since the fermions q belong to the fundamental representation of $SU(M)_L$. There are no other gauge anomalies. Any other mixed anomaly must contain a single SU(N) or $SU(M)_L$ gauge current and must vanish since it is proportional to the trace of the associated symmetry generator.

In the special case where M = 2, which is closer to the Standard Model, one can actually gauge the $SU(2)_{I}$ part directly without inducing any gauge anomalies, provided N is even to avoid Witten's topological anomaly [55]. Hence there is no need to add additional matter in this case. As mentioned above for the generic M > 2 case we add the additional set of fermions (L for leptons) to avoid anomalies. In Table I we summarize the matter content of the theory. The brackets denote the gauge symmetries while $SU(M)_R \times SU(N)$ is the remaining non-Abelian global symmetry. The theory enjoys only a single anomaly free Abelian $U(1)_V$ global symmetry despite the fact that at the classical level it possesses three. However since the gauge group is semisimple the global symmetry current can couple to both SU(N) and $SU(M)_L$ gauge currents respectively. The anomalies have the potential to arise via the diagrams in Fig. 1.

These diagrams, however, vanish provided that the fermions are charged as in the above Table I. There can be no other Abelian anomaly free symmetries. If any one fermion is charged under an Abelian symmetry then all three must be charged due to their gauge symmetry assignments. This can only be the one already given in the table. Lastly we provide the Lagrangian of the system

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} F^{i}_{\mu\nu} F^{i\mu\nu} + \bar{q}^{n_c m_c} \bar{\sigma}^{\mu} (\delta^{n'_c}_{n_c} \delta^{m'_c}_{m_c} \partial_{\mu} - ig_N (T^a)^{n'_c}_{n_c} \delta^{m'_c}_{m_c} G^a_{\mu} - ig_M (S^i)^{m'_c}_{m_c} \delta^{n'_c}_{n_c} A^i_{\mu}) q_{n'_c m'_c} + \bar{\tilde{q}}_{n_c m_f} \bar{\sigma}^{\mu} (\delta^{n_c}_{n'_c} \delta^{m_f}_{m_f'} \partial_{\mu} + ig_N (T^a)^{n_c}_{n'_c} \delta^{m_f}_{m_f'} G^a_{\mu}) \tilde{q}^{n'_c m'_f} + \bar{L}_{n_f m_c} \bar{\sigma}^{\mu} (\delta^{n_f}_{n_f'} \delta^{m_c}_{m'_c} \partial_{\mu} + ig_M (S^i)^{n_f}_{n_f'} \delta^{m_c}_{m'_c} A^i_{\mu}) L^{n'_f m'_c},$$
(1)

TABLE I. Matter content and symmetries.

	[SU(N)]	$[SU(M)_L]$	$SU(M)_R$	SU(N)	$U(1)_V$
q $ ilde{q}$ L	□ □ 1	□ 1 □	1 □ 1	1 1 	$+1 \\ -1 \\ -1$

where n_c , m_c refer to the "color" indices of the gauge groups SU(N), $SU(M)_L$ and n_f , m_f refer to the flavor indices of the global symmetry groups SU(N), $SU(M)_R$. The gauge field strengths, gauge fields and generators of SU(N) and $SU(M)_L$ are denoted as $G^a_{\mu\nu}$, G^a_{μ} , T^a and $F^i_{\mu\nu}$, A^i_{μ} , S^i respectively. The two gauge couplings are denoted as g_N and g_M .

The calculation of the beta functions up to second-loop order for a semisimple gauge theory was first calculated in [56]. It was further generalized to semisimple gauge-Yukawa theories in [57]. In our case with two gauge couplings they are

$$\beta_N(\alpha_N, \alpha_M) = -a_N \frac{\alpha_N^2}{2\pi} - b_N \frac{\alpha_N^3}{(2\pi)^2} - c_N \frac{\alpha_N^2 \alpha_M}{(2\pi)^2}, \qquad (2)$$

$$\beta_M(\alpha_M, \alpha_N) = -a_M \frac{\alpha_M^2}{2\pi} - b_M \frac{\alpha_M^3}{(2\pi)^2} - c_M \frac{\alpha_M^2 \alpha_N}{(2\pi)^2}, \quad (3)$$

where $\alpha_N = \frac{g_N^2}{4\pi}$ and $\alpha_M = \frac{g_M^2}{4\pi}$. The beta function coefficients for this theory can be found in Appendix A. Asymptotic freedom (AF) is dictated by the sign of the first coefficients a_N and a_M . In principle there are four possibilities:

 $a_N > 0$, $a_M > 0$, both couplings are asymptotically free (4)

 $a_N < 0$, $a_M > 0$, only α_M is asymptotically free (5)

 $a_N > 0$, $a_M < 0$, only α_N is asymptotically free (6)

$$a_N < 0, \quad a_M < 0, \quad \text{none are asymptotically free}$$
(7)

However the last situation where both gauge interactions are infrared free cannot be realized since the two conditions $a_N < 0$ and $a_M < 0$ imply $\frac{2}{11} > \frac{M}{N}$ and $\frac{M}{N} > \frac{11}{2}$ which cannot be simultaneously satisfied. Hence we are left with three regions in which either only one of the gauge interactions is asymptotically free or both are asymptotically free.

We plot these three regions in the (N, M) plane in Fig. 2. Region I (red) is bounded by $\frac{2}{11} < \frac{M}{N} < \frac{11}{2}$ while region II (blue) is bounded by $\frac{11}{2} < \frac{M}{N}$ and region III (green) is bounded by $\frac{M}{N} < \frac{2}{11}$. We will now discuss fixed points in each region.

Setting to zero the two coupled beta functions we find that there are nontrivial fixed points located at



FIG. 2. The three regions where (I) both couplings α_N and α_M are asymptotically free (red), (II) where α_M (α_N) is asymptotically (infrared) free (blue), (III) where α_N (α_M) is asymptotically (infrared) free (green). The hatched regions indicate where FP₁ (the upper region) and FP₂ (the lower region) are physical.

$$\alpha_N^* = -2\pi \frac{a_N}{b_N}, \qquad \alpha_M^* = 0 \qquad (\text{FP}_1) \qquad (8)$$

$$\alpha_N^* = 0, \qquad \alpha_M^* = -2\pi \frac{a_M}{b_M} \qquad (FP_2) \qquad (9)$$

$$\alpha_{N}^{*} = -2\pi \frac{a_{N}b_{M} - a_{M}c_{N}}{b_{N}b_{M} - c_{N}c_{M}},
\alpha_{M}^{*} = -2\pi \frac{a_{M}b_{N} - a_{N}c_{M}}{b_{N}b_{M} - c_{N}c_{M}}$$
(FP₃). (10)

In Appendix B we show that the third fixed point does not exist in any of regions I, II or III for this specific theory. Hence we are only left with the first and second nontrivial fixed points. If we switch off the gauge coupling α_N (α_M) then FP₂ (FP₁) is the infrared Banks-Zaks fixed point for α_M (α_N) provided that α_M (α_N) is asymptotically free. This, of course, also implies that they cannot be ultraviolet fixed points in regions II and III respectively and falls in line with the results of [23] where an ultraviolet fixed point is generated perturbatively if the theory contains scalars.

The phase diagram of this specific theory is therefore rather simple. There are no further nontrivial fixed points induced by the mixing of the gauge couplings. On the other hand we will show below that a theory where this is the case indeed does exist. In Fig. 2 we plot the phase diagram in the (N, M) plane. In order for the fixed points to be physically acceptable we demand perturbation theory to hold and therefore require $\alpha_N^* < 1$ and $\alpha_M^* < 1$. This is the hatched region in Fig. 2.



FIG. 3. The left (right) panel shows the phase diagram of the theory with N = 7 and M = 25 (N = 25 and M = 7) in the region where only FP₁ (FP₂) exists. The violet point is the ultraviolet free fixed point while the red point is FP₁ (left) or FP₂ (right).

We now discuss the stability of the two fixed points FP_1 and FP_2 by linearizing the flow around the fixed points and determining the eigenvalues of

$$M = \begin{pmatrix} \frac{\partial \beta_N}{\partial \alpha_N} & \frac{\partial \beta_N}{\partial \alpha_M} \\ \frac{\partial \beta_M}{\partial \alpha_N} & \frac{\partial \beta_M}{\partial \alpha_M} \end{pmatrix}_{|\alpha_N = \alpha_N^*, \alpha_M = \alpha_M^*}.$$
 (11)

If an eigenvalue is negative the fixed point is unstable (relevant direction) while if it is positive it is stable (along their associated eigendirections) and leads to an irrelevant direction. At the two fixed points FP_1 and FP_2 they are

Eigenvalues
$$(M_{\rm FP_1}) = \left(\frac{2(2M - 11N)^2N}{3(13MN^2 - 34N^3 - 3M)}, 0\right)$$
(12)

Eigenvalues
$$(M_{\rm FP_2}) = \left(0, \frac{2(11M - 2N)^2 M}{3(34M^3 + 3N - 13M^2N)}\right).$$
(13)

The nonzero eigenvalue of M_{FP_1} is positive for $\alpha_N^* < 1$ and the nonzero eigenvalue of M_{FP_2} is positive for $\alpha_M^* < 1$. Hence if the fixed points exist then they are attractive along one direction. The stability along the remaining eigendirection cannot be determined. The vanishing of one of the eigenvalues is easily understandable because it is the footprint of the Gaussian behavior with respect to that coupling direction. In Fig. 3 we plot the flow of the couplings for two sets of generic values of N and M. In both plots the trivial ultraviolet fixed point is marked violet. In the left plot the red point is FP₁ while in the right plot the red point is FP₂. We see that for all the asymptotically free trajectories the couplings will grow large as the infrared regime is approached. The only exception in the left (right) plot is the trajectory for which α_M (α_N) is switched off. Here α_N (α_M) just approaches the fixed point in the infrared. If the system evolves along one of the trajectories that come close to either of the two nontrivial fixed points the couplings will exhibit near scale invariant characteristics (i.e. walking) at intermediate scales before blowing up in the deep infrared.

Finally we observe that there is a rather special trajectory which flows directly out of FP₁ in the left plot and directly out of FP₂ in the right plot. In the left plot along this trajectory the fixed point FP₁ acts as a trivial ultraviolet fixed point for α_M but as a nontrivial ultraviolet fixed point for α_N . Hence the coupling α_M is asymptotically free but α_N is asymptotically safe. Such a trajectory is a *safety free* trajectory. The same occurs in the right plot but with the role of α_N and α_M switched.

III. GAUGING AN $SU(2)_L$ SUBGROUP ENRICHES THE PHASE DIAGRAM

We will now gauge only an $SU(2)_L$ subgroup of the $SU(M)_L$ group and remember that the original theory was composed of an SU(N) gauge group with M fundamental Weyl fermions q and M antifundamental fermions \tilde{q} . The original theory has an $SU(M)_L \times SU(M)_R \times U(1)_V$ anomaly free global symmetry. If we only gauge an $SU(2)_L$ subgroup of $SU(M)_L$ we do not run into the problem of inducing gauge anomalies as compared to the above case where we gauged the entire $SU(M)_L$ since all representations are now (pseudo)real. Therefore from the point of

	[SU(N)]	$[SU(2)_L]$	$SU(M-2)_L$	$SU(M)_R$	SU(N)	$U(1)_1$	$U(1)_{2}$	$U(1)_{3}$
\overline{q}			1	1	1	М	2-M	0
\hat{q}'		1		1	1	0	2	-M
$\tilde{\tilde{q}}$	$\overline{\Box}$	1	1	\Box	1	-2	0	M-2
L	1		1	1	Ō	-M	M-2	0

TABLE II. Matter content of the theory with an $SU(2)_L$ gauged subgroup.

view of gauge anomalies we do not need to add additional fermions to the theory.

In the Standard Model the gauge anomalies vanish only due to a nontrivial cancellation between the quark and lepton contributions. However in the Standard Model also an $U(1)_Y \subset SU(M)_R$ Abelian subgroup is gauged and this induces mixed anomalies that can only be canceled by the inclusion of leptons. This is not an issue here.

However since one of the gauge groups is now SU(2) we have to worry about the Witten topological anomaly [55]. So far the theory contains N Weyl doublets and hence only in the specific case where N is even does the Witten anomaly vanish. In general however we can cancel the Witten anomaly by including N lepton doublets such that the theory contains 2N doublets for any N. In Table II we summarize the matter content of the theory. Note that there are three Abelian anomaly free symmetries.

The evolution of the system is characterized to two loops by the coupled beta functions

$$\beta_N = -a_N \frac{\alpha_N^2}{2\pi} - b_N \frac{\alpha_N^3}{(2\pi)^2} - c_N \frac{\alpha_2 \alpha_N^2}{(2\pi)^2}$$
(14)

$$\beta_2 = -a_2 \frac{\alpha_2^2}{2\pi} - b_2 \frac{\alpha_2^3}{(2\pi)^2} - c_2 \frac{\alpha_N \alpha_2^2}{(2\pi)^2}$$
(15)

where the beta function coefficients can be found in Appendix C. Asymptotic freedom is dictated by the sign of the first coefficients a_N and a_2 . This gives four different possibilities

 $a_N > 0$, $a_2 > 0$, both couplings are asymptotically free

$$_N < 0, \quad a_2 > 0, \quad \text{only } \alpha_2 \text{ is asymptotically free (17)}$$

 $a_N > 0$, $a_2 < 0$, only α_N is asymptotically free (18)

$$a_N < 0$$
, $a_2 < 0$, none are asymptotically free. (19)

These four conditions map out four distinct regions in the (N, M) plane. We now proceed to study the fixed point structure of the theory. These are

$$\alpha_N^* = -2\pi \frac{a_N}{b_N}, \qquad \alpha_2^* = 0 \qquad (\text{FP}_1) \qquad (20)$$

$$\alpha_N^* = 0, \qquad \alpha_2^* = -2\pi \frac{a_2}{b_2}$$
 (FP₂) (21)

$$\alpha_N^* = -2\pi \frac{a_N b_2 - a_2 c_N}{b_N b_2 - c_N c_2},
\alpha_2^* = -2\pi \frac{a_2 b_N - a_N c_2}{b_N b_2 - c_N c_2}$$
(FP₃). (22)

Besides the trivial fixed point there are three nontrivial fixed points denoted by FP_1 , FP_2 and FP_3 . In terms of N and M they are

$$\alpha_N^* = -2\pi \frac{2(2MN - 11N^2)}{13MN^2 - 34N^3 - 3M}, \qquad \alpha_2^* = 0 \tag{FP}_1$$

$$\alpha_N^* = -2\pi \frac{4(98MN^2 + 2974N^2 + 198N - 539N^3 - 544MN)}{1274MN^3 + 18469N^3 + 1632M + 27N - 3332N^4 - 7072MN^2 - 294MN},$$

$$\alpha_2^* = -2\pi \frac{4(46MN^3 + 1496N^3 + 132M - 103N^4 - 572MN^2 - 33N^2 - 6MN)}{1274MN^3 + 18469N^3 + 1632M + 27N - 3332N^4 - 7072MN^2 - 294MN}.$$
(FP₃)
(25)

(16)

We discover four different regions bounded by



FIG. 4. The four regions where in region I (IV) both couplings are asymptotically (infrared) free and in region II (III) α_N (α_2) is infrared free while α_2 (α_N) is asymptotically free. The hatched regions mark where the fixed points FP₁ (horizontal lines), FP₂ (diagonal lines) and FP₃ (vertical lines) exist.

Region I,
$$N < 11, M < \frac{11}{2}N$$
, (26)

Region II,
$$N < 11, M > \frac{11}{2}N$$
, (27)

Region III,
$$N > 11, M < \frac{11}{2}N$$
, (28)

Region IV,
$$N > 11, M > \frac{11}{2}N.$$
 (29)

We will require that the values of the fixed points be larger than zero and less than unity $0 < \alpha_N^* < 1$ and $0 < \alpha_2^* < 1$. These requirements will be enforced on all three fixed point solutions FP₁, FP₂ and FP₃. In our analysis each fixed point can only be trusted if it is perturbative.

In Fig. 4 we plot all four regions together with the regions in which there exists either one, two or three fixed points. The black solid lines correspond to N = 11 and $M = \frac{11}{2}N$ and separate the four different regions. The orange lines separate the regions in which one, two or all three fixed points exist from the regions in which none exists. If some or all fixed points are physical the corresponding region is hatched. The theories that lie in a region which is either single, double or triple hatched possess either one, two or all three fixed points respectively. Finally the regions in which FP₁, FP₂ or FP₃ exists are marked with horizontal, diagonal or vertical lines respectively.

One should note that if the third nontrivial fixed point FP_3 exists then so does either FP_1 and FP_2 or only FP_1 . It never exists simultaneously with only FP_2 . The theories that can settle at FP_3 in the deep infrared are the ones lying in the upper right part of region I marked by the black diagonal line and the two orange curved lines in Fig. 4.

The fixed points can be either stable, unstable or metastable. Similar to the above we classify the fixed points according to their stability by linearizing the beta functions around the fixed points and study the eigenvalues of the matrix in (11) where β_M and α_M are now β_2 and α_2 .

Consider the first two fixed points FP₁ and FP₂. Here the stability matrix always has a zero as one of its eigenvalues. The other eigenvalue is always positive at both fixed points with eigenvector $(1,0)^T$ at FP₁ and eigenvector $(0,1)^T$ at FP₂. Therefore FP₁ is always attractive in the direction of α_N while FP₂ is always attractive in the direction of α_2 .

Consider now instead the third fixed point FP_3 . This is the nontrivial fixed point which emerges due to the mixing between the couplings in the beta functions. In the region where the fixed point values of the couplings are positive and less than unity both eigenvalues of the stability matrix are positive implying that the fixed point is infrared attractive from all directions.

There are special renormalization group lines connecting FP_3 to either FP_1 or FP_2 . Along these lines one of the two couplings moves from an infrared fixed point to a Gaussian (asymptotically free) fixed point while the other reaches an interacting ultraviolet (asymptotically safe) fixed point. These are lines of safety free theories. This new phase is an addition to the completely asymptotically free or safe case studied earlier in the literature.

It is illustrative to plot the flow of the couplings for a variety of different theories having either one, two or all three fixed points. As a first example we plot the flow for N = 9 and M = 40 in the left plot of Fig. 5. As can be seen the two fixed points FP₁ and FP₂ are both attractive along one direction. This is the eigendirection with positive eigenvalue of the stability matrix which is in the α_N direction for FP₁ and in the α_2 direction for FP₂. Along the other eigendirection with vanishing eigenvalue both fixed points are repulsive for positive values of the couplings. The eigenvalues at these two fixed points are

Eigenvalues
$$(M_{\rm FP_1}) = \left(\frac{2(2M - 11N)^2N}{3(13MN^2 - 34N^3 - 3M)}, 0\right),$$
(30)

Eigenvalues
$$(M_{\rm FP_2}) = \left(0, \frac{16(N-11)^2}{3(49N-272)}\right).$$
 (31)

There is also the nontrivial fixed point FP_3 which is infrared attractive in all directions. This is the fixed point to which the theory will finally settle in the deep infrared. The



FIG. 5. Coupling flows of theories in region I. Left: The N = 9 and M = 40 theory with fixed points FP₁, FP₂ and FP₃. Right: The N = 8 and M = 30 theory with fixed points FP₁ and FP₃.



FIG. 6. Coupling flows of theories in region I. Left: The N = 6, M = 24 theory with fixed point FP₁. Center: The N = 9, M = 18 theory with fixed point FP₂. Right: The N = 10 and M = 34 theory with fixed points FP₁ and FP₂.

eigenvalues at this fixed point are given below but due to the complicated nature of this eigenvalue in terms of N and M we only show the numerical values with N = 9 and M = 40,

Eigenvalues
$$(M_{\rm FP_3}) = (0.0622, 0.1243).$$
 (32)

The second example is the flow of the theory with N = 8and M = 30. This is the right plot of Fig. 5 and corresponds to the region with double hatched vertical and horizontal lines of Fig. 4. In this theory the fixed points FP₁ and FP₃ exist simultaneously. The flow of the theory for these two fixed points is similar to the flow of the fixed points in the example above; i.e. FP₁ is attractive along one direction and repulsive in the other direction while FP₃ is attractive along all directions. It is important to note that the fixed point FP₂ exists along the direction of α_2 . However we will disregard this fixed point since here the value of the coupling is larger than one. Thus along the trajectories for which the couplings stay below unity the theory will settle at FP₃ in the deep infrared.

The third and fourth examples are of theories where only FP₁ or FP₂ exists. These are the single hatched parts of region I. In Fig. 6 the left plot corresponds to an N = 6 and M = 24 theory with fixed point FP₁ and the center plot is an N = 9 and M = 18 theory with fixed point FP₂. In both of these examples the fixed point FP₁ (FP₂) is attractive along one direction α_N (α_2) and repulsive along the other.

The fifth example is of an N = 10 and M = 34 theory where both fixed points FP₁ and FP₂ exist simultaneously. This is the double hatched part of region I with horizontal and diagonal lines. The fixed point FP₁ (FP₂) is attractive



FIG. 7. Coupling flows of theories in regions II and III. Left: The N = 10 and M = 60 theory with fixed point FP₂. Right: The N = 13 and M = 50 theory with fixed point FP₂.

along the associated eigendirection of α_N (α_2) while being repulsive along the other. Furthermore the fixed point FP₁ is attractive along directions of positive α_2 but repulsive in the direction of negative α_2 . This is of course highly unstable and as the theory eventually flows to negative values of α_2 it is ill defined.

Region II is bounded by N < 11 and $M > \frac{11}{2}N$. Similar to the analysis of region I we shall demand that the fixed points are positive and less than unity in order to trust perturbation theory. In region II the coupling α_2 is asymptotically free while α_N is not. If the first fixed point exists then it must be a UV fixed point. However since the value of α_N^* is negative this does not occur. There is a region in which the second fixed point FP₂ exists. At this point one of the eigenvalues of the stability matrix vanishes while the other is nonzero. Lastly we remark that the third fixed point FP₃ is not positive and hence does exist anywhere in region II.

In the left plot of Fig. 7 we show the flow of the couplings for N = 10 and M = 60. In the far UV the theory sits in the lower right corner of the flow. As the theory evolves towards the IR it develops a perturbative IR fixed point. As can be seen the fixed point happens to be attractive along all the directions of positive values of the couplings and not just along the eigendirection of the nonvanishing eigenvalue.

Similar results exist for region III which is bounded by N > 11 and $M < \frac{11}{2}N$. Here the first fixed point does not exist while the second fixed point exists and is attractive along a single direction. The third fixed point does not exist anywhere in region III. We plot this in the right plot of Fig. 7.

Region IV is bounded by N > 11 and $M > \frac{11}{2}N$. Here none of the couplings is asymptotically free.

IV. SKETCHING OUT PHENOMENOLOGICAL IMPLICATIONS

To better elucidate the plethora of interesting critical phenomena that emerge from our analysis when considering semisimple gauge groups we first provide the actual runnings of the gauge couplings for selected interesting renormalization group trajectories. We then motivate how the unveiled phenomena can spur new ideas for phenomenological applications or be embedded within earlier paradigms.

Let us start with displaying the running in Fig. 8 for a trajectory very close to the fixed point in the left plot of Fig. 2. This running shows α_N and α_M starting in the UV as asymptotically free and then approaching (but not reaching) the fixed point on the α_N axis. Here at intermediate scales both couplings run very slowly and are near conformal; i.e. they walk [58–62] (see [63] for a review). As the deep IR is approached the couplings again grow to larger values. We have, therefore, uncovered an independent mechanism to generate walking theories that uses the gauging of part of their flavor symmetries. Given that the Standard Model is



FIG. 8. The running of α_N and α_M in the $SU(N) \times SU(M)_L$ theory for N = 7 and M = 25 along a trajectory in the left plot of Fig. 3 close to the fixed point on the α_N axis.



FIG. 9. The running of the coupling of α_N and α_2 in the $SU(N) \times SU(2)_L$ theory for N = 9 and M = 40 along a trajectory in the left plot of Fig. 5 from the UV Gaussian fixed point to the central infrared interacting fixed point.

already a semisimple gauge group we find this way of constructing walking theories among the most natural ways explored so far in the literature.

We also show in Fig. 9 the running of the couplings in the $SU(N) \times SU(2)_L$ for N = 9 and M = 40 along a trajectory where both couplings are asymptotically free and simultaneously reach the central interacting IR fixed point of the left plot in Fig. 5.

We now move on to another interesting scenario shown in Fig. 10. Here we observe the phenomenon of *safety free* behavior according to which one coupling is asymptotically safe and the other is asymptotically free while in the infrared they both reach the infrared fixed point. This is only possible in the $SU(N) \times SU(2)_L$ gauge theory scenario. This scenario is quite different from the case of either complete asymptotic freedom or safety. Nevertheless the theory is still well defined both in the UV and in the IR, opening the door to new ways of constructing extensions of the Standard Model where some gauge interactions are asymptotically free and others are asymptotically safe. It is worth stressing that these new phases are possible because of the interplay of two gauge sectors.

An intriguing behavior is the one shown in the left panel of Fig. 11. Here one observes that the interaction strength of one of the two couplings reaches a maximum at some intermediate energy scale while decreasing both in the UV and the IR. This peculiar behavior might turn out to be useful when discussing dark matter properties because if it is realized it could help alleviate phenomenological



FIG. 10. Left: The running of α_N and α_2 in the $SU(N) \times SU(2)_L$ theory for N = 9 and M = 40 along the fine-tuned safety free trajectory from the central infrared fixed point to the ultraviolet fixed point on the α_2 axis. Right: The running of α_N and α_2 of the same theory but now along the fine-tuned safety free trajectory from the central infrared fixed point to the ultraviolet fixed point on the α_N axis.



FIG. 11. Left: The running of α_N and α_2 in the $SU(N) \times SU(2)_L$ theory for N = 9 and M = 40 along a trajectory in the left plot of Fig. 5. The flow starts off near the Gaussian ultraviolet fixed point and then flows close to the interacting fixed point along the α_2 axis but settling at the central infrared interacting fixed point. Right: The running of α_N and α_2 of the same theory but now along a trajectory flowing from the Gaussian ultraviolet fixed point, then flowing close to the infrared interacting fixed point along the α_N axis but settling at the central infrared interacting fixed point.

constraints on symmetric dark matter models along the lines suggested in [64]. The right panel of Fig. 11 displays a theory where both couplings are asymptotically free and both reach an interacting IR fixed point.

Although our analysis seems to suggest that the above phases only emerge when both N and M are considerably large (making realistic model building difficult) we stress that our analysis is bounded by perturbation theory.

Consider supersymmetric QCD with a single SU(N)gauge group and with a variable number of superflavors in the fundamental representation for which the exact conformal window was derived by Seiberg in [2]. Here the lower bound of the conformal window extends well below the boundary set by the value of the coupling constant at the two-loop infrared fixed point reaching unity [3]. In fact the conformal window extends just below the point where the coupling constant at two loops has become arbitrarily large and where one would have naively guessed that no infrared fixed points could have existed. If this is a generic feature then the regions in Fig. 4 bounded by the solid black and orange curves become larger. Specifically the solid orange lines are shifted towards smaller values of N and M. In other words taking into account nonperturbative effects is likely to enlarge the regions in which the above phases exist proving a model template which is more realistic.

Lastly we point out that it is likely that once scalars are included the emerging phase diagram will be more involved. For instance the UV fixed point uncovered in [23] for a specific simple gauge-Yukawa theory can only be seen once the effects from all the couplings in the theory are included. To study the inclusion of Yukawa and scalar selfcouplings in a semisimple gauge-Yukawa theory is the next appropriate step we believe one should take.

V. CONCLUSIONS

We investigated the quantum critical behavior of relevant classes of semisimple fermionic gauge theories resembling the Standard Model. In particular we studied an SU(N)gauge theory with M Dirac flavors where we first gauged an $SU(M)_L$ and then an $SU(2)_L \subset SU(M)_L$ of the original global symmetry $SU(M)_L \times SU(M)_R \times U(1)$ of the theory. Leptonlike particles were added to avoid gauge anomalies. We showed that at the two-loop level an intriguing phase diagram appears. New phases emerged in which one, two or three fixed points have been shown to exist. We also unveiled a phase featuring complete asymptotic freedom and simultaneously an interacting infrared fixed point in both couplings. Intriguingly the analysis further revealed the existence of special renormalization group trajectories along which one coupling displays asymptotic freedom and the other asymptotic safety, while both flow in the infrared to an interacting fixed point. These are the "safety free" trajectories. We further discussed the associated renormalization group flow of the coupling constants. The knowledge of the quantum critical behavior of these types of theories is useful information when constructing beyond Standard Model scenarios. For instance we discovered a new genuine way of producing near-conformal (i.e. walking) theories. This interesting scenario emerges due to a nontrivial interplay between the different gauge couplings and is seen specifically for semisimple gauge theories.

ACKNOWLEDGMENTS

We thank Esben Mølgaard for discussions. This work is partially supported by the Danish National Research Foundation Grant No. DNRF:90.

APPENDIX A: BETA FUNCTION COEFFICIENTS FOR $SU(N) \times SU(M)_{I}$

We here provide the beta function coefficients for the $SU(N) \times SU(M)_L$ theory studied in Sec. II up to two loops:

$$a_N = \frac{11}{3}N - \frac{2}{3}M,$$
 (A1)

$$b_N = \frac{17}{3}N^2 - \frac{5}{3}NM - \left(\frac{N^2 - 1}{2N}\right)M,$$
 (A2)

$$c_N = -\frac{1}{4}(M^2 - 1),$$
 (A3)

$$a_M = \frac{11}{3}M - \frac{2}{3}N,$$
 (A4)

$$b_M = \frac{17}{3}M^2 - \frac{5}{3}NM - \left(\frac{M^2 - 1}{2M}\right)N,$$
 (A5)

$$c_M = -\frac{1}{4}(N^2 - 1). \tag{A6}$$

APPENDIX B: FIXED POINTS IN $SU(N) \times SU(M)_L$

Here we show that the third fixed point FP₃ does not exist for the $SU(N) \times SU(M)_L$ theory. Consider first the one-loop coefficient for a simple gauge group theory and denote it by *a*. We will imagine tuning the number of Weyl fermions N_w around the critical point where asymptotic freedom is lost/gained by parametrizing small departures away from a = 0 via

$$N_w = \frac{11}{2} \frac{C_2(G)}{T(r)} - \epsilon.$$
(B1)

The choice $\epsilon = 0$ corresponds to a = 0. If $\epsilon > 0$ the theory is asymptotically free and if $\epsilon < 0$ the theory is infrared free. Here we assume that N_w can take continuous values which then implies that ϵ is a continuous parameter. This is only for pedagogical reasons and we will later use integer numbers for the matter fields. Now we write the two-loop coefficient b in terms of the parameter ϵ :

$$b = -\frac{7}{2}C_2(G)^2 - \frac{11}{2}C_2(G)C_2(r) + \left(\frac{5}{3}C_2(G) + C_2(r)\right)T(r)\epsilon.$$
 (B2)

We observe that for an infrared free theory, where $\epsilon < 0$, the two-loop coefficient will always be negative. On the other hand for an asymptotically free theory, where $\epsilon > 0$, there exists a region in which *b* is also negative. This can happen since ϵ can be tuned arbitrary small. In this limit the system then has a fixed point which is the usual Banks-Zaks fixed point.

We will now proceed to discuss the $SU(N) \times SU(M)$ theory. Here the fixed point, FP₃, induced by the mixing of the semisimple gauge group structure is in terms of the beta function coefficients given by

$$\alpha_{N}^{*} = -2\pi \frac{a_{N}b_{M} - a_{M}c_{N}}{b_{N}b_{M} - c_{N}c_{M}}, \qquad \alpha_{M}^{*} = -2\pi \frac{a_{M}b_{N} - a_{N}c_{M}}{b_{N}b_{M} - c_{N}c_{M}}.$$
(B3)

Lastly we observe that the mixing coefficients c_N and c_M are always negative.

We will start by discussing the case of regions II and III where either SU(N) or SU(M) is infrared free and then proceed to region I where both groups are asymptotically free.

1. Regions II and III

In these two regions we have either $a_N < 0$ and $a_M > 0$ (region II) or $a_N > 0$ and $a_M < 0$ (region III).

We will first consider region II. From the above considerations we know that $b_N < 0$ since $a_N < 0$ and that b_M can be either positive of negative. Furthermore $c_N < 0$ and $c_M < 0$. Thus the fixed point FP₃ can be written as

$$\begin{aligned} \alpha_N^* &= -2\pi \frac{-|a_N|b_M + |a_M c_N|}{-|b_N|b_M - |c_N c_M|}, \\ \alpha_M^* &= -2\pi \frac{-|a_M b_N| - |a_N c_M|}{-|b_N|b_M - |c_N c_M|}. \end{aligned} \tag{B4}$$

Recall that we require $\alpha_N^* > 0$ and $\alpha_M^* > 0$ for the fixed point to be physical. We now observe that if $b_M \ge 0$ then α_M^* will be overall negative and thus nonphysical. If $b_M < 0$ then for α_N^* to be positive $|b_N b_M| < |c_N c_M|$ has to hold while for α_M^* to be positive $|b_N b_M| > |c_N c_M|$ must be satisfied. This clearly can never happen and the fixed point cannot exist. By the same logic we can reach a similar conclusion for region III. It should then be clear that FP₃ fails to exist in regions II and III.

2. Region I

In region I we have $a_N > 0$ and $a_M > 0$ together with $c_N < 0$ and $c_M < 0$. Whether FP₃ is physical depends then on the sign of the remaining coefficients b_N and b_M . Since both b_N and b_M can be either positive or negative our first task is to find for what specific values of N and M they change sign. Both coefficients are given in Appendix A and the sign is dictated by

$$b_N < 0: \ \frac{34N^2}{13N^2 - 3} < \frac{M}{N} < \frac{11}{2},$$
 (B5)

$$b_N > 0: \ \frac{34N^2}{13N^2 - 3} > \frac{M}{N} > \frac{2}{11},$$
 (B6)

$$b_M < 0: \left(\frac{34M^2}{13M^2 - 3}\right)^{-1} > \frac{M}{N} > \frac{2}{11},$$
 (B7)

$$b_M > 0: \left(\frac{34M^2}{13M^2 - 3}\right)^{-1} < \frac{M}{N} < \frac{11}{2}.$$
 (B8)

First it is easy to check that the situation with $b_N < 0$ and $b_M < 0$ cannot be realized. Hence there are in principle three possibilities for the fixed point to be physical: If $b_N > 0$ and $b_M < 0$ then $|a_M c_N| > |a_N b_M|$ must be satisfied, if $b_N > 0$ and $b_M > 0$ then $|c_N c_M| > |b_N b_M|$ must be satisfied and if $b_N < 0$ and $b_M > 0$ then $|a_N c_M| > |b_N b_M|$ must be satisfied and if $b_N < 0$ and $b_M > 0$ then $|a_N c_M| > |a_M b_N|$ must be satisfied. However one can check by explicit use of the beta function coefficients that this can never occur. Hence the third fixed point solution FP₃ does not exist in regions I, II and III.

APPENDIX C: BETA FUNCTION COEFFICIENTS FOR $SU(N) \times SU(2)_L$

We here provide the beta function coefficients for the $SU(N) \times SU(2)_L$ theory studied in Sec. III up to two loops:

$$a_N = \frac{11}{3}N - \frac{2}{3}M,$$
 (C1)

$$b_N = \frac{17}{3}N^2 - \left(\frac{5}{3}N + \frac{N^2 - 1}{2N}\right)M,$$
 (C2)

$$c_N = -\frac{3}{4},\tag{C3}$$

$$a_2 = \frac{22}{3} - \frac{2}{3}N,$$
 (C4)

$$b_2 = \frac{68}{3} - \frac{49}{12}N,\tag{C5}$$

$$c_2 = \frac{1 - N^2}{4}.$$
 (C6)

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