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Solar System tests of a scalar-tensor gravity with a general potential: Insensitivity of light deflection and Cassini tracking

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In the work of Hohmann *et al.* [Phys. Rev. D 88, 084054 (2013)], the authors worked out the parametrized post-Newtonian (PPN) parameters γ and β of a scalar-tensor theory with an arbitrary coupling function and a generic potential, and they found that these two PPN parameters depend on the radial distance r from the Sun, $\gamma(r)$ and $\beta(r)$. Based on the assumption that measurements on the PPN parameters can be characterized by the shortest distance to the Sun, the authors obtained their best constraints on the model parameters of the scalar-tenor theory by light deflection observation and the Cassini tracking experiment. However, as the authors stated, this approach might not be rigorous. In the present work, we physically model astronomical observations and physical experiments by calculating the null and timelike geodesics in the scalar-tensor theory. We show that, contrary to the results in the previous work, the light deflection and the Cassini tracking cannot distinguish the scalar-tensor theory from general relativity. We also investigate the additional advances in perihelia caused by the largest correction of the scalar field on the Newtonian potential. Since this correction has a Yukawa-like form, we obtain very much improved lower bounds on the model parameters by using current upper limits on the Yukawa parameters.

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I. INTRODUCTION

Scalar-tensor theories of gravitation (see e.g. Ref. [1] for a review) have received much attention in the last few decades, while Einstein's general relativity (GR) has achieved great success in astronomical observations and physical experiments [2–4]. The simplest way to extend GR is by introducing a scalar field Ψ in addition to the metric tensor $g_{\mu\nu}$ [5,6], in order to find a gravitational theory consistent with quantum theory and to look for an explanation for the accelerated expansion of the Universe (e.g., Refs. [7,8]). The action of a generic scalar-tensor theory can contain two arbitrary functions: the coupling function $\omega(\Psi)$ in the kinetic term of the scalar field and the potential $V(\Psi)$ [9–14].

One of the most important research aspects of the scalar-tensor theory is to test it with observations and experiments on various scales of length. In this work, we focus on the Solar System tests. A theoretical framework for such a purpose is the parametrized post-Newtonian (PPN) formalism [2,3], in which values of the PPN parameters represent possible deviations from GR. The scalar-tensor theory has two nonzero PPN parameters, γ and β, which were calculated in some variants (e.g. Refs. [10,15–22]).

Considering a scalar-tensor theory with the coupling function $\omega(\Psi)$ and the potential $V(\Psi)$, the authors of Refs. [21,22] found that its PPN parameters γ and β depend not only on the model parameters of the theory but also on the radial distance r from the Sun, i.e., $\gamma(r)$ and $\beta(r)$. They assumed that the measurements on the standard PPN γ and β can be characterized by the shortest distance to the Sun r_0, and then they estimated the bounds on the model parameters at some specific r_0 by using the measured values of the PPN parameters given by the light deflection observation [23], the Cassini tracking experiment [24], the perihelion precession of Mercury [3] and the lunar laser ranging experiments [25]. However, as the authors stated in Ref. [21], this approach might not be rigorous.

In the present work, we will investigate the Solar System tests of such a generic scalar-tensor theory with a more rigorous methodology. The observations and experiments will be physically modeled: the null geodesic of a light ray is calculated, and the leading contribution of the scalar field on the timelike geodesic of a planet around the Sun is studied. Contrary to the statements of Ref. [21], we find that the observation of light deflection by the Sun and the Cassini tracking experiment cannot distinguish the scalar-tensor theory from GR. Due to the existence of the potential of the scalar field, Newton's inverse-square law has a Yukawa-like correction, which can induce an additional

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periastron advance of a planet. Using the bounds on the Yukawa correction obtained in Ref. [26], we find the constraints on the scalar-tensor theory, which are very much improved over those of Ref. [21].

The rest of the paper is organized as follows. Section II is devoted to briefly reviewing the scalar-tensor theory and its static spherically symmetric solution for completeness. In Sec. III, we calculate the null geodesic of a light ray, which is used to model the light deflection and the Cassini tracking. We investigate the Yukawa-like correction on the timelike geodesics of a planet and constrain the scalar-tensor theory with the available bounds on Newton's inverse-square law in the Solar System in Sec. IV. Finally, in Sec. V, we summarize our results and discuss their implication.

II. THE SCALAR-TENSOR THEORY AND ITS STATIC SPHERICALLY SYMMETRIC SOLUTION

Following the work of Refs. [21,22], we consider a scalar-tensor theory with an arbitrary coupling function $\omega(\Psi)$ and a generic potential $V(\Psi)$, where Ψ is a dynamical scalar field. Its action is defined as

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[\Psi R - \frac{\omega(\Psi)}{\Psi}\partial_\rho\Psi\partial^\rho\Psi - 2\kappa^2 V(\Psi)\right] + S_m[g_{\mu\nu},\chi_m], \tag{1}$$

where we adopt the units $c = 1$ and $\hbar = 1$ and κ^2 is connected to the Newtonian gravitational constant G_N by $\kappa^2 = 8\pi G_N$. $S_m[g_{\mu\nu},\chi_m]$ represents the matter's contribution and χ_m denotes all of the matter fields.

In order to describe astronomical observations and physical experiments in the Solar System, a static spherically symmetric solution of the scalar theory of Eq. (1) in the post-Newtonian approximation is worked out [21,22]. Under isotropic spherical coordinates, the metric has the form

$$\mathrm{d}s^2 = -A(r)^2\mathrm{d}t^2 + B(r)^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2), \tag{2}$$

where r is the radial distance from the origin and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$. The coefficients in Eq. (2) are found as [21]

$$A(r)^2 = 1 - 2\mathcal{U}(r) + \mathcal{O}(\mathcal{U}^2), \tag{3}$$

$$B(r)^2 = 1 + 2\gamma(r)\mathcal{U}(r) + \mathcal{O}(\mathcal{U}^2), \tag{4}$$

where the effective gravitational potential $\mathcal{U}(r)$ and the PPN parameter $\gamma(r)$ are [21]

$$\mathcal{U}(r) = \frac{\mathcal{G}M}{r}\left(1 + \frac{\mathrm{e}^{-m_\psi r}}{2\omega_0 + 3}\right), \tag{5}$$

$$\gamma(r) = \frac{2\omega_0 + 3 - \mathrm{e}^{-m_\psi r}}{2\omega_0 + 3 + \mathrm{e}^{-m_\psi r}}. \tag{6}$$

Here, the effective gravitational constant $\mathcal{G} \equiv G_N/\Psi_0$ and Ψ_0 is the background value of Ψ; ω_0 is the lowest-order value in the Taylor expansion of $\omega(\Psi)$ around Ψ_0; m_ψ is the mass of the scalar field and it reads as

$$m_\psi = 2\kappa\sqrt{\frac{\Psi_0 V_2}{2\omega_0 + 3}}, \tag{7}$$

in which V_2 is the coefficient of the second-order term in the Taylor expansion of $V(\Psi)$ around Ψ_0 (see Ref. [21] for more details). Although higher-order terms of $\mathcal{O}(\mathcal{U}^2)$ are cut off in the metric (2), it is adequate for modeling observations and experiments based on light propagation, e.g. light deflection and the Cassini tracking.

It is worth mentioning that the dependence of the radial distance r makes the parameter $\gamma(r)$ of Eq. (6) beyond the standard framework of the PPN formalism [2,3], in which γ only depends on some model parameters of gravitational theories but not on r. It implies that measurements on the standard PPN γ need to be taken with caution for obtaining constraints on the scalar-tensor theory of Eq. (1). In the work of Ref. [21], the authors assumed the measurements on the standard PPN γ can be characterized by the shortest distance to the Sun r_0, and they estimated the bounds on ω_0 and $m \equiv 2\kappa\sqrt{\Psi_0 V_2}$ at some specific r_0 by using the measured values of the standard PPN γ given by the light deflection observation [23] and the Cassini tracking experiment [24]. However, this method for obtaining the constraints does not model the observation and the experiment in the scalar-tensor theory of Eq. (1). As claimed by the authors in Ref. [21], a rigorous treatment requires a calculation of null geodesics in the Solar System, which will be done in the next section in the present work. We will show that when the null geodesics of light are fully taken into account, the light deflection observation and the Cassini tracking experiment become insensitive to the scalar-tensor theory of Eq. (1) and their outcomes are the same as the prediction by GR. It can be easily checked that, for any $r > 0$,

$$[\gamma(r) + 1]\mathcal{U}(r) = 2\frac{\mathcal{G}M}{r}, \tag{8}$$

which plays a critical role for hiding the scalar field's effects.

III. NULL GEODESICS: MODELS OF LIGHT DEFLECTION AND CASSINI TRACKING

The null geodesic of a photon in the spacetime of Eq. (2) can be expressed as

$$-A(r)^2\left(\frac{\mathrm{d}t}{\mathrm{d}\lambda}\right)^2+B(r)^2\left[\left(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\right)^2+r^2\left(\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}\right)^2\right]=0, \quad (9)$$

where λ is the affine parameter. Since the gravitational field is isotropic, we can consider the orbit of the photon to be confined to the equatorial plane which is $\theta=\pi/2$ (e.g. Ref. [27]). It has two conserved quantities along the light trajectory:

$$E=A(r)^2\frac{\mathrm{d}t}{\mathrm{d}\lambda}, \qquad L=B(r)^2r^2\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}. \quad (10)$$

A. Light deflection

Using Eqs. (9) and (10), we can have the relation between ϕ and r along the light ray as

$$\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{1}{r^2}\frac{A(r)}{B(r)}\left[\frac{1}{b^2}-\frac{A(r)^2}{B(r)^2r^2}\right]^{-1/2}, \quad (11)$$

where $b\equiv L/E$ and the solution with a minus sign is ignored. For the closest approach d, $\mathrm{d}r/\mathrm{d}\phi=0$ leads to

$$b=\frac{B(d)d}{A(d)}. \quad (12)$$

With Eqs. (3) and (4), Eq. (11) can be worked out as

$$\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{d}{r\sqrt{r^2-d^2}}+\frac{dr}{(r^2-d^2)^{3/2}}\{[\gamma(d)+1]\mathcal{U}(d) - [\gamma(r)+1]\mathcal{U}(r)\}+\mathcal{O}(\mathcal{U}^2). \quad (13)$$

With the help of Eq. (8), we can further simplify the above equation as

$$\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{d}{r\sqrt{r^2-d^2}}+\frac{2\mathcal{G}M(r-d)}{(r^2-d^2)^{3/2}}+\mathcal{O}(\mathcal{G}^2). \quad (14)$$

It is obvious that Eq. (14) does not depend on any model parameters of the scalar-tensor theory of Eq. (1) and it predicts the light deflection angle as

$$\Delta\phi=2\int_d^\infty\frac{\mathrm{d}\phi}{\mathrm{d}r}\mathrm{d}r-\pi=4\frac{\mathcal{G}M}{d}+\mathcal{O}(\mathcal{G}^2), \quad (15)$$

which is the same as the one given by GR at the post-Newtonian order in the isotropic coordinates [e.g., see Eq. (6) of Ref. [28]]. It means that observations of the light deflection caused by the Sun cannot distinguish the scalar-tensor theory of Eq. (1) from GR at the post-Newtonian order.

B. Cassini tracking

For convenience, we change the isotropic spherical coordinates in the metric (2) to the isotropic Cartesian coordinates, which leads to

$$\mathrm{d}s^2=-A(r)^2\mathrm{d}t^2+B(r)^2(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2), \quad (16)$$

where $r^2=x^2+y^2+z^2$. We consider a light ray passing the Sun with the closest approach d. In the first-order approximation, its path can be given by $y=d$, $z=0$ and $r=\sqrt{x^2+d^2}$ (see Chap. 11.7 of Ref. [29]). Because of $\mathrm{d}s^2=0$ for a light ray, we find

$$\mathrm{d}t=\frac{B(r)}{A(r)}\mathrm{d}x=\{1+[\gamma(r)+1]\mathcal{U}(r)+\mathcal{O}(\mathcal{U}^2)\}\mathrm{d}x. \quad (17)$$

Again, with the help of Eq. (8), we can simplify the above equation as

$$\mathrm{d}t=\left[1+2\frac{\mathcal{G}M}{\sqrt{x^2+d^2}}+\mathcal{O}(\mathcal{G}^2)\right]\mathrm{d}x, \quad (18)$$

which also does not depend on any model parameters of the scalar-tensor theory of Eq. (1) and predicts the gravitational time delay between $x=0$ and $x=X$ as

$$t(X,d)=\int_0^X\frac{\mathrm{d}t}{\mathrm{d}x}\mathrm{d}x = X+2\mathcal{G}M\ln\left(\frac{X+\sqrt{X^2+d^2}}{d}\right)+\mathcal{O}(\mathcal{G}^2). \quad (19)$$

It is the same as the one given by GR in the isotropic coordinates (see Chap. 11.7 of Ref. [29]), which means that measurements on the gravitational time delay caused by the Sun cannot distinguish the scalar-tensor theory of Eq. (1) from GR at the post-Newtonian order. By substituting $X=\sqrt{r^2-d^2}$, Eq. (19) can be rewritten as

$$t(r,d)=\sqrt{r^2-d^2}+2\mathcal{G}M\ln\left(\frac{r+\sqrt{r^2-d^2}}{d}\right)+\mathcal{O}(\mathcal{G}^2). \quad (20)$$

In the case of superior conjunction (SC), when the receiver is on the opposite side of the Sun as seen from the emitter, by making use of conditions $r_{\mathrm{E}}\gg d$ and $r_{\mathrm{R}}\gg d$, where r_{E} is the distance between the emitter and the Sun and r_{R} is the distance between the reflector and the Sun, we have the time duration of light propagation as

$$\Delta t_{\mathrm{SC}} = 2t(r_{\mathrm{E}}, d) + 2t(r_{\mathrm{R}}, d) = 2(r_{\mathrm{E}} + r_{\mathrm{R}}) + 4\mathcal{G}M \ln\frac{4r_{\mathrm{E}}r_{\mathrm{R}}}{d^2}. \tag{21}$$

In the Cassini tracking experiment [24], what was measured was not the time delay but the relative change in the frequency. Around SC, a ground station transmitted a radio-wave signal with frequency ν_0 to the spacecraft. This signal was coherently transponded by the spacecraft and sent back to the Earth. The two-way fractional frequency fluctuation is

$$y_{\mathrm{SC}} = \frac{\nu(t) - \nu_0}{\nu_0} = \frac{\mathrm{d}\Delta t_{\mathrm{SC}}}{\mathrm{d}t} = -8\frac{\mathcal{G}M}{d}\frac{\mathrm{d}}{\mathrm{d}t}d(t), \tag{22}$$

where Eq. (21) is used and $\mathrm{d}d(t)/\mathrm{d}t$ is approximately equal to the orbital velocity of the Earth, $v_\oplus$, during the experiment. Equation (22) is also exactly the same as the prediction by GR [24,30], which shows that, like the light deflection, the measurements of the Cassini tracking experiment also cannot be taken to constrain the scalar-tensor theory of Eq. (1).

In summary, the observations of light deflection and the Cassini tracking experiment are insensitive to the scalar-tensor theory of Eq. (1) and their outcomes are the same as those of GR. In the next section, we will calculate the timelike geodesics to find bounds on its parameters.

IV. TIMELIKE GEODESIC: ADVANCE OF PERIASTRON

The metric of Eq. (2) is not sufficient for calculating the full post-Newtonian timelike geodesics of a planet around the Sun due to the absence of $\mathcal{O}(\mathcal{U}^2)$ terms, which can be found in Refs. [21,22]. However, we still can obtain the largest contribution of the scalar field in the action of Eq. (1) in the planetary motion by its correction to the Newtonian potential. The effective gravitational potential at the Newtonian limit $\mathcal{U}(r)$ has the Newtonian gravitational potential with an additional Yukawa-like correction, i.e.

$$\mathcal{U}(r) = \mathcal{U}_{\mathrm{N}}(r) + \mathcal{U}_{\mathrm{YK}}(r), \tag{23}$$

where

$$\mathcal{U}_{\mathrm{N}}(r) = \frac{\mathcal{G}M}{r}, \tag{24}$$

$$\mathcal{U}_{\mathrm{YK}}(r) = \alpha\frac{\mathcal{G}M}{r}\exp\left(-\frac{r}{\lambda}\right). \tag{25}$$

In order to connect our results with tests of the Yukawa-like correction (see Ref. [31] for a review), we adopt more conventional notations α and λ in the above equation: α is a dimensionless strength parameter and λ is a length scale for the Yukawa-like correction. It is easy to find that

$$\alpha = \frac{1}{2\omega_0 + 3}, \qquad \lambda = \frac{1}{m_\psi}. \tag{26}$$

It is well known that such a Yukawa-like correction can induce an additional advance in the perihelion of a planet [32,33]

$$\dot{\omega}_{\mathrm{YK}} = \alpha\frac{n_{\mathrm{P}}a_{\mathrm{P}}\sqrt{1 - e_{\mathrm{P}}^2}}{e_{\mathrm{P}}\lambda}\exp\left(-\frac{a_{\mathrm{P}}}{\lambda}\right)I_1\left(\frac{a_{\mathrm{P}}e_{\mathrm{P}}}{\lambda}\right), \tag{27}$$

where a_{P} is the semimajor axis of the planet, e_{P} is the eccentricity, ω_{P} is the argument of periastron and n_{P} is the Keplerian mean motion; $I_1(z) = \mathrm{d}I_0(z)/\mathrm{d}z$ and $I_0(z)$ is the modified Bessel function of the first kind [34]. The accurate ephemerides INPOP10a [35] and EPM2011 [36] were recently adopted in planetary science [37,38] and in detecting gravitational effects and testing modified theories of gravity [39–50]. Using the supplementary advances in the perihelia provided by INPOP10a and EPM2011, the authors of Ref. [26] found the upper bounds on the Yukawa parameters as $\alpha \lesssim 4 \times 10^{-11}$ and $\lambda \lesssim 0.2$ au [51]. It gives the values on ω_0 and m as

$$\omega_0 \gtrsim 10^{10}, \qquad m \gtrsim 8 \times 10^5 m_{\mathrm{au}}, \tag{28}$$

where m is measured in inverse astronomical units $m_{\mathrm{au}} = 1\ \mathrm{au}^{-1}$ [21]. These lower bounds are very much improved over those given by Ref. [21].

V. CONCLUSIONS AND DISCUSSION

We investigated the Solar System tests of the scalar-tensor theory with an arbitrary coupling function and a generic potential with a rigorous methodology. Unlike the work of Ref. [21] based on the assumption that the measurements on the standard PPN parameters can be characterized by the shortest distance to the Sun, we physically modeled the light deflection, the Cassini tracking and the additional advances in perihelia of the Solar System's planets by calculating the null and timelike geodesics.

Contrary to the statements of Ref. [21], we found that the light deflection and the Cassini tracking cannot distinguish the scalar-tensor theory from GR. However, the additional advances in perihelia, which are induced by the largest correction of the scalar field to the Newtonian potential, can provide improved lower bounds on the model parameters.

It demonstrates that using the measured values of the PPN parameters to constrain the model parameters of a gravitational theory should be treated with caution. In fact, it was already pointed out in Ref. [52] that a comparison between the solutions of the field equations and observations involves the solution of the light propagation equations, and it is necessary to present the results in terms of measurable quantities.

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