Saddle point inflation from higher order corrections to Higgs/Starobinsky inflation

Michał Artymowski,[1,*] Zygmunt Lalak,[2,†] and Marek Lewicki[2,‡]

[1]*Jagiellonian University Łojasiewicza 11, 30-348 Kraków, Poland*

[2]*University of Warsaw ul. Pasteura 5, 02-093 Warsaw, Poland*

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We explore two saddle point inflationary scenarios in the context of higher-order corrections related to different generalizations of general relativity. Firstly, we deal with the Jordan frame Starobinsky potential, for which we identify a portion of a parameter space of inflection point inflation, which can accommodate all the experimental results. Secondly, we analyze Higgs inflation and more specifically the influence of nonrenormalizible terms on the standard quartic potential. All results were verified with the Planck 2015 data.

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I. INTRODUCTION

Cosmic inflation [1–3] is a theory of the early Universe which predicts cosmic acceleration and generation of seeds of the large scale structure of the present Universe. It solves problems of classical cosmology and it is consistent with current experimental data [4]. It is instructive to study minimal models of inflation which have the form of small modifications of the canonical models describing features of the visible Universe. The two most prominent examples of such models are, at the moment, $f(R)$ theories of gravity models of Higgs inflation.

The first theory of inflation was the Starobinsky model [5,6], which is an $f(R)$ theory [7] with $R + R^2/6M^2$ Lagrangian density. In such a model the acceleration of space-time is generated by the gravitational interaction itself, without a need to introduce any new particles or fields. The embedding of Starobinsky inflation in no-scale SUGRA has been discussed in Ref. [8]. The Starobinsky inflation can be described as a special case of the Brans-Dicke theory [9] and its predictions are consistent with the so-called Higgs inflation [10], in which inflation is generated by the scalar field nonminimally coupled to gravity. Recently the whole class of generalizations of the Starobinsky and Higgs inflation have been discussed in the literature [11–18], also in the context of the higher-order terms in Starobinsky Jordan frame potential [19–21].

On the other hand, Higgs inflation allows one to generate inflation close to the Higgs sector. The price is the non-minimal coupling to gravity and often additional interactions which allow one to reproduce all features of realistic inflationary scenarios. In particular, corrections to the model can often be represented by higher-order scalar operators. This reminds one of the situation known from $f(R)$ theories, where higher-order corrections to the f(R) function seem to be unavoidable. Usually one assumes that there is a part of the Starobinsky/Higgs inflationary potential where higher-order terms are subdominant, which creates an inflationary plateau long enough to support successful inflation. Nevertheless one can find a part of parameter space where higher-order corrections give significant contribution to the potential for relatively small values of field, which makes the plateau region too short to generate cosmic inflation with at least 60 e-foldings. In this case, a saddle point inflation generated by the higher-order terms may be the only chance to obtain a successful inflationary model, which is the issue we investigate in this paper.

In [21] we proved that the Starobinsky potential with higher-order corrections (i.e. higher powers of the Jordan frame Starobisnky potential) can have a saddle point for some ϕ on the plateau. This requires a certain relation between parameters of the model and leads to the existence of the Starobinsky plateau, a steep slope of exponential potential and a saddle point in between. In this paper, we perform the detailed analysis of such a model in the context of low-scale inflation and generation of primordial inhomogeneities, especially in the context of very big values of higher-order terms. We apply the same approach to the Higgs inflation. In this case the motivation is even more natural: higher-order corrections to the scalar potential (such as ψ^6 or ψ^8, where ψ is a scalar field, but not necessarily the Higgs field itself) are considered to be suppressed by the Planck scale. Consequently, for sufficiently high values of field ψ, one has to take into account the influence of higher-order terms. In particular, higher-order term coefficients could be fine-tuned to create a saddle point (or deflation point) in the Einstein frame scalar potential. In Ref. [22], we have also investigated the saddle point inflation in $f(R)$ theory generated by higher-order corrections to the Starobinsky model. Note that, in this

[*]Michal.Artymowski@uj.edu.pl
[*]Zygmunt.Lalak@fuw.edu.pl
[*]Marek.Lewicki@fuw.edu.pl

paper, we investigate the issue of higher-order corrections to Jordan frame scalar potentials, not to the $f(R)$ function itself.

In what follows we use the convention $8\pi G = M_p^{-2} = 1$, where $M_p = 2.435 \times 10^{18}$ GeV is the reduced Planck mass.

The outline of the paper is as follows. In Sec. II we introduce the form of the potential in modified Strobinsky case and we analyze its features. In Sec. III we analyze the saddle point inflation in this model and the evolution of primordial inhomogeneities. In Sec. IV we investigate the saddle point within the Higgs inflation scenario. Finally, we conclude in Sec. V.

II. STAROBINSKY-LIKE POTENTIAL WITH A SADDLE POINT

A. Jordan frame analysis

Let us consider a Brans-Dicke theory in the flat FRW space-time with the metric tensor of the form $ds^2 = -dt^2 + a(t)^2(d\vec{x})^2$. Any $f(R)$ theory can be expressed in terms of the auxiliary field $\varphi \coloneqq F(R) \coloneqq f'(R)$ with the Jordan frame scalar potential $U = \frac{1}{2}(RF - f)$. The Jordan frame action is of the form

$$S = \int d^4x\sqrt{|g|}(\varphi R - U(\varphi)) + S_{\mathrm{m}}, \tag{2.1}$$

where S_{m} is the action of matter fields. In the context of inflation one can assume that the energy density of the Universe is fully dominated by the inflaton, which gives $S_m = 0$. Then, for the homogeneous field φ, the field's equation of motion and the first Friedmann equation become [7],

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{2}{3}(\varphi U_\varphi - 2U) = 0, \tag{2.2}$$

$$3\left(H + \frac{\dot{\varphi}}{2\varphi}\right)^2 = \frac{3}{4}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 + \frac{U}{\varphi}, \tag{2.3}$$

where $U_\varphi \coloneqq \frac{dU}{d\varphi}$. Let us note that, for $\varphi = 1$, one recovers general relativity (GR). Thus, the $\varphi = 1$ will be denoted as the GR vacuum.

The Starobinsky inflation is a theory of cosmic inflation based on the $f(R) = R + R^2/6M^2$ action, which can be generalized into Brans-Dicke theory with general value of ω_{BD}. The Jordan frame potential of the Starobinsky model takes the following form,

$$U_S = \frac{3}{4}M^2(\varphi - 1)^2, \tag{2.4}$$

where M is a mass parameter, and its value comes from the normalization of the primordial inhomogeneities. For

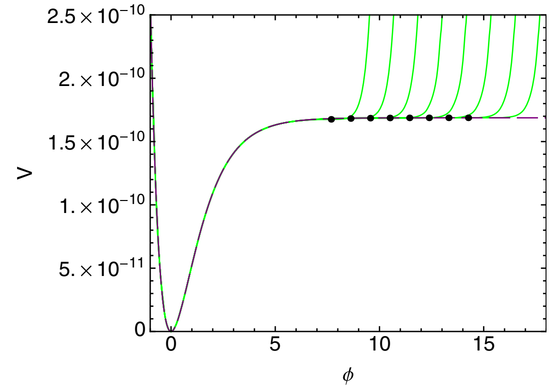

FIG. 1. $V(\phi)$ for Starobinsky inflation and for saddle point inflation (dashed purple line and green lines respectively). Black points represent $\phi = \phi_s$ for $\lambda_1 = 10^k$, where $k \in \{-5, -4, \ldots, 2\}$ (smallest ϕ_s for biggest λ_1).

$\omega = 0$ one finds $M \simeq 1.5 \times 10^{-5}$. The Starobinsky potential is presented in Fig. 1. In this paper, we consider the extension of this model motivated by Ref. [19] and partially analyzed in Ref. [21], namely

$$U = U_S(1 - \lambda_1 U_S + \lambda_2 U_S^2), \tag{2.5}$$

where λ_1, λ_2 are numerical coefficients. In order to avoid $U \to -\infty$ for $\varphi \to \infty$ we assume that $\lambda_2 > 0$. We want to keep λ_i terms as higher-order corrections (i.e. we want to remain in the perturbative regime of the theory); thus, we require $\lambda_1 M^2 \ll 1$ and $\lambda_2 M^4 \ll \lambda_1 M^2$. As we will show, the assumed range of parameters will satisfy these conditions.

B. Einstein frame analysis

The gravitational part of the action may obtain its canonical (minimally coupled to φ) form after transformation to the Einstein frame. Let us assume that $\varphi > 0$. Then for the Einstein frame metric tensor,

$$\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu}, \qquad d\tilde{t} = \sqrt{\varphi}dt, \qquad \tilde{a} = \sqrt{\varphi}a, \tag{2.6}$$

one obtains the action of the form of

$$S[\tilde{g}_{\mu\nu}, \phi] = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - V(\phi)\right], \tag{2.7}$$

where $\tilde{\nabla}$ is the derivative with respect to the Einstein frame coordinates, $\phi = \sqrt{3/2}\log\varphi$, $V(\phi) = U/\varphi^2$ at $\varphi = \varphi(\phi)$, and $\tilde{R}$ is the Ricci scalar of $\tilde{g}_{\mu\nu}$. The GR vacuum appears for $\phi = 0$. Let us define the Einstein frame Hubble parameter as

$$\mathcal{H} \coloneqq \frac{\tilde{a}'}{\tilde{a}}, \quad \text{where } \tilde{a}' \coloneqq \frac{d\tilde{a}}{d\tilde{t}}. \tag{2.8}$$

Then for $\rho_M = P_M = 0$ the first Friedmann equation and the equation of motion of ϕ are the following,

$$3\mathcal{H}^2 = \frac{1}{2}\phi'^2 + V(\phi), \qquad \phi'' + 3\mathcal{H}\phi' + V_\phi = 0, \tag{2.9}$$

where $V_\phi = \frac{dV}{d\phi}$.

C. Saddle point of the Einstein frame potential

In Ref. [21] we showed that the Einstein frame potential of the model from Eq. (2.5) has a saddle point at $\phi = \phi_s$ for $\lambda_1 = \lambda_s$, where

$$\phi_s \simeq \frac{1}{\sqrt{6}}\log\left(\frac{10}{3\lambda_1 M^2}\right), \qquad \lambda_1 = \lambda_s \simeq \frac{5\lambda_2^{3/5}M^{2/5}}{2^{1/5}3^{2/5}},$$
$$V_s \simeq \frac{3}{4}M^2\left(1 - \frac{15}{8}(24M^4\lambda_2)^{1/5}\right), \tag{2.10}$$

where $V_s = V(\phi_s)$. Let us note that, for $\phi \simeq \phi_s$, one obtains a saddle point inflation, which in principle could significantly decrease the scale of inflation. For $\lambda_1 < \lambda_s$ the λ_1 term is always subdominant as compared to the other terms of $V(\phi)$ and it can be neglected in the analysis. Therefore, the potential has no stationary points besides the minimum at $\phi = 0$. On the other hand, for $\lambda_1 > \lambda_s$ one obtains a potential with a local maximum at the plateau, step slope for big ϕ and a minimum in between. This case was analyzed in [21]. Equation (2.10) gives the approximate value of λ_s and ϕ_s, but it predicts all derivatives of V for $\phi = \phi_s$ and $\lambda_1 = \lambda_s$ to be of order of $M^{8/3}\lambda_1^{2/3}$. This is not consistent with the saddlelike point condition $V'(\phi_s)$, $V''(\phi_s) \ll V'''(\phi_s)$ and, therefore, Eq. (2.10) is not accurate enough to calculate power spectra of primordial inhomogeneities.

The analysis presented in this subsection is done under several assumptions and approximations and therefore its accuracy is limited by them. For instance one finds $V_\phi(\phi_s) \sim \lambda_2^{2/5}M^{18/5}$ and $V_{\phi\phi}(\phi_s) \sim \lambda_2^{2/5}M^{8/5}$. We have assumed that $\lambda_1 \gg M^2\lambda_2$ and we have obtained $\lambda_1 \sim \lambda_2^{3/5}M^{2/5}$. Thus, the assumption is satisfied for $(\lambda_2 M^4)^{2/5} \ll 1$, which is also the condition for $V_{\phi\phi}(\phi_s) \ll 1$. One can see that the approximation used in this subsection is far more accurate for the $V_\phi = 0$ condition than for the $V_{\phi\phi} = 0$ condition.

III. INFLATIONARY DYNAMICS AND PRIMORDIAL INHOMOGENEITIES

A. Inflation with and beyond the slow-roll approximation

In this section we will discuss the slow-roll approximation and inflation in the Einstein frame. We want to investigate how nonzero values of λ parameters deviate the inflation from the Starobinsky model. Let us assume that $\phi'' \ll V_\phi$. Then one obtains

$$3\mathcal{H}\phi' + V_\phi \simeq 0, \qquad 3\mathcal{H}^2 \simeq V. \tag{3.1}$$

This approximation holds for nonzero values of V_ϕ, so one cannot use Eq. (3.1) at $\phi = \phi_s$. Thus, we will use the slow-roll equations for $\phi \neq \phi_s$. The cosmic inflation takes place as long as the following slow-roll parameters are much smaller than one:

$$\epsilon \coloneqq \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2, \qquad \eta \coloneqq \frac{V_{\phi\phi}}{V}. \tag{3.2}$$

The number of e-folds generated during the inflation is in the slow-roll approximation equal to

$$N = \int_{t_i}^{t_f}\mathcal{H}d\tilde{t} \simeq \int_{\phi_f}^{\phi_i}\frac{V}{V_\phi}d\phi, \tag{3.3}$$

where indexes i and f refer to initial and final moments of inflation, respectively. Namely, t_i is the first moment when both slow-roll parameters are smaller than 1 and t_f is the moment when any of the slow-roll parameters become bigger than 1. In Fig. 6 of Ref. [21], we showed that during the saddle-point inflation, it is straightforward to generate the number of e-folds significantly bigger than 60, even if the plateau is not present at all.

The power spectra of the superhorizon primordial curvature perturbations and gravitational waves in the Einstein frame are the following:

$$\mathcal{P}_\mathcal{R} = \left(\frac{\mathcal{H}}{2\pi}\right)^2\left(\frac{\mathcal{H}}{\phi'}\right)^2 \simeq \frac{V}{24\pi^2\epsilon}, \qquad \mathcal{P}_h = \left(\frac{\mathcal{H}}{2\pi}\right)^2. \tag{3.4}$$

The $\mathcal{P}_\mathcal{R}$ needs to be normalized at the horizon crossing of scales observed in the CMB. Usually one chooses the normalization moment to be at $N_\star \simeq 50$–60 (where the $\star$ denotes the value of the quantity at the horizon crossing), but in general the scale of normalization strongly depends on the scale of inflation and reheating. In this paper we set $k_\star = 0.002\ \mathrm{Mpc}^{-1}$, where k is the Fourier mode of perturbation, and $N_\star(\lambda_1 = \lambda_2 = 0) = 55$. The value of $N_\star$ decreases for big λ_2, since for lower scale of inflation the comoving Hubble radius will grow less during the post-inflationary era. This procedure sets $\mathcal{P}_\mathcal{R}^{1/2} \sim 5 \times 10^{-5}$.

We have several parameters in the model, namely $\phi_\star$, ϕ_s, M, λ_1 and λ_2; however, since we require the existence of the saddle point, one finds $\lambda_2 = \lambda_2(\lambda_1, M)$ and $\phi_s = \phi_s(\lambda_1, M)$. From its definition, $\phi_\star$ depends on M, λ_1 and λ_2. The normalization is a constraint which we use to calculate the required M, thus decreasing the amount of free parameters to just λ_1. We will use λ_1 to parametrize the deviation from the Starobinsky inflation.

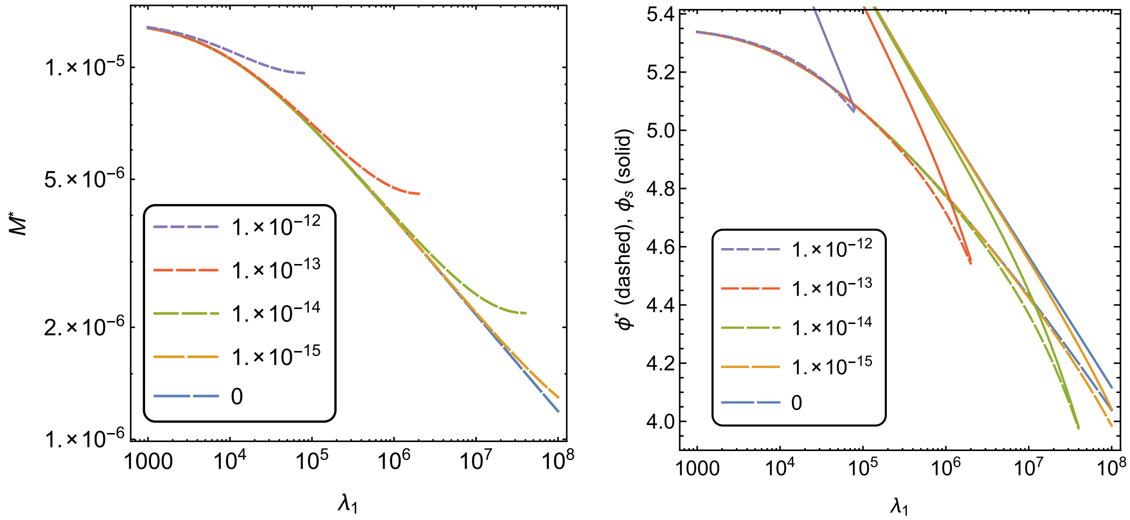

FIG. 2. Left Panel: $M(\lambda_1)$ for several values of $V'(\phi_s)$. As expected, lower $V_\phi(\phi_s)$ gives lower scale of inflation. Right panel: $\phi_\star$ and ϕ_s (dashed and solid lines respectively).

Two parameters of power spectra, which connect theory with the experiment are the tensor-to-scalar ratio r and the scalar spectral index n_s defined by

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\mathcal{R}} \simeq 16\epsilon, \qquad n_s = 1 + \frac{d \log \mathcal{P}_\mathcal{R}}{d \log k} \simeq 1 - 6\epsilon + 2\eta, \tag{3.5}$$

where k is the Fourier mode. In the saddle point inflation, one expects $\epsilon \ll \eta$, so $r_\star \ll 1$ and $n_{s\star} \simeq 1 + 2\eta_\star$. Since $n_{s\star} > 1$ is excluded by the experimental data [4] one needs $\eta < 0$ at the moment of freeze-out. Thus, one requires $\phi_\star < \phi_s$. The total amount of e-folds produced for $\phi < \phi_s$ strongly depends on the length of the Starobinsky plateau and therefore on λ_1. If the plateau for $\phi < \phi_s$ is long enough to generate at least 60 e-folds of inflation then the $\mathcal{P}_\mathcal{R}$ does not significantly deviate from the Starobinsky one. On the other hand for $\lambda_1 \gg 1$ one may decrease the length of the plateau in order to generate e-folds only via the saddle point inflation. As we will show, this may be the way to decrease M and therefore to obtain the low-scale inflation.

In Fig. 2 we present M, $\phi_\star$ and ϕ_s as a function of λ_1. One can see that for $\lambda_1 > 10^3$ the M decreases logarithmically with λ_1 to reach $M \sim 10^{-6}$ at $\lambda_1 \sim 10^8$. For $\lambda_1 > 10^6$

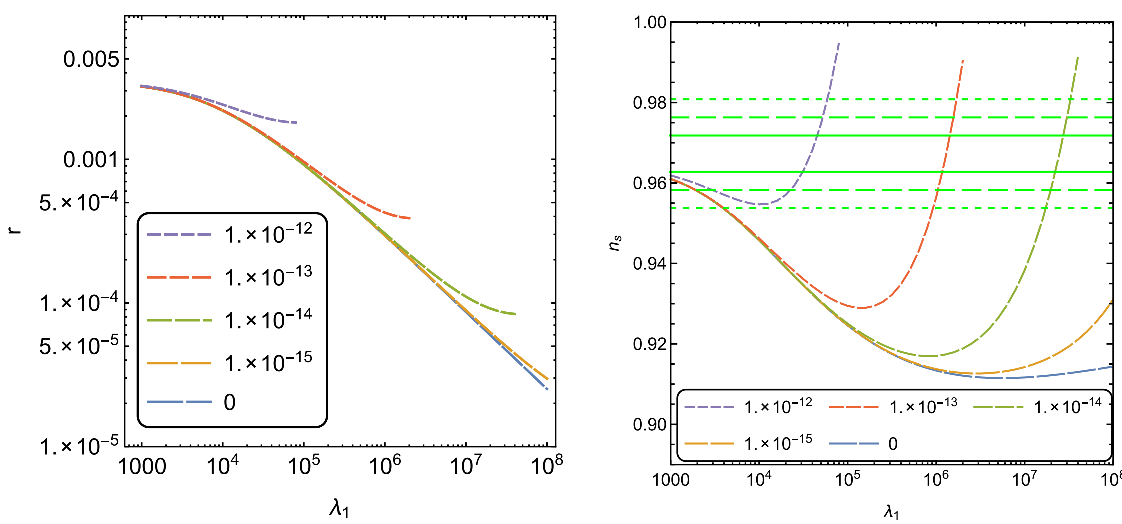

FIG. 3. Left panel: $r(\phi_\star)$ as a function of λ_1 for several values of $V_\phi(\phi_s)$. All obtained values of r are consistent with the Planck data. For $\lambda_1 \gtrsim 10^5$ one obtains $r \lesssim 0.002$, which gives $\Delta\phi < m_p$. Right panel: $n_s(\phi_\star)$ as a function of λ_1 for several values of $V_\phi(\phi_s)$. One, two and three σ regimes of the Planck 2015 best fit of n_s lay between green lines (solid, dashed and dotted respectively). For $\lambda_1 \lesssim 10^3$ one obtains correct value of n_s even for $V_\phi(\phi_s) = 0$. In such a case, the last 60 e-folds of inflation are strongly influenced by the Starobinsky plateau.

the normalization of inhomogeneities happens very close to ϕ_s. Nevertheless, for all considered values of λ_1 we obtained $\phi_\star < \phi_s$. In Fig. 3 we present r and n_s for $\phi = \phi_\star$. For $V_\phi(\phi_s) = 0$ (i.e. for a perfect saddle) and for $\phi_s \simeq \phi_\star$ one finds $n_s < 0.96$ and the model becomes inconsistent with the Planck data. The perfect saddle may still fit the data for $\lambda_1 \lesssim 10^3$; however, then inflation simply occurs on the plateau far from the saddle, and the whole modification is simply irrelevant. Thus, let us consider the case of $V_{\phi\phi} = 0$, $V_\phi \neq 0$ to check if the inflection point inflation gives the correct values of n_s. The value of the field at the inflection will also be denoted as ϕ_s. The results of numerical analysis for saddle and inflection point inflation for different λ_1 and $V_\phi(\phi_s)$ are presented in Figs. 2, 3 and 4. For the inflection point inflation one finds the maximal allowed λ_1 for which the number of e-folds generated for $\phi \leq \phi_s$ is equal to $N_\star$, which is the number of e-folds at the horizon crossing for the normalization scale $k_\star$.

The key result is that inflection point inflation satisfies experimental constraints even for large values of λ_s. This means that it remains a valid solution, and one is not limited to very small modifications of the potential which would simply mean going back to the Starobinsky model. This unfortunately is not true for the pure saddle case in which inflation has to occur on the plateau and the potential can only be modified for very large field values.

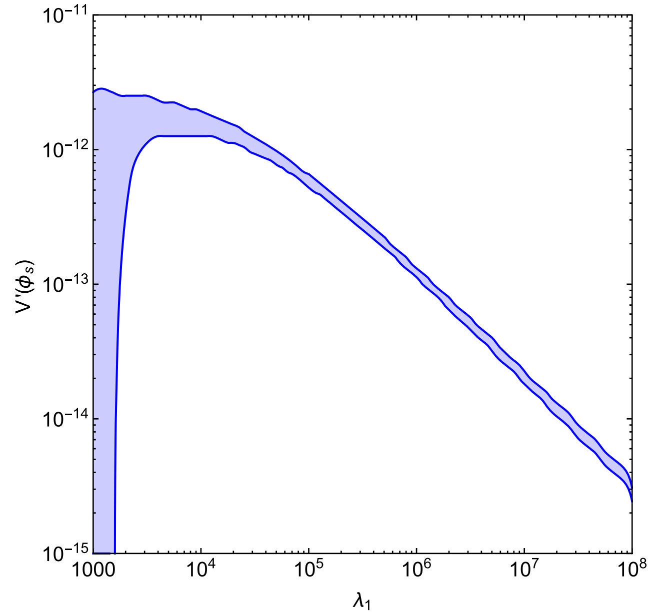

FIG. 4. The blue region represents the part of the $[\lambda_1, V_\phi(\phi_s)]$ parameter space which fits 2σ constrains on n_s from the Planck 2015 data. Smaller $V_\phi(\phi_s)$ corresponds to the smaller scale of inflation. One could extrapolate the blue region for $\lambda_1 > 10^8$, which would give $M^2\lambda_1 \ll 1$ for $\lambda_1 < 10^{20}$. For $\lambda_1 \lesssim 10^3$ any $V_\phi(\phi_s)$ can fit the data, including the perfect saddle point case, however this corresponds to inflation on the plateau far from inflection point.

IV. SADDLE POINT HIGGS INFLATION

A. Higher-order terms to Jordan frame potential

The saddle point inflation can be also obtained from higher-order corrections to the so-called Higgs inflation. Let us define the action of a real scalar field (which in particular may be a Higgs field) with nonminimal coupling to gravity as

$$S[\psi, g_{\mu\nu}] = \int d^4x\mathcal{L} = \int d^4x\sqrt{-g} \times \left[U(\psi)R - \frac{1}{2}(\partial_\mu\psi)(\partial^\mu\psi) - W(\psi)\right] + S_m, \tag{4.1}$$

where $W(\psi)$ is the Jordan frame scalar potential, R is the Ricci scalar, $U(\psi)$ is the function of nonminimal coupling to the gravity and S_m is the action of matter fields, like dust, radiation, additional scalar fields etc. We assume that all fields in S_m are minimally coupled to gravity. For $U \to 1/2$ one restores general relativity. In further parts of this section, we will assume that

$$U(\psi) = \frac{1}{2} + \frac{1}{2}\xi\psi^2, \tag{4.2}$$

where $\xi > 0$ is a dimensionless constant.[1] Thus, the $\psi \to 0$ limit gives the GR vacuum. In the $W \propto \psi^4$ model the slow-roll parameters are proportional to $(\xi\psi^2)^{-1}$, so inflation happens for $\xi\psi^2 \gg 1$. Let us assume that $W(\psi)$ is of the form

$$W(\psi) = W_H + \frac{\lambda_6}{6M_p^2}\psi^6 + \frac{\lambda_8}{8M_p^4}\psi^8, \quad \text{where } W_H = \frac{\lambda}{4}\psi^4, \tag{4.3}$$

where λ_6, $\lambda_8 =$ const, and W_H is a scalar potential which corresponds to the high energy approximation of the Mexican hat potential used e.g. to describe self-interaction of the Higgs field. Terms proportional to λ_6 and λ_8 are the higher-order corrections to this potential.[2] The λ_i constants are dimensionless. As we will show, one needs to require $\lambda_8 > 0$ in order to obtain positive energy density and stability of the potential at very high energies. Nevertheless, the sign of λ_6 remains undetermined.

In order to obtain the canonical form of the action, let us consider the transformation to the Einstein frame, namely

[1] ψ is divided by M_p which is equal to one in our units.
[2] The issue of saddle point Higgs inflation without higher-order corrections has been partially analyzed in Ref. [23].

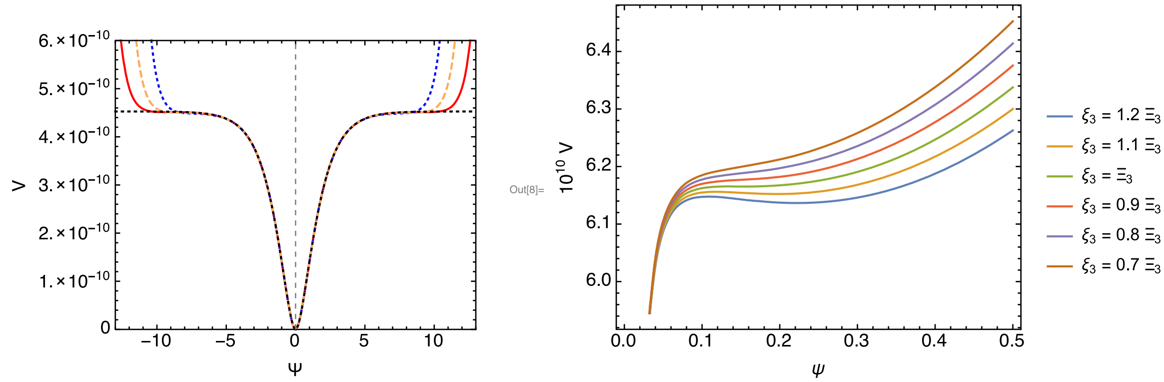

FIG. 5. Left panel: $V(\Psi)$ with saddle point for $\lambda_8 = 1$, $\lambda_6 \propto -\lambda^{3/2}$, $\lambda\xi^{-2} \sim 2\times10^{-9}$ and $\lambda \simeq \{0.15, 0.4, 1\}$ (dotted blue, dashed orange and solid red lines respectively). Dotted black line represents $V(\Psi)$ for $\lambda_6 = \lambda_8 = 0$. The saddle point inflation in this scenario will be analyzed in Sec. IV A. Right panel: V as a function of Jordan frame field ψ for a Higgs inflation with higher-order corrections to W and U. $\xi_3 = \Xi_3$ is an exact saddle. This scenario is briefly analyzed in Sec. IV B.

$$\tilde{g}_{\mu\nu} = 2U g_{\mu\nu} \Rightarrow \tilde{a} = \sqrt{2U}a, \qquad d\tilde{t} = \sqrt{2U}dt, \tag{4.4}$$

$$\left(\frac{d\Psi}{d\psi}\right)^2 = \frac{1}{2}\frac{U + 3U_\psi^2}{U^2}, \qquad V(\Psi) = \frac{W}{4U^2}(\psi = \psi(\Psi)), \tag{4.5}$$

where Ψ is the Einstein frame field and V is the Einstein frame potential. For $\xi\psi^2 \gg 1$ one finds $\psi \simeq \exp(\sqrt{2/3}\Psi)$, which is the result from the Brans-Dicke theory. The potential $V(\Psi)$ for several values of λ_6 and λ_8 parameters has been presented in Fig. 5. Let us note that, for $\lambda_6 = \lambda_8 = 0$, one restores the potential from Ref. [10]. Now the equations of motion take the form

$$\Psi'' + 3\mathcal{H}\Psi' + V_\Psi = 0, \qquad 3\mathcal{H}^2 = \frac{1}{2}\Psi'^2 + V, \tag{4.6}$$

where $V_\Psi = \frac{dV}{d\Psi}$ and $\Psi' = \frac{d\Psi}{d\tilde{t}}$. The potential V has a minimum at $\Psi = 0$, which corresponds to the GR limit of the theory. In the $\xi \gg 1$, $\xi\psi^2 \gg 1$ approximation one finds the following analytical relations, which determine the existence of the saddle point in the Einstein frame:

$$\lambda_6 = -\lambda_s \sim 3\left(\frac{\lambda\lambda_8}{4\xi}\right)^{1/3}, \qquad \psi_s \simeq \sqrt{-\frac{\lambda_6}{3\lambda_8} + \frac{9\sqrt{3}\lambda\lambda_8^{3/2}}{2(-\lambda_6)^{5/2}\xi}}. \tag{4.7}$$

For $\lambda_6 > -\lambda_s$ the potential has a minimum at $\Psi = 0$ and flat plateaus, which ends with step, exponential slopes. The potential $V(\Psi)$ is always growing with $|\Psi|$ and one finds no stationary points. For $\lambda_6 = -\lambda_s$ the only stationary points besides the minimum at $\Psi = 0$ are two saddle points at $\Psi = \pm\Psi_s$. For $\lambda_6 < -\lambda_s$ the potential has additional minima and maxima at some $\Psi = \pm\Psi_{\rm min}$ and $\Psi = \pm\Psi_{\rm max}$, respectively.

We start from five free parameters: λ, λ_6, λ_8, ξ and Ψ_s. We have three constraints: $V_\Psi = 0$, $V_{\Psi\Psi} = 0$ and $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq 5\times10^{-5}$, which we use to determine λ_8, ξ and Ψ_s. In Fig. 6 we show the λ and λ_6 dependence of r, n_s, λ_8 and $\Psi_s/\Psi_\star$. In Fig. 7 we show weak λ_6 dependence of ξ in the allowed part of the parameter space. The λ parameter is naturally limited by 4π. In order to preserve perturbativity of the theory let us assume that $|\lambda_6|$ and λ_8 should not be bigger than $\mathcal{O}(1)$. It is crucial that even including all these constraints a significant part of the parameter space remains valid. Even though inflation occurs some distance away from the saddle ($\Psi_s/\Psi_* \gtrsim 1.26$) and so some influence of the plateau is inevitable, such an extension remains a viable extension of the standard Higgs inflation scenario.

B. Higher-order corrections to nonminimal coupling

So far we have investigated the issue of higher-order corrections to the Jordan frame potential, but another option is considering higher-order terms in the function of nonminimal coupling to gravity. In the most general case one could consider

$$U(\psi) = \frac{1}{2}\left(1 + \sum_{n=2}^{\infty}\xi_n\psi^n\right), \qquad W(\psi) = \sum_{n=2}^{\infty}\lambda_{2n}\psi^{2n}, \tag{4.8}$$

where λ_4 and ξ_2 will be denoted as λ and ξ, respectively. Note that, for any n, the domination of a particular $\lambda_{2n}\psi^{2n}$ term can give an inflationary plateau as long as it is correlated with the $\xi_n\psi^n$ domination in U. Such a generalization of Higgs inflation was already analyzed in [24] and it is consistent with Planck data in the $\xi \gg 1$ limit. Therefore for certain hierarchy of ξ_n and λ_{2n} one can imagine a multiphase inflation, for which the Einstein frame potential is always dominated by only one pair of terms, namely by $U \sim \xi_n\psi^n$ and $W \sim \lambda_{2n}\psi^{2n}$. Increasing the value of ψ would move us to another flat region dominated by ξ_{n+1} and $\lambda_{2(n+1)}$ terms, etc.

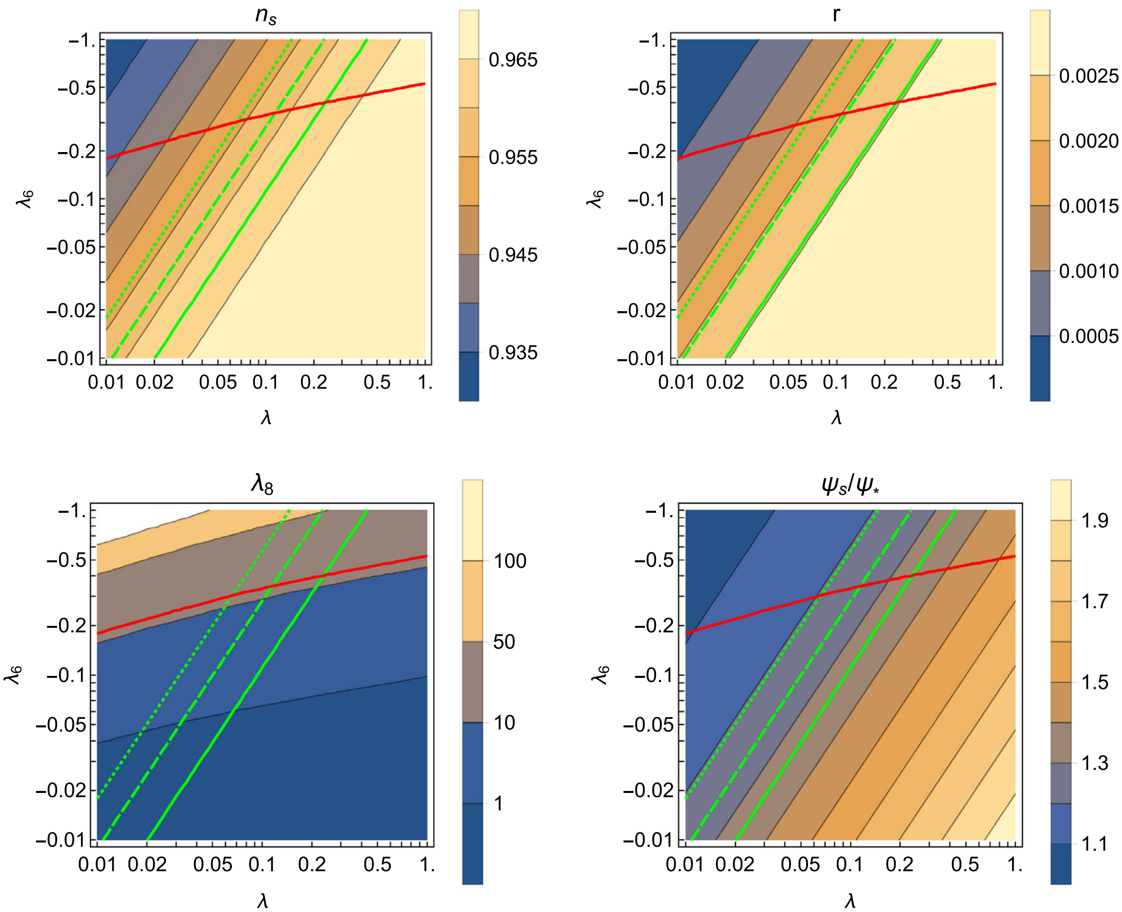

FIG. 6. Left upper panel: spectral index $n_s = n_s(\lambda, \lambda_6)$. One, two and three σ regimes of the Planck 2015 best fit of n_s are below green lines (solid, dashed and dotted, respectively), which correspond to $\lambda_6 = -3.5\lambda^{3/2}$, $\lambda_6 = -8.8\lambda^{3/2}$ and $\lambda_6 = 18\lambda^{3/2}$, respectively. Right upper panel: tensor-to-scalar ratio r. All values of r presented in the plot are consistent with Planck results. Left lower panel: λ_8 as a function of λ and λ_6. In order to obtain perturbative theory one requires $\lambda_8 < 10$, which gives $\lambda_6 < 0.1\lambda^{1/4}$. The allowed region lies under the red line. Right lower panel: $\Psi_s/\Psi_\star$, which determines how close to the saddle point one obtains freeze-out of given scale $k_\star$.

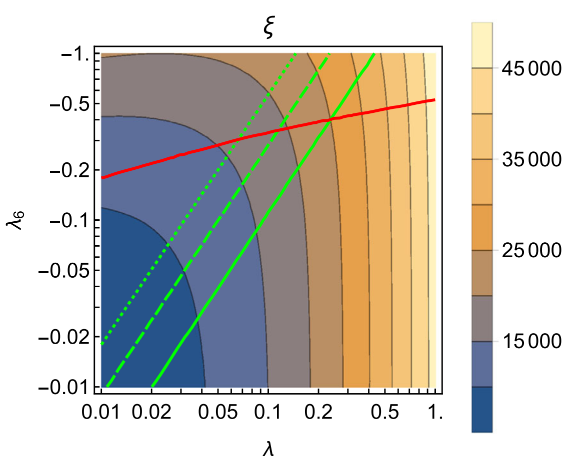

FIG. 7. The nonminimal coupling parameter ξ with constrains on λ_8 and n_s. For $\lambda \gtrsim 0.3$ the λ_6 dependence is very weak and therefore one recovers $\lambda/\xi^2 = \text{const}$. Under the dashed green line (the Planck 2σ regime of n_s) the λ_6 dependence is weak for all λ considered in the plot.

Preliminary results of the analysis of this issue are as follows. For $\lambda_{2n} \simeq \Lambda_{2n} \coloneqq \lambda(\xi_n/\xi)^2$, one can create a series of flat regions in the Einstein frame potential; nevertheless, the model suffers from a problem: flat regions are always separated from each other by local maxima. Therefore, if initially higher-order terms are dominating $V(\Psi)$, the slow-roll evolution cannot be finished by rolling down to the GR vacuum. We would be trapped in one of the local minima of V waiting for the quantum tunneling to $\psi = 0$. In order to avoid this problem one would have to either assume that higher-order terms are negligible up to $\psi = \psi_\star$ or more interestingly, to consider a saddle point inflation scenario. In the latter case, one could consider the $\lambda_6\psi^6$ and $\xi_3\psi^3$ corrections to W and U, respectively. Then the saddle point does exist for

$$\xi_3 = \Xi_3 \coloneqq \frac{\lambda_6}{3\lambda}\sqrt{\frac{\sqrt{3}\sqrt{(8\lambda\xi + 3\lambda_6)^3/\lambda_6}}{\lambda} - \frac{9\lambda_6}{\lambda} + 72\xi}, \qquad (4.9)$$

For $\xi_3 \lesssim \Xi_3$ one obtains the inflection point, which can also be used to generate inflation. Like the case of saddle point Higgs inflation without higher-order corrections to U, we have four free parameters (λ, λ_6, ξ and ξ_3) and two constrains ($\xi_3 \simeq \Xi_3$ and normalization of inhomogeneities), which leaves us with two independent parameters of the model. The Einstein frame potential has a GR minimum at $\psi = \Psi = 0$. We postpone detailed analysis of saddle point inflation with higher-order corrections to W and U to future work.

V. CONCLUSIONS

In this paper we investigated the possibility of generating saddle point inflation from higher-order corrections to theories of modified gravity, namely the Starobinsky model and Higgs inflation. It is crucial that even after including all constraints we discussed, a large portion of the parameter space in both models remains valid. Even though in modified Higgs inflation scenario and Starobinsky model with a pure saddle, inflation has to occur some distance away from the saddle to achieve correct values of n_s, and so some influence of the plateau is inevitable. Inflection or saddle point inflation remain a viable extension of the standard Starobinsky model and Higgs inflation scenarios.

The significant difference between the two is the number of free parameters which in the Higgs inflation case allows solutions with pure saddle to be consistent with all constraints. On the contrary, in modification of the Starobinsky model we had to resort to inflection point inflation rather than a pure saddle in order to achieve new valid results. Note that higher-order corrections do not change the position of Einstein frame vacuum and therefore they do not change the consistency between GR and low energy predictions of discussed theories.

In Sec. II we introduced higher-order corrections to the Jordan frame potential of the Starobinsky theory. We presented the analytical analysis of the existence of the saddle point of the Einstein frame potential.

In Sec. III we analyze features of primordial inhomogeneities generated during saddle point inflation. From the normalization of perturbations and saddle (inflection) point conditions we obtained all parameters of the model (such as M, n_s, r and $\phi_\star$) as functions of λ_1. We found the region in the $[\lambda_1, V_\phi(\phi_s)]$ space which fits the 2σ regime of Planck. We showed how r (and therefore the scale of inflation) can decrease for big λ_1, and we estimated the maximal allowed value of λ_1 to be of order of 10^{20}. The inflation can happen almost exactly at the inflection point, and no Starobinsky plateau is needed to obtain correct shape of the power spectrum. For $\lambda_1 \lesssim 10^3$, consistency with Planck can be obtained even for $V_\phi(\phi_s) = 0$. In such a case, one obtains the Starobinsky plateau for $\phi < \phi_s$, which has significant influence on the generation of primordial inhomogeneities during the last 60 e-folds of inflation.

In Sec. IV we investigated the issue of higher-order corrections to the $\lambda\psi^4$ potential. We found conditions for the existence of a saddle point as well as constrains in the (λ, λ_6) plane, which come from perturbativity of the theory and consistency with Planck. The result is that the saddle point inflation is possible in such a model. However, during the last 60 e-folds of inflation, the interesting part of the potential is also influenced by the $\lambda\psi^4$ term, which generates a Starobinsky-like plateau. This comes from the fact that in the allowed region of parameters, $\Psi_s/\Psi_\star \gtrsim 1.26$, which means that inflation has to proceed on the plateau some distance from the saddle in order to satisfy experimental constraints. We also briefly discuss the possibility of obtaining the saddle point inflation from higher-order corrections to U and W ($\xi_3\psi^3$ and $\lambda_6\psi^6$, respectively). We have derived the analytical relation between parameters of the model which guarantee the existence of the saddle or inflection point. Such a model restores GR in the $\psi \to 0$ limit.

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[1] D. H. Lyth and A. Riotto, Phys. Rep. **314**, 1 (1999).
[2] A. R. Liddle, New Astron. Rev. **45**, 235 (2001).
[3] A. Mazumdar and J. Rocher, Phys. Rep. **497**, 85 (2011).
[4] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **571**, A22 (2014).
[5] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[6] J. D. Barrow and S. Cotsakis, Phys. Lett. B **214**, 515 (1988).
[7] A. De Felice and S. Tsujikawa, Living Rev. Relativity **13**, 3 (2010).
[8] J. Ellis, M. A. G. Garcia, D. V. Nanopoulos, and K. A. Olive, arXiv:1507.02308.
[9] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[10] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).
[11] A. Codello, J. Joergensen, F. Sannino, and O. Svendsen, J. High Energy Phys. 02 (2015) 050.
[12] I. Ben-Dayan, S. Jing, M. Torabian, A. Westphal, and L. Zarate, J. Cosmol. Astropart. Phys. 09 (2014) 005.
[13] M. Artymowski and Z. Lalak, J. Cosmol. Astropart. Phys. 09 (2014) 036.
[14] M. Artymowski, Z. Lalak, and M. Lewicki, J. Cosmol. Astropart. Phys. 06 (2015) 031.
[15] L. Sebastiani, G. Cognola, R. Myrzakulov, S. D. Odintsov, and S. Zerbini, Phys. Rev. D **89**, 023518 (2014).
[16] H. Motohashi, Phys. Rev. D **91**, 064016 (2015).
[17] A. Salvio and A. Mazumdar, Phys. Lett. B **750**, 194 (2015).
[18] Y. Hamada, H. Kawai, K. Y. Oda, and S. C. Park, Phys. Rev. D **91**, 053008 (2015).
[19] B. J. Broy, D. Roest, and A. Westphal, Phys. Rev. D **91**, 023514 (2015).
[20] K. Kamada and J. Yokoyama, Phys. Rev. D **90**, 103520 (2014).
[21] M. Artymowski, Z. Lalak, and M. Lewicki, J. Cosmol. Astropart. Phys. 06 (2015) 032.
[22] M. Artymowski, Z. Lalak, and M. Lewicki, Phys. Lett. B **750**, 595 (2015).
[23] F. Bezrukov and M. Shaposhnikov, Phys. Lett. B **734**, 249 (2014).
[24] R. Kallosh, A. Linde, and D. Roest, Phys. Rev. Lett. **112**, 011303 (2014).