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Yang-Mills condensate as dark energy: A nonperturbative approach

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Models based on the Yang-Mills condensate (YMC) have been advocated for in the literature and claimed as successful candidates for explaining dark energy. Several variations on this simple idea have been considered, the most promising of which are reviewed here. Nevertheless, the previously attained results relied heavily on the perturbative approach to the analysis of the effective Yang-Mills action, which is only adequate in the asymptotically free limit, and were extended into a regime, the infrared limit, in which confinement is expected. We show that if a minimum of the effective Lagrangian in $\theta = -F^a_{\mu\nu}F^{a\mu\nu}/2$ exists, a YMC forms that drives the Universe toward an accelerated de Sitter phase. The details of the models depend weakly on the specific form of the effective Yang-Mills Lagrangian. Using nonperturbative techniques mutated from the functional renormalization-group procedure, we finally show that the minimum in θ of the effective Lagrangian exists. Thus, a YMC can actually take place. The nonperturbative model has properties similar to the ones in the perturbative model. In the early stage of the Universe, the YMC equation of state has an evolution that resembles the radiation component, i.e., $w_y \to 1/3$. However, in the late stage, w_y naturally runs to the critical state with $w_y = -1$, and the Universe transitions from a matter-dominated into a dark energy dominated stage only at latest time, at a redshift whose value depends on the initial conditions that are chosen while solving the dynamical system.

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I. INTRODUCTION

Observational data collected over the last two decades from supernovae type Ia (SN Ia) confirmed that the Universe is undergoing an accelerated phase of expansion. The first piece of evidence for such a behavior was discovered by two independent collaborations and was reported in Ref. [1], by the High-Redshift Supernova Search Team, and in Ref. [2], by the Supernova Cosmology Project Team. Analyses from the combined SN Ia data set (see, e.g., the one reported in Ref. [3]), cosmic microwave background (CMB) radiation, for which we refer, for instance, to Refs. [4–7], and from large scale structures [8,9] have further provided consolidated evidence for the current acceleration of the Universe. The source of late-time cosmic acceleration has been dubbed "dark energy" (DE), an exhaustive theoretical characterization which is still lacking. There have been many attempts so far to determine the origin of DE, but a consensus in the literature has not been reached yet.

A review of the models advocated for thus far within the literature of DE is beyond the purpose of this study, and we prefer to refer the reader to the sizable and rich literature that exists on this subject (see, e.g., Refs. [10–12]). In what follows, we will focus on a rather simple idea which addresses the problem of DE from the perspective of condensation of the Yang-Mills fields. Cosmic acceleration as a source of cosmological inflation was first proposed by Zhang in Ref. [13] and then further developed by the same author and collaborators in the framework of current cosmic acceleration, as a source of DE, in Refs. [14] and [15], respectively, in the perturbative two-loop and three-loop analyses of the effective action of Yang-Mills theory. To be more specific, in Ref. [13] the author has considered a Yang-Mills gauge boson condensate as described by the renormalization-group-improvement action within a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) background. Following Refs. [16–18], the action for the SU(N) Yang-Mills fields has been reshuffled in terms of an effective running coupling constant $g = g(\tau)$, namely,

$$\mathcal{S}_{\rm YM} = \int d^4x \sqrt{-\det(g_{\mu\nu})}\mathcal{L}_{\rm eff},$$
$$\mathcal{L}_{\rm eff} = -\frac{1}{4g^2(\tau)} F^a_{\mu\nu}F^{a\,\mu\nu}, \qquad \tau := \ln\left| -\frac{F^a_{\mu\nu}F^{a\,\mu\nu}}{2\kappa^2}\right|, \qquad (1)$$

in which $g_{\mu\nu}$ stands for the background metric and κ is the square of the renormalization mass scale. From now on, for simplicity of notation, we define the contraction of the field-strength tensors as

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$$\theta := -\frac{1}{2} F^a_{\mu\nu} F^{a\,\mu\nu}, \tag{2}$$

which plays the role of an order parameter for the Yang-Mills condensate (YMC), and this allows us to write the effective Lagrangian in a more compact way, $\tau = \ln|\theta/\kappa^2|$. A further simplification of the studies developed in the literature so far concerns the use of an SU(N) gauge-symmetry group, in which the number of colors N is not fixed *a priori*. This choice connects the physics under investigation to the constituent gauge groups of the standard model of particles.

A. The perturbative expansion

The analysis within Ref. [13] has first focused on a one-loop expansion. This entails considering the effective action

$$\mathcal{L}_{\rm eff} = \frac{1}{2} b\theta \ln\left|\frac{\theta}{e\kappa^2}\right|. \tag{3}$$

The constant b is of order one and depends on the number of colors N of the SU(N) gauge group. Relying on the form of the effective Lagrangian that has been derived for the asymptotically free regime, it has been shown that when the minimum is attained, namely, when $\theta = \kappa^2$, the energy density becomes

$$\rho = \frac{b_0}{2}(E^2 - B^2), \quad B^2 = \frac{1}{2} F^a_{ij} F^{aij}, \quad E^2 = -\frac{1}{2} F^a_{0i} F^{a0i}. \tag{4}$$

The equation of state then reads like the one for dark energy, namely, $p = -\rho$.

The issue of proving that the dark energy behavior of the YMC is stable with respect to higher order loop corrections was addressed in [14] and [15]. In Ref. [14], the analysis has resorted to the two-loop effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{b}{2}\theta\left[\ln\left|\frac{\theta}{\kappa^2}\right| + \eta \ln\left|\ln\left|\frac{\theta}{\kappa^2}\right| + \delta\right|\right], \tag{5}$$

derived within the asymptotically free regime and then extended to the infrared confining regime, in the range of values for θ in which $\mathcal{L}_{\rm eff}$ has a minimum. Again, the coefficients b and δ depend on the number of colors N of the gauge-symmetry group and specifically read $b = \frac{11N}{3(4\pi)^2}$ and $\eta = 2\frac{b_1}{b^2}$, with $b_1 = \frac{17N^2}{3(4\pi)^4}$. The energy density and the pressure from this model are provided by the relations

$$\rho = \frac{b}{2}\theta\left[\tau + 2 + \eta\left(\ln|\tau+\delta| + \frac{2}{\tau+\delta}\right)\right], \tag{6}$$

$$p = \frac{b}{6}\theta\left[\tau - 2 + \eta\left(\ln|\tau+\delta| - \frac{2}{\tau+\delta}\right)\right]. \tag{7}$$

At high energies, when $\tau \gg 1$, the equation of state of the YMC evolves toward the equation of state of radiation

$$w = \frac{p}{\rho} \to \frac{1}{3}. \tag{8}$$

The stability of the system accounting for interactions with matter and electromagnetic radiation, into which the YMC may decay, has been successfully checked. To achieve this goal, in [14] the authors considered the dynamical system provided by the first Friedmann equation in the presence of matter with energy density ρ_m and (electromagnetic) radiation with energy density ρ_r,

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_m + \rho_r), \tag{9}$$

the equations of motion for the energy density component of the YMC, and the matter and radiation that arise from conservation of the total energy-momentum tensor and from the decay of the YMC into matter. In comoving coordinates, in which $H = \frac{\dot a}{a}$, these latter equations read

$$\begin{aligned} \dot\rho + 3H(\rho + p) &= -\Gamma\rho, \\ \dot\rho_m + 3H\rho_m &= \Gamma\rho, \\ \dot\rho_r + 3H(\rho_r + p_r) &= 0. \end{aligned} \tag{10}$$

The decay rate Γ is a free parameter of the model that enters the definition of the dimensionless dynamical system to be solved, Eqs. (9) and (10). The specific value of the rescaled parameter $\gamma = \frac{\Gamma}{H}$ then affects the attractor coefficients in the analysis of the stability of the model. Consistently with big-bang nucleosynthesis [19], with the fractional density observed for dark energy ($\Omega \sim 0.73$), and with the observational constraint on the equivalence between the energy densities of the radiation and matter at the redshift of recombination, we then set the initial conditions of the dynamical variables. Finally, the model has been shown to possess a dark energy tracking solution, and the dynamical system to have a fixed point, which is stable. For $\gamma_0 = 0.5$, the parameter of the equation of state takes the asymptotic value $w = -1.14$, provided that in $\mathcal{L}_{\rm eff}$ one sets $\delta = 3$. A different value of this latter parameter, for instance, $\delta = 7$, would rather entail $w = -1.18$.

A further analysis, developed along the same lines as the one reported above, was further explored by the same authors in [15], and the investigation was extended to the case of the three-loop effective action. Within the latter work, the effective three-loop coupling constant reads

$$g^2(\tau) = \frac{1}{b}\left[\frac{1}{\tau} - \eta\frac{\ln|\tau|}{\tau^2} + \eta^2\frac{\ln^2|\tau| - \ln|\tau| + C}{\tau^3} + O\left(\frac{1}{\tau^3}\right)\right], \tag{11}$$

and the main predictions have been substantially unaffected by the improvement in the perturbative analysis. For different choices of γ_0, which is the decay rate parameter rescaled by H evaluated at the present time, the parameter of the equation of state at the present time takes the values $w_0 = 1.05$ (given that $\gamma_0 = 0.31$) or $w_0 = -1.15$ (given that $\gamma_0 = 0.67$).

We emphasize that, in both cases summarized above, the two-loop expansion within [14] and the three-loop expansion within [15], the dark energy behavior arises from the perturbative computation of the effective Lagrangian in the asymptomatically free limit. The validity of this procedure is then extended to the infrared regime of the Yang-Mills theory, in order to recover a minimum in θ for the effective Lagrangian and derive the equation of state for the Universe, which entails accelerated expansion. But the occurrence of divergences in the effective action may shed some doubts on the validity of the conclusions for the dark energy behavior of the theory.

Finally, we would like to point out that the gauge interactions taken into account here might not necessarily be considered to be the ones constituting the standard model of particle physics. It is interesting to note that an extra "dark sector" might be advocated for to explain the gauge group here involved. Furthermore, it might be tempting to identify the gauge group copies with suitable candidates for dark matter, postulating the existence of "dark copies" of fermions colored under the extra "dark gauge group." The dark matter sector that is then introduced may eventually be connected to the mirror standard model theories [20,21] discussed in the literature.

II. TOWARD A NONPERTURBATIVE INFRARED ANALYSIS

Within previous studies [13–15], the existence and the stability of a dark energy mechanism based on the YMC was investigated. The analyses were developed to move from the ultraviolet results covered in the literature up to three loops in the effective action for Yang-Mills SU(N) fields, and the results were then extended in [13–15] to the infrared regime, in order to derive a physical characterization of an accelerated expansion of the Universe. An important technical issue is one that concerns the stability of the result at higher-loop expansion since the appearance of an additional term in the effective action might indeed spoil the stability, which totally relies on ultraviolet perturbative expansion.

Furthermore, we know that the infrared regime of Yang-Mills SU(N) theories has a completely different behavior than the ultraviolet regime. The latter encodes asymptotic freedom, while the former shows a confinement behavior that depends on the number of gauge colors involved, and even on the number of colored fermions involved in the theory.

Therefore, the main motivation for this study has been to prove that, under mild and general assumptions—which are basically the existence of a minimum in θ in the nonperturbative effective Lagrangian—a dark energy behavior is recovered. Then, by making use of the nonperturbative techniques mutated from the functional renormalization-group procedure, which is more adequate to be used in the confining infrared limit of the theory, it is possible to prove that a such a minimum indeed exists.

For this purpose, we provided the explicit example of the SU(2) Yang-Mills action, deriving the effective Lagrangian for such a model and deepening the cases in which interactions with different forms of matter is considered. The procedure, which might not be completely reliable for the precise determination of the coefficients of the effective nonperturbative Lagrangian, is nevertheless enough to ensure that a minimum of θ exists for $\mathcal{L}_{\rm eff}$, and thus that a YMC works as a reliable candidate to explain the origin of dark energy. In the following sections, we unfold detailed arguments in support of this thesis.

A. Yang-Mills effective action from a nonperturbative approach

Within the perturbative YMC model for dark energy that we reviewed in the previous section, the effective Yang-Mills Lagrangian is the one calculated at one loop in the seminal work by Savvidy [22], namely,

$$\mathcal{L}_{\rm eff} = \frac{1}{2} b\theta \log\left|\frac{\theta}{\kappa^2}\right|, \tag{12}$$

where $b = (11N)/24\pi^2$ for SU(N). Higher-loop corrections have been then deployed in order to check on whether the substance of the physical results remains unchanged, and how the details may vary.

Instead of continuing to consider higher-loop improvements of (12), the main purpose of this work is to start from a general nonperturbative form of the action, i.e.,

$$\mathcal{L}_{\rm eff} = \mathcal{W}(\theta). \tag{13}$$

We may then proceed to determine the form of $\mathcal{W}(\theta)$ by stating some general requirements that must be fulfilled in order to obtain a YMC that can work to explain the origin of dark energy. The function $\mathcal{W}$ will also, in general, be equipped with an energy scale κ in analogy with (1), for dimensional reasons. We notice, indeed, that $\mathcal{W}$ must be a (not necessarily analytic) function of θ satisfying the following properties:

(1) $\mathcal{W}(\theta)$ has a nontrivial minimum in θ.
(2) $\mathcal{W}(\theta)$ possesses a perturbative limit, which reproduces the one-loop result derived by Savvidy [22].
(3) $\mathcal{W}(\theta)$ shows the asymptotic behavior of being at least linear in θ, which in turn is linear to the bare Yang-Mills action. This final requirement can be formalized as follows: in the ultraviolet regime $\theta \gg \kappa^2$, $\mathcal{W}$ must hold the limit

$$\frac{d\ln\mathcal{W}}{d\ln\theta}(\theta)\to 1. \tag{14}$$

In what follows we will consider for simplicity a pure SU(2) Yang-Mills theory, the gauge field of which is not coupled to any other fundamental matter fields. We want to stress that the SU(2) Yang-Mills fields introduced here, as well as the SU(N) Yang-Mills fields dealt with in [13–15], despite being suitable for building a model for cosmic dark energy, should not necessarily be identified as standard model gauge fields. The introduction of an additional copy of SU(N) Yang-Mills fields might, in any case, allow us to make contact with some mirror standard model theories [20,21] that have been advocated for in the dark matter literature. Again, such a link is not necessary for our purposes and will not be analyzed within this investigation of a dark energy YMC. Nevertheless, it might suggest some interesting directions to be followed in forthcoming studies.

B. YMC as cosmological dark energy

In what follows, we discuss the cosmological consequences of the requirements we followed above for the nonperturbative effective action, and we shed light on the behavior of the YMC for the fate of cosmological dark energy. We will assume a flat FLRW universe, the line element of which can be cast in terms of comoving or conformal coordinates, respectively, as follows:

$$ds^2=dt^2-a^2(t)\delta_{ij}dx^idx^j=a^2(\tau)[d\tau^2-\delta_{ij}dx^idx^j], \tag{15}$$

where the cosmological time t and the conformal time τ are related through $dt=ad\tau$. We will consider the simplest case of a universe filled only with the YMC and will assume it to be minimally coupled to the gravity. Then the effective action reads[1]

$$\mathcal{S}=\int\sqrt{-g}\left[-\frac{\mathcal{R}}{16\pi G}+\mathcal{L}_{\rm eff}\right]d^4x. \tag{16}$$

From now on, we will simply denote by g the determinant of the metric $g_{\mu\nu}$. $\mathcal{R}$ is the scalar Ricci curvature, and $\mathcal{L}_{\rm eff}$ is the effective Lagrangian of the YMC, described by Eq. (13). Varying $\mathcal{S}$ with respect to the metric $g^{\mu\nu}$, one obtains the Einstein equation $G_{\mu\nu}=8\pi GT_{\mu\nu}$, where the energy-momentum tensor of the YMC is given by

$$T^{\mu\nu}=\sum_{a=1}^{3}{}^{(a)}T^{\mu\nu}=\sum_{a=1}^{3}g^{\mu\nu}\mathcal{W}(\theta)-2\frac{\partial\mathcal{W}}{\partial\theta}F_a^{\gamma\mu}F^a{}_\gamma{}^\nu. \tag{17}$$

[1]Following the definition in [23], we adopt the sign convention $(-,+,+)$ for the metric, the Riemann tensor, and the Einstein equation, respectively.

We may set up a gauge that preserves the isotropy and the homogeneity of the FLRW background. We write gauge fields as functions of the cosmological time t, and we choose their components so as to satisfy $A_0=0$ and $A_i^a=\delta_i^aA(t)$. This choice indeed ensures that the total energy-momentum tensor $T_{\mu\nu}$ is homogeneous and isotropic. We then introduce the usual definition of the Yang-Mills tensor, cast in terms of the group structure constants f^{abc}, namely,

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+f^{abc}A^b_\mu A^c_\nu. \tag{18}$$

For the SU(2) gauge group to which we are specializing our analysis, the structure constants reduce to $f^{abc}=\epsilon^{abc}$. Furthermore, thanks to the gauge fixing we have selected above and looking at the case of a constant electric field (for simplicity, we will assume in the following a vanishing magnetic field), we can rewrite the Yang-Mills tensor in the simplified form

$$F^{a\mu}{}_\nu=\begin{pmatrix}0 & E_1 & E_2 & E_3\\ -E_1 & 0 & B_3 & -B_2\\ -E_2 & -B_3 & 0 & B_1\\ -E_3 & B_2 & -B_1 & 0\end{pmatrix}=\frac{1}{3}\begin{pmatrix}0 & E & E & E\\ -E & 0 & 0 & 0\\ -E & 0 & 0 & 0\\ -E & 0 & 0 & 0\end{pmatrix}. \tag{19}$$

This allows us to express θ in a very simple form, i.e., $\theta=\sum_{i=1}^3E_i^2=E^2$, and the components of the energy-momentum tensor can then be rewritten as

$${}^{(a)}T^0_\mu=-\frac{1}{3}\mathcal{W}(\theta)\delta^0_\mu+\frac{2}{3}\mathcal{W}'(\theta)E^2\delta^0_\mu, \tag{20}$$

$${}^{(a)}T^i_j=\frac{1}{3}\mathcal{W}(\theta)\delta^i_j-\frac{2}{3}\mathcal{W}'(\theta)E^2\delta^i_j\delta^a_j. \tag{21}$$

These tensors are not yet isotropic, with their values depending on the direction of the color a. Nevertheless, the total energy-momentum tensor $T_{\mu\nu}=\sum_{a=1}^3{}^{(a)}T_{\mu\nu}$ is isotropic, and the corresponding energy density and pressure are given by

$$\rho_{\rm YMC}=-\mathcal{W}(\theta)+2\mathcal{W}'(\theta)\theta,$$
$$p_{\rm YMC}=\mathcal{W}(\theta)-\frac{2}{3}\mathcal{W}'(\theta)\theta. \tag{22}$$

Consequently, the equation of state (EOS) of the YMC is immediately recovered as

$$w_{\mathrm{YMC}} \equiv \frac{p_{\mathrm{YMC}}}{\rho_{\mathrm{YMC}}} = -\frac{\mathcal{W} - \frac{2}{3}\mathcal{W}'\theta}{\mathcal{W} - 2\mathcal{W}'\theta} = -\frac{1 - \frac{2}{3}\frac{\mathcal{W}'}{\mathcal{W}}\theta}{1 - 2\frac{\mathcal{W}'}{\mathcal{W}}\theta}. \tag{23}$$

It is worth discussing the mathematical properties of the EOS w_{YMC}. On one hand, if we require the Yang-Mills theory to condensate, then the function $\mathcal{W}$ must have a nontrivial minimum, as we required in Sec. II A. However, this is equivalent to requiring that $\mathcal{W}'$ vanish at some point θ_0. At this point the YMC has an EOS of the cosmological constant with $w_{\mathrm{YMC}} = -1$. Around this critical point the YMC dark energy models can account either for an EOS characterized by $w_{\mathrm{YMC}} > -1$ or for an EOS characterized by $w_{\mathrm{YMC}} < -1$, thus encoding phantom matter behavior. On the other hand, in the high-energy-scale regime in which $\theta \gg \kappa^2$, one would like to retrieve that the YMC exhibits an EOS of radiation, characterized by $w_{\mathrm{YMC}} = 1/3$, in analogy with the perturbative analysis [13–15]. Within the framework of the nonperturbative action (13), this corresponds to the third requirement listed in Sec. II A. The effective action should then scale for $\theta \gg \kappa^2$, at least like the bare Yang-Mills action, i.e., $d\ln\mathcal{W}(\theta)/d\ln\theta \sim 1$.

In the following sections, we will show in detail that the YMC evolves from an EOS with $w_{\mathrm{YMC}} = 1/3$ to one with $w_{\mathrm{YMC}} = -1$ while the Universe is expanding.

C. A noninteracting YMC model

The cosmological model we are about to analyze in this section entails three different sources for the energy-momentum tensor: (i) the dark energy, the role of which we assume to be played by the YMC; (ii) the matter, including both baryons and dark matter, which is dealt with as nonrelativistic dust with negligible pressure; and (iii) the radiation, the component of which consists of photons and possibly other massless particles, such as neutrinos. We will describe the three components in terms of their EOSs, without accounting for any microscopic treatment in terms of the fundamental matter fields.

Since, from Eq. (15), we assumed *ab initio* the Universe to be flat, fractional densities will sum up to one, namely, $\Omega_{\mathrm{YMC}} + \Omega_m + \Omega_r = 1$. Indeed, the fractional energy densities are defined as $\Omega_{\mathrm{YMC}} \equiv \rho_{\mathrm{YMC}}/\rho_{\mathrm{tot}}$, $\Omega_m \equiv \rho_m/\rho_{\mathrm{tot}}$, and $\Omega_r \equiv \rho_r/\rho_{\mathrm{tot}}$, and the total energy density is given by $\rho_{\mathrm{tot}} \equiv \rho_{\mathrm{YMC}} + \rho_m + \rho_r$. The overall expansion of the Universe is determined by the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\mathrm{YMC}} + \rho_m + \rho_r), \tag{24}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_{\mathrm{YMC}} + 3p_{\mathrm{YMC}} + \rho_m + \rho_r + 3\rho_r), \tag{25}$$

where the dot denotes the d/dt.

As a first preliminary investigation, we will assume that there is no interaction between the three energy components. The dynamical evolution of these is determined by their equations of motion, which in turn follow from imposing the conservation of each component of the energy-momentum tensor,

$$\dot{\rho}_{\mathrm{YMC}} + 3\frac{\dot{a}}{a}(\rho_{\mathrm{YMC}} + p_{\mathrm{YMC}}) = 0, \tag{26}$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = 0, \tag{27}$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0. \tag{28}$$

From Eqs. (27) and (28), we can immediately obtain the standard evolutions of the matter and radiation components, $\rho_m \propto a^{-3}$ and $\rho_m \propto a^{-4}$. A less obvious but still rather simple task is solving the evolution of the YMC component. Inserting (22) into (26), we obtain the following relation:

$$\dot{\theta}(\mathcal{W}' + 2\mathcal{W}''\theta) + 4\frac{\dot{a}}{a}\mathcal{W}'\theta = 0, \tag{29}$$

which is in a quite compact form. This equation is integrable for any regular enough $\mathcal{W}$. In particular, we can easily derive the result

$$\sqrt{\theta}\mathcal{W}'(\theta) = \alpha a^{-2}, \tag{30}$$

where α is a coefficient of proportionality that depends on the initial conditions.

At very high redshift, Eq. (30) entails an increase of the order parameter θ that involves the limit $\theta \gg \kappa^2$. Thus, at very high redshift, the system transitions toward the ultraviolet regime. Within this limit, Eq. (23) encodes the EOS parameter $w_{\mathrm{YMC}} \to 1/3$. The YMC then starts behaving as the radiation component, as one would have expected since the theory is evolving toward asymptotic freedom at high energy. At small redshift, the expansion of the Universe requires the lhs of Eq. (30) to vanish, which occurs for the extremal value of $\theta = \theta_0$, and the EOS' parameter then converges toward $w_{\mathrm{YMC}} = -1$. This ensures that the YMC behaves as an effective cosmological constant.

Finally, notice that we may proceed as in [14,15] and take advantage of the observational constraint on the ratio between the dark energy density and the critical energy density, in order to fix the energy scale κ^2 that appears in the effective Lagrangian $\mathcal{W}(\theta)$. The effective Lagrangian is no more dependent on the parameter κ, and we can rescale θ as in the previous literature.

D. Interacting YMC models

In this section, we generalize the YMC dark energy model and take into account some effective interaction with dust matter. Nevertheless, for the sake of simplicity, at this first stage of the analysis we will disregard the interaction between radiation and the YMC. We will then describe the YMC dark energy and background matter interaction through one additional parameter Q. Notice, however, that the way the interaction is considered here, and the parameter Q is introduced, is merely phenomenological and does not intend to capture any microphysical feature. The latter could be correctly taken into account only through a much more elaborate analysis than the ones that are currently being carried out in the literature.

The equations of the conservation of energy in (26) and (27) should be modified into

$$\dot{\rho}_{\rm YMC} + 3\frac{\dot{a}}{a}(\rho_{\rm YMC} + p_{\rm YMC}) = -\frac{\dot{a}}{a}Q, \qquad (31)$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = +\frac{\dot{a}}{a}Q, \qquad (32)$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0. \qquad (33)$$

The interaction parameter Q, in natural units, has the dimension of $[\text{energy}]^4$ and has been introduced for phenomenological reasons. Its possible form will be addressed later, but here we want to briefly mention that the only meaningful characterization of the right-hand sides of (31) and (32) is by a quantity proportional to the Hubble parameter and to the components of the energy density. If the YMC transfers energy to matter—for instance, if the YMC decays into pairs of matter particles—we should require the parameter Q to be positive. In the opposite case, we should require the parameter Q to be negative.

We can then proceed to the study of the evolution of the system (31) and (32). It is convenient to introduce the so-called e-folding time $N \equiv \ln a$, the derivative with respect to which we will denote with a prime. We will denote as x the dimensionless matter density $x = \rho_m/\kappa^2$. The system (31) and (32) then reads

$$\theta'(\mathcal{W}' + 2\theta\mathcal{W}'') + 4\theta\mathcal{W}' = -Q, \qquad (34)$$

$$x' + 3x = +Q. \qquad (35)$$

Using the above definitions, we can immediately recast the fractional energy densities of the YMC and the dust matter

$$\Omega_{\rm YMC} = \frac{-\mathcal{W} + 2\mathcal{W}'\theta}{-\mathcal{W} + 2\mathcal{W}'\theta + x}, \quad \text{and}$$
$$\Omega_m = \frac{x}{-\mathcal{W} + 2\mathcal{W}'\theta + x}. \qquad (36)$$

It is useful to discuss the general properties of this dynamical system before specifying the form of the interaction term Q. We can seek the fixed points of the system by imposing $\theta' = x' = 0$ in Eqs. (34) and (35), and we then look for the solutions θ_c and x_c of the simplified system derived from Eqs. (34) and (35). The latter reads

$$4\theta_c\mathcal{W}'(\theta_c) = -3x_c, \qquad (37)$$

$$3x_c = +Q(x_c, \theta_c). \qquad (38)$$

The stability of the solutions of these differential equations, and the possible existence of attractor solutions, will be analyzed in the forthcoming subsections, in which we will specialize the form of the coupling Q by assuming different linear combinations of the energy-density components considered so far.

1. $Q \propto \rho_{\rm YMC}$

In this section, we parametrize the interaction as proportional to the YMC energy density, namely, $Q = \alpha\rho_{\rm YMC} = \alpha(-\mathcal{W} + 2\mathcal{W}'\theta)$. The trivial case $\alpha = 0$ reduces to the free YMC dark energy model studied above. We will only consider the simplest case, with α being a nonzero dimensionless constant. The evolution equations (34) and (35) are then recast as

$$\theta'(\mathcal{W}' + 2\theta\mathcal{W}'') + 4\theta\mathcal{W}' = -\alpha(-\mathcal{W} + 2\mathcal{W}'\theta), \qquad (39)$$

$$x' + 3x = +\alpha(-\mathcal{W} + 2\mathcal{W}'\theta). \qquad (40)$$

When the fractional density of the YMC is subdominant in the Universe, we expect the effect on the dust to be small. Only in the latest stage of expansion of the Universe, where the YMC dark energy dominates its evolution, the effect of interaction on the dust component can become important.

The critical point equations (37) and (38) now rewrite as

$$2(2+\alpha)\theta_c\mathcal{W}'(\theta_c) = \alpha\mathcal{W}(\theta_c), \qquad (41)$$

$$3x_c = -4\theta_c\mathcal{W}'(\theta_c). \qquad (42)$$

It is easy to see that the fractional energy densities of the YMC and the EOS at this critical point do not depend on the details of the potential $\mathcal{W}$, but they might rather have a dependence on the parameter α. It is straightforward to verify this from the very definition of an EOS calculated at the fixed point solution of Eq. (41):

$$\Omega_{\rm YMC} = -\frac{1}{w_{\rm YMC}} = \frac{\mathcal{W}(\theta_c) - 2\theta_c\mathcal{W}'(\theta_c)}{\mathcal{W}(\theta_c) - \frac{2}{3}\theta_c\mathcal{W}'(\theta_c)} = \frac{3}{3+\alpha}. \qquad (43)$$

The constraint $0 \leq \Omega_{\rm YMC} \leq 1$ implies that $\alpha > 0$. Data from SN Ia and the CMB considerably restrict the available range of α, allowing us to choose the value $\alpha \simeq 10^{-2}$ that we can approximately infer from the estimate of w and its related systematical errors. This implies that solutions of (40) will differ from the standard behavior $\rho_m \propto a^{-3}$ by a small perturbation. Indeed, the solution of the homogeneous equation will be trivially the same as the standard case, while the homogeneous solution will be of the order α. Internal consistency of the model requires the solution of the system to be stable under perturbations. The next step is then to require the critical point to be an attractor solution. In order to achieve that, we first need to compute the eigenvalues of the linearized system equations (39) and (40) at the critical point:

$$\lambda_1 = -3, \tag{44}$$

$$\lambda_2 = -\frac{1}{(\mathcal{W}'(\theta_c) + 2\theta_c \mathcal{W}''(\theta_c))^2}((4+\alpha)\mathcal{W}'(\theta_c)^2 + (2+\alpha)4\theta_c^2\mathcal{W}''(\theta_c)^2 + 4(3+\alpha)\theta_c\mathcal{W}'(\theta_c)\mathcal{W}''(\theta_c)). \tag{45}$$

The solution is an attractor if

$$(4+\alpha)\mathcal{W}'(\theta_c)^2 + (2+\alpha)4\theta_c^2\mathcal{W}''(\theta_c)^2 + 4(3+\alpha)\theta_c\mathcal{W}'(\theta_c)\mathcal{W}''(\theta_c) > 0. \tag{46}$$

In general, we will need a specific form of $\mathcal{W}$ to further discuss the nature of the critical point.

2. $Q \propto \rho_m$

The next case to be considered hinges on an interaction of the form $Q = \alpha\rho_m = \alpha x$. The evolution equations (34) and (35) now rewrite as

$$\theta'(\mathcal{W}' + 2\theta\mathcal{W}'') + 4\theta\mathcal{W}' = -\alpha x, \tag{47}$$

$$x' + 3x = +\alpha x. \tag{48}$$

If we assume α to be a nonvanishing constant, we easily derive that $\rho_m \propto a^{\alpha-3}$. This result might then lead to observational inconsistencies: the evolution of the dust component conflicts with the evolution of dust in the standard big-bang model. We should then avoid considering an interaction term of such a form in the early stage of evolution of the Universe, at very high redshift.

Nevertheless, even if we insisted on phenomenologically describing at small redshift the interaction between YMC dark matter and matter energy density with a term of the form $Q = \alpha x$, we would find only a trivial critical point $\theta_c = \theta_0$ and $x_c = 0$. Thus, we must conclude that it is impossible to obtain an attractor solution for this kind of system.

3. $Q \propto \rho_{\mathrm{YMC}} + \rho_m$

As a final case, we can discuss a model where $Q = \alpha(\rho_{\mathrm{YMC}} + \rho_m)$. We limit again ourselves to the consideration of α being a dimensionless constant. In the later stage, when the dark energy dominates the evolution, this model reduces to the case we studied in Sec. II D 1, while in the dust dominated stage reduces to the case we studied in Sec. II D 2.

In analogy to the discussion developed in the previous section, if we insisted on applying this model to the description of the early Universe, the evolution of dust would turn out to be incompatible with the prediction of the standard hot big-bang models. Nevertheless, if we want to develop a phenomenological model for small redshift, we may elaborate on this case.

The dynamical equations (34) and (35) become

$$\theta'(\mathcal{W}' + 2\theta\mathcal{W}'') + 4\theta\mathcal{W}' = -\alpha(-\mathcal{W} + 2\mathcal{W}'\theta + x), \tag{49}$$

$$x' + 3x = +\alpha(-\mathcal{W} + 2\mathcal{W}'\theta + x), \tag{50}$$

and the system admits a critical point in

$$3\alpha\mathcal{W} = 2(\alpha+6)\theta_c\mathcal{W}', \qquad x_c = -\frac{4}{3}\theta_c\mathcal{W}'(\theta_c). \tag{51}$$

The fractional energy density and the EOS of the YMC at this critical point now read

$$\Omega_{\mathrm{YMC}} = -\frac{1}{w_{\mathrm{YMC}}} = \frac{3-\alpha}{3}. \tag{52}$$

Notice that the observational constraint of $0 \leq \Omega_{\mathrm{YMC}} \leq 1$ now sets a different range of allowed values for α, namely, $0 < \alpha \leq 3$.

III. A NONPERTURBATIVE EXAMPLE: SU(2)-YMC

We focus now on the analysis of the YMC model we have described in the previous sections, with a specific choice for the IR effective Lagrangian. We keep analyzing a YM theory that enjoys an SU(2) gauge group, and we show at the nonperturbative level that a YMC forms that drives accelerated expansion of the Universe at small redshift. The functional renormalization group (FRG) approach to non-Abelian gauge theories will be particularly fruitful for our purposes, as it allows us to introduce nonperturbative methods that can be treated as much as possible analytically.

A. Functional renormalization group

The FRG approach is a tool developed to study interacting quantum field theory and statistical systems in the nonperturbative regime, where no small coupling exists and perturbative techniques are not applicable. The method is based upon a Wilsonian momentum-shell-wise integration of the path integral: a masslike regulator function $R_k(p)$ suppresses quantum fluctuations with momenta lower than an IR momentum cutoff scale k. This allows us to define a scale-dependent effective action, the flowing action Γ_k, which only contains the effect of quantum fluctuations with momenta greater than k.[2] By changing k, we can interpolate smoothly between the microscopic action $\Gamma_{k\to\infty}$ and the full quantum effective action $\Gamma_{k\to 0}$. The scale dependence of the flowing action is then given by the functional renormalization group equation (FRGE) [24,25]:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}(\Gamma_k^{(2)} + R_k)^{-1}\partial_t R_k. \tag{53}$$

Herein, $\Gamma_k^{(2)}$ denotes the second functional derivative of the flowing action with respect to the fields and constitutes a matrix in field space. The super trace STr includes a summation over all of the discrete indices and the fields, including a negative sign for Grassmann valued fields, i.e., fermions and Faddeev-Popov ghosts. The super trace also includes a summation over the eigenvalues of the Laplacian in the kinetic term. The main technical advantage of the FRGE lies in its one-loop form, which nevertheless takes into account higher-loop effects, as it depends on the full, field-dependent nonperturbative regularized propagator $(\Gamma_k^{(2)} + R_k)^{-1}$ (see [26]). The FRGE has been extensively applied to SU(N) Yang-Mills theories. For further references see [27–31] and, for the application of the FRG to the study of YMC, [32–35].

B. Finding the effective Lagrangian

Solving Eq. (53) exactly is a titanic task. Since we are mainly interested in qualitative and, as much as possible, analytic results, we will deploy some approximations. (We refer the reader interested in the state of the art of a YMC in the FRG framework to the work of Eichhorn, Gies, and Pawlowski [35], in which a numerical extrapolation between the low and high momenta of full propagators was used to compute the gluon condensate.)

First of all, we will replace Γ_k in the rhs of Eq. (53) with the bare action S.[3] Doing so, we are allowed to integrate both sides of the equation:

$$\Gamma_k = -\int \mathcal{L}_{\mathrm{eff}} = -\int \mathcal{W}_k(\theta) = \int dk \frac{1}{2}\mathrm{STr}(S^{(2)} + R_k)^{-1}\partial_t R_k$$
$$= \frac{1}{2}\mathrm{STrLog}(S^{(2)} + R_k) + \mathrm{const.}$$

We select the bare action to be $S = \frac{1}{4}\int \mathrm{d}x F_a^{\mu\nu} F^a_{\mu\nu}$, which corresponds to the UV limit of our effective theory. We will fix the integration constant, requiring the effective action to vanish for vanishing field strength.

To correctly invert the regularized propagator, we need to include in the action a (harmonic) gauge fixing and the associated Faddeev-Popov ghosts:

$$S_{\mathrm{gf}} = \frac{1}{2\alpha}\int \mathrm{d}x \bar{D}_\mu a_\nu^a \bar{D}_\nu a_\mu^a,$$
$$S_{\mathrm{gh}} = \int \mathrm{d}x \bar{D}_\mu \bar{c}_\nu D^\mu c^\nu.$$

In the Landau gauge $\alpha \to 0$, the trace over the gauge field space is restricted to the transverse sector

$$\frac{1}{2}\mathrm{STrLog}(S^{(2)} + R_k) = \frac{1}{2}\mathrm{T}_{\mathrm{rtransv}}\mathrm{Log}(\bar{D}_T^{\mu\nu} + R_k(\bar{D}_T^{\mu\nu})) - \frac{1}{2}\mathrm{T}_{\mathrm{rghost}}\mathrm{Log}(\bar{D}_{gh}^{\mu\nu} + R_k(\bar{D}_{gh}^{\mu\nu})), \tag{54}$$

with operators $\bar{D}_T^{\mu\nu} = \bar{\Box}\delta_{cb}\delta^{\mu\nu} + g\bar{F}^{a\mu\nu} f_{abc}$ and $\bar{D}_{gh}^{\mu\nu} = \eta^{\mu\nu}\bar{\Box}$, in which g is the YM coupling and the barred quantities are made of background fields; for more details on the actions and its variations, see Appendix A. We will employ the simplest possible regulator functions (masslike cutoff)

$$R_k(\mathcal{D}) = k^2, \tag{55}$$

in both the transversal gauge and the ghost sector $\mathcal{D} \to \bar{D}_T^{\mu\nu}, \bar{D}_{gh}^{\mu\nu}$.

We can employ an integral representation[4] of the logarithm in order to find an exactly summed expression,

$$\mathrm{Log}(A) = -\int_0^\infty \frac{ds}{s} e^{-As}. \tag{56}$$

A wise choice of the background allows us to perform the traces as sums over the eigenvalues of the chosen kinetic operators. In general, the eigenvalues of the operator $\bar{D}_T$ are not known, and the only known stable covariantly constant background is the self-dual that was already employed in the FRG context in [35] (the key properties needed for this work are summarized in Appendix B). The effective Lagrangian is finally recovered to be

[2] Notice that k is, in principle, different from the scale κ defined above. The latter is indeed the one-loop renormalization scale, while the former is the cutoff defining scale of the FRG flow. For dimensional reasons, since these two represent the only relevant scale in the YM sector, they turn out to be proportional to each other.

[3] This kind of approximation is usually called "one loop" in the FRG literature because of the similarities between the FRGE and the standard one-loop effective action.

[4] One should actually be more careful with the definition of the integral. A more precise formula is the following:

$$\mathrm{Log}(A) = -\lim_{\epsilon\to 0}\int_\epsilon^\infty \frac{ds}{s}(e^{-As} - e^{-s}).$$

Nevertheless, we will use the naive representation and implicitly regularize the final expression.

$$\mathcal{L}_{\rm eff} = \frac{g^2B^2}{2\pi^2}\int_0^\infty \frac{ds}{s}\sum_{m,n=0}^{\infty}(e^{-2gB(n+m)+k^2} + e^{-2gB(n+m+2)+k^2} - e^{-2gB(n+m+1)+k^2}) = \frac{g^2B^2}{2\pi^2}\int_0^\infty \frac{ds}{s}e^{-k^2 s}\left(\frac{1}{4\sinh^2(gBs)}+1\right). \quad (57)$$

The "magnetic field" B is the only variable of the self-dual background, which is related to the order parameter through $\theta = B^2$. The next step is to remove the constant part from the integral that gets a contribution from the lowest order expansion of the sinh. We then perform a change of variable and reshuffle (57) as

$$\mathcal{L}_{\rm eff} = \mathcal{W}_k(\theta) = \frac{g^2B^2}{2\pi^2}\int_0^\infty \frac{ds}{s}e^{-\frac{k^2}{gB}s}\left(\frac{1}{4\sinh^2(s)}+1-\frac{1}{4s^2}\right) = \frac{g^2\theta}{2\pi^2}\int_0^\infty \frac{ds}{s}e^{-s(\frac{k^4}{g^2\theta})^{1/2}}\left(\frac{1}{4\sinh^2(s)}+1-\frac{1}{4s^2}\right). \quad (58)$$

The asymptotic behavior for the small coupling constant g of the integral is reproduced exactly at the lowest order and matches the one-loop effective action. Furthermore, the (unique) nontrivial minimum for (58) is found to be at $\frac{g^2\theta_0}{k^4} \approx 0.361$, as can be read out from Fig. 1.

C. FRG improved YMC Lagrangian

In order to check on whether the effective Lagrangian calculated in (58) and derived for an SU(2) YM theory can actually explain dark energy, we need to review whether our example satisfies the properties that we discussed in Sec. II A.

(1) From Fig. 1, it is evident that the function (58) has a nonzero global minimum. The exact position of this minimum can be computed numerically and, in terms of dimensionless quantities, is found to be $\frac{g^2\theta_0}{k^4} \approx 0.361$.

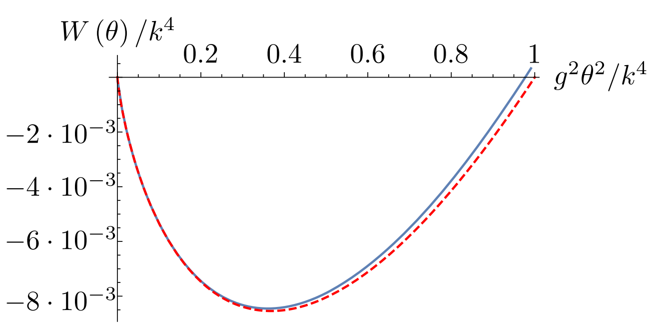

FIG. 1. Plot of the function (58) (the blue solid line) and the one loop (12) (the red dashed line). Notice the presence of a nonzero global minimum for $\frac{g^2\theta_0}{k^4} \approx 0.361$.

(2) It is possible to compute the asymptotic expansion of $\mathcal{W}_k(\theta)$ for small values of the YM coupling constant g. In this limit we are able to reproduce the one-loop result derived by Savvidy [22]: if we perform the Taylor expansion of $1/\sinh^2(s) = \frac{1}{s^2} - \frac{1}{3} + O(s^2)$, we indeed find

$$\mathcal{W}_k(\theta) = \frac{g^2\theta}{2\pi^2}\int_0^\infty \frac{ds}{s}e^{-s(\frac{k^4}{g^2\theta})^{1/2}}\left(-\frac{1}{12}+1\right) + \dots = \frac{11}{24\pi^2}g^2\theta\int_0^\infty \frac{ds}{s}e^{-s(\frac{k^4}{g^2\theta})^{1/2}} + \dots = \frac{1}{2}\frac{11}{24\pi^2}g^2\theta\,{\rm Log}\left(\frac{k^4}{g^2\theta}\right). \quad (59)$$

(3) $\mathcal{W}_k(\theta)$ shows an asymptotic behavior that is at least linear in θ. This means that, for $\theta \gg k^4$, the exponential in the integral tends to 1 and the only θ dependence is the overall one, namely,

$$ {\rm Log}(\mathcal{W}_k(\theta)) = {\rm Log}(\theta) + O(\theta).$$

The condition $\frac{d\ln\mathcal{W}}{d\ln\theta} \to 1$ then follows immediately.

In the following sections, we show in detail that the YMC described by our toy model evolves from a radiation-like component to a dark energy one.

1. A noninteracting YMC model

We have already shown that, in the case of a non-interacting YMC model, the condensate evolution equation is implicitly solvable, and also that the solution $\theta(a)$ can be obtained by inverting the equation

$$\sqrt{\theta}\mathcal{W}_k'(\theta) = \alpha a^{-2}, \quad (60)$$

where α is a coefficient of proportionality that depends on the initial conditions. We can then fix the renormalization scale k by comparing the "predicted" YMC fractional energy density with the measured dark energy fractional energy density ($\Omega_\Lambda = 0.735$), finding for a big range of initial conditions $k \approx 3.2h^{1/2}10^{-3}$ eV. This energy scale, as was already noted in [13–15], is low compared to typical energy scales in particle physics, such as the QCD and the weak-electromagnetic unification, and this prevents the identification of this YMC as a condensate of some SM gauge fields. Then we can study the evolution of the YMC energy density and its EOS for different values of $\alpha = (10^{-7}, 10^{-5})$, still obtaining the same asymptotic values. The results are summarized in Fig. 2.

2. $Q \propto \rho_{\rm YMC}$

For YMC models enjoying an interaction proportional to the YMC energy density, we have already discussed in

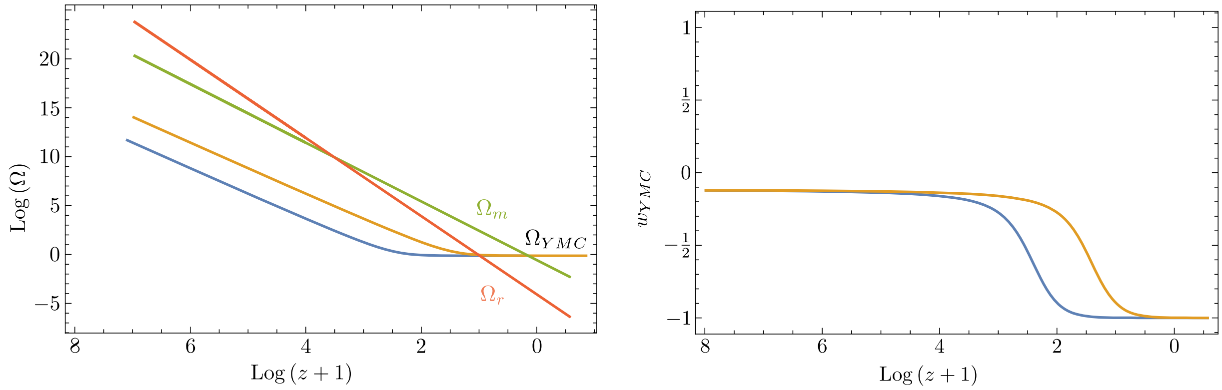

FIG. 2. In the free YMC dark energy model, the evolution of the YMC fractional energy density (left panel) and EOS (right panel) for the models with different initial conditions ($\alpha = 10^{-7}$ on the left and $\alpha = 10^{-5}$ on the right).

Sec. II D 1 the equation defining the existence of a fixed point—derived from the differential equation evolution system (34) and (35)—and the condition to be imposed on the coupling parameter α in order to characterize an attractor solution. Here, we report numerical computations on the position of the fixed point and on the value of the critical exponents at the fixed point. The fixed point exists for every positive value of the coupling parameter and is always attractive, as shown in Fig. 3.

IV. CONCLUDING REMARKS

The query whether the YMC may actually provide a consistent and physically reliable model for dark energy, along the lines of the analysis first developed in Refs. [13–15], has been addressed in this paper within the attempt of finding support to this theoretical hypothesis in the nonperturbative approach to the calculation of the Yang-Mills effective action.

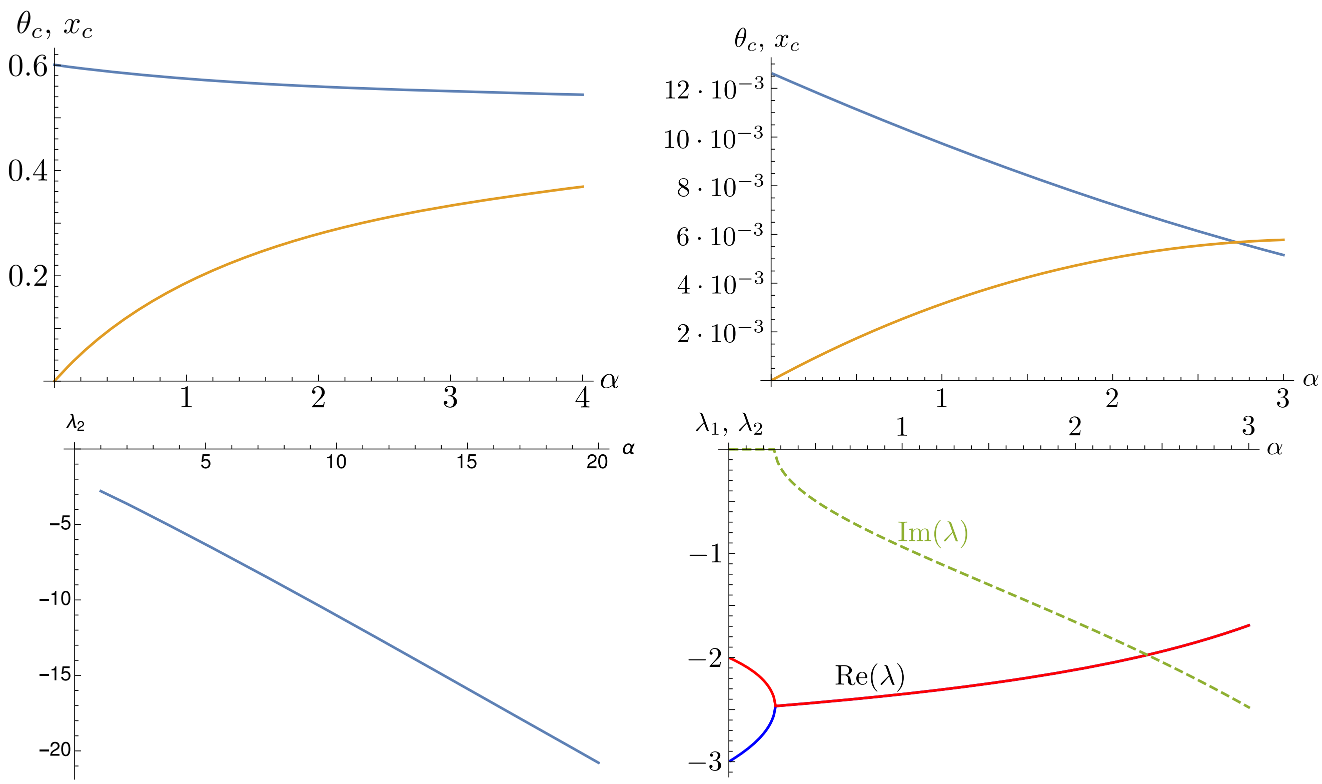

FIG. 3. For the coupled YMC dark energy models, the plot against the coupling parameter α of the critical order parameter and matter density θ_c, x_c (the top panels) and of the critical exponents λ_1, λ_2 at the fixed point (the bottom panels). In the bottom panel on the left is reported the case in which $Q \propto \rho_{\rm YMC}$, and on the right the case in which $Q \propto \rho_{\rm YMC} + \rho_{\rm m}$.

In particular, we have first discussed the properties that the effective Lagrangian $\mathcal{W}(\theta)$ must possess in order to drive the Universe toward a cosmological dark energy phase. If a condensate exists, i.e., if the nonperturbative effective action has a minimum in the YMC order parameter θ, then the model can actually reproduce the dark energy behavior of the expanding Universe at small redshift. If the effective action scales at least like the bare Yang-Mills action for the high-energy scale, at high redshifts it entails the EOS of radiation. Moreover, internal consistency also requires that perturbative one-loop results must still be recovered in the appropriate asymptotic limit.

We have then focused on the particular example provided of the SU(2) Yang-Mills bare action. We have deployed nonperturbative techniques mutated from the FRG method, in order to show that for the SU(2) Yang-Mills a nontrivial minimum indeed exists, and that the high-energy scale regime approaches known perturbative results and yields radiation dominated EOS. Although our conclusions thus far can only be based on this particular example, this successful check of the requirements necessary for having a viable YMC dark energy model seems to us to be extremely encouraging in strengthening the possibility that YMC can work as a model for dark energy.

The improvement of the nonperturbative techniques may allow us in the future to extend the present analysis to the cases of $SU(N)$ gauge groups or, more generally, to other classes of Lie groups. For this purpose, we may either ask ourselves whether condensation can work for any $SU(N)$ group, or whether consistency of the model necessarily predicts a maximal value of N in order for the mechanism to work. Not unconnected to these questions comes the query on the relation between the Yang-Mills fields involved, which are necessary in order for the condensate to form, and the elementary-particle fields advocated for to explain dark matter. Indeed, it would be tempting to try to link YMC dark energy models to other models for dark matter, such as the ones referred to in the literature [36–41] as mirror standard model for dark matter.

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APPENDIX A: ACTION AND VARIATIONS

We consider the bare action of the Yang-Mills theory

$$\mathcal{S} = \frac{1}{4}\int \mathrm{d}x F_a^{\mu\nu} F_{\mu\nu}^a.$$

We split the field A_μ into a background plus a fluctuation $A_\mu = \bar{A}_\mu + a_\mu$. The quadratic part of this action in the fluctuation field is

$$\delta^2 S = \frac{1}{2}\int \mathrm{d}x(\bar{D}_\mu a_\nu^a \bar{D}_\mu a_\nu^a - \bar{D}_\mu a_\nu^a \bar{D}_\nu a_\mu^a + g\bar{F}^{a\mu\nu} f_{abc} a_\nu^b a_\mu^c)$$
$$= \frac{1}{2}\int \mathrm{d}x a_\nu^b (\delta_{cb}\bar{\Box} + \bar{D}^\mu \bar{D}^\nu \delta_{cb} + g\bar{F}^{a\mu\nu} f_{abc}) a_\mu^c,$$

where the bar quantities are made out of the background fields. To compute the inverse propagator, we need to add also a gauge fixing action and the corresponding Faddeev-Popov ghost action:

$$S_{gf} = \frac{1}{2\alpha}\int \mathrm{d}x \bar{D}_\mu a_\nu^a \bar{D}_\nu a_\mu^a, \qquad S_{gh} = \int \mathrm{d}x \bar{D}_\mu \bar{c}_\nu D^\mu c^\nu.$$

The FRGE splits into the trace over the transverse part and the longitudinal part of the connection field and the ghost sector:

$$\frac{1}{2}\mathrm{STr}(S^{(2)} + R_k)^{-1}\partial_t R_k$$
$$= \frac{1}{2}\mathrm{Tr}_T(S^{(2)} + R_k)^{-1}\partial_t R_k + \frac{1}{2}\mathrm{Tr}_L(S^{(2)} + R_k)^{-1}\partial_t R_k$$
$$- \mathrm{Tr}_{gh}(S_{gh}^{(2)} + R_k)^{-1}\partial_t R_k$$
$$= \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k}{\bar{D}_T^{\mu\nu} + R_k} + \frac{1}{2}\mathrm{Tr}\frac{\alpha\partial_t R_k}{\bar{D}_L^{\mu\nu} + \alpha R_k} - \mathrm{T}_{\mathrm{rghost}}\frac{\partial_t R_k}{\Box + R_k}.$$

Calculations are simplified by the fact that, in the Landau gauge $\alpha \to 0$, the longitudinal trace can be dropped out completely.

APPENDIX B: SELF-DUAL FIELD CONFIGURATION

We may choose a background field configuration that allows us to project onto the effective potential $\mathcal{W}(\theta)$. Hence, a covariantly constant field strength with $D_\mu F^{\mu\nu} = 0$ suffices. Since the spectrum of the Laplace-type operators, like $D_T^{\mu\nu} = \bar{\Box}\delta_{cb}\delta^{\mu\nu} + g\bar{F}^{a\mu\nu} f_{abc}$—or at least the heat-kernel trace for these operators—has to be known, we have a limited choice in the possible background field configurations. To avoid problems with tachyonic modes, which indicate the instability of a background, we project onto the only known stable covariantly constant background, which is self-dual, namely, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\rho\sigma} F_{\rho\sigma} = F_{\mu\nu}$. Then we set $F_{\mu\nu} = 0$. Apart from $F_{01} = F_{23} \equiv B = \mathrm{const}$, all other nonzero components follow from the antisymmetry of the field-strength tensor. Because of the enhanced symmetry properties connected to the self-duality, zero modes exist. These carry important information and have to be regularized carefully since the standard choice for R_k is zero on the zero mode subspace:

$$\begin{aligned}\mathrm{spec}(\overline{D}_T^{\mu\nu}) &= 2gB_l(n+m+2) \quad \text{with} \quad n,m\in\mathbb{N}\,\text{and with multiplicity}\,2\,\text{in four dimensions},\\ &= 2gB_l(n+m) \quad \text{with} \quad n,m\in\mathbb{N}\,\text{and with multiplicity}\,2\,\text{in four dimensions.}\\ \mathrm{spec}(\square) &= 2gB_l(n+m+1) \quad \text{with} \quad n,m\in\mathbb{N},\end{aligned}$$

with a degeneracy factor $\frac{B^2}{2\pi^2}$. Herein, $B_l = |\nu_l|B$ and ν_l is given by $\nu_l = \mathrm{spec}\{(T^a n^a)^{bc}|n^2=1\}$, with the generators of the adjoint representation $T^a$and therefore for a general gauge group depends on the direction of the unit vector n.

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