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# Majorana neutrinos with point interactions

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We propose a realistic model with Majorana neutrinos in the framework of unifying the three generations of fermions by point interactions in an extra dimension. This model can simultaneously explain the origin of fermion generations, fermion masses and mixing, and the smallness of the masses of Majorana neutrinos. We show that there are two mechanisms working together to suppress the neutrino masses significantly, so we do not have to introduce a very large extra-dimension cutoff scale. One is the type-I seesaw mechanism and the other is the overlap integration of localized lepton wave functions. A singlet scalar with an exponential-like vacuum expectation value plays a central role in these two mechanisms. For consistency in this model we introduce a $U(1)'$ gauge symmetry, which will be broken by the singlet scalar. Parameters of our model can fit the masses and flavor mixing data well. These parameters can also predict all $CP$ violating phases including the Majorana ones and accidentally rescue the proton from decay.

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## I. INTRODUCTION

The recent discovery of the Higgs boson is a great success for the Standard Model (SM) of particle physics. In the SM, the masses of weak gauge bosons and fermions are generated by the Higgs mechanism, which predicts the existence of a $CP$-even scalar particle, and finally this only scalar boson was discovered at the Large Hadron Collider (LHC) in 2012 [1,2].

However, many people believe that the SM should not be the finale of particle physics. One of the reasons is that it cannot explain the large hierarchy of fermion masses. In the SM, all fermion masses, mixing angles and $CP$ phases are free parameters. If one looks at the mass spectrum of fermions, one will find a significant hierarchy between different generations. The hierarchy between quark sector and lepton sector is even worse.

In the original version of the SM, the neutrino masses are assumed to be zero. However, to explain the oscillation phenomena observed in experiments, the neutrinos have to be massive. Similar to the way used in the SM to give fermions masses, it can make neutrinos massive by introducing right-handed neutrinos which couple to the Higgs field through Yukawa terms. But this way is quite unnatural due to the large hierarchy. A cosmological observation from Planck set a 0.23 eV upper bound for the sum of the three generations of neutrinos [3]. It leads to about 11 order of magnitude hierarchy between the Yukawa coupling of top quark and the neutrinos. This unnaturalness indicates to us a strong motivation to go beyond the SM.

There are three types of canonical seesaw mechanisms to explain the smallness of neutrino masses. The type-I seesaw introduces right-handed neutrinos coupled with the left-handed leptons through Yukawa interactions, and then the Majorana masses of the left-handed neutrinos will be generated by a higher dimensional operator and be suppressed by the heavy Majorana masses of the right-handed ones [4–6]. The type-II seesaw introduces triplet scalars coupled with the left-handed lepton doublets, and the vacuum expectation value (VEV) of the scalar will be suppressed by its large quadratic masses [7–9]. The type-III seesaw is similar to the type-I, but it introduces heavy triplet leptons [10]. All these mechanisms usually need a high seesaw energy scale, for example the grand unification theory (GUT) scale, to suppress the induced Majorana masses of the left-handed neutrinos. Note that there is another popular mechanism that can generate the higher dimensional operator for neutrinos. Rather than generating the operator at tree level, people try to generate the mass for neutrino radiatively through one-loop [11–13], two-loop [14–16] and even three-loops [17].

Besides the seesaw mechanism and radiative generation, an alternative way to explain the mass hierarchy naturally is to enlarge the spacetime dimension. One interesting case is the thick wall model [18], in which fermions have Gaussian wave functions of the fifth dimension coordinate and their locations are determined by their five-dimensional (5D) masses. When two fermion wave functions are separated slightly, their overlap integration with the Higgs VEV profile will be suppressed exponentially, then a large hierarchy structure between fermions can be naturally obtained. Another fascinating case is the Randall-Sundrum model [19,20], in which right-handed neutrinos localize near a hidden brane, while the other fermions and the Higgs field are confined on a visible brane. Thus the right-handed neutrinos interact with the other fields weakly, and they only have tiny masses.

Recently, a new extra-dimension model [21,22] was proposed to unify the three fermion generations. The model introduces 5D fermion fields living in an extra-dimensional interval or circle with several point interactions (i.e.

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0-thickness branes). For each 5D fermion, there are three independent modes between branes. They behave as three generations, and the hierarchy between generations is achieved by coupling the 5D fermion field to a scalar field which has an exponentially increasing extra-dimensional coordinate-dependent VEV. This specific VEV can be generated by imposing Robin's boundary conditions on the scalar at two boundaries of the fifth dimension (see more details on the phase structures in [23]). In addition, a twisted boundary condition is imposed on the Higgs doublet to create $CP$ violating phases for both quark and lepton sectors [24]. In Ref. [22], a 5D singlet neutrino field (which has a right-handed chiral neutrino 0-mode) is introduced to construct Dirac mass terms for neutrinos, and the smallness of neutrino masses is obtained from a proper arrangement of the point interaction positions. A stringent constraint on the model with a set of fitted parameters is to suppress the proton decay rates. By a rough analysis with some baryon number violating dimension-8 operators, the cutoff $\Lambda \sim L^{-1}$ is estimated to be as large as the GUT scale ($10^{15}$ GeV).

In this paper, we discuss a possibility to extend the model of Ref. [22] to a Majorana neutrino case and to avoid the large cutoff scale. To implement this, we need a Majorana mass term of the singlet neutrinos. A naive trial is to write down an explicit Majorana mass term for the singlet neutrino fields and their charge conjugation. However, it fails since the equations of motion for the singlet neutrinos no longer respect the so-called quantum mechanical supersymmetry (QMSUSY) which is important for acquiring chiral zero modes [18,25]. The existence of a Majorana mass term implies that the lepton number is no longer a conserved quantity, and thus a dimension-7 effective operator $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$ may appear in the Lagrangian in principle. Here $L(x,y)$ is the 5D lepton doublet field, $H(x,y)$ is the 5D Higgs doublet, and the power counting is achieved in 5D spacetime. But this effective operator can induce large Majorana masses of the left-handed neutrinos after the electroweak symmetry breaking. To avoid large neutrino masses which violates the experimental bounds, it requires either a high cutoff scale or a very small coupling constant for this term.

To overcome this problem and to forbid the harmful explicit Majorana masses terms at the same time, we introduce a new $U(1)'$ gauge symmetry. If we let the singlet neutrino field $N_R$ and the combination $H^\dagger\sigma^2 L^c$ be $U(1)'$ charged, *none* of these two annoying terms, $\overline{N_R^c}N_R$ and $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$, can survive, since they are doubly $U(1)'$ charged. In other words, we increase the symmetry of the model to prohibit the unwanted operators like $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$.

We need a natural way to realize the experimentally acceptable Majorana masses for neutrinos. Consider another dimension-7 operator $(\Phi^*)^2\overline{N_R^c}N_R + \mathrm{H.c.}$, which is available if the $U(1)'$ charge of the singlet scalar $\Phi$ is assigned to be the same as that of the singlet neutrino $N_R$. Obviously, this term can contribute to a Majorana mass for the right-handed neutrino 0-mode when the scalar $\Phi$ obtains a nonzero VEV and break the $U(1)'$ gauge symmetry. The 5D scalar $\Phi(x,y)$ is initially introduced to realize the hierarchy of the three generations of quarks and leptons, and it is imposed on the Robin's boundary condition to get a VEV, $\langle\Phi(y)\rangle$, as an exponential-like function of the extra-dimensional coordinate $y$. This VEV has the effect of killing two birds with one stone. If the 0-thickness branes' positions of singlet neutrino are chosen appropriately, that is, if the third generation singlet neutrino wave function has a big overlap with the large value side of $\langle\Phi(y)\rangle$, it can obtain a mass which is much larger than the Dirac masses. This large mass turns on the type-I seesaw mechanism to lower the neutrino masses further.

To make our model self-consistent, we set the $U(1)'$ charge of each field agreeing with the anomaly free conditions [26]. We also consider the constraint from the proton decay. By some simple analysis with the dimension-8 baryon number violating operators, we see that for our best-fit parameters the proton will not decay. So it is not necessary to let the cutoff energy be the GUT scale in this model.

An outline of the paper is as follows. In Sec. II, we will summarize some key elements of the model and building a realistic model in the framework. We also discuss the problems of introducing an explicit Majorana mass term. In Sec. III, we discuss how to generalize the model to include Majorana neutrinos and how the seesaw mechanism works with a few TeV extra-dimension energy scale. We will also fit the data of leptons and do some discussion. Section IV is a summary. In Appendixes A, B, and C, we supply some mathematical details of the discussion in Sec. II.

## II. THE MODEL

To begin with, let us summarize some general setups of this model (with some mathematical details reviewed in Appendix A) [21,22]:

(i) The spacetime is extended by a finite size of space-like extra dimension, i.e. an interval or a circle. The mode expansion is made as usual and the lowest modes, i.e., the zero modes, are regarded as the SM particles. The mass gap between the first Kaluza-Klein (KK) modes and the zero modes is roughly the inverse of the fifth dimensional size. In many extra-dimension models, the mass scale is at least around the energy scale of the LHC experiment.

(ii) In the free field limit, there is a quantum mechanical supersymmetry (QMSUSY) between the left-handed and right-handed components of 5D fermion [18,25,27]. This symmetry ensures that the left-handed and right-handed modes at the same level have equal masses. Thus, their 4D parts can be separated from the fifth-dimension-coordinate-dependent parts, and can form a Dirac fermion satisfying the 4D Dirac equation. In particular, for the zero mode, the symmetry together with the

Dirichlet boundary conditions implies that one of the chiral spinors should vanish and the other one is massless. This is the method of generating chiral zero modes.

(iii) An important ingredient for unifying generations is the point interaction [21,22], which can be regarded as a Delta-function-like interaction. This specific interaction is located at a point in the fifth dimension and results in the Dirichlet boundary condition for the 5D fermion. If we introduce two interacting points, then they will separate the interval at extra dimension into three pieces. The modes living in different pieces are independent from each other although they come from the same 5D fermion field. These different modes can be regarded as different generations.

(iv) To achieve the hierarchy among generations, a singlet scalar field $\Phi(x,y)$ is introduced to couple with 5D fermions. A Robin's boundary condition on the 5D scalar will force its VEV$\langle\Phi(y)\rangle$ to be $y$-dependent as

$$\langle\Phi(y)\rangle = \frac{\nu}{\mathrm{cn}(\sqrt{\frac{\lambda}{2}}\frac{\mu}{k}(y-y_0),k)}, \tag{1}$$

where the function $\mathrm{cn}(x,k)$ is the Jacobi elliptic function of $x$ with index $k$, and $k$, $\mu$, $\nu$ are defined as

$$\begin{aligned} k^2 &= \tfrac{\mu^2}{\mu^2+\nu^2} \\ \mu^2 &= \tfrac{M^2}{\lambda}\left(1+\sqrt{1+\tfrac{4\lambda|Q|}{M^4}}\right) \\ \nu^2 &= \tfrac{M^2}{\lambda}\left(\sqrt{1+\tfrac{4\lambda|Q|}{M^4}}-1\right) \end{aligned} \tag{2}$$

with $Q, y_0$ being constants of integration determined by $L_\pm$. A study of this singlet scalar with Robin's boundary condition can be found in Ref. [23]. An important result in their study is that $\Phi(x,y)$ can couple with gauge fields corresponding to some group, such as a $U(1)'$ group. This symmetry will break if $L < L_c = \frac{1}{|M|}\tanh^{-1}(\frac{|M|(L_+ + L_-)}{1+M^2L_+L_-})$ [21,23]. Usually we use the condition $M^2 < \frac{1}{L_{\max}}$, $L_{\max} = \max(L_+, L_-)$, which is sufficient but not necessary.

When we proceed to construct a realistic model comparable with experiments, some special settings are also needed [21,22,24]. The requirements are briefly listed as follows:

(1) The fifth dimension needs to be a circle ($S^1$). This is a part of the requirements from the flavor mixing behavior of the SM. It is also consistent with the twisted boundary condition setting of the Higgs doublet.

(2) We need to specify the 5D matter fields with appropriate boundary conditions. In the quark sector, we should introduce an electroweak $SU(2)$ doublet quark $Q(x,y) = (U_L(x,y)D_L(x,y))^T$, and two singlets quarks $U_R(x,y)$ and $D_R(x,y)$. For the doublet $Q$, we use a Dirichlet boundary condition $P_RQ = 0$ at $y = L_0^{(q)} = 0$, $L_1^{(q)}$, $L_2^{(q)}$ so that its zero modes are left handed, while for the singlets $U_R$ and $D_R$, we use Dirichlet boundary conditions $P_LU_R = 0$ at $y = L_0^{(u)}$, $L_1^{(u)}$, $L_2^{(u)}$ and $P_LD_R = 0$ at $y = L_0^{(d)}$, $L_1^{(d)}$, $L_2^{(d)}$ so that their zero modes are right handed. Note that in general $L_i^{(q)}$ are different from $L_i^{(u)}$ and $L_i^{(d)}$. This is necessary for flavor mixing structure. For the lepton sector, the situation is similar to the quark case. We just replace the quark doublet by a lepton doublet and the up and down-type quark singlet by neutrino and charged lepton singlet.

(3) We need a Higgs doublet $H(x,y)$ to couple with fermion fields through Yukawa couplings. Of course it should acquire nonzero VEV $\langle H\rangle$ to break the electroweak symmetry. A special treatment is to impose a twisted boundary condition on $H(x,y)$ as $H(y+L) = e^{i\theta}H(y)$ [24]. This twisted boundary condition will make the VEV $\langle H\rangle$ get $y$ dependent phase as $\langle H(y)\rangle = \frac{v}{\sqrt{2L}}e^{\frac{i\theta}{L}y}$, then its overlap integration with fermions' wave functions will produce $CP$ phases for Cabibbo-Kobayashi-Maskawa (CKM) or Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices.

As an example, the detailed treatment of the quark sector is presented in Appendix B. We also fit the parameters of quark sector independently and list them in Table IV. The fitting will fix the $M$ parameter from the singlet scalar $\Phi$ and the $\theta$ from the Higgs $H$, and they will be regarded as input data for the lepton case.

Before going to the next section to discuss our treatment of the lepton sector. It will be helpful to ask what is wrong if we just write down an explicit Majorana mass term. We will discuss this briefly as follows, and supply more details in Appendix C.

One problem of this naive trial is that Majorana mass term will modify the equation of the motion for the 5D fermion. This modification breaks the QMSUSY between the left-handed and right-handed components in the equation of motion (E.O.M). As we mentioned previously, generating chiral zero modes rely on this symmetry.

Another problem with this naive trial is that since we are going to break the lepton number conservation explicitly, then in principle we should also include an operator as $\bar{L}\sigma^2H^*H^\dagger\sigma^2L^c$ which has the same dimension with the terms we used to generate the Dirac masses for leptons. After the Higgs acquires a nonzero VEV, this operator will generate Majorana masses for the left-handed neutrino zero modes. Then a fine-tuning is needed when we diagonalize the neutrino mass matrix to obtain sub-eV masses.

## III. THE LEPTON SECTOR

### A. $U(1)'$ symmetry and type-I seesaw

For the lepton sector, we introduce an $SU(2)$ doublet $L = (N_L(x,y), E_L(x,y))^T$, and singlets $N_R(x,y)$, $E_R(x,y)$.

When we consider the structure of our model, the lepton number is not necessary to be preserved. The most famous model which violate lepton number is the type-I seesaw [7]. In type-I seesaw a Majorana mass term for the right-handed neutrino is introduced. If the Majorana mass $M_R$ is extremely large compared to the Dirac mass $m_D^{(\nu)}$, then after diagonalizing the mass matrix, a mass for the three lightest neutrinos taking the form $-m_D^{(\nu)} M_R^{-1} m_D^{(\nu)T}$ will be suppressed significantly. But as we discussed in Sec. II, an explicit Majorana mass term is not allowed to exist. We will assign a $U(1)'$ charge to $N_R$ to forbid such a troublesome term to keep the chiral 0-mode, and then use the VEV of the scalar $\Phi$ to create the Majorana masses for the right-handed neutrino 0-mode.

As we have mentioned in Sec. II, the $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$ operator will bring us a problem of fine-tuning. To solve this problem it will be forbidden by the $U(1)'$ symmetry if we let $\bar{L}i\sigma^2 H^*$ be charged. All of these indicate that it would be better to add the $U(1)'$ symmetry into the model. Then to justify the model, we should put some constraints to the undetermined $U(1)'$ charges.

The gauge group in our model is now $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$. Let us denote the representation of all left-handed zero modes in the form $(N_{c,i},\ N_{w,i},\ Y_i,\ Q_i')$, where $N_{c,i}$ and $N_{w,i}$ denote the dimensions of $SU(3)$ and $SU(2)$ representation (conjugated representation with a bar) of the $i$th field, while $Y_i$ and $Q_i'$ denote the $U(1)$ hypercharge and $U(1)'$ charge of the $i$th field. $N_{c,i}, N_{w,i}, Y_i$ for each type of field are just the same as in the Standard Model. $Q_i'$ s for each type of field are unknown variables and will be determined later. We list the representations for fermions in Table I. Now the covariant derivatives for each field are

$$\begin{aligned}
D_N^{(Q)} &= \partial_N - ig_s G_N^i t^i - igW_N^a T^a - i\frac{1}{6}g' B_N - iQ_q' g_c C_N \\
D_N^{(U)} &= \partial_N - ig_s G_N^i t^i - i\frac{2}{3}g' B_N + iQ_u' g_c C_N \\
D_N^{(D)} &= \partial_N - ig_s G_N^i t^i + i\frac{1}{3}g' B_N + iQ_d' g_c C_N \\
D_N^{(L)} &= \partial_N - igW_N^a T^a + i\frac{1}{2}g' B_N - iQ_l' g_c C_N \\
D_N^{(N)} &= \partial_N + iQ_n' g_c C_N \\
D_N^{(E)} &= \partial_N + ig' B_N + iQ_e' g_c C_N \\
D_N^{(H)} &= \partial_N - igW_N^a T^a - i\frac{1}{2}g' B_N - iQ_h' g_c C_N \\
D_N^{(\Phi)} &= \partial_N - iQ_\phi' g_c C_N,
\end{aligned} \tag{3}$$

where $C_N$ is the gauge field corresponding to $U(1)'$ and $g_c$ is the gauge coupling. There are six constraints of $Q_i'$ come from the consideration of anomaly free [26]. They are as follows

TABLE I. Gauge group representations for fermions.

| Fields | $q$ | $u_R^c$ | $d_R^c$ |
|---|---|---|---|
| Representations | $(3,2,1/6,Q_q')$ | $(\bar{3},1,-2/3,Q_u')$ | $(\bar{3},1,1/3,Q_d')$ |
| Fields | $l$ | $\nu_R^c$ | $e_R^c$ |
| Representations | $(1,2,-1/2,Q_l')$ | $(1,1,0,Q_n')$ | $(1,1,1,Q_e')$ |

$$\begin{aligned}
2Q_q' + Q_u' + Q_d' &= 0 \\
3Q_q' + Q_l' &= 0 \\
6\left(\frac{1}{6}\right)^2 Q_q' + 3\left[-\left(\frac{2}{3}\right)^2 Q_u' + \left(\frac{1}{3}\right)^2 Q_d'\right] & \\
+2\left(-\frac{1}{2}\right)^2 Q_l' + Q_e' &= 0 \\
6Q_q'^3 + 3[Q_u'^3 + Q_d'^3] + 2Q_l'^3 + Q_e'^3 + Q_n'^3 &= 0 \\
6\cdot\frac{1}{6}Q_q'^2 + 3\left[-\frac{2}{3}Q_u'^2 + \frac{1}{3}Q_d'^2\right] + 2\left(-\frac{1}{2}\right)Q_l'^2 + Q_e'^2 &= 0 \\
6Q_q' + 3[Q_u' + Q_d'] + 2Q_l' + Q_e' + Q_n' &= 0.
\end{aligned} \tag{4}$$

It seems that we have six equations for six variables, but actually only four of them are independent. We rewrite $Q_i'$ s in terms of $Q_l'$ and $Q_e'$ as follows:

$$\begin{aligned}
Q_q' &= -\tfrac{1}{3}Q_l' \\
Q_u' &= -\tfrac{2}{3}Q_l' - Q_e' \\
Q_d' &= \tfrac{4}{3}Q_l' + Q_e' \\
Q_n' &= -2Q_l' - Q_e'.
\end{aligned} \tag{5}$$

Then when we choose a set $(Q_l', Q_e')$, all the other variables are determined. For our purpose, we will impose more theoretical constraints on $Q_i'$ s. One is that we need Yukawa terms as

$$\begin{aligned}
&\Phi\bar{Q}(i\sigma^2 H^*)U_R, \quad \Phi^*\bar{Q}HD_R, \\
&\Phi\bar{L}(i\sigma^2 H^*)N_R, \quad \Phi^*\bar{L}HE_R,
\end{aligned} \tag{6}$$

to be gauge invariant. Assign a $U(1)'$ charge $Q_h'$ to $H$ and $Q_\phi'$ to $\Phi$, and using (5) finally we find the only constraint is

$$Q_l' + Q_e' - Q_h' + Q_\phi' = 0. \tag{7}$$

Another important constraint is to let $Q_\phi' = Q_n'$ so that $\Phi^2\overline{N_R^c}N_R$ is gauge invariant, or let $Q_\phi' = -Q_n'$ so that $\Phi^{*2}\overline{N_R^c}N_R$ is gauge invariant. Then we replace $Q_\phi'$ by $\pm Q_n'$ in (7) and use (5), we obtain $Q_h' = -Q_l'$ for $\Phi^2\overline{N_R^c}N_R$ or $Q_h' = 3Q_l' + 2Q_e'$ for $\Phi^{*2}\overline{N_R^c}N_R$. Remember that we want $\bar{L}(i\sigma^2 H^*)$ to be $U(1)'$ charged and it requires that $Q_h' \neq -Q_l'$, so only $Q_h' = 3Q_l' + 2Q_e'$ corresponding to $\Phi^{*2}\overline{N_R^c}N_R$ is allowed. Of course, we should have $Q_n' \neq 0$ to kill the explicit Majorana mass term for singlet neutrino and this requires that $Q'_e \neq -2Q'_l$. The other constraints may come from experimental considerations, for example in Ref. [28], the authors claim that in the hadron collider experiment, from the parametrization in their Eq. (3.8), once the new gauge boson is found in the dilepton decay, one can measure its mass and map it onto the $c_u - c_d$ plane, where $c_u$ and $c_d$ are parameters depending on the charge $Q'_q$ and $Q'_{u,d}$ relatively. Then the gauge coupling for different models with particular charge assignments can be fixed. On the other hand, in this model the gauge coupling is related to the mass of the gauge boson [in Eq. (22), the other parameters should be fixed by fitting the masses and flavor mixing data], therefore it might be able to check which model predicts the right coupling.

There are still many possible choices of $Q'_i$ s and we only list three interesting candidates which are similar to [29,30]:

(1) $U_R$: $Q'_l = Q'_q = 0$, $Q'_u = 1$, $Q'_d = -1$, $Q'_e = -1$, $Q'_n = 1$, $Q'_h = -2$, $Q'_\phi = -1$.

(2) $U_{B-L}$: $Q'_q = \frac{1}{3}$, $Q'_u = Q'_d = -\frac{1}{3}$, $Q'_l = -1$, $Q'_n = Q'_e = 1$, $Q'_h = -1$, $Q'_\phi = -1$.

(3) $U_\chi$: $Q'_q = \frac{1}{5}$, $Q'_u = \frac{1}{5}$, $Q'_d = -\frac{3}{5}$, $Q'_l = -\frac{3}{5}$, $Q'_n = 1$, $Q'_e = \frac{1}{5}$, $Q'_h = -\frac{7}{5}$, $Q'_\phi = -1$.

The mass term of zero-mode leptons will be generated by

$$\begin{aligned}\mathcal{L}_{yuk} = &-\int dy[\mathcal{Y}^{(n)}\Phi(y)\bar{L}(i\sigma^2 H^*)N_R \\ &+ \mathcal{Y}^{(e)}\Phi^*(y)\bar{L}HE_R + \text{H.c.}] \\ &-\frac{1}{2}\int dy[y^{(m)}\Phi^{*2}\overline{N_R^c}N_R + \text{H.c.}],\end{aligned} \tag{8}$$

where $\mathcal{Y}^{(n)}$, $\mathcal{Y}^{(e)}$ and $y^{(m)}$ are couplings with dimension $-2$. After the $U(1)'$ and $SU(2)\times U(1)$ breaking, two terms in the first line generate Dirac mass matrices for charged leptons and neutrinos and the term in the second line generates a Majorana mass matrix for right-handed neutrinos.

Imposing Dirichlet boundary conditions on fermion fields, the twisted boundary condition on the Higgs doublet and the Robin boundary condition on $\Phi$, we can expand fields in modes and finally obtain their profiles:

$$\begin{aligned}L &= \sum_{i=1}^{3}\begin{pmatrix} f_{l_{iL}^{(0)}}(y)\nu_{iL}^{(0)}(x) \\ f_{l_{iL}^{(0)}}(y)e_{iL}^{(0)}(x)\end{pmatrix} + (\text{KKmodes}), \\ E_R &= \sum_{i=1}^{3} f_{e_{iR}^{(3)}}(y)e_{iR}^{(0)}(x) + (\text{KKmodes}), \\ N_R &= \sum_{i=1}^{3} f_{\nu_{iR}^{(3)}}(y)\nu_{iR}^{(0)}(x) + (\text{KKmodes}), \quad N_R^c = C\overline{N_R}^T, \\ f_{l_{iL}^{(0)}}(y) &= N_{iL}^{(l)}e^{M_L(y-L_{i-1}^{(l)})}\theta(y-L_{i-1}^{(l)})\theta(L_i^{(l)}-y), \\ f_{e_{iR}^{(0)}}(y) &= N_{iR}^{(e)}e^{-M_E(y-L_{i-1}^{(e)})}\theta(y-L_{i-1}^{(e)})\theta(L_i^{(e)}-y), \\ f_{\nu_{iR}^{(0)}}(y) &= N_{iR}^{(\nu)}e^{-M_N(y-L_{i-1}^{(\nu)})}\theta(y-L_{i-1}^{(\nu)})\theta(L_i^{(\nu)}-y),\end{aligned} \tag{9}$$

where $N_{iL}^{(l)}$, $N_{iR}^{(e)}$, $N_{iR}^{(\nu)}$ are normalization constants. Substituting these profiles into (8), we get the Dirac mass matrices and Majorana mass matrix:

$$\begin{aligned}m_{ij}^{(e)} &= \int dy\mathcal{Y}^{(e)}\frac{v}{\sqrt{2L}}\langle\Phi(y)\rangle f_{l_{iL}^{(0)}}(y)f_{e_{jR}^{(0)}}(y)e^{\frac{i\theta y}{L}}, \\ m_{D,ij}^{(n)} &= \int dy\mathcal{Y}^{(n)}\frac{v}{\sqrt{2L}}\langle\Phi(y)\rangle f_{l_{iL}^{(0)}}(y)f_{\nu_{jR}^{(0)}}(y)e^{-\frac{i\theta y}{L}}, \\ M_{R,ij} &= y^{(m)}\int_{L_{i-1}^{(\nu)}}^{L_i^{(\nu)}} dy\langle\Phi(y)\rangle^2 f_{\nu_{iR}^{(0)}}(y)f_{\nu_{jR}^{(0)}}(y).\end{aligned} \tag{10}$$

Obviously, $M_R$ is a diagonal matrix since the integration only involves the profile of $N_R$. Now we write the chiral zero modes in Weyl basis:

$$\nu_{iL}^{(0)} \to \nu_{iL,a}, \qquad e_{iL}^{(0)} \to e_{iL,a}, \qquad \nu_{iR}^{(0)} \to \nu_{iR}^{\dagger,\dot{a}}, \tag{11}$$

where $a$, $\dot{a}$ are indices of Weyl spinors. Then for neutrinos we can represent the mass term as

$$\mathcal{L}_{\text{mass}}^{(\nu)} = -\frac{1}{2}(\nu_{iL,\dot{a}}^\dagger\nu_{iR,\dot{a}}^\dagger)\begin{pmatrix} 0 & m_{D,ij}^{(n)} \\ (m_{D,ij}^{(n)})^T & M_{R,ij}\end{pmatrix}\begin{pmatrix}\nu_{jL}^{\dagger,\dot{a}} \\ \nu_{jR}^{\dagger,\dot{a}}\end{pmatrix} + \text{H.c.} \tag{12}$$

Following Xing's parametrization and discussion [31], we introduce a $6\times 6$ unitary matrix $\mathcal{U}$ to transform the mass eigenstates to flavor states. $\mathcal{U}$ can be decomposed into

$$\mathcal{U} = \begin{pmatrix}\mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0\end{pmatrix}\begin{pmatrix} A & R \\ S & B\end{pmatrix}\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1}\end{pmatrix} = \begin{pmatrix} AV_0 & R \\ U_0SV_0 & U_0B\end{pmatrix}, \tag{13}$$

where $V_0$ and $U_0$ are $3\times 3$ unitary matrices and $A$, $B$, $R$, $S$ are $3\times 3$ matrices under the unitary conditions:

$$\begin{aligned}AA^\dagger + RR^\dagger = BB^\dagger + SS^\dagger = \mathbf{1}, \\ AS^\dagger + RB^\dagger = AR^\dagger + S^\dagger B = \mathbf{0}, \\ A^\dagger A + S^\dagger S = B^\dagger B + R^\dagger R = \mathbf{1}.\end{aligned} \tag{14}$$

We can use $\mathcal{U}$ to diagonalize the mass matrix in (12):

$$\mathcal{U}^\dagger\begin{pmatrix} 0 & m_{D,ij}^{(n)} \\ (m_{D,ij}^{(n)})^T & M_{R,ij}\end{pmatrix}\mathcal{U}^* = \begin{pmatrix}\hat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \hat{M}_N\end{pmatrix}, \tag{15}$$

where $\hat{M}_\nu$ and $\hat{M}_N$ are diagonal matrices: $\hat{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ are very small while $\hat{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ should be very large. Finally we can find approximately

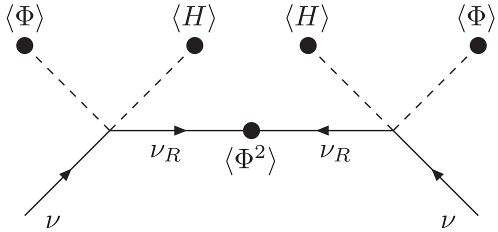


FIG. 1. A diagrammatic description of Eq. (16).

$$\hat{M}_\nu \simeq -V_0^\dagger (m_{D,ij}^{(n)} M_R^{-1} (m_{D,ij}^{(n)})^T) V_0^*. \tag{16}$$

The minus sign can be absorbed into charged lepton basis. Remember that at the beginning of this section, we use the $U(1)'$ symmetry to kill the $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$ dimension-7 operator. When the $U(1)'$ symmetry breaks spontaneously, this term comes back by connecting two Yukawa interactions with an internal Majorana sterile neutrino line. A diagrammatic description of Eq. (16) is shown in Fig. 1. Thus, the smallness of this Majorana mass is natural.

Masses $m_D^{(n)}$ and $M_R$ are determined by model parameters and then we can use Takagi diagonalization with the unitary matrix $V_0$ to diagonalize the symmetric complex matrix $m_{D,ij}^{(n)} M_R^{-1} (m_{D,ij}^{(n)})^T$ [32]. The PMNS matrix $V_0$ can be parametrized as

$$V_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^* c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\widehat{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}, \tag{17}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $\hat{s}_{ij} \equiv e^{i\delta_{ij}} \sin\theta_{ij}$, $\theta_{ij}$s are mixing angles of active neutrino and $\delta_{ij}$s are $CP$ phase angles (three for Majorana neutrinos).

As we know, to suppress the neutrino masses to sub-eV with the seesaw mechanism, we need extremely large $M_R$s. Interestingly, this can be achieved by the exponentially increasing behavior of the VEV $\langle\Phi(y)\rangle$. The matrix element $M_{R,ij}$ can be estimated as follows:

$$\begin{aligned} M_{R,ij} &= y^{(m)} \int_{L_{i-1}}^{L_i} dy \langle\Phi(y)\rangle^2 f_{\nu_{iR}^{(0)}}(y) f_{\nu_{jR}^{(0)}}(y) \\ &\approx y^{(m)} N_{iR}^{(\nu)2} \delta_{ij} \int_{L_{i-1}^{(\nu)}}^{L_i^{(\nu)}} dy v^2 \cosh^2(M(y-y_0)) e^{-2M_N(y-L_{i-1}^{(\nu)})} \\ &\approx \delta_{ij} \frac{y^{(m)}|Q|}{M_N^2 - M^2} \left[ \frac{M_N^2}{M^2} - 1 + \frac{M_N^2}{M^2}\cosh(2M(L_{i-1}^{(\nu)} - y_0)) + \frac{M_N}{M}\sinh[2M(L_{i-1}^{(\nu)} - y_0)] \right. \\ &\quad \left. + e^{-2M_N(L_i^{(\nu)} - L_{i-1}^{(\nu)})} \left( \frac{M_N^2}{M^2} - 1 + \frac{M_N^2}{M^2}\cosh[2M(L_i^{(\nu)} - y_0)] + \frac{M_N}{M}\sinh[2M(L_i^{(\nu)} - y_0)] \right) \right]. \end{aligned} \tag{18}$$

We plot the third element of the diagonal, $M_{R,33}$, as a function in terms of $M_N$ and let $L_3^{(\nu)} \to L$, $L_2^{(\nu)} = 0.65L, 0.7L, 0.75L$ in Fig. 2. This function increases when $L_2^{(\nu)}$ increases or $\tilde{M}_N$ decreases, and we find that if $\tilde{M}_N < 15$, $L_2^{(\nu)} \sim 0.75$ then $M_{R,33}$ can be as large as $500L^{-1} \sim 10000L^{-1}$.

Apparently there are hierarchies $M_{R,11} \ll M_{R,22} \ll M_{R,33}$ and one may worry that some element of matrix $m_D^{(\nu)} M_R^{-1} (m_D^{(\nu)})^T$ is not suppressed by $M_{R,33}$, but by $M_{R,11}$ instead. So we show the explicit expression of $m_D^{(\nu)} M_R^{-1} (m_D^{(\nu)})^T$ as follows:

$$\begin{pmatrix} \frac{m_{11}^2}{M_{R,11}} + \frac{m_{12}^2}{M_{R,22}} + \frac{m_{13}^2}{M_{R,33}} & \frac{m_{11}m_{21}}{M_{R,11}} + \frac{m_{12}m_{22}}{M_{R,22}} + \frac{m_{13}m_{23}}{M_{R,33}} & \frac{m_{11}m_{31}}{M_{R,11}} + \frac{m_{12}m_{32}}{M_{R,22}} + \frac{m_{13}m_{33}}{M_{R,33}} \\ \frac{m_{11}m_{21}}{M_{R,11}} + \frac{m_{12}m_{22}}{M_{R,22}} + \frac{m_{13}m_{23}}{M_{R,33}} & \frac{m_{21}^2}{M_{R,11}} + \frac{m_{22}^2}{M_{R,22}} + \frac{m_{23}^2}{M_{R,33}} & \frac{m_{11}m_{31}}{M_{R,11}} + \frac{m_{12}m_{32}}{M_{R,22}} + \frac{m_{13}m_{33}}{M_{R,33}} \\ \frac{m_{11}m_{31}}{M_{R,11}} + \frac{m_{12}m_{32}}{M_{R,22}} + \frac{m_{13}m_{33}}{M_{R,33}} & \frac{m_{11}m_{31}}{M_{R,11}} + \frac{m_{12}m_{32}}{M_{R,22}} + \frac{m_{13}m_{33}}{M_{R,33}} & \frac{m_{31}^2}{M_{R,11}} + \frac{m_{32}^2}{M_{R,22}} + \frac{m_{33}^2}{M_{R,33}} \end{pmatrix}. \tag{19}$$

Then we see that all terms contain $m_{33}$ (which is assumed to be the largest element of Dirac mass matrix) are suppressed by $M_{R,33}$. Also note that $m_{11}$, $m_{22}$, etc. are usually much smaller than $m_{33}$, so their suppression does not need masses as large as $M_{R,33}$.

In conclusion, thanks to the exponential-like VEV of the scalar, although our scale $L^{-1}$ is only about order of TeV, it is still possible to lower the neutrino mass $m_D^{(\nu)} M_R^{-1} (m_D^{(\nu)})^T$ to sub-eV with the Majorana mass $M_R$.

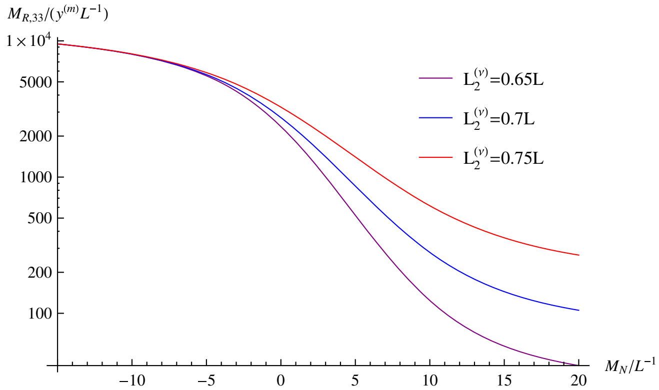


FIG. 2. $M_{R,33}$ vs $M_N$ with $L_3^{(\nu)} \to L$ and $L_2^{(\nu)} = 0.65L$, $0.7L$, $0.75L$.

### B. Numerical results and discussion

Since we have fitted the parameters of the scalar $\Phi$ and $H$ in the quark case (see Appendix B), we set them fixed in the lepton fitting. Although we extend the gauge group in this model, it will not affect the parameters we obtained in the quark case. Note that the parameter $y^{(m)}$ comes into the fitting only in a combination $\frac{\mathcal{Y}^{(n)}}{\sqrt{y^{(m)}}}$, so we will not treat $y^{(m)}$ and $\mathcal{Y}^{(n)}$ separately. In our fitting, we only consider the normal hierarchy of neutrino mass.

The recent experiment data of leptons used in our fitting are listed in the following:

(i) Masses of charged leptons: $m_e = (0.510998928 \pm 1.1 \times 10^{-8})$ MeV, $m_\mu = (105.6583715 \pm 3.5 \times 10^{-6})$ MeV, $m_\tau = (1776.82 \pm 0.16)$ MeV [33].

(ii) Mass squared difference between two generations: $\Delta m_{31}^2 = (2.473 \pm 0.069) \times 10^{-3}$ eV$^2$, $\Delta m_{21}^2 = (7.5 \pm 0.19) \times 10^{-5}$ eV$^2$ [34].

(iii) Mixing angles: $\sin^2\theta_{12} = 0.302 \pm 0.012$, $\sin^2\theta_{23} = 0.413 \pm 0.032$, $\sin^2\theta_{13} = 0.0227 \pm 0.0024$ [34].

Since there are more free parameters than data, we only show one set of the possible parameters. They are listed in Table II. If we assume that $\tilde{y}^{(m)} \sim O(1)$ (a parameter with a tilde means it is scaled by $L$ to be dimensionless), then we can see that the hierarchy between $\mathcal{Y}^{(e)}$ and $\mathcal{Y}^{(n)}$ is about 3 orders of magnitude which is acceptable. Notice that when $L^{-1}$ has larger magnitude such as 10 or 100 TeV, $\frac{\tilde{\mathcal{Y}}^{(n)}v}{\sqrt{2}}$ may get closer to $\frac{\tilde{\mathcal{Y}}^{(e)}v}{\sqrt{2}}$. If we compare the Yukawa couplings with that for the quark sector in Table IV, we will find that $\mathcal{Y}^{(e)}$ has the same order with $\mathcal{Y}^{(d)}$, so no hierarchy of the Yukawa couplings between quarks and leptons. All lepton 5D masses $M_L$, $M_E$ and $M_N$ are O(10) up to the scale $L^{-1}$ which also seemed natural.

TABLE II. Best fit parameters for leptons.

| $L_0^{(l)}$ | $L_1^{(l)}$ | $L_2^{(l)}$ | $M_L$ |
|---|---|---|---|
| $0.378389L$ | $0.670380L$ | $0.908743L$ | $-11.792317L^{-1}$ |
| $L_0^{(n)}$ | $L_1^{(n)}$ | $L_2^{(n)}$ | $M_N$ |
| $0.062289L$ | $0.515437L$ | $0.741436L$ | $13.293167L^{-1}$ |
| $L_0^{(e)}$ | $L_1^{(e)}$ | $L_2^{(e)}$ | $M_E$ |
| $0.317799L$ | $0.448665L$ | $0.701578L$ | $36.580911L^{-1}$ |
| $\frac{\tilde{\mathcal{Y}}^{(e)}v}{\sqrt{2}}$ | $\frac{\tilde{\mathcal{Y}}^{(n)}v}{\sqrt{2\tilde{y}^{(m)}}}\sqrt{\frac{\mathrm{TeV}}{L^{-1}}}$ | ... | ... |
| 0.317575 GeV | 0.000319953 GeV | ... | ... |

This set of parameters will give

(i) Masses of charged leptons: $m_e = 0.510999$ MeV, $m_\mu = 105.65837$ MeV, $m_\tau = 1776.79963$ MeV. They all deviate the experimental value less than 0.01% as the fitting required.

(ii) Masses of neutrinos: $m_1 = 0.005074$ eV, $m_2 = 0.010092$ eV, $m_3 = 0.049868$ eV. Comparing with the data, the mass squared differences between the first and third generation deviates the experimental one about 0.5%, while the mass squared differences between the first and second generation deviates the experimental one about 1.5%.

(iii) Masses of sterile neutrinos: $M_1 = 1.2144\mathrm{GeV}\frac{\tilde{y}^{(m)}L^{-1}}{\mathrm{TeV}}$, $M_2 = 4.9870\mathrm{TeV}\times\frac{\tilde{y}^{(m)}L^{-1}}{\mathrm{TeV}}$, $M_3 = 358.8498\mathrm{TeV}\frac{\tilde{y}^{(m)}L^{-1}}{\mathrm{TeV}}$. Both $\tilde{y}^{(m)}$ and the scale $L^{-1}$ are undetermined. But we can see that if $\tilde{y}^{(m)}L^{-1} \sim O(1\text{–}10\ \mathrm{TeV})$, the lightest sterile neutrino can be produced by the LHC, and since it interacts weakly with other particles, it may only contribute to a little part of the missing $E_t$.

(iv) Mixing angles: $\sin^2\theta_{12} = 0.30315$, $\sin^2\theta_{23} = 0.4359$, $\sin^2\theta_{13} = 0.0221$. They all deviate the experimental value less than 6%.

(v) $CP$ phases: $\delta_{12} = 0.1944$, $\delta_{23} = 1.2796$, $\delta_{13} = 3.0716$.

We can also calculate the effective Majorana mass as

$$\langle m_{\beta\beta}\rangle \equiv \Big|\sum_k m_k U_{ek}^2\Big| = 7.43\ \mathrm{meV}. \tag{20}$$

This quantity is related to the double-beta decay which now has limit $\langle m_{\beta\beta}\rangle \lesssim 120\text{–}250$ meV (90% C.L.) [35]. Not surprisingly, our result is far from the experimental limit since the masses of active neutrinos are all smaller than 100 meV.

We can also estimate the mass of gauge field $C_\mu$ as follows:

$$\frac{1}{2}m_c^2 C^\mu C_\mu = \int dy \frac{1}{L} g_c^2 \langle\Phi\rangle^2 C^\mu C_\mu \tag{21}$$

which implies

$$\begin{aligned} m_c^2 &= 2\tilde{g}_c^2 \int dy \langle\Phi\rangle^2 \\ &= 2\tilde{g}_c^2 \frac{2|Q|}{M}\left(\frac{L}{2} + \frac{L}{4\tilde{M}}(\sinh(2\tilde{M} + 2\tilde{M}\tilde{y}_0) - \sinh 2\tilde{M}\tilde{y}_0)\right) \\ &\approx \frac{\tilde{g}_c^2 |\tilde{Q}| L^{-2}}{2\tilde{M}^2} e^{2\tilde{M}(1-\tilde{y}_0)} \end{aligned} \tag{22}$$

which further leads to

$$m_c \approx \tilde{g}_c \frac{\sqrt{|\tilde{Q}|L^{-1}}}{\sqrt{2}\tilde{M}} e^{\tilde{M}(1-\tilde{y}_0)} \approx (124 \cdot \tilde{g}_c)\ \mathrm{TeV}\left(\frac{L^{-1}}{\mathrm{TeV}}\right). \tag{23}$$

So for $\tilde{g}_c \approx 0.1$–$1$, $L^{-1} \approx 1$–$100$ TeV, we have $m_c \approx$ 10–10000 TeV. Notice that there is another mixing effect if $H$ is $U(1)'$ charged. When electroweak symmetry breaks, there will be a mass term involving $Z$ and $C$ [26], then to obtain the mass eigenvalues we shall diagonalize a mass matrix in $(Z, C)$ basis as

$$\mathcal{M}^2 = \begin{pmatrix} m_Z^2 & \beta m_Z^2 \\ \beta m_Z^2 & m_c^2 \end{pmatrix}, \tag{24}$$

where $\beta$ is a factor about $O(1)$ or less. Since our $m_c^2$ is apparently much larger than $m_Z^2$, so the mixing would not be significant and the $\rho = \frac{m_Z^2}{m_1^2}$ is very closed to 1, where $m_1$ is the smaller mass eigenvalue. Notice that this heavy gauge field will also significantly suppress the effective coupling of some process mediated by it. The effective coupling which is similar to the Fermi constant $G_c \sim \frac{\tilde{g}_c^2}{m_c^2} = \frac{1}{(124L^{-1})^2} \approx \frac{G_F}{(500L^{-1}/\mathrm{TeV})^2}$ is much smaller than $G_F$, so this process will not change the whole amplitude.

Interestingly, given the parameters shown in Tables II and IV, we do not need to worry about the constraints from the proton decay. Following the analysis of [22], the dimension-8 operators leading to proton decay are $QQQL, DUQL, UDEU$ and $QQUE$. We show the domains of the first generation wave functions which involved the operators in Fig. 3. We find that for each operator, at least two domains do not overlap, and thus the integration vanishes.

## IV. SUMMARY

In this paper, we have discussed the possibility to generalize the model constructed in Refs. [21,22] to a Majorana neutrino case. The extra-dimension scale $L^{-1}$ is about several TeV, which seems far from the scale for seesaw mechanism and is unlikely to explain the small neutrino masses naturally. But we note that the smallness of neutrino masses can be a synthesized effect of the type-I

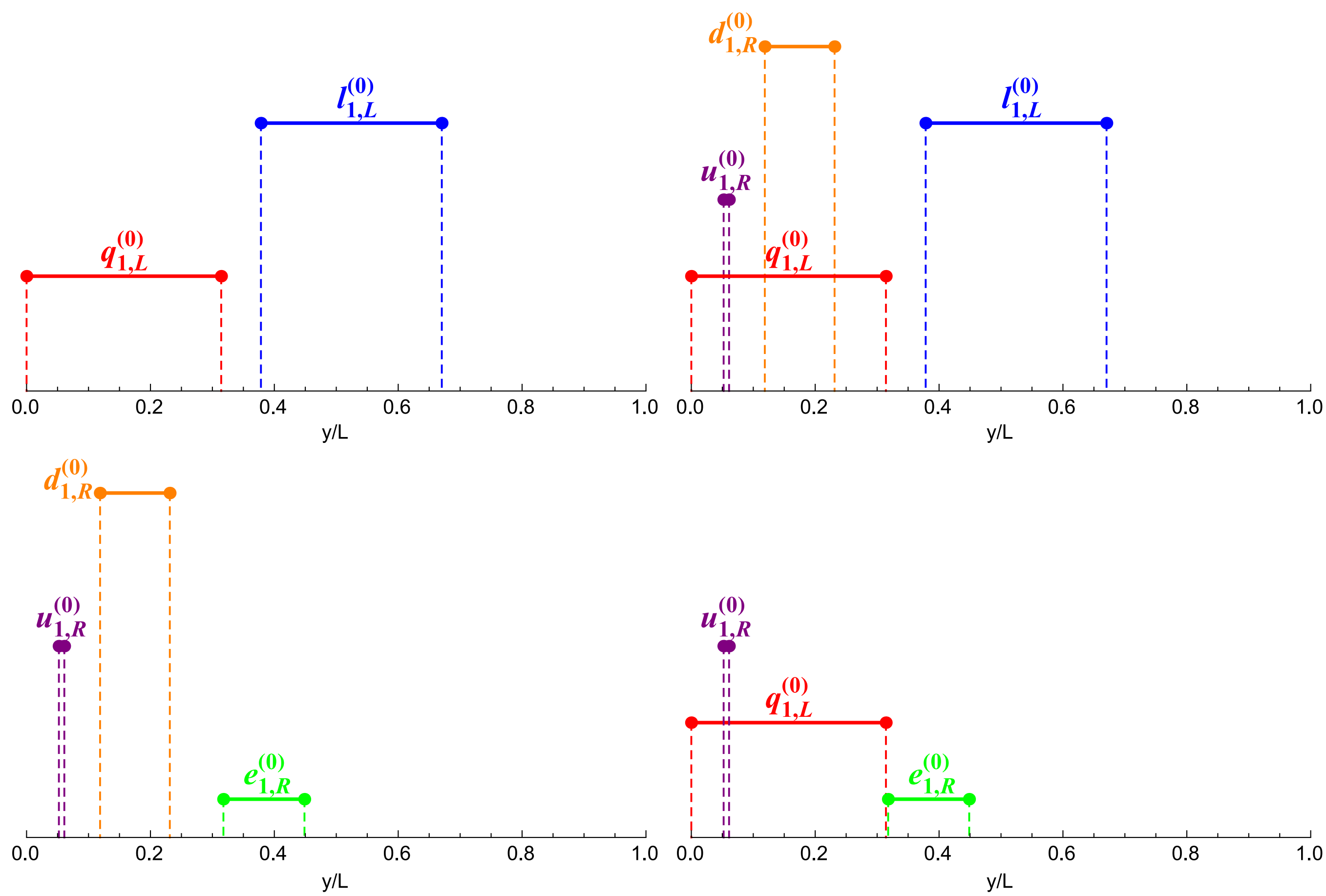


FIG. 3. The domains of the first generation wave functions. The left top is for the operator $QQQL$; the right top is for the $DUQL$; the left bottom is for the $UDEU$; while the right bottom is for the $QQUE$.

seesaw and the overlap integration of the localized lepton wave functions. We find that a 5D scalar $\Phi$ with an exponentially warped VEV, which was initially introduced in Refs. [21,22] to generate a hierarchy between generations, can also be used to generate large Majorana masses for the neutrino right-handed 0-modes. The strategy is to let $\Phi$ couple with the singlet neutrino field in the manner $\Phi^{*2}\bar{N}_R^c N_R$. When $\Phi$ acquires a nonzero vacuum expectation value, $\langle\Phi(y)\rangle^2$, which exponentially depends on the extra-dimension coordinate $y$, will be extremely large near $y = L$ so that the third generation of right-handed neutrino will be very heavy and turn on the seesaw mechanism. At the same time, if the positions of the 0-thickness branes and the 5D bulk mass $M_N$ are properly chosen, the overlap integration of the left-handed and right-handed neutrino wave functions will be also smaller than that of the charged leptons. Both of these effects work together, and they can significantly suppress the neutrino masses.

To justify the model, it is necessary to add a $U(1)'$ gauge symmetry into the model. This symmetry prohibits some troublesome terms like $\bar{L}\sigma^2 H^* H^\dagger \sigma^2 L^c$ and the explicit Majorana terms. When $\Phi$ obtains a nonzero vacuum expectation value, the $U(1)'$ symmetry will break spontaneously. Since the mass of the $U(1)'$ gauge boson is very large, it will not change the prediction significantly. For consistency, we also discuss how the anomaly cancellation conditions constrain the $U(1)'$ charge of each field. The numerical results of our model parameters have no significant hierarchy among them. They can fit all masses and flavor mixing data very well. We use this set of parameters to calculate some observable quantities such as the effective Majorana mass, and we find it is consistent with the double-beta decay experiments. Our parameters also rescue us from the stringent proton-decay constraint on the cutoff scale.

## ACKNOWLEDGMENTS

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## APPENDIX A: THE GENERAL SETUP OF THE FRAMEWORK

In this Appendix we briefly review the extra-dimension model with point interactions. The basic setup is to let all fields live in 5D spacetime and have point interactions with some 0-thickness branes [21,22]. The point interaction means a $\delta$-function-potential-like interaction which vanishes everywhere except at a point in the fifth dimension [21,36,37].

The action of a 5D fermion field $\Psi(x, y)$ is given by [21]

$$S = \int d^4x \int dy \bar{\Psi}(x, y)(i\Gamma^M \partial_M + M_F)\Psi(x, y), \quad \text{(A1)}$$

where $M_F$ is the 5D bulk mass, and the $\Gamma$ matrices obey the Clifford algebra $\{\Gamma_M, \Gamma_N\} = -2\eta_{MN}$ with the 5D metric $\eta_{MN} = \mathrm{diag}\{-1, 1, 1, 1, 1\}$ and the indices $M, N = 0, 1, 2, 3, 5$ and $\mu, \nu = 0, 1, 2, 3$. An explicit representation of the $\Gamma$ matrices is $\Gamma^\mu = \gamma^\mu$ and $\Gamma^y = -i\gamma_5 = \gamma^0\gamma^1\gamma^2\gamma^3$. The variation of the action (A1) is

$$\begin{aligned}\delta S &= \int d^4x \int dy[\delta\bar{\Psi}(i\Gamma^M \partial_M + M_F)\Psi \\ &\quad + \bar{\Psi}(i\Gamma^M \partial_M + M_F)\delta\Psi] \\ &= \int d^4x \int dy[\delta\bar{\Psi}(i\Gamma^M \partial_M + M_F)\Psi \\ &\quad - \bar{\Psi}(i\Gamma^M \overleftarrow{\partial}_M - M_F)\delta\Psi + \partial_M(\bar{\Psi} i\Gamma^M \delta\Psi)]. \quad \text{(A2)}\end{aligned}$$

Thus, $\delta S/\delta\bar{\Psi} = 0$ implies the equation of motion (EOM) for $\Psi$:

$$(i\Gamma^M \partial_M + M_F)\Psi = \begin{pmatrix} -\partial_y + M_F & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu \partial_\mu & \partial_y + M_F \end{pmatrix}\begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = 0, \quad \text{(A3)}$$

where the field $\Psi(x, y)$ has been decomposed into the left-handed and right-handed components $\Psi_{L,R} = P_{L,R}\Psi = [(1\mp\gamma_5)/2]\Psi$ in the chiral representation of Dirac matrices $\gamma^\mu$. Taking the complex conjugate of Eq. (A3) gives the EOM for $\bar{\Psi}$: $\bar{\Psi}(i\Gamma^M \overleftarrow{\partial}_M - M_F) = 0$. Substituting it and (A3) into (C8) and taking $\delta S = 0$, we obtain

$$\begin{aligned}0 &= \int d^4x \int dy \partial_M(\bar{\Psi}\Gamma^M \delta\Psi) \\ &= \int d^4x \int dy[\partial_\mu(\bar{\Psi}\Gamma^\mu \delta\Psi) + \partial_y(\bar{\Psi}\Gamma^y \delta\Psi)]. \quad \text{(A4)}\end{aligned}$$

Since the integral of the 4D total divergence vanishes, $\int d^4x \partial_\mu(\bar{\Psi}\Gamma^\mu \delta\Psi) = 0$, we have

$$\int dy \partial_y(\bar{\Psi}\Gamma^y \delta\Psi) = 0 \quad \text{(A5)}$$

which, as we have seen, is required for the consistency of the EOMs for $\Psi$ and $\bar{\Psi}$.

Now let us consider a toy model, in which the extra one-dimensional space is an interval with length $L$ and in the fifth dimension there are three boundary points assigned as $0$, $L_1(< L)$, $L$, respectively. In this case, Eq. (A5) implies

$$0=\int_0^L dy\partial_y(\bar{\Psi}\Gamma^y\delta\Psi)=\left(\int_0^{L_1-\epsilon}+\int_{L_1+\epsilon}^L\right)dy\partial_y(\bar{\Psi}\Gamma^y\delta\Psi)$$
$$=(\bar{\Psi}\Gamma^y\delta\Psi)|_{y=L}-(\bar{\Psi}\Gamma^y\delta\Psi)|_{y=0}+(\bar{\Psi}\Gamma^y\delta\Psi)|_{y=L_1-\epsilon}$$
$$-(\bar{\Psi}\Gamma^y\delta\Psi)|_{y=L_1+\epsilon}, \tag{A6}$$

where $\epsilon$ is a positive infinitesimal length. A sufficient condition to satisfy Eq. (A6) is to let the term vanish at all the boundary points:

$$\bar{\Psi}\Gamma^y\delta\Psi=i(\Psi_R^\dagger\delta\Psi_L-\Psi_L^\dagger\delta\Psi_R)=0$$
$$(\text{at } y=0,L_1\pm\epsilon,L). \tag{A7}$$

It is sufficient to satisfy Eq. (A7) by imposing the Dirichlet boundary condition

$$\Psi_R=0 \quad \text{or} \quad \Psi_L=0 \quad (\text{at } y=0,L_1\pm\epsilon,L). \tag{A8}$$

More specifically, we can take $\Psi_R=0$ (or $\Psi_L=0$) at all the boundary points to realize the left-handed (or right-handed) fermions in the zero-mode sector, as we will discuss later.

Multiplying the operator $(i\Gamma^N\partial_N-M_F)$ on Eq. (A3) from the left gives

$$(i\Gamma^N\partial_N-M_F)(i\Gamma^M\partial_M+M_F)\Psi$$
$$=\begin{pmatrix}-DD^\dagger+\partial_\mu\partial^\mu & \\ & -D^\dagger D+\partial_\mu\partial^\mu\end{pmatrix}\begin{pmatrix}\Psi_L\\ \Psi_R\end{pmatrix}=0, \tag{A9}$$

where $D\equiv\partial_y+M_F$, $D^\dagger\equiv-\partial_y+M_F$, and $\partial_\mu\partial^\mu\equiv\eta_{\mu\nu}\partial^\mu\partial^\nu=-\partial_t^2+\nabla^2$ with the 4D metric $\eta_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$. Let us separate variables of the solutions of Eq. (A9) as follows:

$$\Psi_L(x,y)=\sum_n\psi_L^{(n)}(x)f_{\psi_L^{(n)}}(y),$$
$$\Psi_R(x,y)=\sum_n\psi_R^{(n)}(x)f_{\psi_R^{(n)}}(y). \tag{A10}$$

For every particular solution of the left-handed wave function, $\Psi_L(x,y)=\psi_L^{(n)}(x)f_{\psi_L^{(n)}}(y)$, we have

$$0=(-DD^\dagger+\partial_\mu\partial^\mu)\psi_L^{(n)}(x)f_{\psi_L^{(n)}}(y)$$
$$=[-DD^\dagger f_{\psi_L^{(n)}}(y)]\psi_L^{(n)}(x)+[\partial_\mu\partial^\mu\psi_L^{(n)}(x)]f_{\psi_L^{(n)}}(y)$$
$$=[(-DD^\dagger+M^2_{\psi^{(n)}})f_{\psi_L^{(n)}}(y)]\psi_L^{(n)}(x), \tag{A11}$$

where we have used the 4D Klein-Gordon equation $(\partial_\mu\partial^\mu-M^2_{\psi^{(n)}})\psi_L^{(n)}(x)=0$. Equation (A11) implies

$$DD^\dagger f_{\psi_L^{(n)}}(y)=M^2_{\psi^{(n)}}f_{\psi_L^{(n)}}(y). \tag{A12a}$$

Likewise, using $(\partial_\mu\partial^\mu-M^2_{\psi^{(n)}})\psi_R^{(n)}(x)=0$, we obtain

$$D^\dagger Df_{\psi_R^{(n)}}(y)=M^2_{\psi^{(n)}}f_{\psi_R^{(n)}}(y). \tag{A12b}$$

In Eqs. (A12a) and (A12b), we have used the fact that the operators $DD^\dagger$ and $D^\dagger D$ are supersymmetric quantum mechanical partners [18,25,27] and thus they have exactly the same eigenvalues except for the lowest zero eigenvalue. It can be easily explained as follows. If $f_{\psi_L^{(n)}}(y)$ is the eigenfunction of $DD^\dagger$ with the eigenvalue $M^2_{\psi^{(n)}}$ and $M^2_{\psi^{(n)}}\neq 0$, then

$$D^\dagger D[D^\dagger f_{\psi_L^{(n)}}(y)]=D^\dagger[DD^\dagger f_{\psi_L^{(n)}}(y)]=M^2_{\psi^{(n)}}[D^\dagger f_{\psi_L^{(n)}}(y)], \tag{A13}$$

that is, $D^\dagger f_{\psi_L^{(n)}}(y)$ is an eigenfunction of $D^\dagger D$ with the same eigenvalue $M^2_{\psi^{(n)}}$. Define $f_{\psi_R^{(n)}}(y)\propto D^\dagger f_{\psi_L^{(n)}}(y)$ and let $f_{\psi_R^{(n)}}(y)$ have the same normalization as $f_{\psi_L^{(n)}}(y)$:

$$\langle f_{\psi_L^{(n)}}(y)|f_{\psi_L^{(n)}}(y)\rangle\equiv\int dy[f_{\psi_L^{(n)}}(y)]^*f_{\psi_L^{(n)}}(y)=1 \tag{A14}$$

which implies

$$\int dy[D^\dagger f_{\psi_L^{(n)}}(y)]^*D^\dagger f_{\psi_L^{(n)}}(y)$$
$$=\int dy[f_{\psi_L^{(n)}}(y)]^*DD^\dagger f_{\psi_L^{(n)}}(y)=M^2_{\psi^{(n)}}. \tag{A15}$$

Then it is sufficient to get $\langle f_{\psi_R^{(n)}}(y)|f_{\psi_R^{(n)}}(y)\rangle=1$ by letting

$$f_{\psi_R^{(n)}}(y)=\frac{1}{M_{\psi^{(n)}}}D^\dagger f_{\psi_L^{(n)}}(y). \tag{A16a}$$

Multiplying the operator $D$ on the above equation from the left gives

$$f_{\psi_L^{(n)}}(y)=\frac{1}{M_{\psi^{(n)}}}Df_{\psi_R^{(n)}}(y). \tag{A16b}$$

Substituting a pair of chiral modes of (A10) into Eq. (A3),

$$\begin{pmatrix}D^\dagger & i\sigma^\mu\partial_\mu\\ i\bar{\sigma}^\mu\partial_\mu & D\end{pmatrix}\begin{pmatrix}\psi_L^{(n)}(x)f_{\psi_L^{(n)}}(y)\\ \psi_R^{(n)}(x)f_{\psi_R^{(n)}}(y)\end{pmatrix}=0, \tag{A17}$$

we have

$$\psi_L^{(n)}(x)[D^\dagger f_{\psi_L^{(n)}}(y)] + [i\sigma^\mu\partial_\mu\psi_R^{(n)}(x)]f_{\psi_R^{(n)}}(y) = 0 \quad \text{(A18a)}$$

$$[i\bar{\sigma}^\mu\partial_\mu\psi_L^{(n)}(x)]f_{\psi_L^{(n)}}(y) + \psi_R^{(n)}(x)[Df_{\psi_R^{(n)}}(y)] = 0 \quad \text{(A18b)}$$

which, together with Eqs. (A16a) and (A16b), lead to

$$i\sigma^\mu\partial_\mu\psi_R^{(n)}(x) + M_{\psi^{(n)}}\psi_L^{(n)}(x) = 0 \quad \text{(A19a)}$$

$$i\bar{\sigma}^\mu\partial_\mu\psi_L^{(n)}(x) + +M_{\psi^{(n)}}\psi_R^{(n)}(x) = 0, \quad \text{(A19b)}$$

that is,

$$\begin{pmatrix} M_{\psi^{(n)}} & i\sigma^\mu\partial_\mu \\ i\bar{\sigma}^\mu\partial_\mu & M_{\psi^{(n)}} \end{pmatrix}\begin{pmatrix} \psi_L^{(n)}(x) \\ \psi_R^{(n)}(x) \end{pmatrix} = 0. \quad \text{(A20)}$$

Thus, the combination $\psi^{(n)}(x) \equiv (\psi_L^{(n)}(x), \psi_R^{(n)}(x))^T$ obeys the 4D Dirac equation $(i\partial\!\!\!/ + M_{\psi^{(n)}})\psi^{(n)}(x) = 0$ and forms a Dirac spinor.

Suppose that the eigenequation (A12a) of $DD^\dagger$ has a zero eigenvalue $M^2_{\psi^{(n)}} = 0$ with the corresponding eigenfunction $f_{\psi_L^{(0)}}(y)$ called the 0-mode. That is, $DD^\dagger f_{\psi_L^{(0)}}(y) = 0$. It is sufficient to satisfy the above relation if $f_{\psi_L^{(0)}}(y)$ is annihilated by $D^\dagger$:

$$D^\dagger f_{\psi_L^{(0)}}(y) = (-\partial_y + M_F)f_{\psi_L^{(0)}}(y) = 0. \quad \text{(A21)}$$

(i) If the Dirichlet boundary condition $\Psi_L(x, y) = 0$ is imposed at $y = 0,\ L_1 \pm \epsilon,\ L$, that is,

$$f_{\psi_L^{(0)}}(y) = 0 \quad (\text{at } y = 0, L_1 \pm \epsilon, L), \quad \text{(A22)}$$

then Eqs. (A21) and (A22) imply that $f_{\psi_L^{(0)}}(y) = 0$ at all points. Thus, the 0-mode eigenfunction of $DD^\dagger$ does not exist in the boundary condition of (A22).

(ii) If the Dirichlet boundary condition $\Psi_R(x, y) = 0$ is imposed at $y = 0,\ L_1 \pm \epsilon,\ L$, that is,

$$f_{\psi_R^{(0)}}(y) = 0 \quad (\text{at } y = 0, L_1 \pm \epsilon, L). \quad \text{(A23)}$$

then this boundary condition has no effect on Eq. (A21), but the setup of the 0-thickness branes' positions itself can split the solutions of (A21) into two independent degenerate modes:

$$f_{\psi_L^{(0)},(1)}(y) = \begin{cases} N_1 e^{M_F y} & (0 \le y < L_1) \\ 0 & (L_1 \le y < L) \end{cases} \quad \text{(A24a)}$$

$$f_{\psi_L^{(0)},(2)}(y) = \begin{cases} 0 & (0 \le y < L_1) \\ N_2 e^{M_F y} & (L_1 \le y < L), \end{cases} \quad \text{(A24b)}$$

where $N_1$ and $N_2$ are normalization constants and, by using (A14), they can be figured out as

$$N_1 = \sqrt{\frac{2M_F}{e^{2M_F L_1} - 1}},$$
$$N_2 = e^{-M_F L_1}\sqrt{\frac{2M_F}{e^{2M_F(L-L_1)} - 1}}. \quad \text{(A25)}$$

Using the Heaviside step function $\theta(y)$, we can also write the two degenerate zero modes as follows:

$$f_{\psi_L^{(0)},(1)}(y) = \sqrt{\frac{2M_F}{e^{2M_F L_1} - 1}}e^{M_F y}[\theta(y)\theta(L_1 - y)] \quad \text{(A26a)}$$

$$f_{\psi_L^{(0)},(2)}(y) = \sqrt{\frac{2M_F}{e^{2M_F(L-L_1)} - 1}}e^{M_F(y-L_1)} \times [\theta(y - L_1)\theta(L - y)]. \quad \text{(A26b)}$$

The 5D wave function of 0-mode $\Psi_L^{(0)}(x, y)$ may be expanded with respect to $f_{\psi_L^{(0)},(1)}(y)$ and $f_{\psi_L^{(0)},(2)}(y)$ as

$$\Psi_L^{(0)}(x, y) = \psi_{1L}^{(0)}(x)f_{\psi_L^{(0)},(1)}(y) + \psi_{2L}^{(0)}(x)f_{\psi_L^{(0)},(2)}(y), \quad \text{(A27)}$$

where the coefficients $\psi_{1L}^{(0)}(x)$ and $\psi_{2L}^{(0)}(x)$ are identified with the 4D wave functions of two generations of left-handed fermions in this toy model.

Likewise, consider the 0-mode eigenfunction $f_{\psi_R^{(0)}}(y)$ of $D^\dagger D$. It obeys the equation $D^\dagger D f_{\psi_R^{(0)}}(y) = 0$. A sufficient condition of this equation is

$$Df_{\psi_R^{(0)}}(y) = (\partial_y + M_F)f_{\psi_R^{(0)}}(y) = 0. \quad \text{(A28)}$$

(i) If the Dirichlet boundary condition (A22) for the left-handed fermion is imposed, then it is the location of the point-interaction positions, rather than Eq. (A22), that affects the solutions of (A28) and splits them into two degenerate modes:

$$f_{\psi_R^{(0)},(1)}(y) = \sqrt{\frac{2M_F}{1 - e^{-2M_F L_1}}}e^{-M_F y}[\theta(y)\theta(L_1 - y)] \quad \text{(A29a)}$$

$$f_{\psi_R^{(0)},(2)}(y) = \sqrt{\frac{2M_F}{1 - e^{-2M_F(L-L_1)}}}e^{-M_F(y-L_1)} \times [\theta(y - L_1)\theta(L - y)]. \quad \text{(A29b)}$$

The expansion of the 5D wave function of 0-mode $\Psi_R^{(0)}(x, y)$ with respect to the two modes is given by

$$\Psi_R^{(0)}(x,y) = \psi_{1R}^{(0)}(x) f_{\psi_R^{(0)},(1)}(y) + \psi_{2R}^{(0)}(x) f_{\psi_R^{(0)},(2)}(y), \tag{A30}$$

where the 4D wave functions $\psi_{1R}^{(0)}(x)$ and $\psi_{2R}^{(0)}(x)$ belong to two generations of right-handed fermions in this toy model.

(ii) If the Dirichlet boundary condition (A23) for the right-handed fermion is imposed, then Eqs. (A28) and (A23) imply that $f_{\psi_R^{(0)}}(y) = 0$ at all points. That is, the 0-mode eigenfunction of $D^\dagger D$ vanishes in this boundary condition.

To sum up, if the boundary condition $\Psi_L = 0$ is imposed at all the 0-thickness branes' positions, then the 5D fermion field $\Psi(x,y)$ has only right-handed 0-modes $\Psi_R^{(0)}(x,y)$ as given in Eq. (A30); instead, if $\Psi_R = 0$ is imposed at all the boundary points, then $\Psi(x,y)$ has only left-handed 0-modes $\Psi_L^{(0)}(x,y)$ as given in Eq. (A27). In a word, the Dirichlet boundary condition $\Psi_{L,R} = 0$ makes the 0-mode wave functions of $\Psi(x,y)$ to be chiral. Including the KK modes (i.e. the modes with $M^2_{\psi^{(n)}} \neq 0$), the expansion of a 5D fermion field $\Psi(x,y)$ in all modes is given by

(i) For $\Psi_L = 0$ at $y = 0,\ L_1,\ L$

$$\begin{aligned}\Psi(x,y) = &\sqrt{\frac{2M_F}{1-e^{-2M_F L_1}}} e^{-M_F y}[\theta(y)\theta(L_1 - y)]\psi_{1R}^{(0)}(x) \\ &+ \sqrt{\frac{2M_F}{1-e^{-2M_F(L-L_1)}}} e^{-M_F(y-L_1)} \\ &\times [\theta(y-L_1)\theta(L-y)]\psi_{2R}^{(0)}(x) \\ &+ (\text{KKmodes}).\end{aligned} \tag{A31}$$

(ii) For $\Psi_R = 0$ at $y = 0,\ L_1,\ L$

$$\begin{aligned}\Psi(x,y) = &\sqrt{\frac{2M_F}{e^{2M_F L_1} - 1}} e^{M_F y}[\theta(y)\theta(L_1 - y)]\psi_{1L}^{(0)}(x) \\ &+ \sqrt{\frac{2M_F}{e^{2M_F(L-L_1)} - 1}} e^{M_F(y-L_1)} \\ &\times [\theta(y-L_1)\theta(L-y)]\psi_{2L}^{(0)}(x) \\ &+ (\text{KKmodes}).\end{aligned} \tag{A32}$$

To realize both left-handed and right-handed 0-mode fermions in this two-generation toy model, we need at least two 5D fermion fields, $\Psi_1(x,y)$ and $\Psi_2(x,y)$. One 5D fermion $\Psi_1(x,y)$ has two left-handed 0-modes due to the boundary condition $P_R\Psi_1(x,y) = 0$ at points $y = 0,\ L_1,\ L$; while another 5D fermion $\Psi_2(x,y)$ has two right-handed 0-modes from the boundary condition $P_L\Psi_2(x,y) = 0$ at points $y = 0,\ L_1',\ L$. The locations of $L_1$ and $L_1'$ are in general not equal. Indeed, it is the inequality of $L_1$ and $L_1'$ that leads to the mixing of the two generations of fermions. A schematic picture of the wave functions of these 0-mode chiral fermions is shown in Fig. 4. To give the chiral fermions masses, we need to introduce an extra 5D scalar field $\Phi(x,y)$, which will acquire a nonzero VEV after the electroweak symmetry breaking. The mixing structure of the Dirac mass matrix is also explained in Fig. 4.

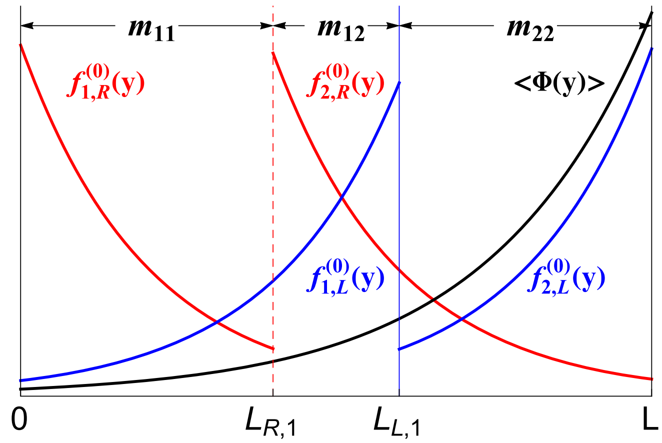


FIG. 4. A schematic diagram of wave functions for chiral 0-mode fermions. The red curves represent the wave functions for two generations of right-handed 0-mode fermions, while the blue curves represent the wave functions for two generations of left-handed 0-mode fermions. The black line is a profile of a scalar Φ's VEV. The overlap integration of the profiles in different intervals gives the corresponding mass matrix element.

In addition, it is worthy to point out that the operators $D \equiv \partial_y + M_F$ and $D^\dagger \equiv -\partial_y + M_F$ can be used to construct a pair of supersymmetric generators, $Q \equiv D\gamma^0 P_L$ and $Q^\dagger \equiv D^\dagger\gamma^0 P_R$, which satisfy the supersymmetric algebra [see the paragraphs between Eqs. (8) and (9) in Ref. [18] for more details]:

$$\begin{aligned}&Q^2 = Q^{\dagger 2} = 0, \qquad \{Q, Q^\dagger\} = 2H, \\ &[Q,H] = [Q^\dagger, H] = 0.\end{aligned} \tag{A33}$$

The Hamiltonian operators (up to a constant factor) is $H \propto \{Q, Q^\dagger\} = DD^\dagger P_R + D^\dagger D P_L$, and the pair of modes $(f_{\psi_L^{(n)}}(y)\psi_L^{(n)}(x), f_{\psi_R^{(n)}}(y)\psi_R^{(n)}(x))^T$ is an eigenstate of $H$ with eigenvalue $M^2_{\psi^{(n)}}$.

## APPENDIX B: QUARK MASSES HIERARCHY AND FLAVOR MIXINGS

The Yukawa terms which generate the masses for quarks are

$$\begin{aligned}\mathcal{L}^{Yuk}_{\text{quarks}} = -\int dy[&\mathcal{Y}^{(u)}\Phi\bar{Q}(i\sigma^2 H^*)U_R \\ &+ \mathcal{Y}^{(d)}\Phi^*\bar{L}HD_R + \text{H.c.}],\end{aligned} \tag{B1}$$

where $\mathcal{Y}^{(u)}$ and $\mathcal{Y}^{(d)}$ are the couplings with dimension $-2$ for the up-type and down-type quarks, respectively.

Note that we will let $\Phi$ be $U(1)'$ charged. Then if we do not want the $U(1)'$ breaks explicitly, we should also make $U_R$, $D_R$, $Q$ and $H$ be $U(1)'$ charged. We have determined the $U(1)'$ charge for each field in Sec. III. We can see that terms as $\bar{Q}(i\sigma^2 H^*)U_R$ and $\bar{L}HD_R$ can be forbidden by the $U(1)'$ symmetry.

After the $U(1)'$ and electroweak symmetry breaking, we obtain Dirac mass terms of quarks. The mixing structure of the mass matrix will be generated by the overlaps of wave functions from different generations. Then we can write down the mass matrices as

$$m^{(u)} = \begin{pmatrix} m^u_{11} & m^u_{12} & m^u_{13} \\ 0 & m^u_2 & m^u_{23} \\ 0 & 0 & m^u_{33} \end{pmatrix}, \quad m^{(d)} = \begin{pmatrix} m^d_{11} & m^d_{12} & m^d_{13} \\ 0 & m^d_2 & m^d_{23} \\ 0 & 0 & m^d_{33} \end{pmatrix} \tag{B2}$$

$$m^{(u)}_{ij} = \mathcal{Y}^{(u)} \int_a^b dy f_{q^{(0)}_{iL}}(y) f_{u^{(0)}_{jR}}(y) \langle\phi(y)\rangle\langle H(y)^*\rangle \tag{B3}$$

$$m^{(d)}_{ij} = \mathcal{Y}^{(d)} \int_a^b dy f_{q^{(0)}_{iL}}(y) f_{d^{(0)}_{jR}}(y) \langle\phi(y)\rangle\langle H(y)\rangle. \tag{B4}$$

The integration range $(a,b)$ represents the overlap region between the profiles $f_{q^{(0)}_{iL}}(y)$ and $f_{u^{(0)}_{jL}}(y)$ or $f_{d^{(0)}_{jL}}(y)$. The integration will contribute to a diagonal element when $i=j$, and an off diagonal element when $i \neq j$. Two Dirac mass matrices $m^{(u)}$ and $m^{(d)}$ are apparently complex and we can diagonalize them with unitary matrices $V^{(u)}_L (V^{(d)}_L)$ and $V^{(u)}_R (V^{(d)}_R)$:

$$m^{(u)}_{\rm diag} = V^{(u)}_L m^{(u)} V^{(u)\dagger}_R, \quad m^{(d)}_{\rm diag} = V^{(d)}_L m^{(d)} V^{(d)\dagger}_R. \tag{B5}$$

Then we can compare the masses with experimental data. Using matrices $V^{(u)}_L$ and $V^{(d)}_L$, we can calculate the CKM matrix which is defined as

$$V_{\rm CKM} = V^{(u)}_L V^{(d)\dagger}_L. \tag{B6}$$

The CKM matrix contains not only information about flavor mixing angles but also information about the $CP$ violation. The $CP$ violation can be characterized by the Jarlskog invariant $\mathcal{J}$ defined as

$$\mathrm{Im}[(V_{\rm CKM})_{ij}(V_{\rm CKM})_{kl}(V^*_{\rm CKM})_{il}(V^*_{\rm CKM})_{kj}] = \mathcal{J}\sum_{m,n=1}^{3}\epsilon_{ikm}\epsilon_{jln} \tag{B7}$$

We list the experimental data used in our fitting as follows:

(i) The up- and down-type quark masses are shown in Table III.

(ii) The absolute values of CKM matrix elements from Ref. [33] are

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.049 \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \end{pmatrix}. \tag{B8}$$

(iii) The Jarlskog invariant from Ref. [33] is $\mathcal{J} = (2.96 \pm 0.18) \times 10^{-5}$.

After fitting the data listed above, we found a set of parameters, which is compatible with the data, and show them in Table IV. We have set $|\tilde{\lambda}| \equiv |\lambda L| = 0.001$, $|\tilde{Q}| \equiv |QL^5| = 0.001$ and $\tilde{y}_0 \equiv y_0 L^{-1} = -0.16$ fixed as Ref. [21] did, so the only free parameter of $\Phi$ is $M$. Since $\Phi$ and $H$ also couple to leptons, the values of $M$ and $\theta$ which are found in the quark case will be set fixed to reduce the number of free parameters in the lepton case. In the

TABLE III. Quark masses from Ref. [33].

| Up-type quark | Mass | Down-type quark | Mass |
|---|---|---|---|
| u | $2.3 \pm 0.6$ MeV | d | $4.8 \pm 0.5$ MeV |
| c | $1.275 \pm 0.025$ GeV | s | $95 \pm 5$ MeV |
| t | $173.5 \pm 1.4$ GeV | b | $4.18 \pm 0.03$ GeV |

TABLE IV. Best fit parameters for quarks

| $L^{(q)}_0$ | $L^{(q)}_1$ | $L^{(q)}_2$ | $M_Q$ |
|---|---|---|---|
| 0 | $0.31423L$ | $0.67665L$ | $9.26018L^{-1}$ |
| $L^{(u)}_0$ | $L^{(u)}_1$ | $L^{(u)}_2$ | $M_U$ |
| $0.05218L$ | $0.06095L$ | $0.56328L$ | $-4.48152L^{-1}$ |
| $L^{(d)}_0$ | $L^{(d)}_1$ | $L^{(d)}_2$ | $M_D$ |
| $0.11866L$ | $0.23128L$ | $0.66636L$ | $5.71010L^{-1}$ |
| $M$ | $\tilde{\mathcal{Y}}^{(u)}v/\sqrt{2}$ | $\tilde{\mathcal{Y}}^{(d)}v/\sqrt{2}$ | $\theta$ |
| $9.36099L^{-1}$ | 3.15684 GeV | 0.20552 GeV | 2.91684 |

following, a parameter with a tilde means it has been scaled to dimensionless by multiplying some power of $L$.

Note that we can calculate $L_\pm$ in the Robin boundary condition by

$$L+ = -\frac{\Phi(0)}{\partial_y\Phi(0)} = -0.118074L$$
$$L_- = \frac{\Phi(L)}{\partial_y\Phi(L)} = 0.104502L. \tag{B9}$$

Then we find that $M = 9.36099 < \frac{1}{L_-} = 9.5692$, which is consistent with the symmetry breaking condition $|M|^2 < \frac{1}{L_{\max}^2}$.

Using the parameters of $\Phi$ we can calculate the tree level mass of the 4D excitation $\phi(x)$. One of its degrees of freedom will be gauged out by the gauge boson of $U(1)'$ when the symmetry breaking occurs. To obtain the mass of $\phi(x)$, we shall consider its excitation around the minimum of potential

$$\mathcal{E}[\Phi] = \int_0^L dy\left\{-\Phi^\dagger\partial_y^2\Phi + M^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4\right\}. \tag{B10}$$

Substituting the zero mode $\Phi^{(0)} = f^{(0)}(y)(\nu+\phi)$, $\nu f^{(0)}(y) = \langle\Phi(y)\rangle$ into $\mathcal{E}[\Phi]$ and using the minimized condition, $-\partial_y^2 f_0(y) + M^2 f_0 + \lambda\nu^2 f_0^3 = 0$, we can get the mass

$$m_\phi^2 = 2\int_0^L dy(2\lambda\langle\Phi(y)\rangle^2 f_0^2) \approx \frac{\lambda|Q|}{M^2}e^{2M(L-y_0)}$$
$$= \frac{\tilde{\lambda}|\tilde{Q}|}{\tilde{M}^2}e^{2\tilde{M}(1-\tilde{y}_0)}L^{-2} \tag{B11}$$

which implies

$$m_\phi \approx \frac{\sqrt{\tilde{\lambda}|\tilde{Q}|}}{\tilde{M}}e^{\tilde{M}(1-\tilde{y}_0)}L^{-1} \approx 5.55\ \text{TeV}\cdot\left(\frac{L^{-1}}{\text{TeV}}\right). \tag{B12}$$

If the scale $L^{-1} \sim \mathrm{O}(1\ \text{TeV})$, this mass is under the energy scale of LHC. But it is unlikely to be detected in the recent experiments, because the $\phi$-fermion-fermion couplings are so weak. This can be seen by estimating the couplings as

$$\zeta_{ij}^{(q)} = \frac{m_{ij}^{(q)}\cdot A}{\nu}, \qquad \zeta_{ij}^{(e)} = \frac{m_{ij}^{(e)}\cdot A}{\nu}, \tag{B13}$$

where

$$A = \frac{\sqrt{2/L}}{\sqrt{1+\sinh(\tilde{M})\cosh(\tilde{M}-2\tilde{M}\tilde{y}_0)/\tilde{M}}} \simeq \sqrt{\frac{2}{L}}\frac{2\sqrt{\tilde{M}}}{e^{\tilde{M}(1-\tilde{y}_0)}} \tag{B14}$$

$$\nu \simeq \frac{\sqrt{2|\tilde{Q}|}}{\tilde{M}}L^{-\frac{3}{2}}. \tag{B15}$$

Using the parameters in our fitting, we find the Yukawa couplings for $\phi$-quark-quark and $\phi$-lepton-lepton are

$$\zeta_{ij}^{(q)} \simeq 0.03\times\frac{m_{ij}^{(q)}}{L^{-1}}, \qquad \zeta_{ij}^{(e)} \simeq 0.03\times\frac{m_{ij}^{(e)}}{L^{-1}}. \tag{B16}$$

Both Yukawa couplings are much weaker than the Yukawa couplings for Higgs-quark-quark and Higgs-lepton-lepton. Since the coupling is proportional to the mass, the strongest Yukawa coupling may be the coupling of $\phi$-top-top which is about $0.03\times 0.17 \approx 0.005$ when $L^{-1} \sim 1$ TeV.

Note that there is a $C|\Phi|^2|H|^2$ term which may lead to some problem with the gauge universality as discussed in Ref. [21]. We will just let $C$ to be small enough (about $10^{-7}$ for $L^{-1} \sim 1$ TeV) to resolve this.

## APPENDIX C: WHY AN EXPLICIT MAJORANA MASS TERM DOES NOT WORK

The 5D charge conjugation operator $C$ is defined as

$$C\Gamma^M C^{-1} = (\Gamma^M)^T \tag{C1}$$

with properties

$$C^T = C^{-1} = C^\dagger = -C. \tag{C2}$$

It is easy to check that $C$ can be written as $C = \gamma^0\gamma^2(i\gamma_5)$ [18]. We can write it in Weyl basis

$$C = \begin{pmatrix} \epsilon_{ab} & \\ & -\epsilon^{\dot{a}\dot{b}} \end{pmatrix}. \tag{C3}$$

The charge conjugation of a 5D fermion is defined as

$$\Psi^c = C\bar{\Psi}^T. \tag{C4}$$

We can also write it down in Weyl basis:

$$\Psi(x,y) = \begin{pmatrix} \xi_a(x,y) \\ \chi^{\dagger\dot{a}}(x,y) \end{pmatrix} \Rightarrow \Psi^c = \begin{pmatrix} \chi_a(x,y) \\ -\xi^{\dagger\dot{a}}(x,y) \end{pmatrix}. \tag{C5}$$

Note that the relation $(\Psi^c)^c = \Psi$ no longer holds in the 5D case and the correct relation is $(\Psi^c)^c = -\Psi$.

Now we consider to add terms as $\bar{\Psi}i\Gamma^M\partial_M\Psi^c + \text{H.c.}$ After several lines of calculation, we can get

$$\bar{\Psi}i\Gamma^M\partial_M\Psi^c = \bar{\Psi}i\Gamma^M\partial_M C\bar{\Psi}^T$$
$$= \partial_M(\bar{\Psi}i\Gamma^M\Psi^c) - \bar{\Psi}i\Gamma^M\partial_M\Psi^c. \tag{C6}$$

This implies that these terms can be absorbed into the boundary terms and do not contribute to the equations of motion.

However, the mass terms as $M_R\bar{\Psi}\Psi^c + \text{H.c.}$ survive and will contribute to the equations of motion. Now let us add the mass terms into the action:

$$S = \int d^4x \int dy[\bar{\Psi}(x,y)(i\Gamma^M\partial_M + M_F)\Psi(x,y) + \frac{1}{2}(M_R\bar{\Psi}\Psi^c + \text{H.c.})]. \tag{C7}$$

The variation of the action (C7) is

$$\begin{aligned}\delta S =& \int d^4x \int dy\Big[\delta\bar{\Psi}(i\Gamma^M\partial_M + M_F)\Psi \\ &+ \bar{\Psi}(i\Gamma^M\partial_M + M_F)\delta\Psi + \frac{1}{2}M_R\delta\bar{\Psi}\Psi^c + \frac{1}{2}M_R\bar{\Psi}\delta\Psi^c \\ &+ \frac{1}{2}M_R\delta\bar{\Psi}^c\Psi + \frac{1}{2}M_R\bar{\Psi}^c\delta\Psi\Big] \\ =& \int d^4x \int dy[\delta\bar{\Psi}(i\Gamma^M\partial_M + M_F)\Psi \\ &- \bar{\Psi}(i\Gamma^M\overleftarrow{\partial}_M - M_F)\delta\Psi \\ &+ \partial_M(\bar{\Psi}i\Gamma^M\delta\Psi) + M_R\delta\bar{\Psi}\Psi^c + M_R\bar{\Psi}^c\delta\Psi].\end{aligned} \tag{C8}$$

Thus, the equation of motion (EOM) becomes

$$\begin{aligned}0 &= (i\Gamma^M\partial_M + M_F)\Psi + M_R\Psi^c \\ &= \begin{pmatrix} -\partial_y + M_F & i\sigma^\mu\partial_\mu \\ i\bar{\sigma}^\mu\partial_\mu & \partial_y + M_F \end{pmatrix}\begin{pmatrix} \xi_a(x,y) \\ \chi^{\dagger\dot{a}}(x,y) \end{pmatrix} \\ &\quad + \begin{pmatrix} M_R & \\ & M_R \end{pmatrix}\begin{pmatrix} \chi_a(x,y) \\ -\xi^{\dagger\dot{a}}(x,y) \end{pmatrix}.\end{aligned} \tag{C9}$$

If we try to separate the field in modes as $\xi_a(x,y) = \sum_n f^{(n)}(y)\xi_a^{(n)}(x)$, $\chi_a = \sum_n g^{(n)}(y)\chi_a^{(n)}(x)$, then the equations for each mode become

$$(-\partial_y + M_F)f^{(n)}(y)\xi_a^{(n)}(x) + M_R g^{(n)}(y)\chi_a^{(n)}(x) + g^{(n)*}(y)i\sigma^\mu\partial_\mu\chi^{(n)\dagger\dot{a}}(x) = 0 \tag{C10}$$

$$(\partial_y + M_F)g^{(n)*}(y)\chi^{(n)\dagger\dot{a}}(x) - M_R f^{(n)*}(y)\xi^{(n)\dagger\dot{a}}(x) + f^{(n)}i\bar{\sigma}^\mu\partial_\mu\xi_a^{(n)}(x) = 0. \tag{C11}$$

Apparently, in a general case, it is impossible to factor out the functions $f^{(n)}(y)$, $g^{(n)}(y)$ from the 4D Dirac equations of spinors $\xi_a(x)$, $\chi_a(x)$. This means a special choice which can achieve this is to let $M_F = 0$ and $\chi^{\dagger\dot{a}} = -\xi^{\dagger\dot{a}}$, then the EOM becomes

$$(\partial_y + M_R)\xi_a(x,y) + i\sigma^\mu_{a\dot{a}}\partial_\mu\xi^{\dagger\dot{a}}(x,y) = 0 \tag{C12}$$

$$(-\partial_y + M_R)\xi^{\dagger\dot{a}}(x,y) + i\bar{\sigma}^{\mu\dot{a}a}\partial_\mu\xi_a(x,y) = 0. \tag{C13}$$

We can recover the 4D Dirac equation for a Majorana fermion by setting $\xi_a(x,y) = A\xi_a(x)$ where $A$ is a constant so the profile is independent of the fifth dimension coordinate $y$. Thus, this fermion has only one mode with a Majorana mass $M_R$. But this solution requires some special choice of the 5D fermion.

If we accept this special pattern of fermion to be the singlet neutrino $N_R$, and generate Dirac masses with the Yukawa interaction, then the seesaw turns on when the Majorana mass is much larger than the Dirac ones. However, an operator as $\bar{L}\sigma^2H^*H^\dagger\sigma^2L^c$ is still allowed and it will contribute to the Majorana masses of left-handed zero modes. Now we have to diagonalize the following mass matrix:

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}. \tag{C14}$$

In the large $M_R$ limit, the light neutrino masses are $m_\nu \approx M_L - M_D M_D^T/M_R$. These masses should be as small as O(0.1 eV) to fit the current neutrino mass bound and imply that either we use an unnaturally small coupling for $\bar{L}\sigma^2H^*H^\dagger\sigma^2L^c$ operator or we fine-tune the parameters to cancel $M_L$ by $M_D M_D^T/M_R$ in high precision.

Actually, in the SM the gauge symmetries and the lepton number conservation do not allow the explicit Majorana mass term and $\bar{L}\sigma^2H^*H^\dagger\sigma^2L^c$ to exist. However, in this model we are going to add a SM-gauge-group singlet neutrino field into the model, and try to violate the lepton number explicitly. Thus, we have to face these annoying terms unless they are also forbidden by some symmetry. The strategy we use in the paper is to forbid both $\bar{L}\sigma^2H^*H^\dagger\sigma^2L^c$ and $M_R\bar{\Psi}\Psi^c + \text{H.c.}$. terms by a U(1)' symmetry. Then the singlet neutrinos have chiral zero modes as any other fermions. Their right-handed Majorana masses are generated by the VEV of $\Phi$ with the same mechanism as their Dirac masses generated by the VEV of $\Phi$ and the Higgs field. Now the mass matrix we need to diagonalize is

$$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \tag{C15}$$

In the large $M_R$ limit, the light neutrino masses are $m_\nu \approx M_D M_D^T/M_R$ which can be naturally suppressed to O(0.1 eV).

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