

Perturbative QCD factorization of $\rho\gamma^* \rightarrow \rho$ Ya-lan Zhang,¹ Shan Cheng,^{2,*} Jun Hua,¹ and Zhen-Jun Xiao^{1,3,†}¹*Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing, Jiangsu 210023, People's Republic of China*²*Theoretische Elementarteilchenphysik, Naturwissenschaftlich Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*³*Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University, Nanjing 210023, People's Republic of China*

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In this paper we first demonstrate step by step that the factorization hypothesis is valid at the next-to-leading order (NLO) for the exclusive process $\rho\gamma^* \rightarrow \rho$ by employing the collinear factorization approach, and then extend this proof to the case of the k_T factorization by taking into account the transversal momentum of the light external quark (antiquark) lines in the ρ meson. At the NLO level, we then show that the soft divergences from different subdiagrams will cancel each other in the quark level, while the remaining collinear divergences can be absorbed into the NLO meson wave functions. The full NLO amplitudes can therefore be factorized as the convolution of the NLO wave functions $\Phi_\rho^{(1)}$ and the infrared-finite leading order (LO) hard kernels $G_{X,II,kl}^0$ in the k_T factorization. We also write down the polarized NLO ρ meson wave functions in the form of nonlocal hadron matrix elements with the gauge factor integral path deviating from the light cone. These NLO ρ meson wave functions can be used to calculate the NLO hard corrections to some relevant exclusive processes, such as $B \rightarrow \rho$ transition.

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I. INTRODUCTION

The factorization theorem [1–3] is a fundamental hypothesis to deal with the QCD involved exclusive processes with various energy scales which may generate infrared divergence [4]. The perturbative QCD (PQCD) factorization approach is constructed based on the so-called k_T factorization theorem [5], where the parton transversal momentum in the denominator of the propagators are picked up in order to remove the endpoint singularity. In the PQCD factorization approach, fortunately, the hard contributions can be calculated perturbatively, while the remaining infrared infinity part could be absorbed into the nonperturbative universal inputs, such as the hadron wave functions. The high-order QCD hard corrections can then be extracted by making the difference of the full quark amplitudes with the convoluted amplitudes of effective wave functions and the hard amplitudes.

Over the past decade, the collinear factorization and the k_T factorization for the exclusive processes $\pi\gamma^* \rightarrow \gamma(\pi)$, $B \rightarrow \gamma(\pi)l\bar{\nu}$ and $\rho\gamma^* \rightarrow \pi$ have been demonstrated at the leading order (LO) and the next-to-leading order (NLO) order [6,7]. And the NLO hard corrections to these processes have also been found in Refs. [8,9]. Particularly, the NLO corrections to the time-like pion electromagnetic form factor are also calculated in the

PQCD approach [10], which at the same time to improve the parameterization precision of the timelike pion form factor [11], also can help us in studies about the pion B meson three-body hadronic decays [12–14].

In this paper, we will consider the ρ meson EM transition process, demonstrate the factorization for the $\rho\gamma^* \rightarrow \rho$ transition at the NLO level, as an extension to the previous work [7] where we verified explicitly that the factorization hypothesis is valid for the $\rho\gamma^* \rightarrow \pi$ process at the NLO level. This work may help us to understand the exclusive processes involving at least a vector hadron at the NLO level [15–19].

With the standard steps to factorize the full quark level amplitudes in the momentum, spin and color spaces, we will find that the soft divergences will cancel in the quark level diagrams and the remaining collinear divergences can be absorbed into the NLO wave functions. And we finally obtain the infrared safe NLO correction amplitudes to the LO hard kernels as expected, and write down these collinear parts in the form of nonlocal two quarks hadron matrix element with the gauge factor Wilson line.

The paper is organized as following. In Sec. II we present the leading order dynamical analysis and the hard amplitudes of $\rho \rightarrow \rho$ transition process. In Sec. III we prove that the factorization hypothesis is valid for $\rho\gamma^* \rightarrow \rho$ transition process at the next-to-leading order, and construct the NLO ρ meson wave functions by means of the nonlocal hadron matrix element with the gauge factor. A short summary will appear in the final section.

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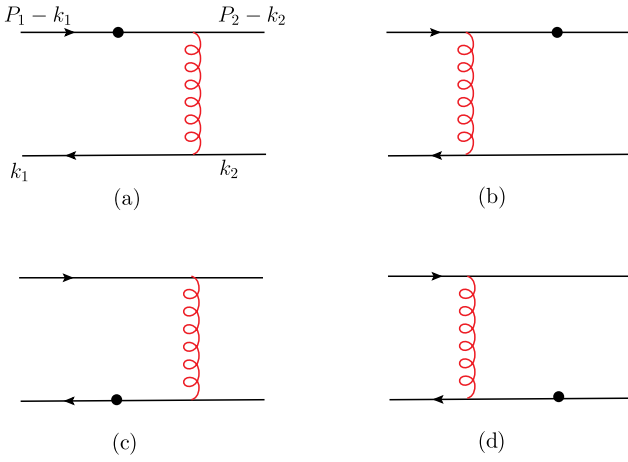


FIG. 1. The leading-order quark diagrams for the $\rho\gamma^* \rightarrow \rho$ form factor, with the virtual photon vertices, denoted by the symbol \bullet , setting at four different positions.

II. LEADING ORDER HARD KERNEL

The leading-order quark diagrams for the $\rho\gamma^* \rightarrow \rho$ transition are depicted in Fig. 1, with the following kinematics defined in the light cone coordinate:

$$p_{1\mu} = \frac{Q}{\sqrt{2}}(1, \gamma_\rho^2, \mathbf{0}_T), \quad k_{1\mu} = \frac{Q}{\sqrt{2}}(x_1, 0, \mathbf{0}_T);$$

$$\epsilon_{1\mu}(L) = \frac{1}{\sqrt{2}\gamma_\rho}(1, -\gamma_\rho^2, \mathbf{0}_T), \quad \epsilon_{1\mu}(T) = (0, 0, \mathbf{1}_T); \quad (1)$$

$$p_{2\mu} = \frac{Q}{\sqrt{2}}(\gamma_\rho^2, 1, \mathbf{0}_T), \quad k_{2\mu} = \frac{Q}{\sqrt{2}}(0, x_2, \mathbf{0}_T);$$

$$\epsilon_{2\mu}(L) = \frac{1}{\sqrt{2}\gamma_\rho}(-\gamma_\rho^2, 1, \mathbf{0}_T), \quad \epsilon_{2\mu}(T) = (0, 0, \mathbf{1}_T), \quad (2)$$

where the transverse momentum of the valance quarks in the initial and final mesons have been neglected. The momentum fraction x_i represents the strength for the corresponding valance quark to carry the longitudinal momentum, and the polarization vectors $\epsilon_{i\mu}(L/T)$ is defined by the normalization conditions $\epsilon_i^2(L/T) = -1$, and the parameter $\gamma_\rho = m_\rho/Q$. With the above kinematics in hand, the evolution behaviors of the ρ meson radiation form factor on the Lorentz invariant argument Q^2 can be obtained.

$$q^2 = (p_1 - p_2)^2 \equiv -Q^2, \quad Q^2 > 0. \quad (3)$$

The longitudinal and transverse ρ meson wave functions are obtained from the light cone expansion around

the small interval $x^2 \sim 0$ and expressed in series of different twists [20]. By employing the kinematics as shown in Eqs. (1), (2) and the dimensionless unit vector $n = (1, 0, \mathbf{0})$ and $v = (0, 1, \mathbf{0})$ we can write the two-particle initial and final ρ meson wave functions as in Ref. [19],

$$\Phi_\rho(p_1, \epsilon_{1T}) = \frac{i}{\sqrt{2N_c}} [M_\rho \epsilon_{1T} \phi_\rho^v(x_1) + \epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)],$$

$$\Phi_\rho(p_1, \epsilon_{1L}) = \frac{i}{\sqrt{2N_c}} [M_\rho \epsilon_{1L} \phi_\rho(x_1) + \epsilon_{1L} \not{p}_1 \phi_\rho^l(x_1) + M_\rho \phi_\rho^s(x_1)], \quad (4)$$

$$\Phi_\rho(p_2, \epsilon_{2T}) = \frac{i}{\sqrt{2N_c}} [M_\rho \epsilon_{2T} \phi_\rho^v(x_2) + \epsilon_{2T} \not{p}_2 \phi_\rho^T(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)],$$

$$\Phi_\rho(p_2, \epsilon_{2L}) = \frac{i}{\sqrt{2N_c}} [M_\rho \epsilon_{2L} \phi_\rho(x_2) + \epsilon_{2L} \not{p}_2 \phi_\rho^l(x_2) + M_\rho \phi_\rho^s(x_2)], \quad (5)$$

where $N_c = 3$ is the color number, the distribution amplitudes (DAs) ϕ_ρ and ϕ_ρ^T are the leading twist-2 (T2) terms, while $\phi_\rho^{a/v}$ and $\phi_\rho^{l/s}$ are the subleading twist-3 (T3) terms.

It is very lucky that the four diagrams in Fig. 1, with the different locations to radiate the virtual photon, have the simple exchanging symmetry of the kinematics if we do not distinguish the flavor ‘‘u’’ and ‘‘d’’ in ρ meson, as claimed in [8,9,19]. For the sake of simplicity, we can just make the calculation for one of these four diagrams in detail and then extend the results to other three figures by simple exchanging symmetry. We first concentrate on the study for Fig. 1(a) in this paper.

In order to exactly show the factorization at NLO level and to obtain the shapes of the NLO ρ meson wave function term by term, we here classify the LO transition amplitude $G_a^{(0)}$ according to the double expansions to the different polarization components and the twists, for the initial and the final ρ meson wave functions. Then the LO amplitude with fixed polarization and twist is expressed as $G_{a,IJ,kl}^{(0)}$, where the first subscript a denotes the amplitude from Fig. 1(a), other subscripts $I, J = L, T$ ($k, l = 2, 3$) represent the polarization (twist) of the initial and the final meson wave functions in the relevant calculation.

The independent LO amplitudes for $\rho\gamma^* \rightarrow \rho$ transition process can then be written in the following form,

$$G_{a,LL,22}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2} \right], \quad (6)$$

$$G_{a,LL,33}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2} \right], \quad (7)$$

$$G_{a,LT,23}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \cdot \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2} \right], \quad (8)$$

$$G_{a,TL,23}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2} \right], \quad (9)$$

$$G_{a,TL,32}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)] \cdot \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2} \right], \quad (10)$$

$$G_{a,TT,33}^{(0)}(x_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2} \cdot \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right], \quad (11)$$

where $C_F = 4/3$ is a color factor. The standard analytical calculations for the LO amplitudes in Eqs. (6)–(11) show that:

- (1) For $G_{a,LL,22}^{(0)}$ and $G_{a,LL,33}^{(0)}$, where both the initial and final ρ meson are longitudinal polarized, only the wave functions with the same twist power (T2 & T2, or T3 & T3) for initial and final states contribute to the radiation amplitude, since there is no helicity flip between outgoing and incoming quarks for such combinations. In the light cone coordinate, the matrix γ_α and γ_μ in Eq. (6) are required to be γ_\perp and γ^+ respectively. While in Eq. (7) with the T3 wave functions contribution, the matrix γ_α is arbitrary and γ_μ is chosen to be γ^- to collect the dominate contribution from p_2^- .
- (2) For the amplitudes with different polarizations of the initial and final ρ meson, the contributions only arise from different twist power of the initial and final wave functions (T2 & T3, or T3 & T2), say the amplitude $G_{a,TL,23}^{(0)}$ and $G_{a,TL,32}^{(0)}$, because of the helicity flip. For other possible such kind of combinations, the amplitude $G_{a,LT,23}^{(0)}$ can provide contribution, but the amplitude $G_{a,LT,32}^{(0)}$ is forbidden due to the zero result of the contraction $\gamma^\alpha \epsilon_{2T} \not{p}_2 \gamma_\alpha$. It is clearly that the matrix γ_α is arbitrary in Eqs. (8), (9), while it should be γ_\perp in Eq. (10).
- (3) For the amplitude with both transverse polarized initial and final ρ meson, say $G_{a,TT,22}^{(0)}$ and $G_{a,TT,33}^{(0)}$, only $G_{a,TT,33}^{(0)}$ provides nonzero contribution, while $G_{a,TT,22}^{(0)}$ does not because of the same reasons as

mentioned above. And the matrix γ_α in Eq. (11) is arbitrary and γ_μ should to be γ^- .

III. FACTORIZATION OF $\rho\gamma^* \rightarrow \rho$ AT NEXT-TO-LEADING ORDER

In this section we will first make the demonstration for the factorization of the exclusive electromagnetic $\rho\gamma^* \rightarrow \rho$ transition process at the NLO level in the collinear and the soft approximations and with the omission of the transverse momentum. At the end of this section, we will discuss how to factorize the infrared contribution from the light cone NLO meson wave functions safely in the k_T factorization frame, where we will pick up the transverse momentum.

We again do not consider the infrared safe self-energy corrections to the far off-shell internal quark line, as we have done for the $\rho\gamma^* \rightarrow \pi$ transition process [7]. In general, the factorization of the long-distance and the short-distance physics in an exclusive QCD process is not trivial. To this end, we should divide the amplitudes simultaneously in the current and momentum spaces based on their infrared properties, and then sum up all the possible Feynman diagrams to collect the color factors and to maintain the gauge invariance. In more detail, we can deal with the factorization process in three steps: (a) First, the eikonal approximation is used to rearrange the singularity propagator into a more clear formula without γ matrix; (b) Second, the Fierz identity in Eq. (12) is used to separate the fermion currents, the different terms on the right-hand side (RHS) of Eq. (12) can be treated as the contributions with different twist, and then the infrared contribution

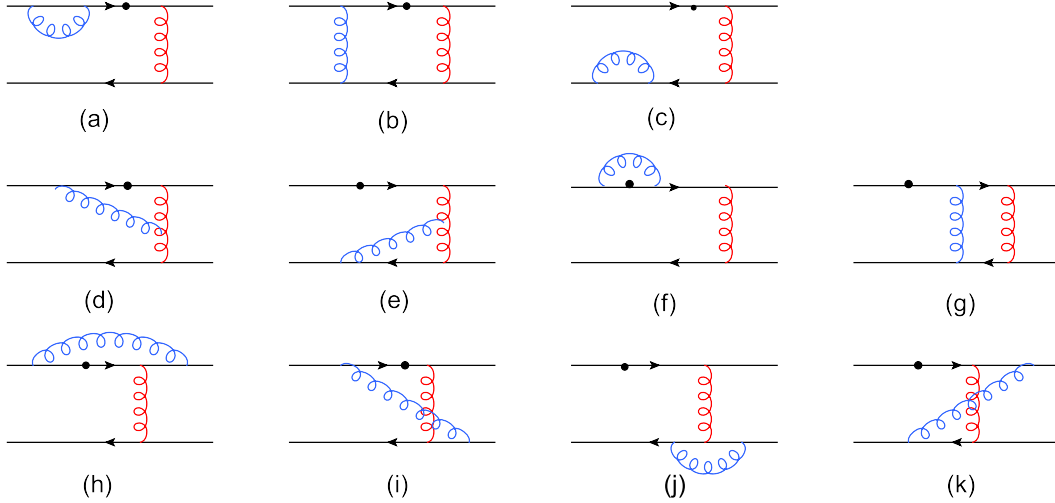


FIG. 2. The Feynman diagrams which provide the NLO QCD corrections to Fig. 1(a) with an additional gluon (blue curves) emitted from the quark (antiquark) lines of the initial ρ meson.

obtained in step-I can be absorbed into the newly defined wave functions; and (c) we finally should consider all the possible diagrams for a gluon radiation, which means that to collect the Feynman-diagrams with all possibilities for the end locations of the radiated gluon.

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma_5)_{ik}(\gamma_5)_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma^\alpha)_{lj} + \frac{1}{4}(\gamma_5\gamma^\alpha)_{ik}(\gamma_5\gamma^\alpha)_{lj} + \frac{1}{8}(\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta})_{lj}. \quad (12)$$

A. Collinear factorization for the NLO($\mathcal{O}(\alpha_s^2)$) corrections to Fig. 1(a)

We here will show the factorization of the NLO corrections for the Fig. 1(a) only, The NLO corrections for the other three relevant Feynman diagrams Figs. 1(b)–1(d) are similar in nature with those from Fig. 1(a) and can be obtained directly from the result of Fig. 1(a) by simple kinematic replacements. The NLO corrections to Fig. 1(a), in principle, can be divided into two parts:

- (1) The first set contains the diagrams where the gluon lines emitted from the valence quark lines of the initial ρ meson and attached to any other possible places, as depicted in Fig. 2.
- (2) The second set has the diagrams where the gluon lines emitted from the valence quark lines of the final ρ meson, very similar with those as illustrated in Fig. 2.

We therefore will first deal with Fig. 2, where the emitted gluon may be parallel to the initial ρ meson momentum p_1 . Even more, in a previous paper [7] the transversely polarized NLO initial ρ meson wave functions $\Phi_\rho^T(x_1)$ (T2) and $\Phi_\rho^{v,a}(x_1)$ (T3) have been defined in a form of nonlocal matrix elements and studied systematically. So here we can consider only for the

longitudinally polarized initial ρ meson wave functions $\Phi_\rho(x_1)$ (T2) and $\Phi_\rho^s(x_1)$ (T3) which entered into Fig. 2 through Eqs. (6), (7). Since the ρ meson wave function $\Phi_\rho^s(x_1)$ (T3) does not contribute through Fig. 1 at the leading order, we cannot get much NLO knowledge about this wave function $\Phi_\rho^s(x_1)$.

In general, there exist two kinds of standard infrared divergence¹ for the Feynman diagrams in Fig. 2. When the emitted gluon is exchanged between two on-shell external lines, the soft divergence will appear when the momentum $l = (l^+, l^-, l_\perp) \sim (\lambda, \lambda, \lambda)$ (here $\lambda \sim \Lambda_{\text{QCD}}$). And when the collinear gluon $l \sim (Q, \lambda^2/Q, \lambda^2)$ is emitted from a massless external quark line, this gluon will generate the collinear divergence [23,24]. In the ρ meson electromagnetic transition process, both the external and internal quark lines are light quarks and can be considered as massless, so the collinear divergence cannot be regularized by the quark mass and have to be absorbed into the meson wave functions, which is one of the main purposes of this paper as mentioned previously.

It is straightforward to write down the NLO amplitudes of Figs. 2(a)–2(c) because the end location of the radiated gluon lines does not attach to the internal lines. Then their radiations do not pollute the LO hard kernel even for the format of the corresponding amplitudes, so that such amplitudes can be factorized by a simple insertion of the Fierz identity as shown in Eq. (12) in the external quark lines,

¹A third kind of infrared divergence arose from the Glauber region where $l \sim (\lambda^2/Q, \lambda^2/Q, \lambda)$ [21], and this Glauber divergence appeared in the NLO spectator amplitudes of B meson two-body nonleptonic decays, can help us to resolve the long-standing $\pi\pi$ puzzle while still surviving the constraints in the standard model [22].

$$\begin{aligned}
G_{2a,LL,22}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \cdot (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'} \right] \\
&= \frac{1}{2} \Phi_{\rho,a}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(x_1; x_2),
\end{aligned} \tag{13}$$

$$\begin{aligned}
G_{2b,LL,22}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma_\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \cdot (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \right] \\
&= \Phi_{\rho,b}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(\xi_1; x_2),
\end{aligned} \tag{14}$$

$$\begin{aligned}
G_{2c,LL,22}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1)^2 l^2} \right] \\
&= \frac{1}{2} \Phi_{\rho,c}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(x_1; x_2),
\end{aligned} \tag{15}$$

$$\begin{aligned}
G_{2a,LL,33}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right] \\
&= \frac{1}{2} \Phi_{\rho,a}^{(1),s}(x_1, \xi_1) \otimes G_{a,LL,33}^{(0)}(x_1; x_2),
\end{aligned} \tag{16}$$

$$\begin{aligned}
G_{2b,LL,33}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma^{\rho'}}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right] \\
&= \Phi_{\rho,b}^{(1),s}(x_1, \xi_1) \otimes G_{a,LL,33}^{(0)}(\xi_1; x_2),
\end{aligned} \tag{17}$$

$$\begin{aligned}
G_{2c,LL,33}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1)^2 l^2} \right] \\
&= \frac{1}{2} \Phi_{\rho,c}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,33}^{(0)}(x_1; x_2),
\end{aligned} \tag{18}$$

where the NLO wave functions $\Phi_{\rho,i}^{(1)}$ and $\Phi_{\rho,i}^{(1),s}$ with $i = (a, b, c)$ will absorb all the infrared singularities from those reducible subdiagrams Figs. 2(a)–2(c), and can be written in the form of

$$\begin{aligned}
\Phi_{\rho,a}^{(1)}(x_1, \xi_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- \gamma_\rho^+ (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right], \\
\Phi_{\rho,b}^{(1)}(x_1, \xi_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- \gamma^{\rho'} (k_1 - l) \gamma_\rho^+ (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right], \\
\Phi_{\rho,c}^{(1)}(x_1, \xi_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma_\rho^+}{(k_1 - l)^2 (k_1)^2 l^2} \right],
\end{aligned} \tag{19}$$

$$\begin{aligned}
\Phi_{\rho,a}^{(1),s}(x_1, \xi_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{(\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right], \\
\Phi_{\rho,b}^{(1),s}(x_1, \xi_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma^{\rho'} (k_1 - l) (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right], \\
\Phi_{\rho,c}^{(1),s}(x_1, \xi_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1}{(k_1 - l)^2 (k_1)^2 l^2} \right].
\end{aligned} \tag{20}$$

The QCD dynamics requires that the soft gluon could not resolve the color structure of the initial ρ meson, so it's reasonable to find that the soft divergences in these reducible amplitudes $G_{2a,LL,kl}^{(1)}$, $G_{2b,LL,kl}^{(1)}$, $G_{2c,LL,kl}^{(1)}$ with $k = (l, 2, 3)$, corresponding to the Figs. 2(a)–2(c), will be canceled from each other. And the remaining collinear divergences can then be absorbed into the NLO wave functions $\Phi_{\rho,j}^{(1)}$ and $\Phi_{\rho,j}^{(1),s}$ with $j = (a, b, c)$.

We then consider the NLO correction to the LO amplitude $G_{a,LL,22}^{(0)}$ in Eq. (6). The irreducible subdiagrams Figs. 2(d)–2(g) will generate collinear divergences only, since the soft contributions, generated when the end locations of the radiated gluons are connected to one of the internal propagators, are highly suppressing by the

factor $1/Q^2$. What is more, the infrared divergence in Figs. 2(f) and 2(g) are suppressed also by the kinematics. We therefore can set their amplitudes $G_{2f,LL,22}^{(1)}$ and $G_{2g,LL,22}^{(1)}$ as zero safely in the factorization theorem.

$$G_{2d,LL,22}^{(1)} = -eg_s^4 \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma^\beta (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma^\gamma F_{\alpha\beta\gamma}}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 l^2} \right] \\ \sim \frac{9}{8} \Phi_{\rho,d}^{(1)}(x_1, \xi_1) \otimes [G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2) - G_{a,LL,22}^{(0)}(\xi_1; x_2)], \quad (21)$$

$$G_{2e,LL,22}^{(1)} = eg_s^4 \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\gamma (k_1 - l) \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma^\beta (\not{p}_2 - k_1) \gamma_\mu F_{\alpha\beta\gamma}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - k_2 - l)^2 (k_1 - l)^2 l^2} \right] \\ \sim \frac{9}{8} \Phi_{\rho,e}^{(1)}(x_1, \xi_1) \otimes [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2)], \quad (22)$$

$$G_{2f,LL,22}^{(1)} = \frac{-eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma^{\rho'} (\not{p}_2 - k_1 + l) \cdot \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_2 - k_1 + l)^2 l^2 (p_1 - k_1 + l)^2} \right] \sim 0, \quad (23)$$

$$G_{2g,LL,22}^{(1)} = \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_{\rho'} \cdot (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (k_1 - l)^2 l^2 (p_2 - k_1 + l)^2} \right] \sim 0, \quad (24)$$

The new LO hard amplitudes in Eqs. (21), (22) with different variables x_1 , ξ_1 , and x_2 are collected as follows,

$$G_{a,LL,22}^{(0)}(\xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \right], \quad (25)$$

$$G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \right], \quad (26)$$

$$G_{a,LL,22}^{(0)'}(x_1, \xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\not{\epsilon}_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \right]. \quad (27)$$

And it is easy to find the relation $G_{a,LL,22}^{(0)}(x_1; \xi_1, x_2) = G_{a,LL,22}^{(0)'}(x_1; \xi_1, x_2)$ in the collinear region $l \parallel p_1$. For the tensor $F_{\alpha\beta\gamma} = g_{\alpha\beta}(2k_2 - 2k_1 + l)_\gamma + g_{\beta\gamma}(k_1 - k_2 - 2l)_\alpha + g_{\gamma\alpha}(k_1 - k_2 + l)_\beta$ in Eq. (21), only the terms proportional to $g_{\alpha\beta}$ provide the corrections to the LO hard kernel $G_{a,LL,22}^{(0)}$. After applying the eikonal approximation to divide the collinear divergence out of the several LO hard kernels $G^{(0)}$, the NLO twist-2 longitudinal ρ meson wave function $\Phi_{\rho,d}^{(1)}$, with the gluon radiated from the left-up quark line, can be written as

$$\Phi_{\rho,d}^{(1)}(x_1, \xi_1) = \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_\rho^- \gamma_\rho^+](\not{p}_1 - k_1 + l) \gamma^{\rho'} v_{\rho'}}{(p_1 - k_1 + l)^2 l^2 (v \cdot l)} \right]. \quad (28)$$

Similarly, for the tensor $F_{\alpha\beta\gamma} = g_{\alpha\beta}(2k_2 - 2k_1 + l)_\gamma + g_{\beta\gamma}(k_1 - k_2 + l)_\alpha + g_{\gamma\alpha}(k_1 - k_2 - 2l)_\beta$ in Eq. (22), only the first term contributes to the LO hard kernel $G_{a,LL,22}^{(0)}$. And we can also write the NLO twist-2 longitudinal ρ meson wave function $\Phi_{\rho,e}^{(1)}$, in which the additional gluon is emitted from the left-down antiparton line, in the following form,

$$\Phi_{\rho,e}^{(1)}(x_1, \xi_1) = \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_\rho^-] \gamma^{\rho'} (k_1 - l) [\gamma_\rho^+] v_{\rho'}}{(k_1 - l)^2 l^2 (v \cdot l)} \right]. \quad (29)$$

The remaining subdiagrams in Figs. 2(h)–2(k) may include the collinear divergence as well as the soft divergence. The corresponding NLO amplitudes can be written in the following standard forms,

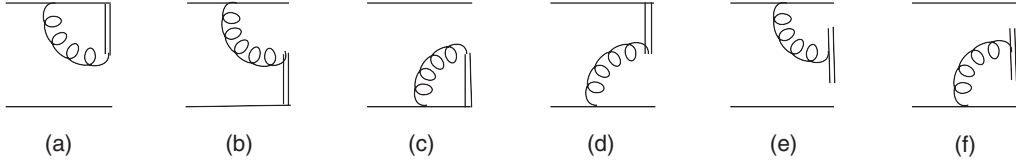


FIG. 3. $\mathcal{O}(\alpha_s)$ effective diagrams for the initial longitudinal ρ meson wave function. The vertical double line denotes the Wilson line along the light cone, whose Feynman rule is $v_\rho/(v \cdot l)$ as described in Eqs. (36), (50), (51), (52), (53).

$$G_{2h,LL,22}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon'_{2L} M_\rho \phi_\rho(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_2 - k_2 + l)^2 l^2 (p_1 - k_1 + l)^2} \cdot (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \right] \\ \sim -\frac{1}{8} \Phi_{\rho,d}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2), \quad (30)$$

$$G_{2i,LL,22}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 + l) \gamma^{\rho'} [\epsilon'_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 l^2 (k_2 + l)^2} \cdot (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \right] \\ \sim \frac{1}{8} \Phi_{\rho,d}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(\xi_1; x_2), \quad (31)$$

$$G_{2j,LL,22}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\epsilon'_{2L} M_\rho \phi_\rho(x_2)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_2 - l)^2 l^2 (k_1 - l)^2} \cdot \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\ \sim -\frac{1}{8} \Phi_{\rho,e}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(x_1; x_2), \quad (32)$$

$$G_{2k,LL,22}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma^\alpha [\epsilon'_{2L} M_\rho \phi_\rho(x_2)] \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (k_1 - l)^2 l^2 (p_2 - k_2 - l)^2} \cdot (\not{p}_2 - k_2 - l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\ \sim \frac{1}{8} \Phi_{\rho,e}^{(1)}(x_1, \xi_1) \otimes G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2). \quad (33)$$

To investigate the collinear factorization in the NLO level while keeping the gauge invariance, we should sum up all the irreducible amplitudes with the same gluon radiation starting-point together. The summed amplitudes for the Feynman diagrams with the gluon radiated from the left-up quark line can be written as

$$G_{2up,LL,22}^{(1)}(x_1; x_2) = \sum_{m=d,f,h,i} G_{2m,LL,22}^{(1)}(x_1; x_2) = \Phi_{\rho,d}^{(1)}(x_1, \xi_1) \otimes [G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2) - G_{a,LL,22}^{(0)}(\xi_1; x_2)]. \quad (34)$$

While the summed amplitudes for the gluon radiated from the left-down antiquark line is written as

$$G_{2down,LL,22}^{(1)}(x_1; x_2) = \sum_{n=e,g,j,k} G_{2n,LL,22}^{(1)}(x_1; x_2) = \Phi_{\rho,e}^{(1)}(x_1, \xi_1) \otimes [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1, \xi_1; x_2)]. \quad (35)$$

There exist no soft divergences in these summed amplitudes. With a simple deformation of the amplitudes in the soft kinematic region, the soft singularities in Eqs. (30), (31) and Eqs. (32), (33) cancel by themselves. The remaining collinear divergences, fortunately, can be absorbed into the NLO $T 2\rho$ meson Wave function $\Phi_\rho^{(1)}$ by redefinition of the nonlocal hadronic matrix element with the spin structure $\gamma^-/2$:

$$\Phi_\rho^{(1)} = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} e^{-ix p_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma^-}{2} (-ig_s) \\ \times \int_0^{y^-} dz v A(zv) q(0) | \rho(p_1) \rangle. \quad (36)$$

The relevant effective diagrams for the NLO initial ρ meson wave function are shown in Fig. 3, in which all the collinear divergences from the rho meson radiation in the irreducible NLO quark diagrams Figs. 2(d)–2(k) are collected. Figures 3(a) and 3(c) describe the radiative corrections with the gluon momentum not flow into the LO hard kernel. Figures 3(b) and 3(d) [Figs. 3(e) and 3(f)] lead to the radiative corrections with the gluon momentum flow into the LO hard kernel completely (partially). For the NLO vector wave functional $\Phi_\rho^{(1)}$ discussed in this paper, only the subdiagrams Figs. 3(b), 3(c), 3(e), and 3(f) give contributions. This is because, as described in Eqs. (34), (35), the gluon momentum started at the up-line quark flow into the LO hard kernel (no matter completely or partially),

while the gluon momentum started at the down-line anti-quark can flow into the LO hard kernel partially or does not flow into the LO hard kernel. Then the collinear factorization hypothesis is valid for the NLO corrections to the LO amplitude $G_{a,LL,22}^{(0)}$ in Eq. (6) with the longitudinally polarized and the leading twist initial and final states wave functions Φ_ρ .

For the NLO corrections to the LO amplitude $G_{a,LL,33}^{(0)}$ in Eq. (7), the infrared divergences in the NLO amplitudes corresponding to Figs. 2(d), 2(f), 2(h), and 2(i) are all forbidden by kinematics. Then only the subdiagrams Figs. 2(e), 2(g), 2(j), and 2(k) provide the infrared corrections to the LO amplitude $G_{a,LL,33}^{(0)}$. The relevant NLO amplitudes are of the following form,

$$G_{2d,LL,33}^{(1)} = eg_s^4 \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\beta (\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma^\nu F_{\alpha\beta\gamma}}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (k_1 - k_2 - l)^2 l^2} \right] \sim 0, \quad (37)$$

$$G_{2e,LL,33}^{(1)} = -eg_s^4 \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\nu (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\beta (\not{p}_2 - k_1) \gamma^\mu F_{\alpha\beta\gamma}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1 - k_2 - l)^2 l^2} \right] \\ \sim \frac{9}{16} \Phi_{\rho,e}^{(1),s}(x_1, \xi_1) \otimes [G_{a,LL,33}^{(0)}(x_1; x_2) - G_{a,LL,33}^{(0)}(x_1, \xi_1; x_2)], \quad (38)$$

$$G_{2f,LL,33}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma^{\rho'} (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (p_2 - k_1 + l)^2 l^2} \right] \sim 0, \quad (39)$$

$$G_{2g,LL,33}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^{\rho'} (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (k_1 - l)^2 l^2} \right] \\ \sim \Phi_{\rho,e}^{(1),s}(x_1, \xi_1) \otimes [G_{a,LL,33}^{(0)}(x_1, \xi_1; x_2) - G_{a,LL,33}^{(0)}(\xi_1; x_2)]. \quad (40)$$

$$G_{2h,LL,33}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(k_1 - k_2)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (p_2 - k_2 + l)^2 l^2} \right] \sim 0, \quad (41)$$

$$G_{2i,LL,33}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha (k_2 + l) \gamma^{\rho'} [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (k_2 + l)^2 l^2} \right] \sim 0, \quad (42)$$

$$G_{2j,LL,33}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(k_1 - k_2)^2 (p_2 - k_1)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \right] \\ \sim -\frac{1}{8} \Phi_{\rho,e}^{(1),s}(x_1, \xi_1) \otimes G_{a,LL,33}^{(0)}(x_1; x_2), \quad (43)$$

$$G_{2k,LL,33}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 - l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(k_1 - k_2 - l)^2 (p_2 - k_1)^2 (k_1 - l)^2 (p_2 - k_2 - l)^2 l^2} \right] \\ \sim \frac{1}{8} \Phi_{\rho,e}^{(1),s}(x_1, \xi_1) \otimes G_{a,LL,33}^{(0)}(x_1, \xi_1; x_2), \quad (44)$$

where the NLO ρ meson wave function $\Phi_\rho^{(1),s}$ is of the form

$$\Phi_{\rho,e}^{(1),s}(x_1, \xi_1) = \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{(k_1 - l) \gamma^\rho v_{\rho'}}{(k_1 - l)^2 l^2 (v \cdot l)} \right], \quad (45)$$

which absorbs all the collinear divergences. The new LO hard amplitudes $G_{a,LL,33}^{(0)}$ appeared in Eqs. (38), (40), (43), (44) are of the following form,

$$G_{a,LL,33}^{(0)}(\xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \right], \quad (46)$$

$$G_{a,LL,33}^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \right], \quad (47)$$

$$G_{a,LL,33}^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[M_\rho \phi_\rho^s(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \right]. \quad (48)$$

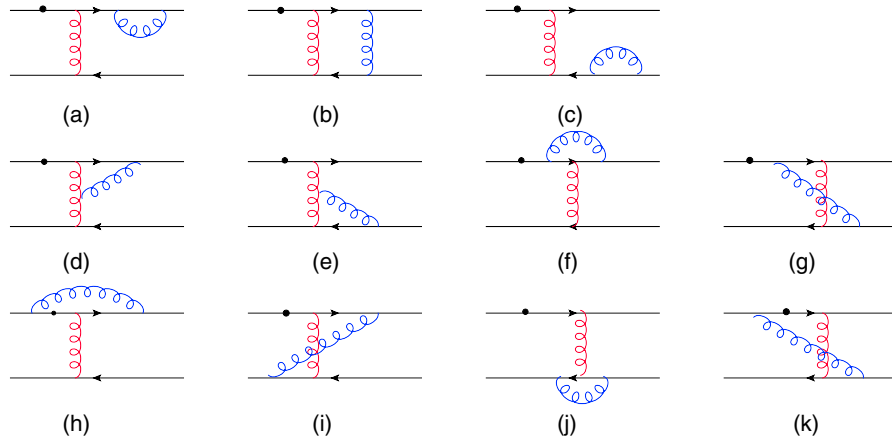


FIG. 4. Analogous to Fig. 2, but with the additional gluon (blue curves) emitted from the quark (antiquark) lines of the final state ρ meson.

Because there is no NLO infrared corrections to LO hard amplitude $G_{a,LL,33}^{(1)}$ from the gluon radiated from the up quark line, so we can just sum up the infrared contribution generated from the gluon radiated from the down antiquark line:

$$G_{2\text{down},LL,33}^{(1)}(x_1; x_2) = \sum_{n=e,g,j,k} G_{2n,LL,33}^{(1)}(x_1; x_2) \\ = \Phi_{\rho,e}^{(1)}(x_1, \xi_1) \otimes \left[\frac{7}{16} G_{LL,33}^{(0)}(x_1; x_2) - \frac{1}{16} G_{LL,33}^{(0)}(x_1, \xi_1; x_2) - \frac{7}{8} G'_{LL,33}^{(0)}(x_1, \xi_1; x_2) \right]. \quad (49)$$

It is manifest that the soft divergences generated from Figs. 2(j) and 2(k) only and will be canceled by each other, while the collinear divergence can also be absorbed into the NLO T3 longitudinal meson DA $\Phi_{\rho}^{(1),s}$, just like what we have done for the collinear divergences in the NLO corrections to $G_{a,LL,22}^{(1)}$ above. The nonlocal hadronic matrix element of $\Phi_{\rho}^{(1),s}$ with the scalar structure $I/2$ can be written as

$$\Phi_{\rho}^{(1),s} = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} e^{-ix p_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{1}{2} (-ig_s) \\ \times \int_0^{y^-} dz v \cdot A(zv) q(0) | \rho(p_1) \rangle. \quad (50)$$

Here only the three subdiagrams Figs. 3(c), 3(d), and 3(g), where the gluon was radiated from the down antiquark line, will contribute.

Following the similar procedures, we can also calculate the NLO infrared corrections to the remaining four LO hard amplitudes $G_{a,LT,23}^{(0)}(x_1; x_2)$, $G_{a,TL,23}^{(0)}(x_1; x_2)$, $G_{a,TL,32}^{(0)}(x_1; x_2)$ and $G_{a,TT,33}^{(0)}(x_1; x_2)$ as listed Eqs. (8)–(11). The explicit expressions for the NLO corrections to these four LO amplitudes and their factorization are all given in detail in the Appendix.

After absorbing the collinear divergences, the NLO transverse ρ meson wave functions can also be written in the nonlocal matrix element with the gluon momentum along the light cone and flow in a finite interval $(0, y^-)$:

$$\Phi_{\rho}^{(1),T} = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} e^{-ix p_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma_{\perp} \gamma^+}{2} (-ig_s) \\ \times \int_0^{y^-} dz v \cdot A(zv) q(0) | \rho(p_1) \rangle, \quad (51)$$

$$\Phi_{\rho}^{(1),v} = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} e^{-ix p_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma_{\perp}}{2} (-ig_s) \\ \times \int_0^{y^-} dz v \cdot A(zv) q(0) | \rho(p_1) \rangle, \quad (52)$$

$$\Phi_{\rho}^{(1),a} = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} e^{-ix p_1^+ y^-} \langle 0 | \bar{q}(y^-) \frac{\gamma_{\perp} \gamma_5}{2} (-ig_s) \\ \times \int_0^{y^-} dz v \cdot A(zv) q(0) | \rho(p_1) \rangle. \quad (53)$$

It is reasonable to assume that the NLO transverse ρ meson wave functions in the above equations have the same forms as those we got for the $\rho \gamma^* \rightarrow \pi$ transition process [7], which satisfies the universality requirement of the non-perturbative infrared physics.

Now let us turn our attentions to the infrared contributions in $\mathcal{O}(\alpha_s^2)$ radiative corrections to Fig. 1(a) with the gluon emitted from the final ρ meson as shown explicitly in Fig. 4, where the infrared divergences would appear when the radiated (blue) gluon is parallel with the final ρ meson momentum p_2 . Why we do this is to show that factorization

is still exactly work at the NLO for the gluon radiation from the final state ρ meson, even though there exist asymmetry for the initial and final ρ meson in Fig. 1(a) because of the choice of the weak vector position. Right here, as an example, we will simply present the detailed NLO corrections for the LO amplitudes $G_{a,LL,22}^{(0)}$ only, extract the NLO

final ρ meson wave function $\Phi_\rho^{(1)}(\xi_2, x_2)$, but do not present the NLO corrections for other LO amplitudes for the sake of the concision of this paper because the calculations and the factorization processes are very similar.

The NLO amplitudes of reducible subdiagrams Figs. 4(a)–4(c) are listed as

$$\begin{aligned} G_{4a,LL,22}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_{\rho'} (\not{p}_2 - k_2 + l) \gamma^{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_2 - k_2)^2 (p_2 - k_2 + l)^2 l^2} \cdot (\not{p}_2 - k_2) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\ &= \frac{1}{2} G_{a,LL,22}^{(0)}(x_1; x_2) \otimes \Phi_{\rho,a}^{(1)}(\xi_2, x_2), \end{aligned} \quad (54)$$

$$\begin{aligned} G_{4b,LL,22}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 - l) \gamma_{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2 + l)^2 (p_2 - k_2 + l)^2 (k_2 - l)^2 l^2} \cdot (\not{p}_2 - k_2 + l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\ &= G_{a,LL,22}^{(0)}(x_1; x_2) \otimes \Phi_{\rho,b}^{(1)}(\xi_2, x_2), \end{aligned} \quad (55)$$

$$\begin{aligned} G_{4c,LL,22}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha k_2 \gamma_{\rho'} (k_2 - l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_2 - l)^2 (k_2)^2 l^2} \right] \\ &= \frac{1}{2} G_{a,LL,22}^{(0)}(x_1; x_2) \otimes \Phi_{\rho,c}^{(1)}(\xi_2, x_2), \end{aligned} \quad (56)$$

with the NLO wave functions $\Phi_{\rho,i}^{(1)}$, $i = a, b, c$

$$\begin{aligned} \Phi_{\rho,a}^{(1)}(\xi_2, x_2) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- \gamma_\rho^+ \gamma_{\rho'} (\not{p}_2 - k_2 + l) \gamma^{\rho'} (\not{p}_2 - k_2)}{(p_2 - k_2)^2 (p_2 - k_2 + l)^2 l^2} \right], \\ \Phi_{\rho,b}^{(1)}(\xi_2, x_2) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- (k_2 - l) \gamma_{\rho'} \gamma_\rho^+ \gamma_{\rho'} (\not{p}_2 - k_2 + l)}{(p_2 - k_2 + l)^2 (k_2 - l)^2 l^2} \right], \\ \Phi_{\rho,c}^{(1)}(\xi_2, x_2) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_\rho^- k_2 \gamma^{\rho'} (k_2 - l) \gamma_{\rho'} \gamma_\rho^+}{(k_2 - l)^2 (k_2)^2 l^2} \right], \end{aligned} \quad (57)$$

Amplitudes for the irreducible subdiagrams Figs. 4(d)–4(k) are collected with the similar treatments as for Fig. 2:

$$\begin{aligned} G_{4d,LL,22}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^\gamma (\not{p}_2 - k_2 + l) \gamma_\beta (\not{p}_2 - k_1) \gamma_\mu F_{\alpha\gamma\beta}}{(p_2 - k_2 + l)^2 (p_2 - k_1)^2 (k_1 - k_2 + l)^2 (k_1 - k_2)^2 l^2} \right] \\ &\sim \frac{9}{8} [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1; \xi_2)] \otimes \Phi_{\rho,d}^{(1)}(\xi_2, x_2), \end{aligned} \quad (58)$$

$$\begin{aligned} G_{4e,LL,22}^{(1)} &= eg_s^4 \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 - l) \gamma^\gamma [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^\beta (\not{p}_2 - k_1) \gamma_\mu F_{\alpha\gamma\beta}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - k_2 + l)^2 (k_2 - l)^2 l^2} \right] \\ &\sim \frac{9}{8} [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1; \xi_2)] \otimes \Phi_{\rho,e}^{(1)}(\xi_2, x_2), \end{aligned} \quad (59)$$

$$\begin{aligned} G_{4f,LL,22}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_2 - k_1 + l)^2 l^2 (p_2 - k_2 + l)^2} \cdot (\not{p}_2 - k_1 + l) \gamma_{\rho'} (\not{p}_2 - k_1) \gamma_\mu \right] \\ &\sim -\frac{1}{8} [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2)] \otimes \Phi_{\rho,d}^{(1)}(\xi_2, x_2), \end{aligned} \quad (60)$$

$$G_{4g,LL,22}^{(1)} = \frac{-eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha}{(p_2 - k_1)^2 (k_1 - k_2 + l)^2 (k_2 - l)^2 l^2 (p_2 - k_1 - l)^2} \cdot (\not{p}_2 - k_1 - l) \gamma_{\rho'} (\not{p}_2 - k_1) \gamma_\mu \right] \\ \sim \frac{1}{8} [G_{a,LL,22}^{(0)}(x_1; \xi_2) - G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2)] \otimes \Phi_{\rho,e}^{(1)}(\xi_2, x_2), \quad (61)$$

$$G_{4h,LL,22}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_2 - k_2 + l)^2 l^2 (p_1 - k_1 + l)^2} \cdot (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \right] \\ \sim -\frac{1}{8} G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2) \otimes \Phi_{\rho,d}^{(1)}(\xi_2, x_2), \quad (62)$$

$$G_{4i,LL,22}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 + l) \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2 + l)^2 (p_2 - k_2 + l)^2 l^2 (k_1 + l)^2} \cdot (\not{p}_2 - k_2 + l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\ \sim \frac{1}{8} G_{a,LL,22}^{(0)}(x_1; \xi_2) \otimes \Phi_{\rho,d}^{(1)}(\xi_2, x_2), \quad (63)$$

$$G_{4j,LL,22}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_2 - l)^2 l^2 (k_1 - l)^2} \right] \\ \sim -\frac{1}{8} G_{a,LL,22}^{(0)}(x_1; x_2) \otimes \Phi_{\rho,e}^{(1)}(\xi_2, x_2), \quad (64)$$

$$G_{4k,LL,22}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha}{(p_2 - k_1 - l)^2 (k_1 - k_2 + l)^2 (k_2 - l)^2 l^2 (p_1 - k_1 - l)^2} \cdot (\not{p}_2 - k_1 - l) \gamma_\mu (\not{p}_1 - k_1 - l) \gamma_{\rho'} \right] \\ \sim \frac{1}{8} G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2) \otimes \Phi_{\rho,e}^{(1)}(\xi_2, x_2), \quad (65)$$

where the NLO final state ρ meson longitudinal wave function $\Phi_{\rho,d}^{(1)}$, $\Phi_{\rho,e}^{(1)}$ obtained from the NLO amplitudes $G_{4d,LL,22}^{(1)}$, $G_{4e,LL,22}^{(1)}$ respectively are of the following form:

$$\Phi_{\rho,d}^{(1)}(\xi_2, x_2) = -\frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_\rho^- \gamma_\rho^+] \gamma^{\rho'} (\not{p}_2 - k_2 + l) n_{\rho'}}{(p_2 - k_2 + l)^2 l^2 (n \cdot l)} \right]. \quad (66)$$

$$\Phi_{\rho,e}^{(1)}(\xi_2, x_2) = \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_\rho^- \gamma_\rho^+] (k_2 - l) \gamma^{\rho'} n_{\rho'}}{(k_2 - l)^2 l^2 (n \cdot l)} \right], \quad (67)$$

and the modified LO hard kernels with the gluon momentum flow, partly flowing into the original LO hard kernel $G_{a,LL,22}^{(0)}(x_1; x_2)$, as in the case for the NLO correction from the initial meson radiations, are defined as,

$$G_{a,LL,22}^{(0)}(x_1; \xi_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2 + l)^2} \right], \quad (68)$$

$$G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \right], \quad (69)$$

$$G_{a,LL,22}^{(0)}(x_1; x_2, \xi_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1 - l)^2 (k_1 - k_2 + l)^2} \right]. \quad (70)$$

Summing up the infrared amplitudes sorted by the gluon radiated either from the right-up [Figs. 4(d), 4(f), 4(h), and 4(i)] or from the right-down [Figs. 4(e), 4(g), 4(j), and 4(k)] quark line, we obtain their total factorization formulation for the final state gluon corrections to the LO amplitudes $G_{a,LL,22}^{(0)}$:

$$G_{4\text{up},LL,22}^{(1)}(x_1; x_2) = \sum_{m=d,f,h,i} G_{4m,LL,22}^{(1)}(x_1; x_2) = \frac{7}{8} [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1; \xi_2)] \otimes \Phi_{\rho,d}^{(1)}(\xi_2, x_2). \quad (71)$$

$$G_{4\text{down},LL,22}^{(1)}(x_1; x_2) = \sum_{n=e,g,j,k} G_{4n,LL,22}^{(1)}(x_1; x_2) = \frac{7}{8} [G_{a,LL,22}^{(0)}(x_1; x_2) - G_{a,LL,22}^{(0)}(x_1; \xi_2)] \otimes \Phi_{\rho,e}^{(1)}(\xi_2, x_2). \quad (72)$$

The NLO corrections to the LO amplitude $G_{a,LL,33}^{(0)}$ from the final state radiations can be treated in the same way. So far the factorizations of Fig. 1(a) at the NLO for both the initial- and the final state radiation are explicitly shown above, and we can also write down the longitudinal NLO final ρ meson wave function $\Phi_{\rho}^{(1)}$, $\Phi_{\rho}^{(1),s}$ in the similar formula as the NLO initial meson wave function in Eqs. (36), (45).

B. k_T factorization of the NLO corrections to $\rho\gamma^* \rightarrow \rho$

In order to eliminate the endpoint singularity which may appear in the small x region under collinear factorization framework, the k_T factorization frame is developed by picking up previously dropped transversal momentum k_T for the external quark lines. And a Sudakov factor $e^{-S(t)}$, which indicating the probability there is no gluon radiation in the localized phase space for exclusive final products, is emerged with the resummation of the large divergences logarithms and furthermore, could suppress the soft-dynamics effectively [25].

In the PQCD approach, because the Sudakov factor increase transversal momentum k_T and longitudinal momentum fraction x simultaneously, the transversal momentum k_T in the initial and final meson bound state remain far less than the energy scales in the LO hard kernel, i.e., $k_{iT}^2 \ll k_1 \cdot k_2$. This argument apparently support our expectation that the eikonal approximation used in the collinear factorization are also valid for the k_T factorization. The collinear factorization operating for $\rho\gamma^* \rightarrow \rho$ in the previous subsection can be extended into the k_T factorization directly. The only difference is that we should take care of the transverse momentum \mathbf{l}_T in the denominator of the gluon propagators. Of course, we can also retrieve the Feynman-rule of the Wilson line by including \mathbf{l}_T through the Fourier transformation for the gauge field from $A(zv)$ to $\tilde{A}(l)$ for those NLO wave functions in Eqs. (36), (50), (51), (52), (53):

$$\begin{aligned} \int_0^\infty dzv \cdot A(zv) &\rightarrow \int_0^\infty dz \int dl e^{iz(v \cdot l + i\epsilon)} v \cdot \tilde{A}(l) \\ &= i \int dl \frac{v_\rho}{v \cdot l} \tilde{A}^\rho(l), \end{aligned} \quad (73)$$

$$\begin{aligned} \int_0^{y^-} dzv \cdot A(zv) &\rightarrow \int_0^{y^-} dz \int dl e^{[iz(v \cdot l + i\epsilon) - i\mathbf{l}_T \cdot \mathbf{b}]} v \cdot \tilde{A}(l) \\ &= -i \int dl \frac{v_\rho}{v \cdot l} e^{[il^+ y^- - i\mathbf{l}_T \cdot \mathbf{b}]} \tilde{A}^\rho(l), \end{aligned} \quad (74)$$

where the factor $\exp[il^+ y^-]$ in the integration in Eq. (74) will generate the delta function $\delta(\xi_1 - x_1 + \frac{l^+}{p_1})$, which imply that the gluon momentum is flowing into the LO hard kernel. The other factor $\exp[-i\mathbf{l}_T \cdot \mathbf{b}]$ represents the transversal momentum flowing into the LO hard kernel, accompanied with the function $\delta(\xi_1 - x_1 + \frac{l^+}{p_1})$. And this modification can be understood graphically as the deviation of the second half integration for Wilson line by a transversal interval \mathbf{b} from the light cone direction, as illustrated by Figs. 5(a) and 5(b).

It is clear to see that the Wilson lines, whether along the light cone as shown in Fig. 5(a) or parallel to the light cone as illustrated in Fig. 5(b), will generate light cone singularity. The simple method to resolve this problem is to move the Wilson line a little away from the light cone, as depicted in Fig. 5(c), and regularize the nonphysical light cone singularity by the logarithm of n^2 with $n^2 \neq 0$. Even the scheme-dependence induced by this variation has been resummed in Ref. [26] and found that such effect is small in size. One problem that resulted from such operation is the appearance of the pinched infrared divergence in the self-energy correction of the Wilson line. In order to avoid such kind of infrared divergence, the transverse momentum dependent distribution (TMD) with a soft subtraction was proposed in Ref. [27], and another new definition with the choice of two orthogonal gauge vectors for off light cone Wilson links also be discussed in Ref. [28]. But such topic is apparently beyond the scope of this paper. In this paper, we are aim to factorize the infrared NLO corrections to those LO amplitudes, so we still use the variation method to avoid the light cone singularity.

To finish this section, we write down the NLO ρ meson functions obtained from $\rho\gamma^* \rightarrow \rho$ at leading and subleading twist level in k_T factorization as the following form,

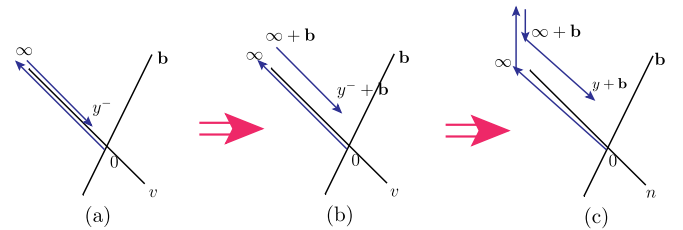


FIG. 5. The deviation of the integral (Wilson link) from the light cone by \mathbf{b} in the coordinate space for the two-parton meson wave function.

$$\Phi_\rho^{(1),T}(x_1, \xi_1; \mathbf{b}_1) = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} \frac{d\mathbf{b}_1}{(2\pi)^2} e^{-ixp_1^+ y^- + ik_{1T} \cdot \mathbf{b}_1} \cdot \langle 0 | \bar{q}(y^-) \frac{\gamma_\perp^b \gamma^+}{4} (-ig_s) \int_0^y dzn \cdot A(zn) q(0) | \rho(p_1) \rangle, \quad (75)$$

$$\Phi_\rho^{(1),v}(x_1, \xi_1; \mathbf{b}_1) = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} \frac{d\mathbf{b}_1}{(2\pi)^2} e^{-ixp_1^+ y^- + ik_{1T} \cdot \mathbf{b}_1} \cdot \langle 0 | \bar{q}(y^-) \frac{\gamma_\perp}{2} (-ig_s) \int_0^y dzn \cdot A(zn) q(0) | \rho(p_1) \rangle, \quad (76)$$

$$\Phi_\rho^{(1),a}(x_1, \xi_1; \mathbf{b}_1) = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} \frac{d\mathbf{b}_1}{(2\pi)^2} e^{-ixp_1^+ y^- + ik_{1T} \cdot \mathbf{b}_1} \cdot \langle 0 | \bar{q}(y^-) \frac{\gamma_5 \gamma_\perp}{2} (-ig_s) \int_0^y dzn \cdot A(zn) q(0) | \rho(p_1) \rangle; \quad (77)$$

$$\Phi_\rho^{(1)}(x_1, \xi_1; \mathbf{b}_1) = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} \frac{d\mathbf{b}_1}{(2\pi)^2} e^{-ixp_1^+ y^- + ik_{1T} \cdot \mathbf{b}_1} \cdot \langle 0 | \bar{q}(y^-) \frac{\gamma^-}{2} (-ig_s) \int_0^y dzn \cdot A(zn) q(0) | \rho(p_1) \rangle \quad (78)$$

$$\Phi_\rho^{(1),s}(x_1, \xi_1; \mathbf{b}_1) = \frac{1}{2N_c P_1^+} \int \frac{dy^-}{2\pi} \frac{d\mathbf{b}_1}{(2\pi)^2} e^{-ixp_1^+ y^- + ik_{1T} \cdot \mathbf{b}_1} \cdot \langle 0 | \bar{q}(y^-) \frac{1}{2} (-ig_s) \int_0^y dzn \cdot A(zn) q(0) | \rho(p_1) \rangle. \quad (79)$$

With these gauge invariant NLO wave functions, we can then calculate the NLO hard corrections to $\rho\gamma^* \rightarrow \rho$, $B \rightarrow \rho$ transition processes with the factorization hypothesis.

IV. SUMMARY

In this paper we first investigated the ρ meson electromagnetic transition process and wrote down its hard amplitudes according to the polarizations and the light cone expansion power of the initial and final meson wave functions. Next we made an explicit demonstration for the collinear factorization of the NLO corrections to the LO amplitudes for the Feynman diagrams with a gluon radiated from the initial ρ meson. With the successful separation in collinear factorization, we then extend these results to the case of the k_T factorization.

From our analytical evaluations we found the following points:

- (a) For the considered $\rho\gamma^* \rightarrow \rho$ transition process, there are six LO amplitudes with the fixed polarization and the twist, say $G_{X,IJ,kl}^{(0)}$ with $X = (a, b, c, d)$, $I, J = (L, T)$ (polarization) and $k, l = (2, 3)$ (twist), for each sub-diagram ‘‘X’’ in Fig. 1. Between the four subdiagrams in Fig. 1 there exists a simple kinematic exchanging symmetry. Taking Fig. 1(a) as an example, its LO hard amplitudes have been sorted in terms of polarization and twist of the meson wave functions and given in Eqs. (6), (7), (8), (9), (10), (11).
- (b) We show explicitly that the factorization hypothesis is valid at the NLO level for $\rho\gamma^* \rightarrow \rho$ transition process, even if we include the transversal momentum of the light external quark lines. At the NLO level, the soft divergence from different subdiagrams will cancel each other in the quark level and the

remaining collinear divergences can be absorbed into the NLO meson wave functions $\Phi_\rho^{(1)}$.

- (c) The NLO ρ meson wave functions, which absorb the collinear divergences, can be written in a nonlocal two quarks hadron matrix elements as given in Eqs. (36), (50)–(53), (56)–(60), in which the integration path of the gauge factor represent the direction of the collinear gluon momentum.

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APPENDIX: FACTORIZATION OF THE NLO AMPLITUDES

In this Appendix we supply the explicit expressions for the NLO corrections to the four LO amplitudes $G_{a,LT,23}^{(0)}$, $G_{a,TL,23}^{(0)}$, $G_{a,TL,32}^{(0)}$ and $G_{a,TT,33}^{(0)}$ and the relevant factorization. The infrared divergences are absorbed into the NLO wave functions $\Phi_{\rho,i}^{(1)}$.

1. The NLO amplitudes for $G_{a,LT,23}^{(0)}$

For all the eleven subdiagrams in Fig. 2, the corresponding NLO amplitudes for $G_{a,LT,23}^{(0)}$ are the following

$$\begin{aligned}
G_{2a,LT,23}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{\gamma^\alpha (\not{p}_2 - k_1) \gamma_\mu (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&= \frac{1}{2} \Phi_{\rho,a}^{(1)} \otimes G_{a,LT,23}^{(0)}(x_1; x_2), \tag{A1}
\end{aligned}$$

$$\begin{aligned}
G_{2b,LT,23}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma^{\rho'} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma^\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&= \Phi_{\rho,b}^{(1)} \otimes G_{a,LT,23}^{(0)}(\xi_1, x_2), \tag{A2}
\end{aligned}$$

$$\begin{aligned}
G_{2c,LT,23}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1)^2 l^2} \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&= \frac{1}{2} \Phi_{\rho,c}^{(1)} \otimes G_{a,LT,23}^{(0)}(x_1; x_2), \tag{A3}
\end{aligned}$$

$$\begin{aligned}
G_{2d,LT,23}^{(1)} &= eg_s^4 \text{Tr} \left[\frac{\gamma_\beta (\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma^\nu F_{\alpha\beta\gamma} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (k_1 - k_2 - l)^2 l^2} \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&\sim \frac{9}{16} \Phi_{\rho,d}^{(1)} \otimes [G_{a,LT,23}^{(0)}(x_1; \xi_1; x_2) - G_{a,LT,23}^{(0)}(\xi_1; x_2)], \tag{A4}
\end{aligned}$$

$$\begin{aligned}
G_{2e,LT,23}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{\gamma_\beta (\not{p}_2 - k_1) \gamma^\mu F_{\alpha\beta\gamma} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\nu (k_1 - l) \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1 - k_2 - l)^2 l^2} \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&\sim \frac{9}{16} \Phi_{\rho,e}^{(1)} \otimes [G_{a,LT,23}^{(0)}(x_1; x_2) - G_{a,LT,23}^{(0)}(x_1; \xi_1; x_2)], \tag{A5}
\end{aligned}$$

$$\begin{aligned}
G_{2f,LT,23}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1) \gamma^{\rho'} (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (p_2 - k_1 + l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \sim \Phi_{\rho,d}^{(1)} \otimes [G_{a,LT,23}^{(0)}(x_1; x_2) - G_{a,LT,23}^{(0)}(x_1; \xi_1; x_2)], \tag{A6}
\end{aligned}$$

$$\begin{aligned}
G_{2g,LT,23}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^{\rho'} (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \sim \Phi_{\rho,e}^{(1)} \otimes [G_{a,LT,23}^{(0)}(x_1; \xi_1; x_2) - G_{a,LT,23}^{(0)}(\xi_1; x_2)]. \tag{A7}
\end{aligned}$$

$$\begin{aligned}
G_{2h,LT,23}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha}{(k_1 - k_2)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (p_2 - k_2 + l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \sim -\frac{1}{8} \Phi_{\rho,d}^{(1)} \otimes G_{a,LT,23}^{(0)}(x_1; \xi_1; x_2), \tag{A8}
\end{aligned}$$

$$\begin{aligned}
G_{2i,LT,23}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha (k_2 + l) \gamma^{\rho'}}{(k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (k_2 + l)^2 l^2} \right. \\
&\quad \left. \cdot [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \sim \frac{1}{8} \Phi_{\rho,d}^{(1)} \otimes G_{a,LT,23}^{(0)}(\xi_1; x_2), \tag{A9}
\end{aligned}$$

$$G_{2j,LT,23}^{(1)} = \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'}}{(k_1 - k_2)^2 (p_2 - k_1)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \cdot [\epsilon'_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon'_{2T} v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\ \sim -\frac{1}{8} \Phi_{\rho,e}^{(1)} \otimes G_{a,LT,23}^{(0)}(x_1; x_2), \quad (\text{A10})$$

$$G_{2k,LT,23}^{(1)} = \frac{-eg_s^4}{9} \text{Tr} \left[\frac{\gamma^{\rho'} (\not{p}_2 - k_2 - l) \gamma_\alpha (\not{p}_2 - k) \gamma_\mu [\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha}{(k_1 - k_2 - l)^2 (p_2 - k_1)^2 (k_1 - l)^2 (p_2 - k_2 - l)^2 l^2} \cdot [\epsilon'_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon'_{2T} v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\ \sim \frac{1}{8} \Phi_{\rho,e}^{(1)} \otimes G'_{a,LT,23}^{(0)}(x_1; \xi_1; x_2), \quad (\text{A11})$$

where the NLO ρ meson wave function $\Phi_\rho^{(1)}$ has the same expressions as those for the NLO corrections to the amplitude $G_{a,LL,22}^{(0)}$ in Eqs. (19), (28), (29), and the new LO hard kernels that appeared in the above equations are collected as follows,

$$G_{a,LT,23}^{(0)}(\xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon'_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon'_{2T} v^\rho n^\sigma \phi_\rho^a(x_2)]}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \cdot \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu [\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha \right], \quad (\text{A12})$$

$$G_{a,LT,23}^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon'_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon'_{2T} v^\rho n^\sigma \phi_\rho^a(x_2)]}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \cdot \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu [\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha \right], \quad (\text{A13})$$

$$G'_{a,LT,23}^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\epsilon'_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon'_{2T} v^\rho n^\sigma \phi_\rho^a(x_2)]}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \cdot \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon'_{1L} M_\rho \phi_\rho(x_1)] \gamma^\alpha \right]. \quad (\text{A14})$$

2. The NLO amplitudes for $G_{a,TL,23}^{(0)}$

The first three NLO amplitudes $G_{X,TL,23}^{(1)}$ for subdiagrams Figs. 2(a)–2(c) are the following

$$G_{2a,TL,23}^{(1)} = \frac{1-eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{(\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon'_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right] \\ = \frac{1}{2} \Phi_{\rho,a}^{(1),T} \otimes G_{a,TL,23}^{(0)}(x_1; x_2), \quad (\text{A15})$$

$$G_{2b,TL,23}^{(1)} = \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon'_{1T} M_\rho \phi_\rho^v(x_1)] \gamma^{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma^{\rho'}}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right] \\ = \Phi_{\rho,b}^{(1),T} \otimes G_{a,TL,23}^{(0)}(\xi_1; x_2), \quad (\text{A16})$$

$$G_{2c,TL,23}^{(1)} = \frac{1-eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\epsilon'_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1)^2 (k_1 - l)^2 l^2} \right] \\ = \frac{1}{2} \Phi_{\rho,c}^{(1),T} \otimes G_{a,TL,23}^{(0)}(x_1; x_2), \quad (\text{A17})$$

with the NLO twist-2 transverse wave functions

$$\begin{aligned}
\Phi_{\rho,a}^{(1),T} &= \frac{-ig_s^2 C_F}{8} \text{Tr} \left[\frac{\gamma_\perp^a \gamma^- \gamma_\perp^a \gamma^- (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right], \\
\Phi_{\rho,b}^{(1),T} &= \frac{ig_s^2 C_F}{8} \text{Tr} \left[\frac{\gamma_\perp^a \gamma^- \gamma^{\rho'} (k_1 - l) \gamma_{\perp,a} \gamma^+ (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(k_1 - l)^2 (p_1 - k_1 + l)^2 l^2} \right], \\
\Phi_{\rho,c}^{(1),T} &= \frac{-ig_s^2 C_F}{8} \text{Tr} \left[\frac{\gamma_\perp^a \gamma^- \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma_\perp^a \gamma^-}{(k_1 - l)^2 (k_1)^2 l^2} \right].
\end{aligned} \tag{A18}$$

The NLO infrared amplitudes $G_{x,TL,23}^{(1)}$ for the other eight subdiagrams Figs. 2(d)–2(k) are of the form of

$$\begin{aligned}
G_{2d,TL,23}^{(1)} &= eg_s^4 \text{Tr} \left[\frac{(\not{p}_1 - k_1 + l) \gamma^\nu F_{\alpha\beta\gamma} [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\beta (\not{p}_2 - k_1 + l) \gamma^\mu}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (k_1 - k_2 - l)^2 l^2} \right] \\
&\sim \frac{9}{16} \Phi_{\rho,d}^{(1),T}(x_1) \otimes [G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2) - G_{a,TL,23}^{(0)}(\xi_1; x_2)],
\end{aligned} \tag{A19}$$

$$\begin{aligned}
G_{2e,TL,23}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{(\not{p}_2 - k_1) \gamma^\mu F_{\alpha\beta\gamma} [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\nu (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\beta}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1 - k_2 - l)^2 l^2} \right] \\
&\sim \frac{9}{16} \Phi_{\rho,e}^{(1),T}(x_1) \otimes [G_{a,TL,23}^{(0)}(x_1; x_2) - G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2)],
\end{aligned} \tag{A20}$$

$$\begin{aligned}
G_{2f,TL,23}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma^{\rho'}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (p_2 - k_1 + l)^2 l^2} \right] \\
&\sim \Phi_{\rho,d}^{(1),T}(x_1) \otimes [G_{a,TL,23}^{(0)}(x_1; x_2) - G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2)],
\end{aligned} \tag{A21}$$

$$\begin{aligned}
G_{2g,TL,23}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - k_1 + l) \gamma^{\rho'} (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (k_1 - l)^2 l^2} \right] \\
&\sim \Phi_{\rho,e}^{(1),T}(x_1) \otimes [G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2) - G_{a,TL,23}^{(0)}(\xi_1; x_2)].
\end{aligned} \tag{A22}$$

$$\begin{aligned}
G_{2h,TL,23}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha}{(k_1 - k_2)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (p_2 - k_2 + l)^2 l^2} \right] \\
&\sim -\frac{1}{8} \Phi_{\rho,e}^{(1),T}(x_1) \otimes G_{a,TL,23}^{(0)}(x_1, \xi_1; x_2),
\end{aligned} \tag{A23}$$

$$\begin{aligned}
G_{2i,TL,23}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma^\alpha (k_2 + l) \gamma^{\rho'} [M_\rho \phi_\rho^s(x_2)] \gamma_\alpha}{(k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (k_2 + l)^2 l^2} \right] \\
&\sim \frac{1}{8} \Phi_{\rho,d}^{(1),T}(x_1) \otimes G_{a,TL,23}^{(0)}(\xi_1; x_2).
\end{aligned} \tag{A24}$$

$$\begin{aligned}
G_{2j,TL,23}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [M_\rho \phi_\rho^s(x_2)]}{(k_1 - k_2)^2 (p_2 - k_1)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \right] \sim -\frac{1}{8} \Phi_{\rho,e}^{(1),T}(x_1) \otimes G_{a,TL,23}^{(0)}(x_1; x_2),
\end{aligned} \tag{A25}$$

$$\begin{aligned}
G_{2k,TL,23}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - k_2 - l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu [\epsilon_{1T} \not{p}_1 \phi_\rho^T(x_1)] \gamma_{\rho'} (k_1 - l) \gamma^\alpha [M_\rho \phi_\rho^s(x_2)] \gamma^{\rho'}}{(k_1 - k_2 - l)^2 (p_2 - k_1)^2 (k_1 - l)^2 (p_2 - k_2 - l)^2 l^2} \right] \\
&\sim \frac{1}{8} \Phi_{\rho,e}^{(1),T}(x_1) \otimes G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2),
\end{aligned} \tag{A26}$$

with the NLO twist-2 transverse ρ meson wave functions,

$$\begin{aligned}\Phi_{\rho,d}^{(1),T}(x_1) &= \frac{-ig_s^2 C_F}{8} \text{Tr} \left[\frac{\gamma_{\perp}^{\alpha} \gamma^{-} \gamma_{\perp}^{\alpha} \gamma^{-} (\not{p}_1 - k_1 + l) \gamma^{\rho} v_{\rho}}{(p_1 - k_1 + l)^2 l^2 (v \cdot l)} \right], \\ \Phi_{\rho,e}^{(1),T}(x_1) &= \frac{ig_s^2 C_F}{8} \text{Tr} \left[\frac{\gamma_{\perp}^{\alpha} \gamma^{-} \gamma^{\rho} (k_1 - l) \gamma_{\perp}^{\alpha} \gamma^{-} v_{\rho}}{(k_1 - l)^2 l^2 (v \cdot l)} \right],\end{aligned}\quad (\text{A27})$$

which could be written as a nonlocal matrix element with the Wilson lines.

The newly defined LO hard amplitudes $G_{a,TL,23}^{(0)}$, $G_{a,TL,23}^{(0)}$, and $G_{a,TL,23}^{(0)}$, as shown in Eqs. (A15)–(A17), (A19)–(A22), (A24)–(A25), are of the following form

$$G_{a,TL,23}^{(0)}(\xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} \not{p}_1 \phi_{\rho}^T(x_1)] \gamma^{\alpha} [M_{\rho} \phi_{\rho}^s(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \gamma_{\mu}}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \right], \quad (\text{A28})$$

$$G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} \not{p}_1 \phi_{\rho}^T(x_1)] \gamma^{\alpha} [M_{\rho} \phi_{\rho}^s(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \gamma_{\mu}}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \right], \quad (\text{A29})$$

$$G_{a,TL,23}^{(0)}(x_1; \xi_1; x_2) = \frac{-ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} \not{p}_1 \phi_{\rho}^T(x_1)] \gamma^{\alpha} [M_{\rho} \phi_{\rho}^s(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1) \gamma_{\mu}}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \right]. \quad (\text{A30})$$

3. The NLO amplitudes for $G_{a,TL,32}^{(0)}$

The eleven NLO amplitudes $G_{X,TL,32}^{(1)}$ for subdiagrams Figs. 2(a)–2(k) are the following

$$\begin{aligned}G_{2a,TL,32}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i\epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right. \\ &\quad \cdot \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1) \gamma_{\mu} (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'} \left. \right] \\ &= \frac{1}{2} \Phi_{\rho,a}^{(1),v}(x_1) \otimes G_{a,TL,32}^{(0),v}(x_1; x_2) + \frac{1}{2} \Phi_{\rho,a}^{(1),a} \otimes G_{a,TL,32}^{(0),a}(x_1; x_2),\end{aligned}\quad (\text{A31})$$

$$\begin{aligned}G_{2b,TL,32}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i\epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\ &\quad \cdot \gamma^{\rho'} (k_1 - l) \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \gamma_{\mu} (\not{p}_1 - k_1 + l) \gamma^{\rho'} \left. \right] \\ &= \Phi_{\rho,b}^{(1),v}(x_1) \otimes G_{a,TL,32}^{(0),v}(\xi_1; x_2) + \Phi_{\rho,b}^{(1),a} \otimes G_{a,TL,32}^{(0),a}(\xi_1; x_2),\end{aligned}\quad (\text{A32})$$

$$\begin{aligned}G_{2c,TL,32}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i\epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1)^2 l^2} \cdot \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1) \gamma_{\mu} \right. \\ &= \frac{1}{2} \Phi_{\rho,c}^{(1),v}(x_1) \otimes G_{a,TL,32}^{(0),v}(x_1; x_2) + \frac{1}{2} \Phi_{\rho,c}^{(1),a} \otimes G_{a,TL,32}^{(0),a}(x_1; x_2),\end{aligned}\quad (\text{A33})$$

$$\begin{aligned}G_{2d,TL,32}^{(1)} &= eg_s^4 \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i\epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (k_1 - k_2 - l)^2 l^2} \right. \\ &\quad \cdot \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\beta} (\not{p}_2 - k_1 + l) \gamma^{\mu} (\not{p}_1 - k_1 + l) \gamma^{\nu} F_{\alpha\beta\gamma} \left. \right] \sim 0,\end{aligned}\quad (\text{A34})$$

$$\begin{aligned}G_{2e,TL,32}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i\epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1 - k_2 - l)^2 l^2} \cdot \gamma^{\nu} (k_1 - l) \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\beta} (\not{p}_2 - k_1) \gamma^{\mu} F_{\alpha\beta\gamma} \right. \\ &\quad \sim \frac{9}{8} \Phi_{\rho,e}^{(1),v} \otimes [G_{a,TL,32}^{(0),v}(x_1; x_2) - G_{a,TL,32}^{(0),v}(x_1; \xi_1; x_2)] + \frac{9}{8} \Phi_{\rho,e}^{(1),a} \otimes [G_{a,TL,32}^{(0),a}(x_1; x_2) - G_{a,TL,32}^{(0),a}(x_1; \xi_1; x_2)],\end{aligned}\quad (\text{A35})$$

$$\begin{aligned}
G_{2f,TL,32}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (p_2 - k_1 + l)^2 l^2} \right. \\
&\quad \cdot \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma^{\rho'} (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \left. \right] \\
&\sim \Phi_{\rho,d}^{(1),v} \otimes [G_{a,TL,32}^{(0),v}(x_1; x_2) - G_{a,TL,32}^{(0),v}(x_1; \xi_1; x_2)] + \Phi_{\rho,d}^{(1),a}(x_1) \otimes [G_{a,TL,32}^{(0),a}(x_1; x_2) - G_{a,TL,32}^{(0),a}(x_1; \xi_1; x_2)],
\end{aligned} \tag{A36}$$

$$\begin{aligned}
G_{2g,TL,32}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\
&\quad \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^{\rho'} (\not{p}_2 - k_1) \gamma_\mu \left. \right] \sim 0.
\end{aligned} \tag{A37}$$

$$\begin{aligned}
G_{2h,TL,32}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (p_2 - k_2 + l)^2 l^2} \right. \\
&\quad \cdot \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 + l) \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma^\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \left. \right] \sim 0,
\end{aligned} \tag{A38}$$

$$\begin{aligned}
G_{2i,TL,32}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (k_2 + l)^2 l^2} \right. \\
&\quad \cdot \gamma^\alpha (k_2 + l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1 + l) \gamma_\mu (\not{p}_1 - k_1 + l) \gamma_{\rho'} \left. \right] \sim 0.
\end{aligned} \tag{A39}$$

$$\begin{aligned}
G_{2j,TL,32}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2)^2 (p_2 - k_1)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \right] \\
&\sim -\frac{1}{8} \Phi_{\rho,d}^{(1),v}(x_1) \otimes G_{a,TL,32}^{(0),v}(x_1; x_2) - \frac{1}{8} \Phi_{\rho,d}^{(1),a}(x_1) \otimes G_{a,TL,32}^{(0),a}(x_1; x_2),
\end{aligned} \tag{A40}$$

$$\begin{aligned}
G_{2k,TL,32}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{[\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2 - l)^2 (p_2 - k_1)^2 (k_1 - l)^2 (p_2 - k_2 - l)^2 l^2} \right. \\
&\quad \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha [\epsilon_{2L} M_\rho \phi_\rho(x_2)] \gamma^{\rho'} (\not{p}_2 - k_2 - l) \gamma_\alpha (\not{p}_2 - k_1) \gamma_\mu \left. \right] \\
&\sim \frac{1}{8} \Phi_{\rho,e}^{(1),v}(x_1) \otimes G_{a,TL,32}^{(0),v}(x_1; \xi_1; x_2) + \frac{1}{8} \Phi_{\rho,e}^{(1),a}(x_1) \otimes G_{a,TL,32}^{(0),a}(x_1; \xi_1; x_2),
\end{aligned} \tag{A41}$$

where the NLO twist-3 transverse ρ meson wave functions are the following

$$\begin{aligned}
\Phi_{\rho,a}^{(1),v}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_\rho^\perp] (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right], \\
\Phi_{\rho,a}^{(1),a}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_5] [\gamma_5 \gamma_\rho^\perp] (\not{p}_1 - k_1) \gamma^{\rho'} (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right],
\end{aligned} \tag{A42}$$

$$\begin{aligned}
\Phi_{\rho,b}^{(1),v}(x_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma^{\rho'} (k_1 - l) \gamma_{\perp\rho} (\not{p}_1 - k_1 + l) \gamma_{\rho'}]}{(p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right], \\
\Phi_{\rho,b}^{(1),a}(x_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_5] \gamma^{\rho'} (k_1 - l) [\gamma_5 \gamma_\rho^\perp] (\not{p}_1 - k_1 + l) \gamma_{\rho'}}{(p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right],
\end{aligned} \tag{A43}$$

$$\begin{aligned}\Phi_{\rho,c}^{(1),v}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{\gamma_{\perp\rho} \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma_{\perp}^{\rho'}}{(k_1 - l)^2 (k_1)^2 l^2} \right], \\ \Phi_{\rho,c}^{(1),a}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_5] \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma_5 \gamma_{\perp}^{\rho'}}{(k_1 - l)^2 (k_1)^2 l^2} \right].\end{aligned}\quad (\text{A44})$$

$$\begin{aligned}\Phi_{\rho,d}^{(1),v}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_{\rho'}^{\perp}] (\not{p}_1 - k_1 + l) \gamma^{\rho'} v_{\rho'}}{(p_1 - k_1 + l)^2 l^2 (v \cdot l)} \right], \\ \Phi_{\rho,d}^{(1),a}(x_1) &= \frac{-ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_5] [\gamma_5 \gamma_{\rho'}^{\perp}] (\not{p}_1 - k_1 + l) \gamma^{\rho'} v_{\rho'}}{(p_1 - k_1 + l)^2 l^2 (v \cdot l)} \right].\end{aligned}\quad (\text{A45})$$

$$\begin{aligned}\Phi_{\rho,e}^{(1),v}(x_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho}] \gamma^{\rho'} (k_1 - l) [\gamma_{\perp}^{\rho'}] v_{\rho'}}{(k_1 - l)^2 l^2 (v \cdot l)} \right], \\ \Phi_{\rho,e}^{(1),a}(x_1) &= \frac{ig_s^2 C_F}{4} \text{Tr} \left[\frac{[\gamma_{\perp\rho} \gamma_5] \gamma^{\rho'} (k_1 - l) [\gamma_5 \gamma_{\perp}^{\rho'}] v_{\rho'}}{(k_1 - l)^2 l^2 (v \cdot l)} \right].\end{aligned}\quad (\text{A46})$$

The new LO hard amplitudes $G_{a,TL,32}^{(0)}$ and $G_{a,TL,32}'^{(0)}$ that appeared in Eqs. (A31)–(A41) are the following

$$G_{a,TL,32}^{(0)}(\xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \cdot \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \gamma_{\mu} \right], \quad (\text{A47})$$

$$G_{a,TL,32}^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \cdot \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \gamma_{\mu} \right], \quad (\text{A48})$$

$$G_{a,TL,32}'^{(0)}(x_1, \xi_1; x_2) = -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \cdot \gamma^{\alpha} [\not{\epsilon}_{2L} M_{\rho} \phi_{\rho}(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1) \gamma_{\mu} \right]. \quad (\text{A49})$$

4. The NLO amplitudes for $G_{a,TT,33}^{(0)}$

The eleven NLO amplitudes $G_{X,TT,32}^{(1)}$ for subdiagrams Figs. 2(a)–2(k) are the following

$$\begin{aligned}G_{2a,TT,33}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{(\not{p}_1 - k_1 + l) \gamma_{\rho'} [\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)] \gamma^{\alpha}}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1)^2 (p_1 - k_1 + l)^2 l^2} \right. \\ &\quad \cdot \left. [\not{\epsilon}_{2T} M_{\rho} \phi_{\rho}^v(x_2) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu} v^{\rho} n^{\sigma} \phi_{\rho}^a(x_2)] \gamma^{\alpha} (\not{p}_2 - k_1) \gamma_{\mu} (\not{p}_1 - k_1) \gamma^{\rho'} \right] \\ &= \frac{1}{2} \Phi_{\rho,a}^{(1),v}(x_1) \otimes G_{a,TT,33}^{(0),v}(x_1; x_2) + \Phi_{\rho,a}^{(1),a} \otimes G_{a,TT,33}^{(0),a}(x_1; x_2),\end{aligned}\quad (\text{A50})$$

$$\begin{aligned}G_{2b,TT,33}^{(1)} &= \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{\gamma_{\mu} (\not{p}_1 - k_1 + l) \gamma^{\rho'} [\not{\epsilon}_{1T} M_{\rho} \phi_{\rho}^v(x_1) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu} n^{\rho} v^{\sigma} \phi_{\rho}^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2 (p_1 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\ &\quad \cdot \left. \gamma^{\rho'} (k_1 - l) \gamma^{\alpha} [\not{\epsilon}_{2T} M_{\rho} \phi_{\rho}^v(x_2) + M_{\rho} i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu} v^{\rho} n^{\sigma} \phi_{\rho}^a(x_2)] \gamma_{\alpha} (\not{p}_2 - k_1 + l) \right] \\ &= \Phi_{\rho,b}^{(1),v}(x_1) \otimes G_{a,TT,33}^{(0),v}(\xi_1, x_2) + \Phi_{\rho,b}^{(1),a} \otimes G_{a,TT,33}^{(0),a}(\xi_1, x_2),\end{aligned}\quad (\text{A51})$$

$$\begin{aligned}
G_{2c,TT,33}^{(1)} &= -\frac{1}{2} \frac{eg_s^4 C_F^2}{2} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - \not{k}_1) \gamma_\mu [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^{\rho'} (k_1 - l) \gamma_{\rho'} k_1 \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \right] \\
&= \frac{1}{2} \Phi_{\rho,c}^{(1),v}(x_1) \otimes G_{a,TT,33}^{(0),v}(x_1; x_2) + \frac{1}{2} \Phi_{\rho,c}^{(1),a}(x_1) \otimes G_{a,TT,33}^{(0),a}(x_1; x_2), \tag{A52}
\end{aligned}$$

$$\begin{aligned}
G_{2d,TT,33}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{(\not{p}_1 - \not{k}_1 + l) \gamma^\gamma F_{\alpha\beta\gamma} [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (k_1 - k_2 - l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\beta (\not{p}_2 - \not{k}_1 + l) \gamma^\mu \right] \sim 0, \tag{A53}
\end{aligned}$$

$$\begin{aligned}
G_{2e,TT,33}^{(1)} &= -eg_s^4 \text{Tr} \left[\frac{(\not{p}_2 - \not{k}_1) \gamma^\mu F_{\alpha\beta\gamma} [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (k_1 - l)^2 (k_1 - k_2 - l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^\gamma (k_1 - l) \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\beta \right] \\
&\sim \frac{9}{16} \Phi_{\rho,e}^{(1),v}(x_1) \otimes [G_{a,TT,33}^{(0),v}(x_1; x_2) - G_{a,TT,33}^{\prime(0),v}(x_1; \xi_1; x_2)] \\
&\quad + \frac{9}{16} \Phi_{\rho,e}^{(1),a}(x_1) \otimes [G_{a,TT,33}^{(0),a}(x_1; x_2) - G_{a,TT,33}^{\prime(0),a}(x_1; \xi_1; x_2)], \tag{A54}
\end{aligned}$$

$$\begin{aligned}
G_{2f,TT,33}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - \not{k}_1 + l) \gamma_\mu (\not{p}_1 - \not{k}_1 + l) \gamma_{\rho'} [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2)^2 (p_1 - k_1 + l)^2 (p_2 - k_1 + l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - \not{k}_1) \gamma^{\rho'} \right] \sim 0, \tag{A55}
\end{aligned}$$

$$\begin{aligned}
G_{2g,TT,33}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - \not{k}_1 + l) \gamma^{\rho'} (\not{p}_2 - \not{k}_1) \gamma_\mu [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (k_1 - l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha \right] \\
&\sim \Phi_{\rho,e}^{(1),v}(x_1) \otimes [G_{a,TT,33}^{\prime(0),v}(x_1; \xi_1; x_2) - G_{a,TT,33}^{(0),v}(\xi_1; x_2)] + \Phi_{\rho,e}^{(1),a}(x_1) \otimes [G_{a,TT,33}^{\prime(0),a}(x_1; \xi_1; x_2) - G_{a,TT,33}^{(0),a}(\xi_1; x_2)]. \tag{A56}
\end{aligned}$$

$$\begin{aligned}
G_{2h,TT,33}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma^\mu (\not{p}_1 - \not{k}_1 + l) \gamma_{\rho'} [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (p_2 - k_2 + l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^\alpha [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma^{\rho'} (\not{p}_2 - \not{k}_2 + l) \gamma_\alpha (\not{p}_2 - \not{k}_1 + l) \right] \sim 0, \tag{A57}
\end{aligned}$$

$$\begin{aligned}
G_{2i,TT,33}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\mu (\not{p}_1 - \not{k}_1 + l) \gamma_{\rho'} [\epsilon_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^{\nu'} n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2 - l)^2 (p_2 - k_1 + l)^2 (p_1 - k_1 + l)^2 (k_2 + l)^2 l^2} \right. \\
&\quad \left. \cdot \gamma^\alpha (k_2 + l) \gamma^{\rho'} [\epsilon_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^{\nu'} v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - \not{k}_1 + l) \right] \sim 0. \tag{A58}
\end{aligned}$$

$$\begin{aligned}
G_{2j,TT,33}^{(1)} &= \frac{eg_s^4}{9} \text{Tr} \left[\frac{\gamma_\alpha (\not{p}_2 - \not{k}_1) \gamma_\mu [\not{\epsilon}_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2)^2 (p_2 - k_1)^2 (k_1 - l)^2 (k_2 - l)^2 l^2} \right. \\
&\quad \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha (k_2 - l) \gamma^{\rho'} [\not{\epsilon}_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \left. \right] \\
&\sim -\frac{1}{8} \Phi_{\rho,e}^{(1),v}(x_1) \otimes G_{a,TT,33}^{(0),v}(x_1; x_2) - \frac{1}{8} \Phi_{\rho,e}^{(1),a}(x_1) \otimes G_{a,TT,33}^{(0),a}(x_1; x_2), \tag{A59}
\end{aligned}$$

$$\begin{aligned}
G_{2k,TT,33}^{(1)} &= \frac{-eg_s^4}{9} \text{Tr} \left[\frac{(\not{p}_2 - \not{k}_2 - l) \gamma_\alpha (\not{p}_2 - \not{k}) \gamma_\mu [\not{\epsilon}_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(k_1 - k_2 - l)^2 (p_2 - k_1)^2 (k_1 - l)^2 (p_2 - k_2 - l)^2 l^2} \right. \\
&\quad \cdot \gamma_{\rho'} (k_1 - l) \gamma^\alpha [\not{\epsilon}_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma^{\rho'} \left. \right] \\
&\sim \frac{1}{8} \Phi_{\rho,e}^{(1),v}(x_1) \otimes G_{a,TT,33}^{(0),v}(x_1; \xi_1; x_2) + \frac{1}{8} \Phi_{\rho,e}^{(1),a}(x_1) \otimes G_{a,TT,33}^{(0),a}(x_1; \xi_1; x_2), \tag{A60}
\end{aligned}$$

where the NLO twist-3 transverse ρ meson wave functions ($\Phi_{\rho,i}^{(1),v}$, $\Phi_{\rho,i}^{(1),a}$) with $i = (a, b, c, e)$ are the same ones as those in Eqs. (A42)–(A46). The new LO hard amplitudes that appeared in Eqs. (A50)–(A54) are also defined in a similar way as those in a previous subsection:

$$\begin{aligned}
G_{a,TT,33}^{(0)}(\xi_1; x_2) &= -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2 - l)^2} \right. \\
&\quad \cdot \gamma^\alpha [\not{\epsilon}_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - \not{k}_1 + l) \gamma_\mu \left. \right], \tag{A61}
\end{aligned}$$

$$\begin{aligned}
G_{a,TT,33}^{(0)}(x_1, \xi_1; x_2) &= -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1 + l)^2 (k_1 - k_2)^2} \right. \\
&\quad \cdot \gamma^\alpha [\not{\epsilon}_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - \not{k}_1 + l) \gamma_\mu \left. \right], \tag{A62}
\end{aligned}$$

$$\begin{aligned}
G_{a,TT,33}^{(0)}(x_1, \xi_1; x_2) &= -\frac{ieg_s^2 C_F}{2} \text{Tr} \left[\frac{[\not{\epsilon}_{1T} M_\rho \phi_\rho^v(x_1) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma^{\mu'} \gamma_5 \epsilon_{1T}^\nu n^\rho v^\sigma \phi_\rho^a(x_1)]}{(p_2 - k_1)^2 (k_1 - k_2 - l)^2} \right. \\
&\quad \cdot \gamma^\alpha [\not{\epsilon}_{2T} M_\rho \phi_\rho^v(x_2) + M_\rho i \epsilon_{\mu'\nu\rho\sigma} \gamma_5 \gamma^{\mu'} \epsilon_{2T}^\nu v^\rho n^\sigma \phi_\rho^a(x_2)] \gamma_\alpha (\not{p}_2 - \not{k}_1) \gamma_\mu \left. \right]. \tag{A63}
\end{aligned}$$

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