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Implications of a light "dark Higgs" solution to the $g_\mu - 2$ discrepancy

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A light scalar ϕ with mass $\lesssim$1 GeV and muonic coupling $\mathcal{O}(10^{-3})$ would explain the 3.5σ discrepancy between the Standard Model (SM) muon $g-2$ prediction and experiment. Such a scalar can be associated with a light remnant of the Higgs mechanism in the "dark" sector. We suggest $\phi \to l^+l^-$ bump hunting in $\mu \to e\nu\bar{\nu}\phi$, $\mu^- p \to \nu_\mu n\phi$ (muon capture), and $K^\pm \to \mu^\pm \nu\phi$ decays as direct probes of this scenario. In a general setup, a potentially observable muon electric dipole moment $\lesssim 10^{-23}$ e cm and lepton-flavor-violating decays $\tau \to \mu(e)\phi$ or $\mu \to e\phi$ can also arise. Depending on parameters, a deviation in BR $(H \to \mu^+\mu^-)$ from SM expectations, due to Higgs coupling misalignment, can result. We illustrate how the requisite interactions can be mediated by weak-scale vector-like leptons that typically lie within the reach of future LHC measurements.

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I. INTRODUCTION

The well-established existence of cosmic dark matter (DM)—a form of matter that does not significantly interact with ordinary atoms—furnishes us with clear evidence for physics beyond the Standard Model (SM). In many models, such as supersymmetry, DM naturally fits in extensions of the electroweak sector that attempt to explain properties of the Higgs potential. However, more generally, the dominance of cosmic DM over visible matter could argue for an entirely new sector of particle physics—the "dark sector"—endowed with its own forces and particles, largely decoupled from the SM [1]. The dark sector might only have faint interactions with our visible sector, mediated by the so-called portal [2–5] states that reside in both worlds.

In this work, we examine the possibility that a SM singlet light scalar ϕ residing primarily in the dark sector can account for the long-standing 3.5σ discrepancy between the SM prediction and measured value of the muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$,

$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 276(80) \times 10^{-11}, \tag{1}$$

which we have updated to include next-to-next-to-leading-order hadronic vacuum polarization effects [6,7]. We consider scalar masses $m_\phi \lesssim 1$ GeV, in the framework of a "dark photon" [8] scenario with a simple ultraviolet (UV) completion, i.e. dark weak-scale vector-like leptons and one extra Higgs doublet with hypercharge. All new particles (modulo the dark photon) carry a dark $U(1)_d$ charge, leading, via mixing, to a low-energy theory with the muonic couplings to ϕ necessary to explain Eq. (1). In our scenario, ϕ is associated with $U(1)_d$ breaking in the dark sector, i.e. it is a "dark Higgs" remnant of "dark" symmetry breaking.

A dark sector $U(1)_d$ force, with an associated dark vector boson $\gamma_d \lesssim$ GeV scale, has been motivated for some time from various astrophysical signals ascribed to DM [9]. A "dark Higgs mechanism" can be invoked as a primary source of γ_d mass. Kinetic mixing [10] between $U(1)_d$ and $U(1)_Y$ of hypercharge can allow γ_d to couple to the SM electromagnetic current, where γ_d is then often referred to as a "dark photon." If the kinetic mixing is sufficient, γ_d may itself play an important role in explaining $g_\mu - 2$ [11]; however, we do not consider that possibility here. Instead, we assume the ϕ is responsible for the bulk of the discrepancy.

The dark photon model can be generalized by assuming that γ_d and the SM Z boson couple to a dark second Higgs doublet that induces $\gamma_d - Z$ mass mixing, in which case the resulting light Z_d (a linear $\gamma_d - Z$ combination) acts much like a light "dark Z" [8], with interesting additional implications, such as changes in the low-q^2 running of the weak mixing angle [12,13], and rare decays of K, B and H particles into final states with Z_d's [8].

The aforementioned kinetic mixing can naturally arise in the dark photon scenario from quantum loops of heavy vector fermions that carry both $U(1)_d$ and $U(1)_Y$ charges [10,14]. In principle, such fermions could occur near the weak scale ~250 GeV, especially if they play a role in electroweak symmetry breaking. As precision electroweak and collider bounds generally disfavor states that carry $SU(2)_L$ or color $SU(3)$ charges, one may assume for illustration that in its simplest version the lightest

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vector-like fermions have the quantum numbers of the SM right-handed charged leptons. Therefore, on general grounds, vector-like leptons, as well as SM singlet and doublet scalars that carry $U(1)_d$ charges are well-motivated ingredients underlying the dark Z model [8].

A direct low-energy probe of our framework is ϕ bump hunting in $\mu^\pm \to e^\pm \nu\bar{\nu}\phi$, or $\mu^- p \to \nu_\mu n \phi$ (muon capture on nuclei), and $K^\pm \to \mu^\pm \nu_\mu \phi$, with ϕ decaying into lepton pairs or invisibly to light dark particles after being radiated by the muon. Typically, we may also expect new sources of CP and lepton flavor violations to arise in our scenario due to mass-scalar coupling misalignment, leading to a potentially detectable muon *electric dipole moment* $\mathcal{O}(10^{-23}\ e\,\mathrm{cm})$ and leptonic decays $\ell \to \ell' \phi$, respectively. In the Appendix, we provide a simple high-scale model that can typically accommodate dominant lepton flavor-diagonal couplings with sufficiently suppressed flavor-violating ones for ϕ to evade experimental constraints, some of which we later discuss. This model can also support a realistic neutrino mass matrix, as briefly discussed in the Appendix.

An important ingredient of the above setup is that dark and visible sector Higgs interactions together yield a new source of SM charged lepton masses. Hence, depending on the parameters in the dark sector, one could, in addition to ϕ effects, also expect departures in the 125 GeV Higgs branching fractions into e, μ, τ pairs from SM expectations. Such deviations may be measurable at the LHC in the coming years as Higgs decay statistics continue to improve.

II. LEPTON DIPOLE MOMENTS

In this work, the main motivation for introducing the scalar ϕ is its potential role as a new contribution to $g_\mu - 2$. Furthermore, we will also assume that ϕ can contribute to both the magnetic and electric dipole moments of leptons, through CP-conserving and -violating couplings.

In general, the flavor-diagonal Yukawa couplings can be parametrized (relative to the real CP-conserving charged lepton mass matrix) as

$$\mathcal{L}_{\phi\ell\ell} = -\phi\bar{\ell}(\lambda_S^\ell + i\lambda_P^\ell \gamma_5)\ell \tag{2}$$

where $\ell = e, \mu, \tau$, and λ_S^ℓ (λ_P^ℓ) is the CP-even (-odd) dark Yukawa coupling. At one-loop level these couplings induce additional contributions to the dipole moments of leptons, as shown in Fig. 1. We find they imply [15,16]

$$\Delta a_\ell = \frac{\lambda_S^{\ell 2}}{8\pi^2} r^{-2} \int_0^1 \mathrm{d}z \frac{(1+z)(1-z)^2}{r^{-2}(1-z)^2 + z} - \frac{\lambda_P^{\ell 2}}{8\pi^2} r^{-2} \int_0^1 \mathrm{d}z \frac{(1-z)^3}{r^{-2}(1-z)^2 + z} \tag{3}$$

and for the lepton electric dipole moment

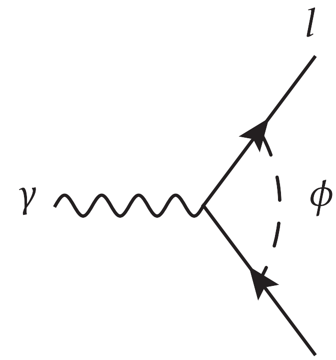

FIG. 1. One-loop ϕ contribution to lepton dipole moments.

$$d_\ell = \frac{\lambda_S^\ell \lambda_P^\ell}{4\pi^2} \frac{e}{2m_\ell} r^{-2} \int_0^1 \mathrm{d}z \frac{(1-z)^2}{r^{-2}(1-z)^2 + z} \tag{4}$$

where $r = m_\phi / m_\ell$. We present analytic expressions for these integrals in the Appendix. In the limit $r \to 0$ (i.e., light ϕ) we have

$$\Delta a_\ell = \frac{1}{16\pi^2}(3\lambda_S^{\ell 2} - \lambda_P^{\ell 2}) \tag{5}$$

$$d_\ell = \frac{\lambda_S^\ell \lambda_P^\ell}{4\pi^2} \frac{e}{2m_\ell}. \tag{6}$$

In Fig. 2 we illustrate (ignoring λ_P^μ) the region of $\lambda_S^{\mu 2}$, m_ϕ favored by Eq. (1) with one-sigma uncertainty.

In the electron case, there is no significant deviation from the SM $g_e - 2$ prediction. However, if Δa_μ is taken to be 276×10^{-11} and we assume $\lambda_S^e \sim \frac{m_e}{m_\mu} \lambda_S^\mu$, (for negligible λ_P^e effects), we find that $|\Delta a_e| < 10^{-13}$ for all m_ϕ, well below the current experimental constraint $\Delta a_e = (-0.91 \pm 0.82) \times 10^{-12}$ [17]. Hence, Δa_e consistent with

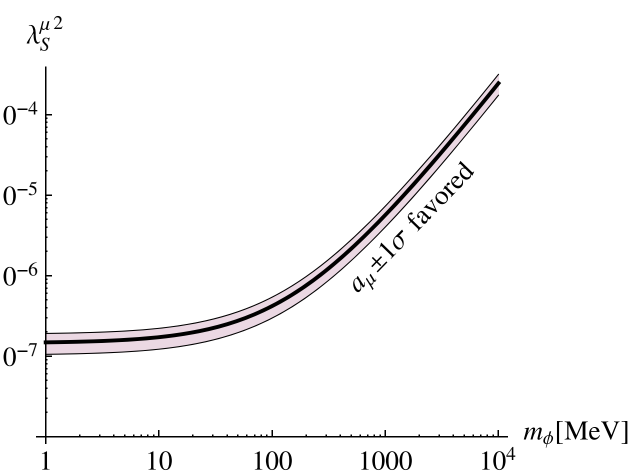

FIG. 2. Central values and one-sigma band of λ_S^μ, required by the measured value of Δa_μ in Eq. (1).

zero is easily accommodated in our scenario for reasonable couplings.

The ratio between electric and anomalous magnetic moments in the $r \to 0$ limit is

$$\frac{d_\ell}{\Delta a_\ell} = \frac{e}{2m_\ell}\frac{4\tan\theta_\ell}{3-\tan^2\theta_\ell} \approx \frac{e}{2m_\ell}\frac{4}{3}\tan\theta_\ell \qquad (7)$$

where we define $\tan\theta_\ell = \lambda_P^\ell/\lambda_S^\ell$. Note that under the opposite $r \to \infty$ limit both Δa_ℓ and d_ℓ vanish (for earlier related work, see Ref. [18]). In principle one should also add the two-loop Barr-Zee contribution [19] to d_ℓ. However, for the muon, we expect it to be subdominant. For the electron it is potentially more important.

The one-loop induced electric dipole moment of a lepton can be written as $d_\ell = 2.36 \times 10^{-15} \lambda_S^\ell \lambda_P^\ell (m_\mu/m_\ell)$ e cm. To estimate the size of the muon electric dipole moment, we assume that the $g_\mu - 2$ central anomaly can be solely explained by the scalar contribution to a_μ from Eq. (3). (The required $\lambda_s^{\mu 2}$ central values as a function of m_ϕ are given in Fig. 2 with a one-sigma spread.) This will determine the dark Yukawa couplings λ_S^μ, up to the CP-violating phase θ_μ. For any given value of $\tan\theta_\mu$, we can compute d_μ as a function of the ϕ mass. Results are shown in Fig. 3, for $\tan\theta_\mu = 0.2$, 0.1, and 0.03. We see that for reasonable values of $\tan\theta_\mu$, the muon electric dipole moment can reach about 10^{-22}–10^{-23} e cm. That is to be compared with the current bound $|d_\mu| < 1.8 \times 10^{-19}$ e cm [20]. Possible muon storage ring measurements of d_μ with sensitivity 10^{-24}–10^{-25} e cm have been envisioned, but for now none are planned [21,22]. In principle, they could explore down to $\tan\theta_\mu \sim 0.0003$ in our scenario.

It is possible that Eq. (4) could also lead to a detectable electric dipole moment for the electron. Intuitively, we might expect that $\lambda_{S,P}^\ell \propto m_\ell$, i.e. proportional to the relative chiral-symmetry-breaking mass scale, even though this is model dependent. Assuming this relation we expect that d_μ/d_e may be of order 10^5–10^6 for m_ϕ within the range [10, 1000] MeV. That means d_e could turn out to be 10^{-28}–10^{-29} e cm, which is to be compared with the current bound [23]

$$|d_e| < 8.7 \times 10^{-29}\ e\,\text{cm}. \qquad (8)$$

Hence d_e could potentially be within the reach of future experiments which are expected to probe down to $|d_e| \sim \mathcal{O}(10^{-30})$ e cm.

III. DIRECT SIGNALS IN RARE LEPTON-FLAVOR-PRESERVING PROCESSES

A direct consequence of our solution to $g_\mu - 2$ is the possibility of ϕ emission in rare lepton-flavor-preserving processes involving initial- or final-state muons. In what follows, we will consider muon and kaon interactions that could offer promising search avenues for our scenario.

Muon decay: We first consider $\mu \to e\phi\bar{\nu}_e\nu_\mu$, whose branching ratio is given in Fig. 4, for λ_S couplings that accommodate $g_\mu - 2$. This $e\phi$ + "invisible" signal can be probed with intense muon sources such as Mu3e; see for example Ref. [24] for a discussion based on the similar case of dark photons. Here, assuming $m_\phi \lesssim m_\mu$ and an $\mathcal{O}(1)$ branching fraction for $\phi \to e^+e^-$, we may expect sensitivity to λ_S^μ similar to that for a dark photon with kinetic mixing parameter ε, where $\varepsilon e \to \lambda_S^\mu$. We note that while the presence of the e^+e^- mode is not strictly required in our scenario, the assumed muon coupling does imply a nonzero loop-induced branching fraction for $\phi \to \gamma\gamma$, which may not have detection prospects similar to that of the e^+e^- final state, depending on the experimental setup (such as the use of a nonzero magnetic field for event selection). While the current bounds are not very constraining for our scenario, future measurements, such as those discussed in Ref. [24] can potentially probe $\lambda_S^\mu \lesssim 10^{-4}$ for $m_\phi \sim 20$–80 MeV, in

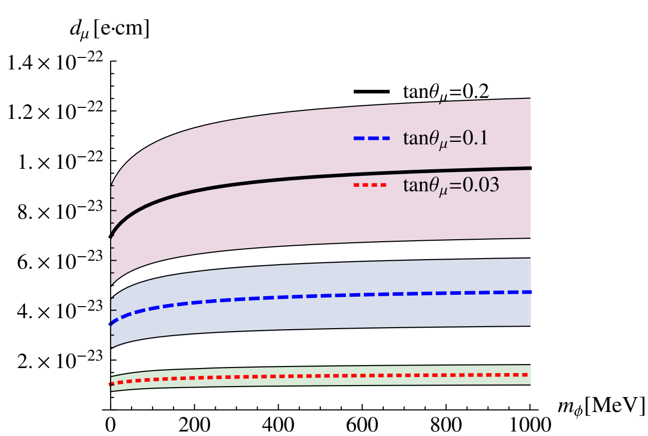

FIG. 3. Muon electric dipole moment, for various CP-violating phases, assuming that Δa_μ agrees with the measured value of $g_\mu - 2$ within one sigma.

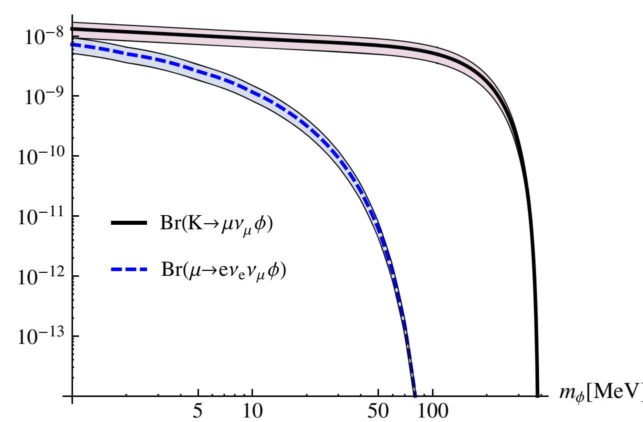

FIG. 4. Branching ratios of $\mu \to e\nu\bar{\nu}\phi$ and $K^+ \to \mu^+\nu\phi$, assuming that Δa_μ agrees with the measured value of $g_\mu - 2$ within one sigma.

the case of $\phi \to e^+e^-$ dominance, which would cover much of the parameter space relevant for $m_\phi \lesssim m_\mu$ that resolves the $g_\mu - 2$ discrepancy.

Muon capture: The decay $\mu^- \to e^-\nu\bar{\nu}\phi$ with $\phi \to e^+e^-$ can also be searched for in bound muon decay studies such as Mu2e at Fermilab and Comet at J-Parc where more than 10^{17} muons are expected to be stopped in an Al target where they form μ^-Al atoms. About half of those stopped muons undergo ordinary muon decay $\mu \to e\nu\bar{\nu}$ in orbit while the other half undergo capture $\mu^-\text{Al} \to \nu_\mu\text{Mg}$. The capture process and decay in orbit are both potential ϕ sources. Using either mode to search for ϕ's, decay or capture, represents an interesting extension of the muon conversion experiments getting underway. Their viability will depend on reconfiguring the detectors to observe e^+e^- pairs from ϕ decays with invariant mass m_ϕ above background.

Kaon decay: The scalar ϕ can, in principle, also be radiated from muons in $K^+ \to \mu^+\nu_\mu$ decays, via the ϕ-muon coupling [25–27]. The branching ratio is given in Fig. 4. The rate is almost an order of magnitude below the current bounds [6]

$$\mathrm{BR}(K^+ \to \mu\nu\phi, \phi \to e^+e^-) < 7 \times 10^{-8}, \tag{9}$$

$$\mathrm{BR}(K^+ \to \mu\nu\phi, \phi \to \mu^+\mu^-) < 4.1 \times 10^{-7}. \tag{10}$$

However, the ongoing NA62 experiment at CERN could be potentially sensitive to this decay mode, up to $m_\phi \sim 300$ MeV. Particularly interesting is the region $m_\phi > 2m_\mu \sim 200$ MeV where a $\mu^+\mu^-$ final state with mass m_ϕ opens up.

Pion decay: For light $\phi \lesssim 30$ MeV, one can look for the decay $\pi^+ \to \mu^+\nu_\mu\phi$, $\phi \to e^+e^-$ by searching for a e^+e^- bump at high-intensity charged pion sources such as NA62 or beam-dump experiments. Exploring that possibility is interesting and worthy of study. However, addressing current bounds requires thorough background studies and depends on the promptness of the decay, which is model dependent. For that reason, further discussion is beyond the scope of this paper.

IV. A CONCRETE UV MODEL

Here, we provide a possible UV completion of our low-energy effective theory, which leads to the assumed coupling in Eq. (2). This model can also provide the requisite ingredients for a potentially viable "dark" Z model (see, for example, Ref. [8]). In this UV framework, all new particles are assumed to be charged under $U(1)_d$ with the same dark charge, unless otherwise stated, and hence we will only identify their SM charges. Let X^ℓ—where $\ell = e$, μ, τ is a flavor index—be vector-like fermions with the quantum numbers of right-handed SM leptons ℓ_R [i.e. $SU(2)$ singlets], and masses $m_X^\ell \gtrsim \text{few} \times 100$ GeV. We also introduce a new Higgs scalar doublet H_d and a complex scalar singlet ϕ. We will assume that H_d and ϕ have nonzero vacuum expectation values which spontaneously break $U(1)_d$. As mentioned before, these ingredients can be motivated within a dark Z model [8]. One can then write down the following $\mathrm{SM} \times U(1)_d$ invariant interactions:

$$-\mathcal{L}_1 = m_X^{\ell\ell'}\bar{X}^\ell X^{\ell'} + \lambda_1\phi\bar{X}_L^\ell \ell_R + \lambda_2 H_d\bar{L}^\ell X_R^\ell + y_\ell H\bar{L}^\ell \ell_R + \text{H.c.}, \tag{11}$$

where L^ℓ and H refer to SM lepton and Higgs doublets, respectively. The above interactions respect lepton flavor conservation up to soft breaking by (small) off-diagonal masses $m_X^{\ell\ell'}$, which we will assume are the only sources of lepton flavor violation. In the Appendix, we illustrate how the above can be realized in a model with flavor symmetries that allow for a realistic neutrino mass matrix. A vacuum expectation value for H_d followed by charged lepton mass matrix diagonalization could result in misaligned ϕ and H lepton couplings which lead to interesting consequences, as outlined below. In the case of extension to quarks, our scenario maintains H and H_d alignment with the mass matrix and avoids quark flavor-changing current constraints at the tree level.

V. LEPTON-FLAVOR-VIOLATING DECAYS

A possible signal of our UV model is the appearance of lepton-flavor-violating (LFV) interactions of the form $\lambda_S^{ij}\phi\bar{l}_i l_j$ (pseudoscalar couplings are also possible, but will not be considered here). In particular, they can give rise to $\mu \to \phi e$ and $\tau \to \phi l$, with $l = \mu, e$. The constraints on these interactions depend sensitively on the dominant ϕ decay channels. Generally speaking, these constraints are quite a bit weaker when $\phi \to$ "invisible" is the dominant decay mode [6,28][1]; we will have more comments on this case later. Instead, let us consider the case of a visible ϕ with $\phi \to \mu^+\mu^-$ or e^+e^- (below the dimuon threshold).

The current upper bound on the $\mu \to 3e$ branching fraction is [6]

$$\mathrm{BR}(\mu \to e\phi, \phi \to e^+e^- \text{ prompt}) < 10^{-12} \tag{12}$$

which corresponds to limits on $|\lambda_S^{\mu e}|$

$$\begin{aligned} |\lambda_S^{\mu e}| &< 1.2 \times 10^{-14} \quad \text{for } m_\phi = 10 \text{ MeV}, \\ |\lambda_S^{\mu e}| &< 1.5 \times 10^{-14} \quad \text{for } m_\phi = 50 \text{ MeV}, \\ |\lambda_S^{\mu e}| &< 1.1 \times 10^{-13} \quad \text{for } m_\phi = 100 \text{ MeV}. \end{aligned} \tag{13}$$

[1]In a scenario where the branching ratio of ϕ decay into invisible is 100%, we found that the bounds on the off-diagonal couplings, depending on flavor, are in general 2–4 orders of magnitude weaker.

In the case of $\tau \to 3l$, roughly speaking, the bounds on the corresponding branching fractions are much weaker, $\sim$few $\times 10^{-8}$ [6]

$$\mathrm{BR}(\tau \to e\phi, \phi \to e^+e^- \text{ prompt}) < 2.7 \times 10^{-8}, \quad (14)$$

$$\mathrm{BR}(\tau \to e\phi, \phi \to \mu^+\mu^- \text{ prompt}) < 2.7 \times 10^{-8}, \quad (15)$$

$$\mathrm{BR}(\tau \to \mu\phi, \phi \to e^+e^- \text{ prompt}) < 1.8 \times 10^{-8}, \quad (16)$$

$$\mathrm{BR}(\tau \to \mu\phi, \phi \to \mu^+\mu^- \text{ prompt}) < 2.1 \times 10^{-8}. \quad (17)$$

These correspond to limits on $|\lambda_S^{\tau l}|$

$$\begin{aligned} |\lambda_S^{\tau l}| &< 1.0 \times 10^{-9} \quad \text{for } m_\phi = 50 \text{ MeV}, \\ |\lambda_S^{\tau l}| &< 1.2 \times 10^{-9} \quad \text{for } m_\phi = 500 \text{ MeV}, \\ |\lambda_S^{\tau l}| &< 3.5 \times 10^{-9} \quad \text{for } m_\phi = 1500 \text{ MeV}. \end{aligned} \quad (18)$$

A rough estimate yields $m_X^{\tau l} \lesssim 10$ keV (assuming vector lepton masses $m_X \sim 100$ GeV). A simple model of flavor, presented in the Appendix, can accommodate such a degree of LFV, while providing Dirac masses $\sim$0.1 eV for neutrinos. The more constraining bound on $\mu \to e\phi$ can be taken to imply a phenomenological preference for $m_\phi \gtrsim 100$ MeV, so that muon decays to on-shell ϕ final states are not kinematically allowed.[2]

Adhering to the types of bounds in Eqs. (13) and (18) will also suppress loop-induced LFV decays such as $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$. However, a detailed study of such effects is likely to require a more complete two-loop analysis [29], which is beyond the scope of this paper.

VI. $H \to l^+l^-$ MISALIGNMENT

If in addition to the SM Higgs mechanism, there exist other contributions to lepton masses, then some misalignment between the charged lepton mass matrix and $H\ell^+\ell^-$ couplings can also be expected. In our framework, a significant source of muon mass can originate from its interactions with ϕ, assuming that $\langle\phi\rangle$ is $\mathcal{O}(100$ GeV$)$.[3] Ignoring flavor-changing effects, which are interesting (especially for $H \to \mu\tau$) but beyond the scope of this study, one can parametrize the misalignment by a $H\ell^+\ell^-$ coupling factor relative to the SM value by [30]

$$\kappa_\ell(\cos\theta_\ell^H + i\gamma_5 \sin\theta_\ell^H) \quad (19)$$

where κ_ℓ scales the relative magnitude of the coupling and θ_ℓ^H allows for a CP-violating component. The latter effect is potentially very interesting for the electron, where the recent bound on the electron electric dipole moment, as given in Eq. (8), already leads to the rather prohibitive constraint [31]

$$|\sin\theta_e^H| < 0.017/\kappa_e. \quad (20)$$

That sensitivity is expected to further improve by as much as 2 orders of magnitude in the future as experiments probe $|d_e| \sim 10^{-30}$ e cm.

Recently, the $H \to \tau^+\tau^-$ decay has been measured by the ATLAS and CMS collaborations at the LHC at better than 5 sigma. The observed branching ratio leads to [32]

$$\kappa_\tau = 0.90^{+0.14}_{-0.13} \quad (21)$$

consistent with the SM expectation $\kappa_\tau^{\mathrm{SM}} = 1$. Further precision is expected from Run II. Measurement of κ_μ will be more difficult but potentially doable in Run II of the collider if $\kappa_\mu \sim 1$ or even larger. Run I searches for $H \to \mu^+\mu^-$ have so far been negative, leading to the constraint [32]

$$\kappa_\mu = 0.2^{+1.2}_{-0.2}. \quad (22)$$

Although still consistent with the SM expectation $\kappa_\mu^{\mathrm{SM}} = 1$, the central value in Eq. (22) reminds us that an enhancement or (perhaps more likely) a suppression of $H \to \mu^+\mu^-$ is very possible. That would be an exciting discovery, confirming misalignment. In the case of $H \to e^+e^-$, the SM branching ratio of $\sim 5 \times 10^{-9}$ is very suppressed, making that decay mode highly unlikely to be observable unless $\kappa_e \gg 1$, which would seem to be somewhat contrived in our scenario.

VII. ADDITIONAL PHENOMENOLOGY

Finally, we would like to discuss potential signals of our scenario, based on the UV model assumed in Eq. (11). To do so, we adopt somewhat specific values for parameters, in order to highlight some typical possibilities for the implied general phenomenology. As illustrated in the following discussion, a wide variety of possibilities can ensue from our underlying theory and, depending on specific choices of parameters, a number of interesting signals can arise in high-energy experiments. A more detailed examination of such possibilities, while quite interesting and instructive, will exceed the intended scope of our current work.

As previously mentioned, our underlying assumption regarding the coupling of ϕ to muons also suggests deviations in the Yukawa coupling of the muon to the

[2]Alternatively, one may consider $m_X^{\mu e} \ll m_X^{\tau l}$, assuming for example that the e-flavor-breaking parameter $S^e \ll S^{\mu,\tau}$. This would imply that $m_X^{\tau e}$ and, consequently, $\tau \to \phi e$ are also suppressed, suggesting that one of the neutrinos is much lighter than the other two (which is currently allowed by all data).

[3]This possibility can be motivated in phenomenologically viable "dark" Z models [8], as a means of suppressing $Z - Z_d$ mass mixing.

observed 125 GeV Higgs, because of a secondary source for $m_\mu \simeq 106$ MeV provided via the dimension-five operator

$$\lambda_1 \lambda_2 \frac{\phi H_d \bar{L}_\mu \mu_R}{m_X}, \qquad (23)$$

with $m_X \equiv m_X^{\mu\mu}$, for notational simplicity. Since H_d is a weak isodoublet, $\langle H_d \rangle$ contributes to electroweak symmetry breaking (EWSB). However, electroweak measurements currently seem to show good agreement with the SM predictions. Hence, it is well motivated to assume that H_d has a su-dominant role in EWSB: $\langle H_d \rangle \ll \langle H \rangle$. Let us assume, for illustration, that $\langle H_d \rangle \sim 30$ GeV and $m_X \sim 300$ GeV, as reasonable values.

Note that for m_ϕ not far from ~100 MeV, a typical value in this work, we have $\lambda_S^\mu \simeq \lambda_1 \lambda_2 \langle H_d \rangle / m_X \sim 10^{-3}$ in order to account for $g_\mu - 2$ for $\lambda_P^\mu \ll \lambda_S^\mu$ (small θ_μ). If $\langle \phi \rangle \sim$ 100 GeV (as in typical dark Z models where such a setup can provide the requisite suppression of $Z - Z_d$ mass mixing [8]) our choices of parameters then imply $\lambda_1 \lambda_2 \sim 10^{-2}$. The coupling of ϕ to the X^μ can also induce a large quantum-loop-generated scalar mass $\delta m_\phi \sim \lambda_1 m_X/(4\pi)$, which motivates the assumption $\lambda_1 < \lambda_2$ and hence we may choose, for example, $\lambda_2 \sim 0.2$ and $\lambda_1 \sim 0.05$. The choice $\lambda_2 \sim 0.2$ implies a $X^\mu - \mu$ mixing $\lambda_2 \langle H_d \rangle / m_X \sim 0.02$, which is roughly consistent with precision bounds.

The above discussion of parameters has interesting implications for the phenomenology of our underlying model, aspects of which we will briefly consider. For one thing, the coupling of the scalar doublet H_d to muons $y_\mu^d = \lambda_S \langle \phi \rangle / \langle H_d \rangle \sim 3 \times 10^{-3}$. This is roughly a factor of ~5 larger than the muon-Higgs Yukawa coupling in the SM! Thus, we have a scenario where the 125 GeV Higgs may have suppressed couplings to muons, whereas the second "dark" doublet H_d may have considerably enhanced interactions with muons. It may, therefore, be interesting to consider the potential resonant production of H_d at a future weak-scale $\mu^+\mu^-$ collider. The same consideration also applies to the production of ϕ at a low-energy $\mu^+\mu^-$ collider, given its assumed relatively large coupling to muons in order to explain $g_\mu - 2$. We note that the small ratio $\langle H_d \rangle / \langle H \rangle$ suppresses the couplings of H_d to quarks, relative to H quark couplings.

The vector-like leptons X^ℓ, employed in our model to induce the $\phi \bar{\ell} \ell$ couplings, can be pair produced in Drell-Yan processes at the LHC; see Table I for examples of typical cross sections. However, their discovery signals depend on the dominant branching fractions. Let us focus on X^μ for definiteness, which can decay in a variety of ways. However, it has only three "direct" channels that are not mediated by mixing: $X^\mu \to \phi\mu$, $X^\mu \to H_d^0 \mu$, and $X^\mu \to H^\pm \nu_\mu$. Of these, given the assumed relation $\lambda_2^2 \gg \lambda_1^2$,

TABLE I. Cross sections for pair production of X^ℓ particle at the LHC (in fb).

m_X [GeV]	200	300	400	600
8 TeV	33	5.9	1.5	0.18
13 TeV	79	16.7	5.1	0.82

the latter two channels are expected to be dominant in our underlying model. Here, H_d^0 denotes the neutral scalar from the H_d doublet and $H^\pm$ are the associated charged Higgs states, whose main decay modes are subject to various assumptions about the parameters of the two-Higgs-doublet potential (see, for example, Ref. [33] for a discussion of $H^\pm$ decays in the context of dark Z models). The exact exclusion limit on X is model dependent. However, as a rough estimate, Ref. [34] suggests that $m_X \lesssim 200$ GeV may already be excluded by the LHC 8 TeV run. On the other hand, according to Table. I, $m_X \gtrsim 300$ still seems viable, and given that m_ϕ naturalness prefers a relatively lighter X, there may be a chance to observe X pair production at the LHC Run II.

The light scalar ϕ may also be an interesting target for low-energy experiments, wherever an intense muon beam is available. The production of ϕ from a muon beam is set by $\lambda_S^\mu \sim 10^{-3}$, making it a "$\mu$-philic" scalar analogue of a dark photon coupled to charged particles via kinetic mixing. Within our setup, for $m_\phi > 2m_\mu$ (but below $2m_\tau$) we can expect a 100% branching fraction for $\phi \to \mu^+\mu^-$. For $m_\phi < 2m_\mu$ (but above ~1 MeV), we may have $\phi \to e^+e^-$ or $\phi \to \gamma\gamma$. Without further assumptions it is not clear which one of these two modes will dominate the low-mass ϕ decays. However, if we assume that the entire mass of the electron is generated by an operator of the type in Eq. (23), then one can expect $\lambda_S^e \sim (m_e/m_\mu)\lambda_S^\mu$. In that case, $\phi \to e^+e^-$ will be the main decay mode in this mass range.

For completeness, we also mention that "dark" sector states may typically have $\mathcal{O}(1)$ couplings to ϕ. If such states are lighter than $m_\phi/2$, then $\phi \to$ "invisible" may be the dominant decay mode of ϕ. However, in this case we may expect $\langle \phi \rangle \lesssim m_\phi$, so that the dark states do not become heavy and can furnish on-shell invisible decay final states.

$H \to \phi\phi$: The SM Higgs could mix with the scalar ϕ via the following term:

$$\kappa(\phi^\dagger \phi)(H^\dagger H). \qquad (24)$$

[$\phi(H)^2$ is not allowed because ϕ has dark charge.] Potentially this would lead to $H \to \phi\phi$ decay. However, requiring m_ϕ to stay in the mass range we consider constrains κ to be $\lesssim 10^{-5}$. This value for κ is stable under quantum corrections, because the only way to induce such a coupling would be through a lepton loop, and for a τ lepton the Yukawa couplings for H and ϕ are of order $\sim 10^{-2}$, and so the induced contribution is tiny. With such a small coupling, $H \to \phi\phi \to 4l$ or $\to$ invisible should be negligible. (Similar consideration also applies to the H_d doublet, whose vacuum expectation value could be much smaller than that of the SM Higgs doublet.)

VIII. SUMMARY AND CONCLUSIONS

In this work, we considered the possibility that a light “dark” Higgs ϕ from a hidden sector can be responsible for the measured 3.5σ deviation of $g_\mu - 2$ from its SM value. We explored the mass range $m_\phi \lesssim 1$ GeV, which provides a counterpart to low-energy “dark” vector-boson models that have been similarly invoked to address $g_\mu - 2$. In fact, one can assume that our dark Higgs ϕ is associated with the mechanism responsible for generating dark vector-boson masses.

A direct consequence of our scenario is the possibility of ϕ emission in decays that include a muon; we briefly discussed $\mu \to e\nu\bar{\nu}\phi$, $\mu^- p \to \nu_\mu n\phi$, and $K \to \mu\nu\phi$ as examples of promising search modes that may lead to signals in future experiments. Also, the generic assumption of CP-violating couplings of ϕ with muons can lead to interesting values of the muon electric dipole moment, perhaps as large as $\sim 10^{-23}$ e cm, which could potentially be measured at a future dedicated storage ring experiment, though a concrete proposal is not currently at hand. Similarly, d_e may be within reach of future experiments. We also discussed that one may anticipate, within a generic parameter space, manifestations of lepton flavor violation in decays that include a ϕ, such as $\tau \to \mu\phi$, with $\phi \to e^+e^-$ or $\mu^+\mu^-$.

If $\langle\phi\rangle \neq 0$, as generally assumed here, we expect a new source of mass for muons (and perhaps other leptons) in our low-energy model. While we did not specify the value of $\langle\phi\rangle$ in our setup, we pointed out the interesting possibility that for $\langle\phi\rangle \sim 100$ GeV, all or much of the muon mass may originate from $\phi\mu^+\mu^-$ couplings that explain the muon $g_\mu - 2$ anomaly. Hence, a potential signal of our scenario could be a misalignment of the 125 GeV Higgs coupling to muons, which may be observable in $H \to \mu^+\mu^-$ at the LHC, over the next few years.

We provided a simple UV completion of our scenario, comprising weak-scale vector leptons and additional “dark” singlet and doublet Higgs scalars. The high-scale model can lead to interesting additional signals at the LHC in its Run II, whose generic features were briefly discussed.

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APPENDIX:

1. Analytic expressions for Eqs. (3) and (4)

After integrating over z we find

$$\Delta a_\ell = \frac{\lambda_S^{\ell 2}}{8\pi^2}\left[\frac{3}{2} - r^2 - r^2(3 - r^2)\log r - (1 - r^2)(4 - r^2)f(r)\right] + \frac{\lambda_P^{\ell 2}}{8\pi^2}\left[-\frac{1}{2} - r^2 - r^2(1 - r^2)\log r + r^2(3 - r^2)f(r)\right] \tag{A1}$$

and

$$d_\ell = \frac{\lambda_S^\ell \lambda_P^\ell}{4\pi^2}\frac{e}{2m_\ell}[1 - r^2\log r - (2 - r^2)f(r)] \tag{A2}$$

where

$$f(r) = \begin{cases} \cos^{-1}(\frac{r}{2})(4r^{-2} - 1)^{-\frac{1}{2}} & : r < 2, \\ 1 & : r = 2, \\ \cosh^{-1}(\frac{r}{2})(1 - 4r^{-2})^{-\frac{1}{2}} & : r > 2. \end{cases} \tag{A3}$$

Note that for small r, $f(r) \approx \pi r/4$.

2. Flavor symmetry

Here, we will present a simple realization of the flavor symmetry that leads to Eq. (11), largely as an illustrative example. Let us consider three separate parities Z_2^ℓ, $\ell = e$, μ, τ, broken by $\langle S^\ell\rangle$, where S^ℓ is a scalar that is Z_2^ℓ odd. We do not specify the underlying dynamics for S^ℓ condensation, since we are only interested in depicting the general symmetry structure. If desired, that physics can be straightforwardly added to the high-energy theory. Here, X^ℓ, as well as SM leptons L^ℓ and ℓ_R are all assumed odd under their respective parity. The usual Yukawa coupling for charged leptons $y_\ell H\bar{L}^\ell\ell_R$ can be written down under our assumptions and it will be diagonal in flavor. The first term in Eq. (11) can be written as a result of spontaneous symmetry breaking with

$$m_X^{\ell\ell'} = \frac{\langle S^\ell\rangle\langle S^{\ell'}\rangle}{M}, \tag{A4}$$

where M is a high mass scale. This is the only source of flavor violation in our setup. In order to write down a generic neutrino mass matrix, let us introduce three right-handed neutrinos ν_R^a, with $a = 1, 2, 3$, that are neutral under

Z_2^ℓ (assuming three massive neutrinos). Then, we can have the neutrino mass matrix

$$\frac{S^\ell}{M} H \bar{L}^\ell \nu_R^a + \mathrm{H.c.} \tag{A5}$$

For $m_\nu \sim 0.1$ eV and M at the Planck scale $M_P \sim 10^{19}$ GeV, we then find $\langle S^\ell \rangle \sim 10^7$ GeV. This implies that $m_X^{\ell\ell'} \sim 10$ keV, which can have the right order of magnitude, given the constraints from flavor-violating decay bounds on $\ell \to \ell' \phi$ (see the text for further details).

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